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To the memory of Igor Batalin

Problem of time in quantum cosmology and origin of the Universe

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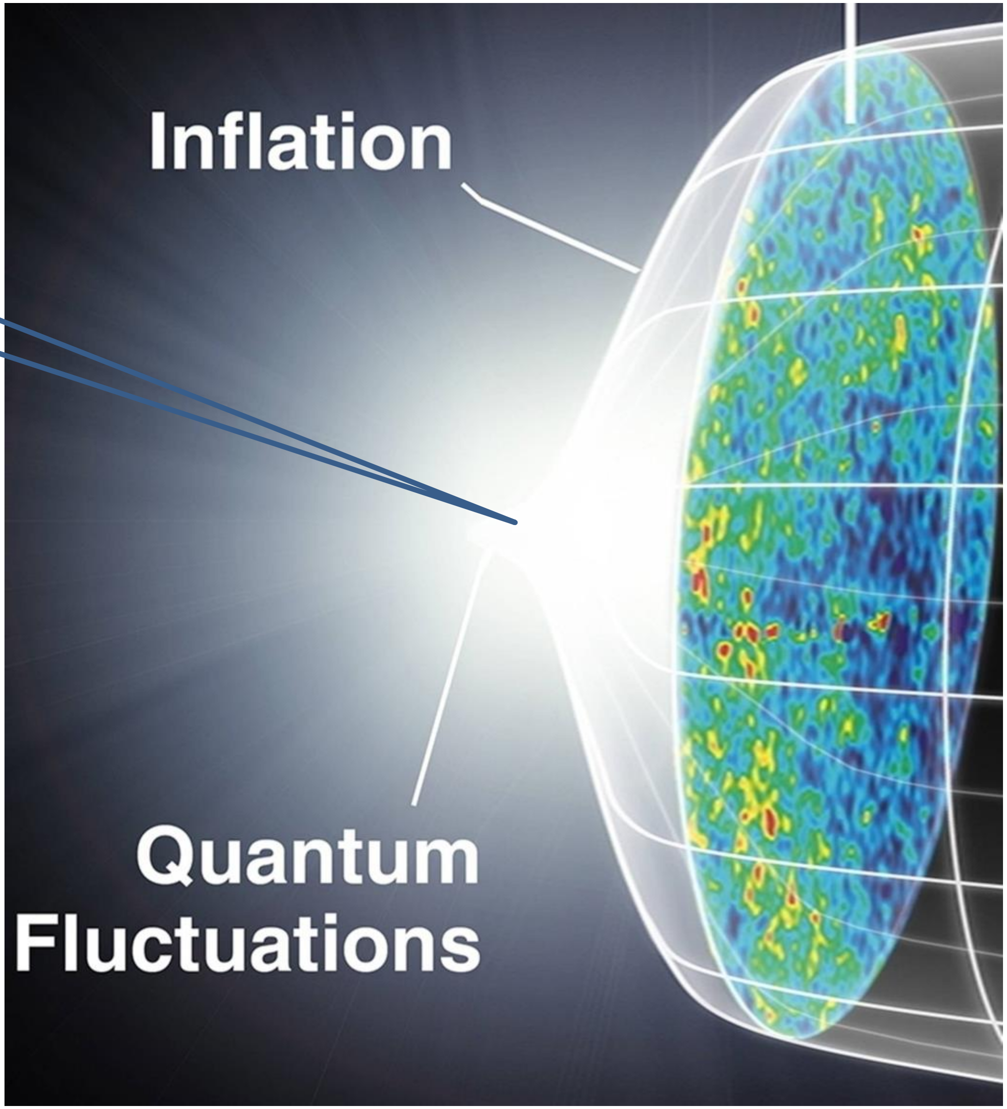
What was at the beginning?

The space and time had both one beginning . The space was made not in time but simultaneously with time.

Saint Augustin of Hippo



Outline

Problem of time in quantum gravity: Schroedinger equation vs Wheeler-DeWitt equation(s)

No-boundary (Hartle-Hawking) vs tunneling wavefunction

Cosmological initial conditions: microcanonical density matrix of the Universe

Batalin-Fradkin-Vilkovisky (BFV) formalism: application to the density matrix of the Universe

CFT driven cosmology: quasi-thermal cosmological instantons and UV bounded range of Λ

New type of hill-top inflation, $\Lambda \to V(\phi)$ – selection of inflaton potential $V(\phi)$ maxima

Mechanism of hill-top potential: origin of non-minimal Higgs inflation and R^2 gravity

Conformal higher spin fields (CHS): solution of hierarchy problem; justification of semiclassical expansion

Problem of time in quantum gravity and cosmology

Canonical ADM action

$$S[\,g_{ij}, p^{ij}, N^{\perp}, N^{i}\,] = \int dt \int d^3\mathbf{x}\,\left\{p^{ij}\dot{g}_{ij} - N^{\perp}H_{\perp} - N^{i}H_{i} + \text{surface terms}\right\}$$

lapse and shift

$$N^{\mu} = N^{\perp}(\mathbf{x}), N^{i}(\mathbf{x})$$

Hamiltonian and momentum constraints

$$H_{\mu} = H_{\perp}(\mathbf{x}), H_{i}(\mathbf{x})$$

$$\frac{\delta S}{\delta N^{\mu}} = -H_{\mu} = 0$$

$$H = \int d^3\mathbf{x}\,\left\{N^{\perp}H_{\perp} + N^{i}H_{i}\right\} + \text{surface terms} = \underbrace{\text{spatial surface terms}}_{\textit{in closed cosmology } \boldsymbol{=0!}}$$

⇩ ***Quantization***

$$\hat{H} = 0, \qquad \frac{d}{dt}|\,\Psi\,\rangle = 0 \quad \boldsymbol{?}$$

Total system:
- q – *semiclassical variable*
- ϕ -- *quantum variable*

$|\,\Psi(q,t)\,\rangle$ *coordinate representation in the sector of q-variables*

$|\,\ldots\,\rangle$ *Hilbert space of* ϕ

$$i\hbar\frac{\partial}{\partial t}|\,\Psi(q,t)\,\rangle = \hat{H}_{\rm tot}|\,\Psi(q,t)\,\rangle$$

$$\hat{H}_{\rm tot} = H_q + \hat{H}_\phi, \qquad H_q = H\Big(q, \frac{\hbar}{i}\frac{\partial}{\partial q}\Big)$$

$$|\,\Psi(q,t)\,\rangle = \underbrace{\exp\Big(\frac{i}{\hbar}S(q,t)\Big)||\,\Psi(q,t)\,\rangle\rangle}_{\text{Semiclassical ansatz}}, \qquad \frac{\partial S}{\partial t} + H_q\Big(q, \frac{\partial S}{\partial q}\Big) = 0$$

$$\Big(-\cancel{\frac{\partial S}{\partial t}} + i\hbar\frac{\partial}{\partial t}\Big)||\,\Psi(q,t)\,\rangle\rangle =$$

$$\Big(\cancel{H_q(q, \frac{\partial S}{\partial q})} + \underbrace{\frac{\hbar}{i}\frac{\partial H_q}{\partial p}\Big|_{p=\frac{\partial S}{\partial q}} \times \frac{\partial}{\partial q}} + \hat{H}_\phi + \ldots\Big)||\,\Psi(q,t)\,\rangle\rangle$$

q - loops$\sim\hbar^2$

$$\downarrow$$

$$q \to q_0(t)$$

Classical solution of the ***q*** -- subsystem

$$-i\hbar\,\dot{q}_0(t)\frac{\partial}{\partial q}$$

transport term

$$|\,\Psi(q,t)\,\rangle = \exp\left(\tfrac{i}{\hbar}S(q,t)\right)||\,\Psi(q,t)\,\rangle\rangle$$

QFT of a total system

$$\Downarrow$$

$$||\,\Psi(t)\,\rangle\rangle = ||\,\Psi(q_0(t),t)\,\rangle\rangle$$

$$i\hbar\frac{d}{dt}||\,\Psi(t)\,\rangle\rangle = \hat{H}_\phi||\,\Psi(t)\,\rangle\rangle + q\text{-loops}$$

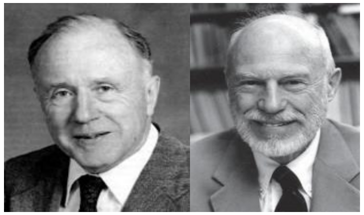
QFT at the classical background of q-subsystem

$$||\,\Psi(t)\,\rangle\rangle = ||\,\Psi(q_0(t),t)\,\rangle\rangle$$

contributes to the evolution of a subsystem state

In spatially closed cosmology physical time entirely originates from the correlation of one subsystem with its complement. For the subsystem as a semiclassically treated gravitational field background this leads to the derivation of QFT from the system of Wheeler-DeWitt equations.

Schroedinger equation vs Wheeler-DeWitt equation(s)



$$\hat H_\mu|\Psi\rangle = 0$$

$$\underbrace{\hat H_\mu = \hat H_\perp(\mathbf{x}),\ \hat H_i(\mathbf{x})}$$

Hamiltonian and momentum quantum Dirac constraints, ***NO TIME !***

Semiclassical gravity factor

$$\hat H_\mu = \hat H_\mu^{grav} + \hat H_\mu^{matter}, \qquad |\Psi\rangle = \Psi[g_{ij},\phi] = e^{iS[g_{ij}]}\,\Psi_{matter}[g_{ij},\phi]$$

Quantum matter wave function in external gravitational field

$$||\Psi(t)\rangle\rangle = \Psi_{matter}[g_{ij}(t),\phi]$$

Solution of classical vacuum Einstein eqs. with ADM lapse and shift $N^\perp(t)$, $N^i(t)$

$$\Downarrow$$

$$i\frac{\partial}{\partial t}||\Psi(t)\rangle\rangle = \hat H_{matter}||\Psi(t)\rangle\rangle + \text{graviton loops}$$

$$\hat H_{matter} = \int d^3x\,\left(N^\perp \hat H_\perp^{matter} + N^i \hat H_i^{matter}\right)$$

V.Lapchinsky & V.Rubakov, Acta Phys. Polon. B10 (1979) 1041-1048

Time as subsystem variable in "timeless" formalism

No-boundary (Hartle-Hawking) vs tunneling wavefunction

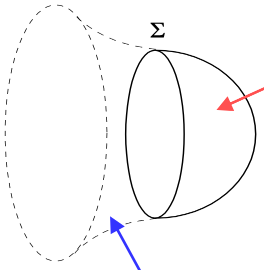
Hyperbolic nature of the Wheeler-DeWitt equation

$$\Psi_\pm\Big(\varphi,\Phi(\mathbf{x})\Big) = \exp\Big(\mp\tfrac{1}{2}S_E(\varphi)\Big)\,\Psi_{matter}\Big(\varphi,\Phi(\mathbf{x})\Big)$$

inflaton

other fields

Euclidean action of quasi-de Sitter instanton with the effective Λ (slow roll):



Euclidean spacetime

no tunneling, really:
“birth from nothing”

$$\Lambda \simeq \frac{V(\varphi)}{M_P^2}$$

FRW $$ds^2 = N^2\,d\tau^2 + a^2\,d\Omega_{(3)}^2$$

$$a_0(\tau) = \frac{1}{H}\sin(H\tau),\quad H = \sqrt{\frac{\Lambda}{3}}$$

$$S_E(\varphi) \simeq -\frac{24\pi^2 M_P^4}{V(\varphi)} < 0$$

Analytic continuation – Lorentzian signature dS geometry:

$$\tau = \pi/2H + it$$

$$a_L(t) = \frac{1}{H}\cosh(Ht)$$

Hartle-Hawking no-boundary wavefunction

$$\Psi_{HH} \sim \exp(-S_E) = \exp\left(12\pi^2 \frac{M_P^4}{V(\varphi)}\right) \to \infty$$

$$\frac{V(\varphi)}{M_P^2} = \Lambda_{eff} \to 0$$

Most probable ***at the minimum***
of inflaton potential ***¤$_{eff}$→ 0***
-- insufficient *amount of inflation*

Tunneling wavefunction

$$\Psi_T \sim \exp(+S_E)$$

Cosmology debate:
no-boundary vs tunneling

Questionable status of both states within unitarity approach to quantum gravity

Microcanonical density matrix of the Universe

$$\hat{\rho} = \sum_{\text{all } |\boldsymbol{\Psi}\rangle} w_{\Psi} |\, \boldsymbol{\Psi} \,\rangle\langle\, \boldsymbol{\Psi} \,|, \qquad w_{\Psi} = 1$$

sum over "everything" that satisfies the Wheeler-DeWitt equation $\widehat{H}_{\mu}\, |\boldsymbol{\Psi}\rangle = 0$

Projector onto the subspace of quantum gravitational constraints

A.O.B., Phys. Rev. Lett. 99, 071301 (2007)

$$\widehat{H}_{\mu} \equiv \underbrace{\widehat{H}_{\perp\mathbf{x}},\; \widehat{H}_{i\,\mathbf{x}}}$$

local operators of the Wheeler-DeWitt equations

$$\widehat{\rho} = \frac{1}{Z} \prod_{\mu} \delta(\widehat{H}_{\mu}), \qquad Z = \mathrm{Tr} \prod_{\mu} \delta(\widehat{H}_{\mu})$$

$$\mu = \big(\perp\mathbf{x},\; i\mathbf{x}\big)$$

Motivation:

A simplest analogy in unconstrained system with a conserved Hamiltonian $\widehat{H}$ *Is the microcanonical density matrix with a fixed energy* ***E***

$$\widehat{\rho} \sim \delta(\widehat{H} - E)$$

Spatially closed cosmology does not have freely specifiable constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ*, all having a particular value --- zero*

An ultimate equipartition in the full set of states of the theory --- "Sum over Everything". Creation of the Universe from Everything is conceptually more appealing than creation from Nothing, because the democracy of the microcanonical equipartition better fits the principle of Occam razor, preferring to drop redundant assumptions, than the selection of a concrete state.

Batalin-Fradkin-Vilkovisky formalism: application to the density matrix of the Universe

Classically

$$\prod_\mu \delta\Big(H_\mu(q,p)\Big) = \int dN\; \mathrm{exp}\Big(-\,iN^\mu H_\mu(q,p)\Big),$$

but at the quantum level:

$$\{H_\mu, H_\nu\} = U^\lambda_{\mu\nu} H_\lambda \qquad \Rightarrow \qquad [\hat{H}_\mu, \hat{H}_\nu] = i\hat{U}^\lambda_{\mu\nu}\hat{H}_\lambda \neq 0 \qquad \prod_\mu \delta(\hat{H}_\mu) = ?$$

Solution --- BFV formalism:

Relativistic phase space

$$Q^I, P_I = \overbrace{q^i, p_i;}^{\textit{Metric \& matter}}\; N^\mu, \pi_\mu;\; \underbrace{C^\mu, \mathcal{P}_\mu;\; \bar{C}_\mu, \bar{\mathcal{P}}^\mu}_{\textit{ghosts (Grassmann)}}, \qquad [Q^I, P_J]_\pm = i\,\delta^I_J$$

$||\,\boldsymbol{\Psi}\,\rangle\rangle$ pseudo-Hilbert space of states, $\langle\langle\, Q\,||\,\boldsymbol{\Psi}\,\rangle\rangle = \boldsymbol{\Psi}(Q)$

BRST charge

$$\hat{\Omega} = \pi_\alpha \bar{\mathcal{P}}^\alpha + C^\mu \hat{H}_\mu + \frac{1}{2} C^\nu C^\mu \hat{U}^\lambda_{\mu\nu} \mathcal{P}_\lambda, \qquad \hat{\Omega}^\dagger = \hat{\Omega}, \qquad \hat{\Omega}^2 = 0$$

Physical states

$$\hat{\Omega}||\,\boldsymbol{\Psi}\,\rangle\rangle = 0$$

gauge fermion (gauge fixing)

unitarizing Hamiltonian and unitary evolution operator

$$\hat{\mathcal{H}}_\Phi = \frac{1}{i}[\,\hat{\Phi},\hat{\Omega}\,], \qquad \hat{\boldsymbol{U}}_\Phi(t_+,t_-) = \mathbb{T}\,\mathrm{exp}\Big(-i\int_{t_-}^{t_+} dt\,\hat{\mathcal{H}}_\Phi\Big)$$

$$\hat{\Omega}\,||\,\boldsymbol{\Psi}_{1,2}\rangle\rangle = 0 \quad\Rightarrow\quad \delta_\Phi\langle\langle\,\boldsymbol{\Psi}_1||\,\hat{\boldsymbol{U}}_\Phi(t_+,t_-)\,||\,\boldsymbol{\Psi}_2\rangle\rangle = 0$$

Truncation to Dirac quantization: $Q,P \to q,p$ *(3-metric and matter field sector)*

$$U(q_+,q_-) = \int dN_+\,dN_-\,\boldsymbol{U}_\Phi(\,t_+,Q_+|\,t_-,Q_-)\,\Big|_{C_\pm=\bar{C}_\pm=0}$$

$$\delta_\Phi U(q_+,q_-) = 0, \qquad \frac{\partial}{\partial t_\pm}U(q_+,q_-) = 0$$

Gauge and $t_\pm$ independence

$$[\,\hat{\Omega},\hat{\boldsymbol{U}}_\Phi(t_+,t_-)\,] = 0$$

$$\Rightarrow \qquad \underbrace{\hat{H}_\mu U(q,q') = 0}_{\textit{WDW equations}}$$

$$\Downarrow$$

$$U(q,q') \sim \langle\,q\,|\prod_\mu \delta(\hat{H}_\mu)|\,q'\,\rangle$$

Canonical path integral representation:

$$U(q_+,q_-) = \int\limits_{\substack{q(t_\pm)=q_\pm\\ C_\pm,\bar{C}_\pm=0}} D[Q,P]\,\mathrm{exp}\left\{i\int_{t_-}^{t_+} dt\,\Big(P_I\dot{Q}^I - \mathcal{H}_\Phi(Q,P)\Big)\right\}$$

lapse and shift $N_\pm$ integrated over

Promotion of classical delta function to the quantum level as the path integral on the segment of “time”:

$$\prod_\mu \delta(H_\mu) = \int dN\,\mathrm{exp}(-iN^\mu H_\mu),$$

⇕

$$\langle\, q_+\,|\prod_\mu \delta(\hat{H}_\mu)|\, q_-\,\rangle = \int\limits_{q(t_\pm)=q_\pm} DqDpDN\,\mathrm{exp}\underbrace{\left\{i\int\limits_{t_-}^{t_+} dt\,\Big(p_i\dot{q}^i - N^\mu H_\mu\Big)\right\}}_{\textit{classical ADM action}}\times\underbrace{\int D[\mathrm{ghosts}]\,(...)}_{\textit{quantum measure}}$$

Origin of time t *in the action entirely as the operator ordering parameter of the noncommutative algebra of quantum Dirac constraints*

Transition to Lagrangian path integral $\int DQ\,DP \to \int Dg_{\mu\nu}\,D\Phi$ *(the choice of Batalin-Marnelius gauge fermion & background covariant gauge conditions)*

Matrix element of the cosmological density matrix

Faddeev-Popov path integral measure

$$\boldsymbol{\rho}\underbrace{(q_+,q_-)}_{\uparrow} = \frac{1}{Z}\int D[\,g_{\mu\nu},\Phi\,]\; e^{iS[\,g_{\mu\nu},\Phi\,]}\Big|_{g_{ij}(t_\pm)=g^\pm_{ij},\;\;\Phi(t_\pm)=\Phi_\pm}$$

$$q_\pm = (g^\pm_{ij},\Phi_\pm)$$

↑

Lorentzian

↓

$$Z = \int dq\,\boldsymbol{\rho}(q,q) = \int\limits_{\text{periodic}} D[\,g_{\mu\nu},\Phi\,]\; e^{iS[\,g_{\mu\nu},\Phi\,]}$$

Partition function

Absence of periodic Lorentzian histories and rotation of integration contours over fields and time

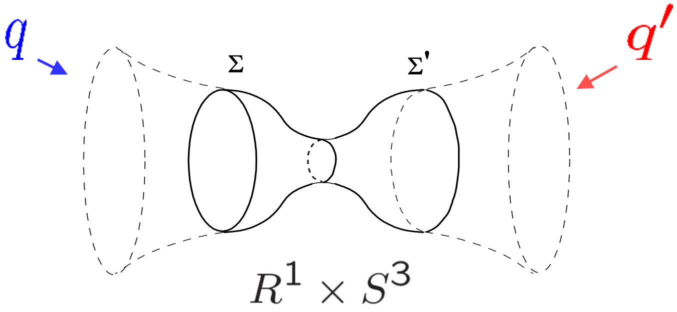
↓

Euclidean path integral and its saddle points

$$Z = \int\limits_{\text{periodic}} D[\,g_{\mu\nu},\Phi\,]\; e^{-S[\,g_{\mu\nu},\Phi\,]}$$

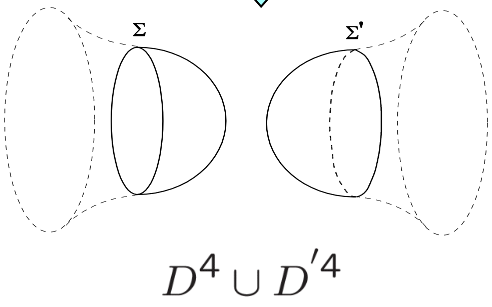
Hartle-Hawking state as a vacuum member of the microcanonical ensemble:

$$\hat{\rho}_{\text{mixed}} = \rho_{\text{mixed}}(q, q')$$



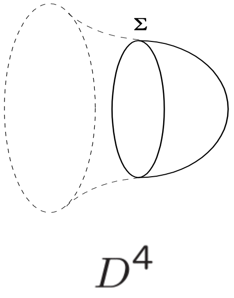
pinching a tubular spacetime

$$|\Psi_{HH}\rangle\langle\Psi_{HH}| = \rho_{HH}(q, q')$$



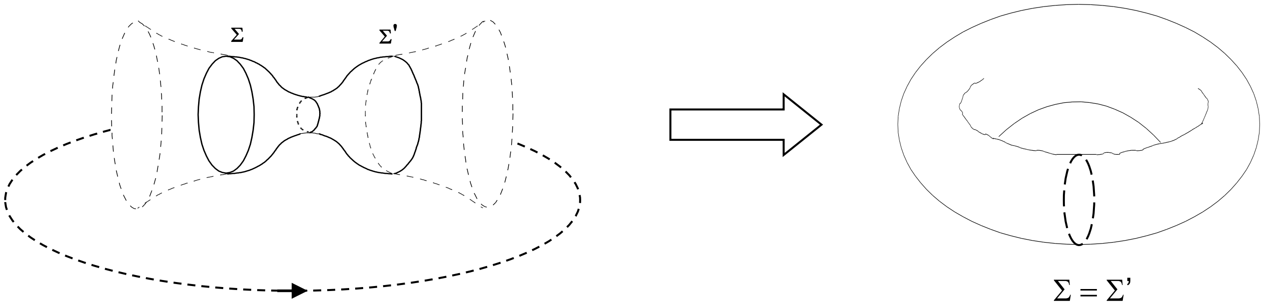
⟺

$$|\Psi_{HH}\rangle = \Psi_{HH}(q)$$



density matrix representation of a pure Hartle-Hawking state

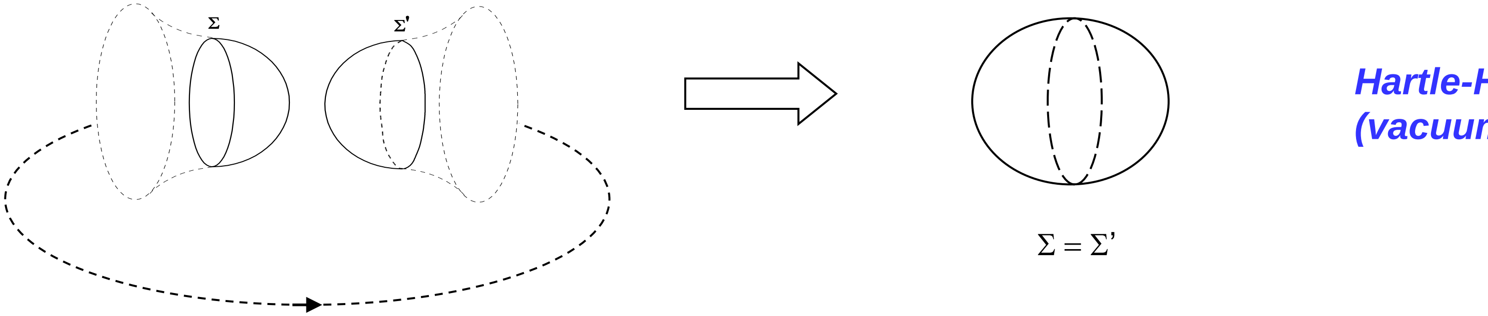
Transition to statistical sums



$R^1 \times S^3$

$S^1 \times S^3$

thermal instantons



$D^4 \cup D'^4$

S^4

Hartle-Hawking (vacuum) instanton

Inflationary model driven by the trace anomaly of Weyl invariant fields --- CFT driven cosmology

$$S[\,g_{\mu\nu},\Phi\,] = -\frac{M_P^2}{2}\int d^4x\, g^{1/2}\,(R-2\Lambda) + S_{CFT}[\,g_{\mu\nu},\Phi\,]$$

Λ -- primordial cosmological constant

Omission of graviton loops

$$S_{\rm eff}[g_{\mu\nu}] = -\frac{M_P^2}{2}\int d^4x\, g^{1/2}(R-2\Lambda) + \Gamma_{CFT}[g_{\mu\nu}],$$

$$e^{-\Gamma_{CFT}[\,g_{\mu\nu}]} = \int D\Phi\, e^{-S_{CFT}[\,g_{\mu\nu},\Phi\,]}$$

Recovery of Γ_{CFT} from the conformal anomaly on a static Einstein Universe (anomaly, Casimir energy and free energy contributions)

$$g_{\mu\nu}\frac{\delta\Gamma_{CFT}}{\delta g_{\mu\nu}} = \frac{1}{64\pi^2}g^{1/2}\left(\beta E + \alpha\Box R + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

Gauss –Bonnet term

Weyl

$\beta = \sum_s \beta_s\,\mathbb{N}_s,\quad \mathbb{N}_s$ − # of spin s fields, β_s − spin-dependent coefficients

β -- critically important parameter (overall coefficient of Gauss-Bonnet term in conformal anomaly)

Minisuperspace (FRW) ansatz
for the saddle point

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = N^2(\tau)\,d\tau^2 + a^2(\tau)\,d^2\Omega^{(3)}$$

$$S_{\mathrm{eff}}[\,g_{\mu\nu}\,] = S_{\mathrm{eff}}[\,a, N\,]$$

Effective Friedmann equation for
saddle points of the path integral:

$$\frac{\delta S_{\mathrm{eff}}[\,a, N\,]}{\delta N(\tau)} = 0$$

$$\frac{1}{a^2} - \frac{\dot{a}^2}{a^2} = \frac{\varepsilon}{3M_{\pm}^2(\varepsilon)},$$

Friedmann equation

$$M_{\pm}^2(\varepsilon) = \frac{M_P^2}{2}\left(1 \pm \sqrt{1 - \frac{\beta}{6\pi^2 M_P^4}\,\varepsilon}\,\right),$$

Effective Planck mass

$$\varepsilon = M_P^2 \Lambda + \frac{1}{2\pi^2 a^4}\sum_{\omega}\frac{\omega}{e^{\eta\,\omega} - 1},$$

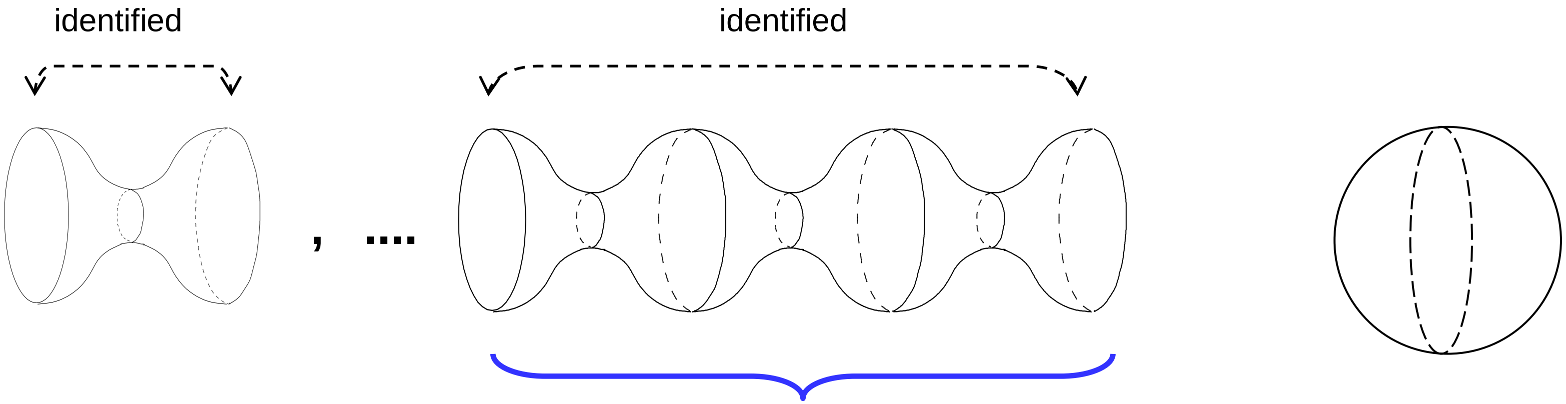
Energy density=¤ + radiation of CFT particles --
sum over field oscillators with frequencies ***!***
(eigenvalues of Laplacian on ***S^3****)*

$$\eta = \int_{S^1}\frac{d\tau N}{a}$$

Inverse temperature in units of conformal
time period on ***S^1***

Existence of the quasi-thermal stage preceding the inflation

Saddle point solutions --- set of periodic (thermal) garland-type instantons with oscillating scale factor ($S^1 \times S^3$) and the vacuum Hartle-Hawking instantons (S^4)



1- fold, k=1

k- folded garland, k=1,2,3,…

S^4

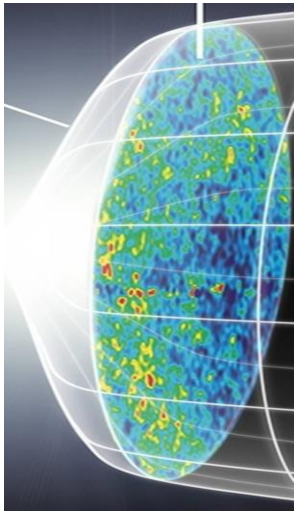
does not contribute: ruled out by infinite positive Euclidean action (effect of conformal anomaly)

UV bounded cosmological constant range:

$$\boxed{\Lambda_{min} < \Lambda < \Lambda_{max} = \frac{12\pi^2}{\beta} M_P^2}$$

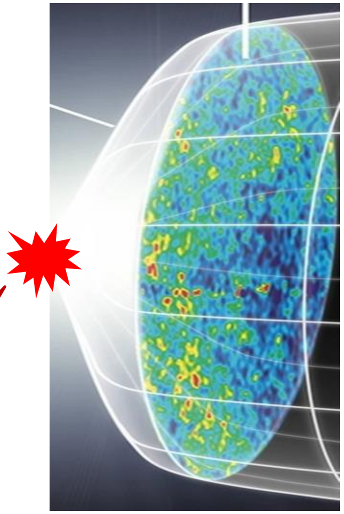
Initial thermal state with the primordial temperature T_{prim} of matter

Standard inflation scenario versus Density matrix scenario



Vacuum,
absolute zero
temperature

Inflation, hot
big-bang
→ relic radiation



Thermal state,
primordial
temperature T_{prim}

Inflation, $T_{prim} \to 0$,
hot big-bang
→ relic radiation

“SOME LIKE IT HOT” (SLIH) scenario

Known inflation paradigm retracted the BB concept by replacing it with the initial vacuum state.

“SOME LIKE IT HOT” (SLIH) scenario recovers a new incarnation of Hot Big Bang -- it incorporates effectively thermal state at the onset of the cosmological evolution.

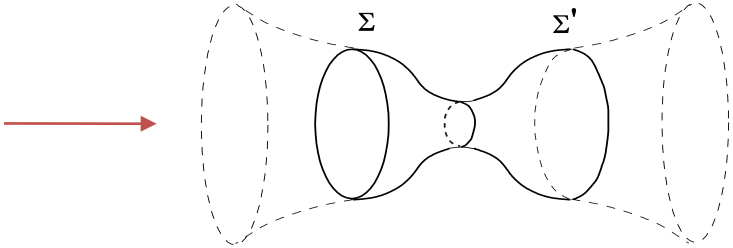
So how does SLIH scenario matches with inflation?

SLIH inflation

1) Generalization to Λ as a composite operator – inflaton potential in "slow roll" regime

$$\Lambda \to \frac{\rho_\phi}{M_P^2}, \qquad \rho_\phi = V(\phi) - \frac{\dot\phi^2}{2} \simeq V(\phi)$$

2) Lorentzian Universe with initial conditions set by the instanton. Analytic continuation of the instanton solutions:

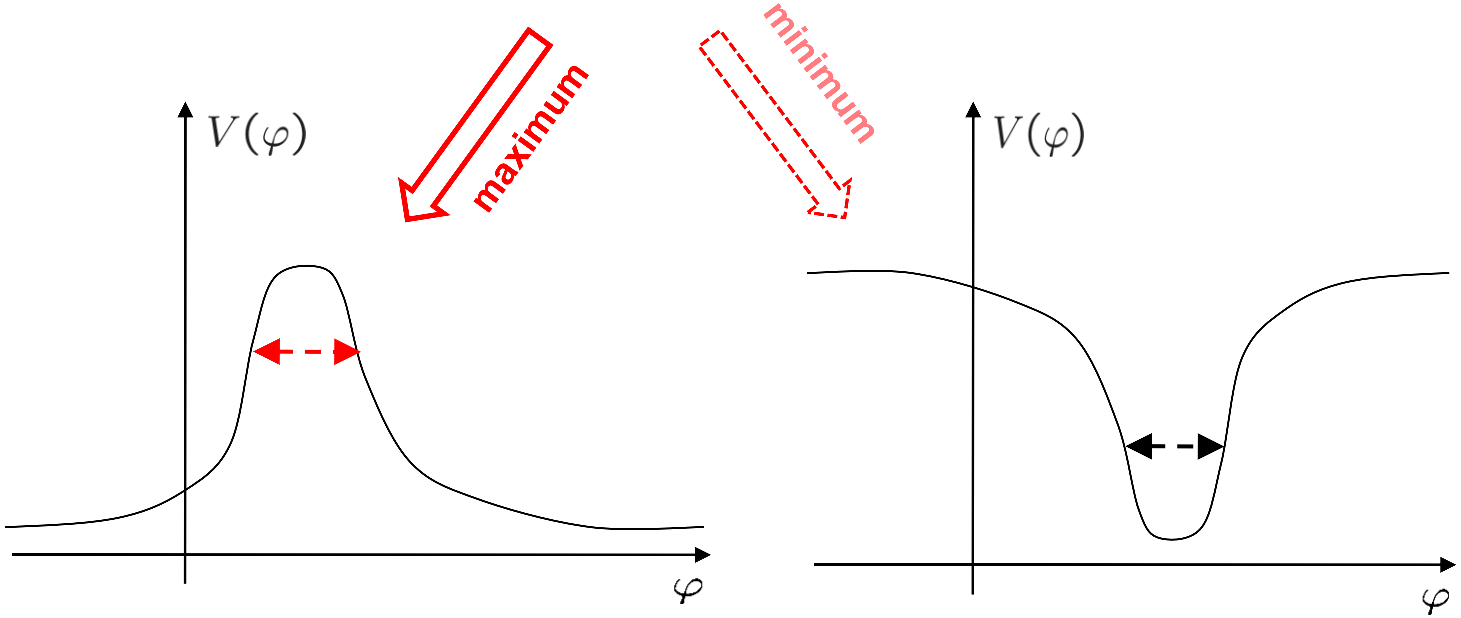


$$\tau = \tau_* + it, \;\; a_L(t) = a(\tau_* + it)$$

3) Expansion and quick dilution of primordial radiation, decay of a composite Λ, exit from inflation and particle creation of conformally non-invariant matter and its thermalization

Selection of inflaton potential *maxima* as initial conditions for inflation

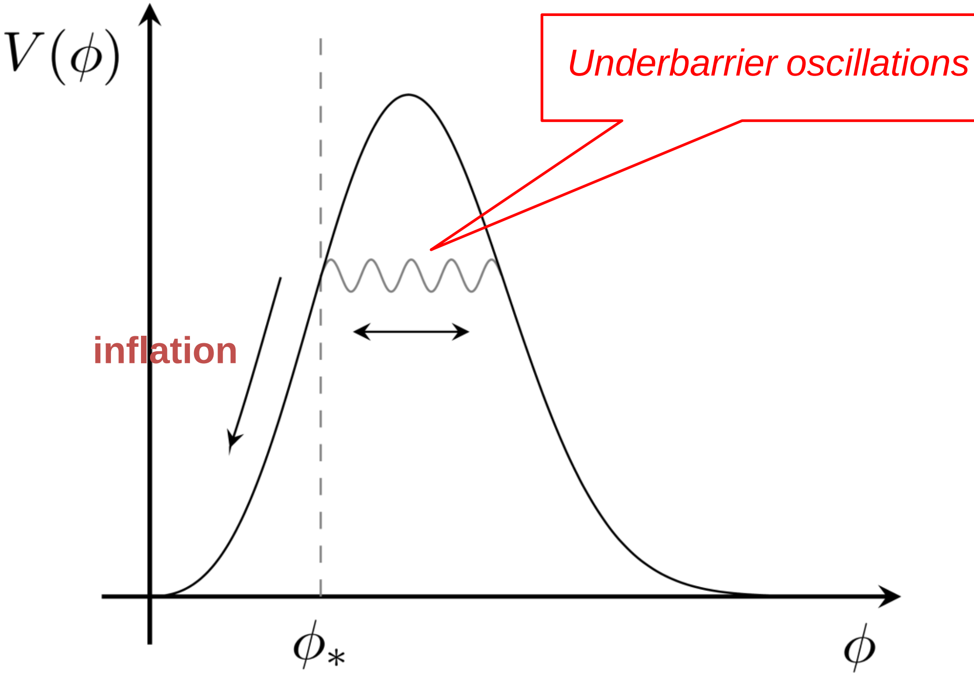
Critical feature: $$\frac{d}{d\tau}a^3\dot{\phi} = a^3\frac{\partial V}{\partial \phi} \Rightarrow \oint d\tau\, a^3\frac{\partial V}{\partial \phi} = 0 \Rightarrow \frac{\partial V}{\partial \phi} \gtrless 0$$ **Potential extremum ``inside” instanton**



classically forbidden
(underbarrier)
oscillation

classically allowed (overbarrier) oscillation --- ruled out because of *underbarrier* oscillations of scale factor

Hill-top inflation



nucleation point of Lorentzian signature cosmology from Euclidean spacetime

Approximation of two coupled oscillators → slow roll parameters typical of Higgs and R^2 inflation:

$$P_\zeta(k) = 2.2 \times 10^{-9} \left(\frac{k}{k_0}\right)^{n_s-1}, \quad k_0 = 0.05 Mpc^{-1}, \quad n_s = 0.965 \pm 0.005$$

$$n_s = 1 - 6\epsilon + 2\eta, \quad \epsilon = \frac{1}{2}\left(M_P \frac{V_*'}{V_*}\right)^2, \quad \eta = M_P^2 \frac{V_*''}{V_*}$$

$$\epsilon \sim \eta^2 \ll |\eta|, \quad \eta < 0$$

Mechanism of hill-top potential: origin of non-minimal Higgs inflation and R^2 gravity

B.Spokoiny 1986, A.Kamenshchik & A.B 1991,
Bezrukov,Shaposhnikov 2008
A.Kamenshchik, A.Starobinsky & A.B 2008

Higgs field H *non-minimally coupled to curvature:*

$$\varphi^2 \equiv H^\dagger H$$

$$S_{EH+SM}[g_{\mu\nu}, H, ...] = \int d^4x\, g^{1/2} \left(\frac{\lambda\varphi^4}{4} - \frac{M_P^2 + \xi\varphi^2}{2} R + \frac{1}{2}(\nabla\varphi)^2 + ... \right)$$

Starobinsky model of R^2 gravity:

$$S_\xi^{\rm Star}[g_{\mu\nu}, \varphi] = \int d^4x\, g^{1/2} \left\{ -\frac{M_P^2}{2} R - \frac{\xi}{4} R^2 \right\}$$

$$\Updownarrow$$

$$S_\xi^{\rm Star}[g_{\mu\nu}, \varphi] = \int d^4x\, g^{1/2} \left\{ -\frac{M_P^2}{2} \left(1 + \xi\frac{\varphi^2}{M_P^2}\right) R + \frac{\xi\varphi^4}{4} \right\}$$

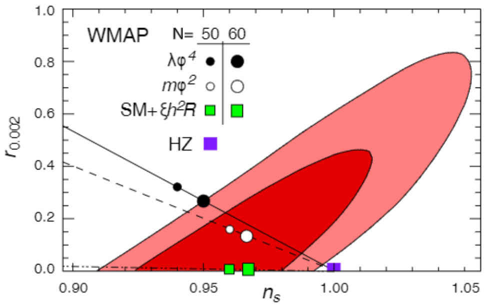


Fig. 2. The allowed WMAP region for inflationary parameters (r, n). The green boxes are our predictions supposing 50 and 60 e–foldings of inflation. Black and white dots are predictions of usual chaotic inflation with $\lambda\phi^4$ and $m^2\phi^2$ potentials, HZ is the Harrison-Zeldovich spectrum.

$\xi \sim 10^4 \gg 1 \Longrightarrow$ **Higgs inflation with** $\frac{\Delta T}{T} \sim 10^{-5},\; n_s \simeq 0.96,\; r \simeq 0.003$

$M_{\rm Higgs} \simeq 126 GeV$

Mechanism of hill-top inflaton potential– quantization in the Jordan frame and transition to Einstein frame:

Not in Einstein frame, no shift symmetry, IR instability and breakdown of grad. expansion!

non-minimal coupling

$$\Gamma[\,g_{\mu\nu},\varphi\,]=\int d^4x\,g^{1/2}\left(V(\varphi)-\overset{\swarrow}{U}(\varphi)\,R(g_{\mu\nu})+\frac{1}{2}G(\varphi)\,(\nabla\varphi)^2+...\right)$$

$$V_{\text{loop}}(\varphi)\sim\varphi^4\ln\frac{\varphi^2}{\mu^2},\qquad U_{\text{loop}}(\varphi)\sim\varphi^2\ln\frac{\varphi^2}{\mu^2},,\qquad G_{\text{loop}}(\varphi)\sim\ln\frac{\varphi^2}{\mu^2}$$

Transition to the Einstein frame:

$$V(\varphi)\to V_{EF}(\phi)=\frac{M_P^4}{4}\,\frac{V(\varphi)}{U^2(\varphi)}\sim\frac{\cancel{\ln\frac{\varphi}{\mu}}}{\ln^{\cancel{2}}\frac{\varphi}{\mu}}\sim\frac{1}{\ln\frac{\varphi}{\mu}}\to 0,\qquad \varphi\to\infty$$

Any l-th loop order:

$$\frac{\ln^l\frac{\varphi}{\mu}}{\ln^{2l}\frac{\varphi}{\mu}}\sim\frac{1}{\ln^l\frac{\varphi}{\mu}}\to 0,\qquad \varphi\to\infty$$

Resummation by RG confirms this.

Justification of semiclassical expansion and hierarchy problem

Starobinsky R^2*-model and non-minimal Higgs inflation model*

$$10^{-11}M_P^4 \simeq V_{\text{inflation}} \sim \Lambda_{max} = \frac{12\pi^2}{\beta}M_P^2 \quad \Longrightarrow \quad \boxed{\beta \simeq 10^{13}}$$

Impossible in Standard model with low spins s=0,1/2,1 and N_s *» 100*

$$\beta = \frac{1}{180}\left(\mathbb{N}_0 + 11\mathbb{N}_{1/2} + 62\mathbb{N}_1\right)$$

Hidden sector of conformal higher spin (CHS) fields

$$S^{(s)}_{CHS} = \int d^4x \left(h^{\mu_1\ldots\mu_s}\Box^s h_{\mu_1\ldots\mu_s} + \ldots\right), \qquad \beta_s \sim s^6$$

Giombi, Klebanov, Pufu, Safdi, and Tarnopolsky 2013; Tseytlin 2013 arXiv:1309.0785

$$\beta = \sum_{s=1}^{S}\beta_s \simeq S^7$$

$$\mathbb{N} = \sum_{s=1}^{S} N_s \sim S^3 \; - \text{total number of polarizations (species)}$$

1/N-expansion and effective field theory below the gravitational cutoff $\Lambda_{grav} = \frac{M_P}{\sqrt{\mathbb{N}}}$

$$\Lambda_{max} \sim \frac{M_P}{\sqrt{\beta}} \sim \frac{M_P}{S^3} \ll \Lambda_{grav} = \frac{M_P}{\sqrt{\mathbb{N}}} \sim \frac{M_P}{S^{3/2}}$$

Thermal corrections to primordial power spectrum

$$n_s(k) = n_s^{\mathrm{vac}}(k) + \Delta n_s^{\mathrm{thermal}}(k)$$

additional red tilt of the CMB spectrum

This number of hidden sector fields gives a red tilted thermal correction to CMB spectral index in the third (potentially observable) decimal order:

$$\Delta n_s^{\mathrm{thermal}} \sim -0.001$$

A.B, arXiv:1308.4451
JCAP 1310 (2013) 059

Microcanonical state of CFT driven cosmology scenario works only for closed Universe with k=+1

99% C.L. evidence for positive spatial curvature (k=+1) of the closed Universe with Ω_k' -0.04 --- Hubble tension discordances

E. Di Valentino, A. Melchiorri and J. Silk,
Nature Astron. 4, 196 (2019);
W. Handley, Phys. Rev. D 103 (2021) L041301,
arXiv:1908.09139

Conclusions

Time in quantum cosmology: *origin of physical time from timeless formalism*

Cosmological initial conditions: *microcanonical density matrix of the Universe*

BFV construction of the density matrix

CFT driven cosmology: *suppression of no-boundary instantons; quasi-thermal stage preceding inflation and UV bounded range of its energy scale*

New type of hill-top inflation, $\Lambda \to V(\phi)$ *-- selection of inflaton potential V(*ϕ*) maxima*

Mechanism of hill-top potential: *origin of non-minimal Higgs inflation and* R^2 *gravity*

Conformal higher spin fields (CHS): *solution of hierarchy problem -- origin of the Universe is the subplanckian phenomenon; justification of semiclassical expansion and 1/N-expansion*