Two-loop divergences in $6D, \mathcal{N} = (1, 1)$ SYM

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Aims:

- Construction of the background superfield method for $6D, \mathcal{N} = (1, 0)$ interacting non-Abelian vector multiplet with hypermultiplet
- Calculation of the one-loop off-shell divergences in vector multiplet and hypermultiplet sectors for arbitrary $\mathcal{N} = (1, 0)$ gauge theory
- Analysis of divergences in the $\mathcal{N} = (1, 1)$ SYM theory
- Two-loop divergences

The talk is based on:

I.L. Buchbinder, E.A. Ivanov, K.V. Stepanayantz, B.M., *JHEP 2305 (**2023**)*; *Phys.Lett.B 820 (**2021**)*; *Nucl.Phys.B 921 (**2017**)*; *JHEP 1701 (**2017**)*; *Phys.Lett.B 763 (**2016**)*.

The modern interest to $6D$ supersymmetric gauge theories is stipulated by the following reasons:

- The problem of describing the quantum structure of six-dimensional supersymmetric gauge theories dimensionally reduced from superstrings and the connection of effective action for the D5-branes at low energies with maximally supersymmetric Yang-Mills theory in six dimensions. [**N.Seiberg (1996), E. Witten (1996); N. Seiberg, (1997)**].
- Lagrangian description of the interacting multiple $M5$-branes is related to $6D$, $\mathcal{N} = (2, 0)$ supersymmetric gauge theory. The theory includes self-dual non-Abelian antisymmetric tensor and it is not constructed still (see e.g. review [**J. Bagger, N. Lambert, S. Mikhu, C. Papageorgakis (2013)**]).

Motivations

▶ The problem of miraculous cancellation of on-shell divergences in higher dimensional maximally supersymmetric gauge theories (theories with 16 supercharges). All these theories are non-renormalizable by power counting.

- Field limit of superstring amplitude shows that $6D, \mathcal{N} = (1,1)$ SYM theory is on-shell finite at one-loop [**M.B. Green, J.H. Schwarz, L. Brink, (1982)**].
- Analysis based on on-shell supersymmetries, gauge invariance and field redefinitions [**P.S. Howe, K.S. Stelle, (1984), (2003); G. Bossard, P.S. Howe, K.S. Stelle, (2009)**].
- Direct one-loop and two-loop component calculations (mainly in bosonic sector and mainly on-shell) [**E.S. Fradkin, A.A. Tseytlin, (1983); N. Marcus, A. Sagnotti, (1984), (1985)**].
- Direct calculations of scattering amplitudes in $6D$ theory up to five loops and in $D8, 10$ theories up to four loops [**L.V. Bork, D.I. Kazakov, M.V. Kompaniets, D.M. Tolkachev, D.E. Vlasenko, (2015).**]

Results: On-shell divergences in $6D$ theory start at three loops.

Purpose

To study one- and two-loop divergences of the superfield effective action in $\mathcal{N} = (1,1)$ SYM theory.

Properties

$6D, \mathcal{N} = (1,1)$ SYM theory possesses some properties close or analogous to $4D, \mathcal{N} = 4$ SYM theory.

- The $6D, \mathcal{N} = (1,1)$ SYM theory can be formulated in harmonic superspace as well as the $4D, \mathcal{N} = 4$ SYM theory.
- The $6D, \mathcal{N} = (1,1)$ SYM theory possesses the manifest $\mathcal{N} = (1,0)$ supersymmetry and additional hidden $\mathcal{N} = (0,1)$ supersymmetry analogous to $4D, \mathcal{N} = 4$ SYM theory where there is the manifest $\mathcal{N} = 2$ supersymmetry and additional hidden $\mathcal{N} = 2$ supersymmetry.
- The $6D, \mathcal{N} = (1,1)$ SYM theory is anomaly free as well as the $4D, \mathcal{N} = 4$ SYM theory and satisfies some non-renormaization theorems.

$4D$

A.Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky, E. Sokatchev, (1984).

A.Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, *Harmonic Superspace*, **(2001).**

General purpose:
to formulate $\mathcal{N} = 2$ models in terms of unconstrained $\mathcal{N} = 2$ superfields.
General idea:
to use the parameters $u^{\pm i}(i = 1, 2)$ (harmonics) related to $SU(2)$ automorphism group of the $\mathcal{N} = 2$ superalgebra and parameterizing the 2-sphere, $u^{+i}u_i^- = 1$.

It allows to introduce the $\mathcal{N} = 2$ superfields and formulate the theory with manifest $\mathcal{N} = 2$ supersymmetry in harmonic superspace. Price for this is a presence of extra bosonic variables, harmonics $u^{\pm i}$.

$6D$
P.S. Howe, K.S. Stelle, P.C. West, (1985).
B.M. Zupnik, (1986); (1999).
G. Bossard, E. Ivanov, A. Smilga, (2015).

Note! Pure spinor approach to describe $6D$ SYM theories, [**M. Cederwall, (2018)**].

- The classical action of $6D$, $\mathcal{N}=(1,1)$ SYM model $S_0[q^+, V^{++}]$ in the harmonic superspace is written as

$$S_0 = \frac{1}{\mathrm{f}^2}\sum_{n=2}^{\infty}\frac{(-i)^n}{n}\mathrm{tr}\int d^{14}z\, du_1 \ldots du_n \frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)} - \frac{1}{2\mathrm{f}^2}\mathrm{tr}\int d\zeta^{(-4)}\, q^{+A}\nabla^{++}q_A^+.$$

- Gauge transformations

$$V^{++\prime} = -ie^{i\lambda}D^{++}e^{-i\lambda} + e^{i\lambda}V^{++}e^{-i\lambda}, \qquad q^{+\prime} = e^{i\lambda}q^+$$

Background field method

Aim:gauge invariant effective action, (see, e.g., [**B.DeWitt (1965)**]).

- Background-quantum splitting

$$V^{++} \to V^{++} + \mathrm{f}v^{++}, \qquad q^{+} \to Q^{+} + q^{+}$$

- The gauge-fixing function

$$\mathcal{F}^{(+4)} = \nabla^{++} v^{++}$$

- Faddev-Popov procedure
  Analogous to one in $4D, \mathcal{N} = 2$ SYM theory[**I.L.Buchbinder, E.I. Buchbinder, S.M. Kuzenko, B.A. Ovrut, (1998)**].

The one-loop approximation is given by the quadratic action S_2:

$$
\begin{aligned}
S_2 \quad = \quad & \frac{1}{2}\int d\zeta^{(-4)} du\, v^{++\,A}\, \widehat{\square}^{AB}\, v^{++\,B} + \int d\zeta^{(-4)} du\, \mathbf{b}^A (\nabla^{++})^{2\,AB} \mathbf{c}^B \\
& + \frac{1}{2}\int d\zeta^{(-4)} du\, \varphi^A (\nabla^{++})^{2\,AB} \varphi^B - \int d\zeta^{(-4)} du\, \tilde{q}^{+\,m} (\nabla^{++})_m{}^n q_n^+ \\
& - i\mathrm{f} \int d\zeta^{(-4)} du \Big\{ \widetilde{Q}^{+\,m} (v^{++})^C (T^C)_m{}^n q_n^+ + \tilde{q}^{+\,m} (v^{++})^C (T^C)_m{}^n Q_n^+ \Big\},
\end{aligned} \tag{1}
$$

We consider the special change of hypermultiplet variables [**I.L. Buchbinder, N.G. Pletnev,** *JHEP* 0704; **S. M. Kuzenko, S. J. Tyler,** *JHEP* 0705] in the one-loop effective action

$$
q_n^+(1) \to q_n^+(1) - \mathrm{f} \int d\zeta_2^{(-4)} du_2\, G^{(1,1)}(1|2)_n{}^p\, i v^{++\,C}(2)\, (T^C)_p{}^l\, Q_l^+(2)\,, \tag{2}
$$

where $G^{(1,1)}$ is the hypermultiplet Green function.

According to the general analysis[**G. Bossard, E. Ivanov, A. Smilga,**(2015)] the one-loop logarithmic divergences have a structure

$$\Gamma^{(1)}_{\rm ln} = \int d\zeta^{(-4)}\, du \left[c_1 (F^{++A})^2 + i c_2 F^{++A} (\widetilde{q}^{+})^m (T^A)_m{}^n (q^+)_n + c_3 \Big((\widetilde{q}^{+})^m (q^+)_m \Big)^2 \right], \tag{3}$$

where c_1, c_2, and c_3 are numerical real coefficients.

Superficial degree of divergence ω

- One can prove that any supergraph for effective action can be written through the integrals over full $\mathcal{N}=(1,0)$ superspace and contains only a single integral over $d^8\theta$ (non-renormalization theorem).
- One-loop approximation $\omega_{1-\rm loop}(G) = 2 - N_Q$
- Two-loop approximation $\omega_{2-\rm loop}(G) = 4 - N_Q$
- The possible divergences correspond to $\omega_{1-\rm loop} = 2$ and $\omega_{1-\rm loop} = 0$

Calculations of ω are analogous to ones in $4D, \mathcal{N}=2$ gauge theory [**I.L. Buchbinder, S.M. Kuzenko, B.A. Ovrut, (1998)**].

The one-loop quantum correction $\Gamma^{(1)}[V^{++}, Q^+]$ to the classical action, which has the following formal expression

$$\begin{aligned}\Gamma^{(1)}[V^{++}, Q] \quad &= \quad \frac{i}{2}\mathsf{Tr}\ln\Big\{ \widehat{\Box}^{AB} - 4\mathrm{f}^2 \widetilde{Q}^{+\,m}\big(T^A G T^B\big)_m{}^n Q^+_n \Big\} - \frac{i}{2}\mathsf{Tr}\ln \widehat{\Box}_{\mathrm{Adj}} \\ &\quad -i\mathsf{Tr}\ln(\nabla^{++})^2_{\mathrm{Adj}} + \frac{i}{2}\mathsf{Tr}\ln(\nabla^{++})^2_{\mathrm{Adj}} + i\mathsf{Tr}\ln\nabla^{++}_{\mathrm{R}}\,. \end{aligned} \tag{4}$$

The divergent contributions read

$$\Gamma^{(1)}_{F^2} = \frac{C_2 - T(R)}{6(4\pi)^3\varepsilon} \int d\zeta^{(-4)} du\, (F^{++A})^2\,. \tag{5}$$

$$\begin{aligned}\Gamma^{(1)}_{QFQ}[V^{++}, Q^+] \quad &= \quad -\frac{2i\mathrm{f}^2}{(4\pi)^3\varepsilon} \int d\zeta^{(-4)} du \\ &\qquad \widetilde{Q}^{+\,m}(C_2\delta^l_m - C(R)_m{}^l)(F^{++})^A\, (T^A)_l{}^n\, Q^+_n. \end{aligned} \tag{6}$$

In general, the off-shell two-loop divergent contributions to effective action may include following terms

$$\Gamma^{(2)}_{\rm div} = {\rm tr}\int d\zeta^{(-4)}\Big(c_1 F^{++}\,\widehat{\Box}\,F^{++} + c_2 i\,F^{++}\,\widehat{\Box}\,[Q^{+A},Q^{+}_{A}] + c_3\,[Q^{+A},Q^{+}_{A}]\,\widehat{\Box}\,[Q^{+B},Q^{+}_{B}]\Big) + \text{terms vanishing on eq.o.m. for } Q^{+}. \quad (7)$$

Inexplicit $\mathcal{N}=(0,1)$ supersymmetry restricts the structure of the divergent contribution (7) and leads to the absence of two-loop divergences on-shell. Hence we obtain non-trivial constraints on the coefficients:

$$2c_2 + 4c_3 = c_1\,.$$

In case $Q=0$, the calculation was provided in [**I.L. Buchbinder, E.A. Ivanov, K.V. Stepanayantz, B.M., (2021)**]

$$c_1 = \frac{\mathrm{f}^2 (C_2)^2}{8(2\pi)^6\varepsilon^2}\,, \qquad \varepsilon\to 0\,, \quad (8)$$

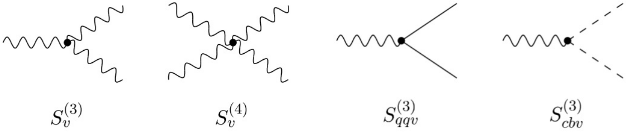

Figure: Standard vertices

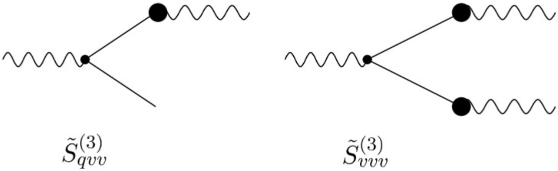

Figure: New type vertices

Two-loop diagrams

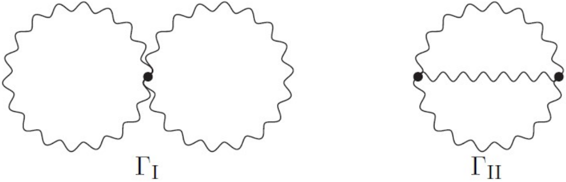

Figure: Two-loop Feynman supergraphs with the gauge self-interactions vertices.

Figure: Two-loop Feynman supergraphs with the hypermultiplet and ghosts vertices.

Two-loop diagrams

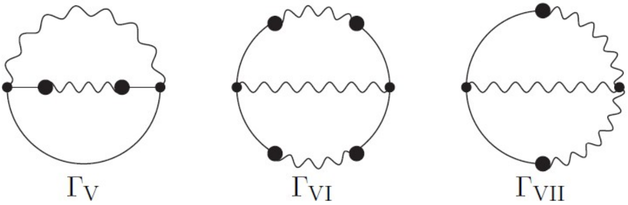

Figure: Two-loop Feynman supergraphs with the new 'non-local' vertices.

Summing up the divergent contributions which contain $F^{++}Q^+Q^+$ from $\Gamma_{\rm I}$, $\Gamma_{\rm V}$ and $\Gamma_{\rm VII}$ we obtain

$$\begin{aligned}
&\Gamma^{\rm FQQ}_{\rm I,\,div} + \Gamma^{\rm FQQ}_{\rm V,\,div} + \Gamma^{\rm FQQ}_{\rm VII,\,div} \qquad (9)\\
&= -\frac{i{\rm f}^2 (C_2)^2}{8(2\pi)^6\,\varepsilon^2}{\rm tr}\int d^{14}z\,\frac{du_1 du_2 du_3}{(u^+_1 u^+_2)(u^+_3 u^+_1)} F^{++}_{1,\tau}[Q^{+A}_{2,\tau}, Q^+_{3A,\tau}]\\
&+\frac{2i{\rm f}^2 (C_2)^2}{(4\pi)^6\varepsilon^2}{\rm tr}\int d^{14}z\frac{du_1 du_2}{(u^+_1 u^+_2)^2} F^{++}_{1,\tau}[Q^{+A}_{2,\tau}, Q^+_{2A,\tau}]\\
&-\frac{4i{\rm f}^2 (C_2)^2}{(4\pi)^6\varepsilon^2}{\rm tr}\int d^{14}z\frac{du_1 du_2 du_3 (u^-_1 u^+_2)}{(u^+_1 u^+_2)(u^+_2 u^+_3)} F^{++}_{1,\tau}[Q^{+A}_{2,\tau}, Q^+_{3A,\tau}] \qquad (10)
\end{aligned}$$

Final expression for the divergent part of the two-loop effective action in the $\mathcal{N}=(1,1)$ SYM theory reads

$$\Gamma^{(2)}_{\rm div} = \frac{{\rm f}^2(C_2)^2}{8(2\pi)^6\varepsilon^2}{\rm tr}\int d\zeta^{(-4)}\Big(F^{++}\,\widehat{\Box}\,F^{++} - \frac{i}{2}\,F^{++}\,\widehat{\Box}\,[Q^{+A},Q^{+}_{A}] + \frac{1}{2}\,[Q^{+A},Q^{+}_{A}]\,\widehat{\Box}\,[Q^{+B},Q^{+}_{B}]\Big) + \text{terms proportional to eq.o.m. for } Q^{+}.$$

The last two coefficient have the form

$$c_2 = -\frac{{\rm f}^2(C_2)^2}{16(2\pi)^6\varepsilon^2}\,, \qquad c_3 = \frac{{\rm f}^2(C_2)^2}{16(2\pi)^6\varepsilon^2}\,, \qquad \varepsilon\to 0\,. \tag{11}$$

Summury

- Background field method in $\mathcal{N} = (1, 0)$ harmonic superspace was developed.
- Superficial degree of divergence was evaluated and structure of one and two-loop counterterms were studied.
- The one-loop divergences in the $6D, \mathcal{N} = (1, 0)$ SYM theory were calculated *off-shell.*
- Two-loop divergences in the $6D, \mathcal{N} = (1, 1)$ SYM theory were find.

Outlook

- 3-loop calculation

Thank you for your attention!