$\mathcal{N} = 2$ superconformal higher-spin supermultiplets

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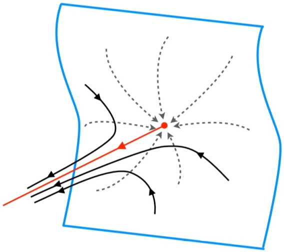
Conformal symmetry I: RG

- Conformal transformations:

$$x^m \to x'^m(x): \qquad g_{mn}(x) \to \Omega(x) g_{mn}(x).$$

- Fixed points of renormalization group are scale(?= conformal) invariant field theories:



- Any quantum field theory can be recovered as a deformation of some conformal field theory.
- Interacting conformal theories (almost always) are in strong coupling regime and are non-Lagrangian theories (see Wilson-Fisher fixed poin, 2D minimal CFT, etc.)
- Higher-spin conformal bootstrap??

Conformal symmetry II: (super)conformal compensators

- Conformal minimal coupling of scalar to gravity:

$$S[\phi, g_{mn}] = \int d^4x\sqrt{-g}\,\phi\left(\Box + \frac{1}{6}R\right)\phi.$$

Action is invariant under under diffeomorphisms and Weyl transformations:

$$\delta g_{mn}(x) = 2a(x)g_{mn}(x), \qquad \delta\phi(x) = -a(x)\phi(x).$$

- In flat limit $g_{mn} = \eta_{mn}$ action (kinetic term have *wrong sign*!)

$$S[\phi] = \int d^4x\ \phi\Box\phi$$

is invariant rigid conformal group $SO(2,4)$:

$$x^m \quad\to\quad x'^m \simeq x^m + \underbrace{a^m}_{translations} + \underbrace{\omega^{[mn]}x_n}_{rotations} + \underbrace{ax^m}_{scale} \underbrace{-k^m x^2 + 2k^n x_n x^m}_{\text{special conformal}},$$

$$\phi(x) \quad\to\quad \phi'(x') = \left|\frac{\partial x'}{\partial x}\right|^{-\frac{1}{4}}\phi(x) \simeq (1 - a - 2k^r x_r)\,\phi(x),$$

$$\eta_{mn} \quad\to\quad g'_{mn}(x') = \left|\frac{\partial x'}{\partial x}\right|^{\frac{1}{2}}\eta_{mn} \simeq (1 + 2a + 4k^r x_r)\,\eta_{mn}.$$

Conformal symmetry II: (super)conformal compensators

- If we fix $\phi = const$, we explicitly break Weyl invariance. Action for conformally coupled scalar will reduced to Einstein-Hilbert gravity action:

$$S[\phi, g_{mn}] = \int d^4x \sqrt{-g}\, \phi \left(\Box + \frac{1}{6} R \right) \phi \quad \xrightarrow{\phi = const} \quad S[g_{mn}] = \frac{1}{6} \int d^4x \sqrt{-g}\, R.$$

- This example admit generalization. If one couple conformal-invariant matter action to gravity in Weyl-invariant way and "compensate" Weyl transformation, one will obtain gravity action:

$$S_{conformal}[\phi] \quad \xrightarrow[gravity]{Weyl} \quad S_{Weyl\ coupling}[\phi, g_{mn}] \quad \xrightarrow[compensator]{conformal} \quad S_{grav}[g_{mn}].$$

- This method is used to construct theories of supergravity. Different versions of supergravity differ in the choice of a superconformal compensator.

$$S_{superconformal} \quad \xrightarrow[multiplet]{Weyl} \quad S_{superWeyl\ coupling} \quad \xrightarrow[compensator]{superconformal} \quad S_{supergrav}.$$

Conformal higher spins ($s > 2$)

- Anti-de Sitter space [Fradkin, Vasiliev 1987], Vasiliev equations [Vasiliev 1990].
- Conformal higher spin gravities are examples of (covariant) actions for interacting higher spin gravities on flat background.
- Free conformal higher spins [Fradkin, Tseytlin 1985].
- Effective action approach [Tseytlin 2002], [Bekaert, Joung, Mourad 2010], ...
- Worldline model and deformation quantization [Segal 2003], [Basile, Grigoriev, Skvortsov 2022], ...
- Conformal interactions between matter and higher-spin (super)fields [Fradkin, Linetsky 1987], [Kuzenko ...], ...
- Fronsdal [1978] (free non-conformal higher spins):

$$\delta\Phi^{\alpha(s)\dot\alpha(s)} \sim \partial^{(\alpha(\dot\alpha} a^{\alpha(s-1))\dot\alpha(s-1))}, \qquad \delta\Phi^{\alpha(s-2)\dot\alpha(s-2)} \sim \partial_{\beta\dot\beta} a^{(\beta\alpha(s-2))(\dot\beta\dot\alpha(s-2))}$$

- Fradkin-Tseytlin [1985] (free conformal higher spins):

$$\delta\Phi^{\alpha(s)\dot\alpha(s)} \sim \partial^{(\alpha(\dot\alpha} a^{\alpha(s-1))\dot\alpha(s-1))}, \qquad \delta\Phi^{\alpha(s-2)\dot\alpha(s-2)} \sim \underbrace{\zeta^{\alpha(s-2)\dot\alpha(s-2)}}_{higher-spin\ Weyl\ transformation}$$

- Using $\zeta^{\alpha(s-2)\dot\alpha(s-2)}$ (higher-spin analogue of Weyl symmetry) one can impose gauge $\Phi^{\alpha(s-2)\dot\alpha(s-2)} = 0$.

Supersymmetry

Supersymmetry is “unique” extension of Poincaré group [Haag–Łopuszański–Sohnius, 1975]:

$$\left\{Q^i_\alpha, \bar{Q}_{\dot\alpha j}\right\} = 2\delta^i_j \sigma^m_{\alpha\dot\alpha} P_m, \quad \{Q^i_\alpha, Q^j_\beta\} = \epsilon_{\alpha\beta} Z^{[ij]}, \quad \{\bar{Q}_{\dot\alpha i}, \bar{Q}_{\dot\beta j}\} = \epsilon_{\dot\alpha\dot\beta} \bar{Z}_{[ij]}, \qquad i = 1, 2 \ldots \mathcal{N}.$$

- In field theories with linearly realized supersymmetry fields are unified in supermultiplets. Supersymmetry mix bosonic and fermionic fields:

$$\delta B \sim \epsilon F, \qquad \delta F \sim \epsilon \partial B.$$

- Form of supersymmetry transformation is model dependent if SUSY realized on physical d.o.f. In interacting theory supersymmetry transformations are non-linear in fields.
- On-shell there are equal number of bosonic and fermionic d.o.f.
- Auxiliary fields are needed for three purposes:
  1. Supersymmetry transformations are model-independent and manifestly linear.
  2. There are equal number of fermionic and bosonic d.o.f. off-shell.
  3. Super-Poincaré algebra is closed off-shell (otherwise, constraints must be imposed during quantization).
- The problem of searching for auxiliary fields is not-trivial and are known only for some $\mathcal{N} = 1, 2, 3$ theories. May be some theories do not admit off-shell formulation, e.g. $\mathcal{N} = 4$ SYM, $\mathcal{N} = 8$ SUGRA.

Superconformal symmetry

- Superconformal symmetry is the extension of supersymmetry if we deal with special massless theories. In superconformal fields theories there are additional conformal supersymmetry generators

$$P_m, L_{mn}, Q^i_\alpha, \bar{Q}_{\dot{\alpha} i}, \quad D, K_m, S^i_\alpha, \bar{S}_{\dot{\alpha} i}, I^i_j$$

  with $SU(2,2|\mathcal{N})$ algebraic structure:

$$\left\{ S_{\alpha j}, \bar{S}^i_{\dot{\alpha}} \right\} = 2\delta^i_j \sigma^m_{\alpha\dot{\alpha}} K_m, \quad \left\{ Q^i_\alpha, S^\beta_j \right\} = -\delta^j_i l_{(\alpha}{}^{\beta)} - 4i\delta^\beta_\alpha I^i_j - 2i\delta^\beta_\alpha \delta^i_j D + \frac{2(4-\mathcal{N})}{\mathcal{N}} \delta^\beta_\alpha \delta^i_j R, \ldots$$

- All lower-spin massless supermultiplets are field representations of SC symmetry.
- There are many constructions of superconformal field theories that do not necessarily admit Lagrangian descriptions, such as class $\mathcal{S}$ theories [Gaiotto, Moore, Neitzke 2009] and Argyres-Douglas theories [Argyres, Douglas 1995].
- "Gauging" of superconformal symmetry lead to Weyl multiplet (conformal supergravity multiplet).

- The most natural and useful way describe off-shell supersymmetric theories is superspace. Superspace is natural generalization of Minkowski space, which is necessary for construction of manifestly Poincare-invariant theories.
- Minkowski space-time can be realized as coset space:

$$\mathbb{M}^4 := \frac{ISO(1,3)}{SO(1,3)} = \frac{\{L_{nm}, P_n\}}{\{L_{nm}\}} = (x^m).$$

Coset construction can be easily generalized to supersymmetry:

- Real superspace:

$$\mathbb{R}^{4|4\mathcal{N}} := \frac{\mathcal{SP}(1,3|\mathcal{N}) \times SU(\mathcal{N})}{SO(1,3) \times SU(\mathcal{N})} = \frac{\{L_{nm}, su(\mathcal{N}), P_n, Q^i_\alpha, \bar{Q}_{\dot{\alpha} i}\}}{\{L_{nm}, su(\mathcal{N})\}} = \left(x^m, \theta^\alpha_i, \bar{\theta}^{\dot{\alpha} i}\right).$$

- Chiral superspace:

$$\mathbb{C}^{4|2\mathcal{N}} := \frac{\{L_{nm}, su(\mathcal{N}), P_n, Q^i_\alpha, \bar{Q}_{\dot{\alpha} i}\}}{\{L_{nm}, su(\mathcal{N}), \bar{Q}_{\dot{\alpha} i}\}} = (x^m, \theta^\alpha_i).$$

- (Super)Poincare group have natural realization on such (super)manifolds:

$$x^m \to x^m + a^m + L^m{}_n x^n, \qquad \theta^\alpha_i \to \theta^\alpha_i + \epsilon^\alpha_i + l^{(\alpha}{}_{\beta)} \theta^\beta_i, \qquad \bar{\theta}^{\dot{\alpha} i} \to \bar{\theta}^{\dot{\alpha} i} + \bar{\epsilon}^{\dot{\alpha} i} + \bar{l}^{(\dot{\alpha}}{}_{\dot{\beta})} \bar{\theta}^{\dot{\beta} i}.$$

One can easily define (Super)Poincare-covariant fields on $\mathbb{M}^4$ ($\mathbb{R}^{4|4\mathcal{N}}, \mathbb{C}^{4|2\mathcal{N}}$) and construct (Super)Poincare-invariant actions.

- Superspaces $\mathbb{R}^{4|4}$ and $\mathbb{C}^{4|2}$ are inevitable for description of $\mathcal{N} = 1$ theories: chiral multiplet, vector multiplet, supergravity multiplet, higher-spin off-shell multiplets have formulation in terms of unconstrained superfields.

Harmonic superspace

- Harmonic superspace [Galperin, Ivanov, Kalitsyn, Ogievetsky, Sokatchev 1984] is efficient approach of dealing with supersymmetric theories with 8 real SUSY generators in a manifestly covariant manner.
- In harmonic superspace there are auxiliary coordinates – harmonics $u_i^{\pm}$:

$$\mathbb{HR}^{4+2|8} := \frac{\{P_m, L_{mn}, Q^i_\alpha, su(2)\}}{\{L_{mn}, u(1)\}} = \mathbb{R}^{4|8} \times S^2 = \{x^m, \theta^{\alpha i}, u^{\pm i}\}$$

- Harmonic superspace allow off-shell hypermultiplet with infinitely many auxiliary fields.
- Harmonics satisfy relations: $u_i^- = (u^{+i})^*$ and $u^{i+} u_i^- = 1$.
- Superfields in HSS: $Q^{(n)} = Q^{(n)}(x^a, \theta^i_\alpha, u^{\pm i})$ have infinitely many components.
- Using harmonics one can convert $su(2)$ indices to $u(1)$:

$$Q^i_\alpha \quad \to \quad Q^+_\alpha = Q^i_\alpha u_i^+, \quad Q^-_\alpha = Q^i_\alpha u_i^- .$$

- Harmonic superspace have new invariant subspace containing only half of the original Grassmann variables. Analytic superspace ($\mathcal{N} = 2$ analog of chiral superspace in $\mathcal{N} = 1, d = 4$):

$$\mathbb{HA}^{4+2|4} := \frac{\{L_{mn}, P_m, Q^\pm_\alpha, \bar{Q}^\pm_{\dot\alpha}, su(2)\}}{\{L_{mn}, Q^+_\alpha, \bar{Q}^+_{\dot\alpha}, u(1)\}} = \underbrace{(x^m, \theta^+_\alpha, \bar\theta^+_{\dot\alpha}, u^{\pm i})}_{\zeta} .$$

Hypermultiplet in harmonic superspace

- Hypermultiplet is the unique $\mathcal{N}=2$ matter multiplet.
- On-shell hypermultiplet contain doublet of complex scalars $f^i(x)$ and a pair of singlet spinors $\psi_\alpha(x), \kappa_\alpha(x)$.
- A finite set of auxiliary fields does not exist for hypermultiplet ("no-go theorems" in extended SUSY).
- **Hypermultiplet** in HSS is described by an unconstrained analytic superfield $q^+(\zeta)$. It contains a doublet of complex scalars f^i and a pair of singlet spinors $\psi_\alpha, \kappa_\alpha$, as well as an infinite set of auxiliary fields which comes from the harmonic S^2 expansions:

$$q^+(\zeta) = f^i u_i^+ + \theta^{+\alpha}\psi_\alpha + \bar{\theta}^+_{\dot{\alpha}}\bar{\kappa}^{\dot{\alpha}} + \text{auxiliary fields}.$$

- The free hypermultiplet action has the form:

$$S_{free} = -\int d\zeta^{(-4)}\tilde{q}^+\mathcal{D}^{++}q^+ = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\mathcal{D}^{++}q^+_a.$$

Here $q^{+a} = (\tilde{q}^+, q^+)$, $q^+_a = \epsilon_{ab}q^{+b} = (q^+, -\tilde{q}^+)$, $d\zeta^{(-4)} := d^4x d^4\theta^+ du$.

- Here we used covariant harmonic derivative:

$$\mathcal{D}^{++} = \underbrace{\partial^{++}}_{u^{+i}\frac{\partial}{\partial u^{-i}}} - 4i\theta^{+\beta}\bar{\theta}^{+\dot{\beta}} \underbrace{\partial_{\beta\dot{\beta}}}_{\frac{1}{2}\sigma^m_{\beta\dot{\beta}}\partial_m} + \theta^{+\hat{\beta}}\partial^+_{\hat{\beta}}.$$

- Equation of motion $\mathcal{D}^{++}q^+ = 0$ allow to exclude all auxiliary fields and lead to free Klein-Gordon and Weyl equations: $\Box f^i = 0$, $\partial^{\alpha\dot{\alpha}}\bar{\psi}_{\dot{\alpha}} = 0$, $\partial^{\alpha\dot{\alpha}}\bar{\kappa}_{\dot{\alpha}} = 0$.

- $\mathcal{N}=2$ superconformal symmetry realized on hypermultiplet as:

$$\delta q^{+a} = -\hat{\Lambda} q^{+a} - \frac{1}{2}\Omega q^{+a},$$

$$\hat{\Lambda} := \lambda^M \partial_M = \lambda^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \lambda^{+\hat{\alpha}}\partial^-_{\hat{\alpha}} + \lambda^{++}\partial^{--},$$
$$\Omega := (-1)^{P(M)}\partial_M \lambda^M = \partial_{\alpha\dot{\alpha}}\lambda^{\alpha\dot{\alpha}} - \partial^-_{\hat{\alpha}}\lambda^{+\hat{\alpha}} + \partial^{--}\lambda^{++}.$$

- This is most general one-derivative transformations of hypermultiplet. If parameters satisfy equation

$$[\mathcal{D}^{++}, \hat{\Lambda}] = \lambda^{++}\mathcal{D}^0,$$

free hypermultiplet action is invariant.

- Most general solution of equation are $\mathcal{N}=2$ superconformal transformations:

$$\begin{cases} \lambda^{\alpha\dot{\alpha}}_{sc} = & a^{\alpha\dot{\alpha}} - 4i\left(\epsilon^{\alpha i}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\bar{\epsilon}^{\dot{\alpha} i}\right)u^-_i + x^{\dot{\alpha}\rho}k_{\rho\dot{\rho}}x^{\dot{\rho}\alpha} + b x^{\alpha\dot{\alpha}} \\ & -4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\lambda^{(ij)}u^-_i u^-_j - 4i\left(x^{\alpha\dot{\rho}}\eta^i_{\dot{\rho}}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\eta^i_\rho x^{\rho\dot{\alpha}}\right)u^-_i, \\ \lambda^{+\alpha}_{sc} = & \epsilon^{\alpha i}u^+_i + \frac{1}{2}\theta^{+\alpha}(b+i\gamma) + x^{\alpha\dot{\beta}}k_{\beta\dot{\beta}}\theta^{+\beta} + x^{\alpha\dot{\alpha}}\eta^i_{\dot{\alpha}}u^+_i \\ & +\theta^{+\alpha}\left(\lambda^{(ij)}u^+_i u^-_j + 4i\theta^{+\rho}\eta^i_\rho u^-_i\right), \\ \bar{\lambda}^{+\dot{\alpha}}_{sc} = & \epsilon^{\dot{\alpha} i}u^+_i + \frac{1}{2}\bar{\theta}^{+\dot{\alpha}}(b-i\gamma) + x^{\dot{\alpha}\beta}k_{\beta\dot{\beta}}\bar{\theta}^{+\dot{\beta}} + x^{\alpha\dot{\alpha}}\eta^i_\alpha u^+_i \\ & +\bar{\theta}^{+\dot{\alpha}}\left(\lambda^{(ij)}u^+_i u^-_j - 4i\bar{\theta}^{+\dot{\rho}}\eta^i_{\dot{\rho}}u^-_i\right), \\ \lambda^{++}_{sc} = & \lambda^{ij}u^+_i u^+_j + 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}k_{\alpha\dot{\alpha}} + 4i\left(\theta^{+\alpha}\eta^i_\alpha + \eta^i_{\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}}\right)u^+_i. \end{cases}$$

- $\mathcal{N} = 2$ conformal supergravity can be obtained by gauging $\mathcal{N} = 2$ superconformal group [Galperin, Ivanov, Ogievetsky, Sokatchev 1987].
- Supergravity prepotentials are analytical superfields and appear as gauge fields in covariant analytic harmonic derivative $\mathcal{D}^{++}$:

$$\mathcal{D}^{++} \qquad \to \qquad \mathfrak{D}^{++} = \mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{s=2},$$

$$\hat{\mathcal{H}}^{++}_{s=2} = h^{++\beta\dot{\beta}}\partial_{\beta\dot{\beta}} + h^{++\beta+}\partial^{-}_{\beta} + h^{++\dot{\beta}+}\partial^{-}_{\dot{\beta}} + h^{(+4)}\partial^{--},$$

$$\delta^{full}_{\lambda}\hat{\mathcal{H}}^{++}_{(s=2)} = [\mathcal{D}^{++} + \kappa_2\hat{\mathcal{H}}^{++}_{(s=2)}, \hat{\Lambda}_{(s=2)}].$$

- Component content of $\mathcal{N} = 2$ Weyl multiplet:

$$\begin{cases} h^{++\alpha\dot{\alpha}} = -4i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\Phi^{\alpha\dot{\alpha}}_{\rho\dot{\rho}} + (\bar{\theta}^{+})^2\theta^{+\rho}\psi^{\alpha\dot{\alpha}i}_{\rho}u^{-}_{i} + (\theta^{+})^2\bar{\theta}^{+\dot{\rho}}\bar{\psi}^{\alpha\dot{\alpha}i}_{\dot{\rho}}u^{-}_{i} \\ \qquad\qquad + (\theta^{+})^2(\bar{\theta}^{+})^2 V^{\alpha\dot{\alpha}(ij)}u^{-}_{i}u^{-}_{j}, \\ h^{++\alpha+} = (\theta^{+})^2\bar{\theta}^{+}_{\dot{\alpha}}P^{\alpha\dot{\alpha}} + \left(\bar{\theta}^{+}\right)^2\theta^{+}_{\beta}T^{(\alpha\beta)} + (\theta^{+})^2(\bar{\theta}^{+})^2\chi^{\alpha i}u^{-}_{i}, \\ h^{++\dot{\mu}+} = \widetilde{h^{++\mu+}}, \\ h^{(+4)} = (\theta^{+})^2(\bar{\theta}^{+})^2 D. \end{cases}$$

- $\mathbf{32_B} + \mathbf{32_F}$ off-shell degrees of freedom.

- General form of cubic vertex of hypermultiplet with higher spin superfields have form:

$$S^{(s)}_{sc-cubic,s} = -\frac{\kappa_s}{2}\int d\zeta^{(-4)}\, q^{+a} h^{++M_1\dots M_{s-1}}\partial_{M_{s-1}}\dots\partial_{M_1}\,(J)^{P(s)}\, q^+_a$$
$$+ \text{lower derivative terms}\,,$$

Here we denote:

$$\partial_M := \left(\partial_{\alpha\dot\alpha}, \partial^-_\alpha, \partial^-_{\dot\alpha}, \partial^{--}\right), \quad J\tilde q^+ = i\tilde q^+, \quad Jq^+ = -iq^+, \qquad P(s) := \frac{1-(-1)^s}{2}.$$

- Superconformal invariance of cubic interaction require inclusion of all possible combination of derivatives and presence terms with $s-1$, $s-3$, ... derivatives:

$$S_{free} + S_{sc-cubic,s} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + \hat{\mathbb{H}}^{++}_{(s)}(J)^{P(s)}\right) q^+_a\,,$$

where we introduced the analytic differential operator of degree $s-1$ with the odd number of superspace derivatives:

$$\hat{\mathbb{H}}^{++}_{(s)} := h^{++M_1\dots M_{s-1}}\partial_{M_{s-1}}\dots\partial_{M_1} + h^{++M_1\dots M_{s-3}}\partial_{M_{s-3}}\dots\partial_{M_1} + \dots .$$

- $\mathcal{N}=2$ superconformal group linearly realized on $h^{++M_1\dots}$ superfields:

$$\delta_{sc}\hat{\mathbb{H}}^{++}_{(s)} = [\hat{\mathbb{H}}^{++}_{(s)}, \hat\Lambda] + \frac{1}{2}[\hat{\mathbb{H}}^{++}_{(s)}, \Omega].$$

Gauge freedom

- Total hypermultiplet action with cubic coupling:

$$S_{free} + S_{sc-cubic,s} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + \hat{\mathbb{H}}^{++}_{(s)}(J)^{P(s)}\right) q^{+}_{a}.$$

- Constructed action have enormous gauge freedom ($k = s, s-2, \dots$):

$$\delta^{(s,k)}_{\lambda} q^{+}_{a} = -\frac{\kappa_s}{2}\left\{\hat{\Lambda}^{M_1\dots M_{k-2}} + \frac{1}{2}\Omega^{M_1\dots M_{k-2}}, \partial_{M_{k-2}}\dots\partial_{M_1}\right\}_{AGB}(J)^{P(s)} q^{+}_{a},$$

$$\delta^{(s,k)}_{\lambda}\hat{\mathbb{H}}^{++}_{(s)} = \frac{1}{2}\left[\mathcal{D}^{++}, \left\{\hat{\Lambda}^{M_1\dots M_{k-2}}, \partial_{M_{k-2}}\dots\partial_{M_1}\right\}_{AGB}\right],$$

$$\hat{\Lambda}^{M_1\dots M_{k-2}} := \lambda^{M_1\dots M_{k-2}N}\partial_N, \qquad \Omega^{M_1\dots M_{k-2}} := (-1)^{P(N)}\partial_N\lambda^{NM_1\dots M_{k-2}}.$$

- The different values of k correspond to the gauge freedom for different spin contributions appearing in the operator $\hat{\mathbb{H}}^{++}_{(s)}$.
- It is also possible to extend cubic coupling to arbitrary $\mathcal{N} = 2$ conformal supergravity background:

$$\mathcal{D}^{++} \quad \to \quad \mathfrak{D}^{++}.$$

- Using gauge freedom one can impose Wess-Zumino-type gauge:

$$
\begin{aligned}
h^{++\alpha(s-1)\dot{\alpha}(s-1)}_{WZ} &= -4i\theta^{+}_{\rho}\bar{\theta}^{+}_{\dot{\rho}}\Phi^{(\rho\alpha(s-1))(\dot{\rho}\dot{\alpha}(s-1))} - (\bar{\theta}^{+})^{2}\theta^{+}_{\rho}\psi^{(\rho\alpha(s-1))\dot{\alpha}(s-1)i}u^{-}_{i} \\
&\quad - (\theta^{+})^{2}\bar{\theta}^{+}_{\dot{\rho}}\bar{\psi}^{\alpha(s-1)(\dot{\alpha}(s-1)\dot{\rho})i}u^{-}_{i} + (\theta^{+})^{2}(\bar{\theta}^{+})^{2}V^{\alpha(s-1)\dot{\alpha}(s-1)ij}u^{-}_{i}u^{-}_{j}\,, \\
h^{++\alpha(s-1)\dot{\alpha}(s-2)+}_{WZ} &= (\theta^{+})^{2}\bar{\theta}^{+}_{\dot{\nu}}P^{\alpha(s-1)(\dot{\alpha}(s-2)\dot{\nu})} + (\bar{\theta}^{+})^{2}\theta^{+}_{\nu}T^{(\alpha(s-1)\nu)\dot{\alpha}(s-2)} \\
&\quad + (\theta^{+})^{4}\chi^{\alpha(s-1)\dot{\alpha}(s-2)i}u^{-}_{i}\,, \\
h^{++\alpha(s-2)\dot{\alpha}(s-1)+}_{WZ} &= \widetilde{h^{++\alpha(s-1)\dot{\alpha}(s-2)+}_{WZ}}\,, \\
h^{(+4)\alpha(s-2)\dot{\alpha}(s-2)}_{WZ} &= (\theta^{+})^{2}(\bar{\theta}^{+})^{2}D^{\alpha(s-2)\dot{\alpha}(s-2)}\,.
\end{aligned}
$$

- All other potentials are purely gauge degrees of freedom. In the gauge that we have fixed, they are set to zero.
- In principle, it is possible to initially fix the gauge in which these fields are zero, but in this case the superconformal group will be implemented non-linearly. This situation is typical for the conformal theory of higher spins, see, e.g., [Kuzenko, Ponds, Raptakis 2022].

Gauge freedom of components

• Bosonic fields $\Phi^{\alpha(s)\dot\alpha(s)}$, $V^{\alpha(s-1)\dot\alpha(s-1)ij}$, $P^{(\alpha(s-1)\dot\alpha(s-1))}$, $D^{\alpha(s-2)\dot\alpha(s-2)}$ are conformal Fradkin-Tseytlin fields [Fradkin, Tseytlin 1985] with gauge freedom of the form:

$$\delta\Phi^{\alpha(s)\dot\alpha(s)} \sim \partial^{(\alpha(\dot\alpha}a^{\alpha(s-1))\dot\alpha(s-1))}, \quad \delta V^{\alpha(s-1)\dot\alpha(s-1)ij} \sim \partial^{(\alpha(\dot\alpha}v^{\alpha(s-2))\dot\alpha(s-2))(ij)}, \dots .$$

• Complex field $T^{\alpha(s)\dot\alpha(s-2)}$ have the gauge freedom:

$$\delta T^{\alpha(s)\dot\alpha(s-2)} \sim \partial^{(\alpha(\dot\alpha}t^{\alpha(s-1))\dot\alpha(s-3))}.$$

This is more general type of conformal field. In the simplest non-trivial $s = 3$ case this fields is equivalent to the traceless hook gauge field $T^{a[bc]}$ with the algebraic symmetries corresponding to the simplest hook Young diagram .

• Fields $\psi^{\alpha(s)\dot\alpha(s-1)i}$ and $\chi^{\alpha(s-1)\dot\alpha(s-2)i}$ are conformal fermionic gauge fields:

$$\delta\psi^{\alpha(s)\dot\alpha(s-1)i} \sim \partial^{(\alpha(\dot\alpha}\xi^{\alpha(s-1))\dot\alpha(s-2))i}, \quad \delta\chi^{\alpha(s-1)\dot\alpha(s-2)i} \sim \partial^{\alpha\dot\alpha}\zeta^{\alpha(s-2))\dot\alpha(s-3))i}.$$

• Finally, the $\mathcal{N} = 2$ superconformal spin **s** multiplet is encompassed by $\mathbf{8(2s-1)_B + 8(2s-1)_F}$ off-shell degrees of freedom.

• For $s \geq 3$ all field are gauge fields, so there are no auxiliary fields!

• The simplest $s = 1, 2$ multiplets (in contrast to $s \geq 3$) contain auxiliary fields and comprise $\mathcal{N} = 2$ Maxwell and $\mathcal{N} = 2$ conformal supergravity (Weyl) gauge multiplets.

Non-conformal (Fronsdal) $\mathcal{N}=2$ higher spins

[Buchbinder, Ivanov, N.Z. 2021]

$$
\begin{aligned}
h_{WZ}^{++\alpha(s-1)\dot\alpha(s-1)} &= -4i\theta^+_\beta\bar\theta^+_{\dot\beta}\Phi^{(\beta\alpha(s-1))(\dot\beta\dot\alpha(s-1))} - 4i\theta^{+(\alpha}\bar\theta^{+(\dot\alpha}\Phi^{\alpha(s-2))\dot\alpha(s-2))} \\
&\quad + (\bar\theta^+)^2\theta^{+\beta}\psi_\beta^{\alpha(s-1)\dot\alpha(s-1)i}u_i^- + (\theta^+)^2\bar\theta^{+\dot\beta}\bar\psi_{\dot\beta}^{\alpha(s-1)\dot\alpha(s-1)i}u_i^- \\
&\quad + (\theta^+)^2(\bar\theta^+)^2 V^{\alpha(s-1)\dot\alpha(s-1)(ij)}u_i^-u_j^- , \\
h_{WZ}^{++\alpha(s-2)\dot\alpha(s-2)} &= -4i\theta^+_\beta\bar\theta^+_{\dot\beta}C^{(\beta\alpha(s-2))(\dot\beta\dot\alpha(s-2))} - 4i\theta^{+(\alpha}\bar\theta^{+(\dot\alpha}C^{\alpha(s-3))\dot\alpha(s-3))} \\
&\quad + (\bar\theta^+)^2\theta^{+\beta}\rho_\beta^{\alpha(s-2)\dot\alpha(s-2)i}u_i^- + (\theta^+)^2\bar\theta^{+\dot\beta}\bar\rho_{\dot\beta}^{\alpha(s-2)\dot\alpha(s-2)i}u_i^- \\
&\quad + (\theta^+)^2(\bar\theta^+)^2 S^{\alpha(s-2)\dot\alpha(s-2)(ij)}u_i^-u_j^- , \\
h_{WZ}^{++\alpha(s-1)\dot\alpha(s-2)+} &= (\theta^+)^2\bar\theta^+_{\dot\beta}P^{\alpha(s-1)\dot\alpha(s-2)\dot\beta} + \left(\bar\theta^+\right)^2\theta^+_\beta T^{\dot\alpha(s-2)\alpha(s-1)\beta} \\
&\quad + (\theta^+)^2(\bar\theta^+)^2\chi^{\alpha(s-1)\dot\alpha(s-2)i}u_i^- , \\
h_{WZ}^{++\dot\alpha(s-1)\alpha(s-2)+} &= \widetilde{\left(h_{WZ}^{++\alpha(s-1)\dot\alpha(s-2)+}\right)} .
\end{aligned}
$$

- Fields $(\Phi^{\alpha(s)\dot\alpha(s)}, \Phi^{\alpha(s-2)\dot\alpha(s-2)})$ and $(C^{\alpha(s-1)\dot\alpha(s-1)}, C^{\alpha(s-3)\dot\alpha(s-3)})$ correspond to the massless Fronsdal spin s and $s-1$ fields:

$$\delta\Phi^{\alpha(s)\dot\alpha(s)} \sim \partial^{(\alpha(\dot\alpha}a^{\alpha(s-1))\dot\alpha(s-1))}, \qquad \delta\Phi^{\alpha(s-2)\dot\alpha(s-2)} \sim \partial_{\beta\dot\beta}a^{(\beta\alpha(s-2))(\dot\beta\dot\alpha(s-2))};$$

$$\delta C^{\alpha(s-1)\dot\alpha(s-1)} \sim \partial^{(\alpha(\dot\alpha}b^{\alpha(s-2))\dot\alpha(s-2))}, \qquad \delta C^{\alpha(s-3)\dot\alpha(s-3)} \sim \partial_{\beta\dot\beta}a^{(\beta\alpha(s-3))(\dot\beta\dot\alpha(s-3))}.$$

- Fields $V^{\alpha(s-1)\dot\alpha(s-1)(ij)}, S^{\alpha(s-2)\dot\alpha(s-2)(ij)}$ are real bosonic auxiliary fields.

- Fields $P^{\alpha(s-1)\dot\alpha(s-2)\dot\mu},\ T^{\alpha(s-1)\nu\dot\alpha(s-2)}$ are complex bosonic auxiliary fields.

- Fields $\left(\psi_\beta^{\alpha(s-1)\dot\alpha(s-1)i}, \bar\rho_{\dot\beta}^{\alpha(s-2)(\dot\alpha(s-3)\dot\beta)i}\right)$ possess gauge freedom characteristic of the doublet of massless spin $s-\frac{1}{2}$ Fang-Fronsdal fields:

$$\delta\psi_\beta^{\alpha(s-1)\dot\alpha(s-1)} \sim \partial_\beta^{(\dot\alpha}\xi^{\alpha(s-1)\dot\alpha(s-1))}, \qquad \delta\bar\rho_{\dot\beta}^{\alpha(s-2)(\dot\alpha(s-3)\dot\beta)} \sim \partial_{\beta\dot\beta}\xi^{(\alpha(s-2)\beta)(\dot\alpha(s-3)\dot\beta)}.$$

- Fields $\rho^{\alpha(s-1)\dot\alpha(s-2)i}, \chi^{\alpha(s-1)\dot\alpha(s-2)i}$ are auxiliary fermionic fields.

- $\mathcal{N}=2$ spin s supermultiplet involves $\mathbf{8(s^2+(s-1)^2)}_B + \mathbf{8(s^2+(s-1)^2)}_F$ off-shell degrees of freedom.

- In the simplest $s=2$ case one reproduces the off-shell multiplet of the "minimal" $\mathcal{N}=2$ Einstein supergravity [Fradkin, Vasiliev 1979], [de Wit, van Holten 1979].

$$\hat{\mathbb{H}}^{++} := \sum_{s=1}^{\infty} \kappa_s \hat{\mathbb{H}}^{++}_{(s)} (J)^{P(s)}.$$

- Action of infinite tower of integer $\mathcal{N}=2$ superconformal higher spins on arbitrary $\mathcal{N}=2$ conformal supergravity background:

$$S_{full} = -\frac{1}{2} \int d\zeta^{(-4)}\, q^{+a} \left(\mathfrak{D}^{++} + \hat{\mathbb{H}}^{++} \right) q^{+}_{a}.$$

- Then by proper transformation of $\hat{\mathbb{H}}^{++}$, one can obtain gauge invariance in any order in couplings constant.
- We denote sum of hypermultiplet gauge transformations for all spins as

$$\delta_\lambda q^{+a} := -\hat{\mathcal{U}} q^{+a} = -\sum_{s=1}^{\infty} \kappa_s \hat{\mathcal{U}}_s q^{+a}.$$

- For set of gauge fields we obtain non-abelian transformation law:

$$\delta_\lambda \hat{\mathbb{H}}^{++} = -\left[\mathfrak{D}^{++} + \hat{\mathbb{H}}^{++}, \hat{\mathcal{U}} \right].$$

This transformations acts linearly on hypermultiplet and mix superfields with different spins.

Conclusions

- We considered the off-shell hypermultiplet model in $\mathcal{N}=2$, $4D$ harmonic superspace and described its rigid superconformal symmetries. For invariance of the cubic higher-spin vertices $(\mathbf{s}, \frac{1}{2}, \frac{1}{2})$ under these symmetries it proved necessary to properly modify the superconformal vertex of the hypermultiplet by the corresponding superconformal gauge superfields;
- To this end, we introduced the complete set of $\mathcal{N}=2$, $4D$ unconstrained analytic spin $\mathbf{s}$ superconformal higher-spin potentials, defined their superconformal and gauge transformations and discussed the physical field contents of the corresponding $\mathcal{N}=2$ higher-spin Weyl supermultiplets;
- As a result, we have derived the manifestly $\mathcal{N}=2$ superconformal cubic vertex of the hypermultiplet coupled to superconformal higher spin external gauge superfields. Generically, the vertex has the structure: *higher spin superconformal gauge superfields* $\times$ *superconformal hypermultiplet supercurrents*. The corresponding supercurrents can be explicitly constructed in terms of the hypermultiplet superfields;
- As generalization, we have constructed the off-shell $(\mathbf{s}, \frac{1}{2}, \frac{1}{2})$ vertices on the arbitrary background of $\mathcal{N}=2$ conformal supergravity background.

Outlook

- *Free dynamical manifestly $\mathcal{N} = 2$ actions for higher-spin superconformal multiplets*
- *Superconformal current superfield and rigid higher-spin superconformal symmetries*
- *Induced actions*
- *Higher-spin conformal compensators*
- *AdS background*
- *Construction of more general interactions*

Thank you for your attention!