Higher-Spin Gauge Theory: From Basics to Recent Results

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Symmetries

Usual lower-spin symmetries:

- Relativistic theories: Poincaré and $(A)dS$ symmetry: P_a and M_{ab}

- SUSY: $P_a, M_{ab} \longrightarrow P_a, M_{ab}, \mathbf{Q}_\alpha\,, \qquad \alpha = 1,2,3,4$

- Inner symmetries: generators T_i are space-time invariant

$$[T_i\,, (P_a, M_{ab})] = 0$$

- Conformal (super)symmetries

Is it possible to go to higher HS symmetries?

HS gauge theory: theory of maximal symmetries

What are physical motivations for their study and possible outputs?

Fronsdal Fields

All $m = 0$ HS fields are gauge fields C.Fronsdal 1978

$\varphi_{a_1 \dots a_s}$ **is a rank s symmetric tensor obeying** $\varphi^{c}{}_{c}{}^{b}{}_{b a_5 \dots a_s} = 0$

Gauge transformation:

$$\delta \varphi_{a_1 \dots a_s} = \partial_{(a_1} \varepsilon_{a_2 \dots a_s)}\,, \qquad \varepsilon^{b}{}_{b a_3 \dots a_{s-1}} = 0$$

Field equations: $G_{a_1 \dots a_s}(x) = 0$ $G_{a_1 \dots a_s}(x)$: **Ricci-like tensor**

$$G_{a_1 \dots a_s}(x) = \Box \varphi_{a_1 \dots a_s}(x) - s \partial_{(a_1} \partial^{b} \varphi_{a_2 \dots a_s b)}(x) + \frac{s(s-1)}{2} \partial_{(a_1} \partial_{a_2} \varphi^{b}{}_{a_3 \dots a_s b)}(x)$$

Action

$$S = \int_{M^d} \left(\frac{1}{2} \varphi^{a_1 \dots a_s} G_{a_1 \dots a_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{b}{}^{b\, a_3 \dots a_s} G^{c}{}_{c\, a_3 \dots a_s}(\varphi) \right)$$

No-go and the Role of $(A)dS$

In 60th it was argued (Weinberg, Coleman-Mandula) that HS symmetries cannot be realized in a nontrivial local field theory in Minkowski space

In 70th it was shown by Aragone and Deser that HS gauge symmetries are incompatible with GR if expanding around Minkowski space

Green light: AdS background with $\Lambda \neq 0$ Fradkin, MV, 1987

In agreement with no-go statements the limit $\Lambda \to 0$ is singular

HS Holography

AdS_4 HS theory is dual to $3d$ vectorial conformal models

Klebanov-Polyakov (2002), Sezgin–Sundell (2002); Giombi and Yin (2009)

HS symmetry in AdS_{d+1}: maximal symmetry of a d-dimensional free conformal field(s)=singletons, usually, scalar and/or spinor.

Symmetries of KG equation in Minkowski space

Shaynkman, MV 2001 $3d$; Shapovalov, Shirokov 1992, Eastwood 2002 $\forall d$

Construction simplifies at $d=3$ within spinor formalism

$3d$ Lorentz algebra: $o(2,1) \sim sp(2,R) \sim sl_2(R)$.

Unfolded massless equations of the form

$$\left(\frac{\partial}{\partial x^{\alpha\beta}} + \frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right) C(y|x) = 0\,, \qquad C(y|x) = \sum_{n=0}^{\infty} C^{\alpha_1 \dots \alpha_{2n}}(x) y_{\alpha_1} \dots y_{\alpha_{2n}}$$

are invariant under $\delta C(y|x) = \epsilon(y, \frac{\partial}{\partial y}|x) C(y|x)$

$$\epsilon(y, \frac{\partial}{\partial y}|x) = \exp\left[-x^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right]\epsilon_{gl}(y, \frac{\partial}{\partial y}) \exp\left[x^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right]$$

$\epsilon_{gl}(y, \frac{\partial}{\partial y})$ describes global HS transformations

NonAbelian HS Algebra

$3d$ **Conformal HS symmetry** $=$ AdS_4 **HS symmetry**

HS gauge fields: $\omega(Y|x)$ **1986**

$Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$, $\alpha, \dot\alpha = 1, 2$ **two-component spinor indices**

$$\omega(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} \omega_{\alpha_1 \ldots \alpha_n, \dot\alpha_1 \ldots \dot\alpha_m}(x) y^{\alpha_1} \ldots y^{\alpha_n} \bar{y}^{\dot\alpha_1} \ldots \bar{y}^{\dot\alpha_m}$$

HS curvature and gauge transformation

$$R(Y|x) = d\omega(Y|x) + \omega(Y|x) * \wedge \omega(Y|x)$$

$$\delta\omega(Y|x) = D\epsilon(Y|x) = d\epsilon(Y|x) + [\omega(Y|x), \epsilon(Y|x)]_*$$

$$[y_\alpha, y_\beta]_* = 2i\varepsilon_{\alpha\beta}, \qquad [\bar{y}_{\dot\alpha}, \bar{y}_{\dot\beta}]_* = 2i\varepsilon_{\dot\alpha\dot\beta}$$

Star product is nonlocal in Y^A !

$$(f * g)(Y) = f(Y) \exp [i \overleftarrow{\partial_A} \overrightarrow{\partial_B} C^{AB}] g(Y)$$

Global symmetry of bosonic HS theory **Fradkin, MV 1986, MV 1988**

Properties of HS Algebras

Let T_s be a homogeneous polynomial of degree $2(s-1)$

$$[T_{s_1}, T_{s_2}] = T_{s_1+s_2-2} + T_{s_1+s_2-4} + \ldots + T_{|s_1-s_2|+2}\,.$$

Once spin $s>2$ appears, the HS algebra contains an infinite tower of higher spins: $[T_s, T_s]$ gives rise to T_{2s-2} as well as T_2 of $o(3,2)\sim sp(4)$.

Usual symmetries: spin-$s \leq 2$ $u(1)\oplus o(3,2)$: maximal finite-dimensional subalgebra of $hu(1,0|4)$. $u(1)$ is associated with the unit element.

HS symmetries do not commute with space-time symmetries

$$[T^a, T^{HS}] = T^{HS}\,, \qquad [T^{ab}, T^{HS}] = T^{HS}$$

HS transformations map gravitational fields (metric) to HS field:

Riemann geometry is not appropriate for HS theory:

concept of local event may become illusive!

HS Gauge Theory and Quantum Gravity

HS symmetry is in a certain sense maximal relativistic symmetry. Hence, it cannot result from spontaneous breakdown of a larger symmetry: HS symmetries are manifest at ultrahigh energies above any scale including Planck scale

- HS gauge theory should capture effects of Quantum Gravity: restrictive HS symmetry versus unavailable experimental tests
- Lower-spin theories as low-energy limits of HS theory: lower-spin symmetries: subalgebras of HS symmetry
- String Theory as spontaneously broken HS theory?! $(s > 2, m > 0)$

Space-Time and Spin

Space-time M is where symmetry $G = O(d-1,2)$ acts

Spin s: different G-modules V_s where fields $\phi^A(x)$ are valued.

V_s contain ground (primary) fields $\phi^A(x)$ along with their derivatives $\partial_{n_1} \ldots \partial_{n_k} \phi^A(x)$ (descendants)

HS vertices contain higher derivatives Bengtsson, Bengtsson, Brink (1983), Berends, Burgers and H. Van Dam (1984), (1985), Fradkin, MV; Metsaev,...

HS symmetries Fradkin, MV 1986 are infinite dimensional extesions of G

Infinite towers of spins $\Rightarrow$ infinite towers of derivatives.

How (non)local is HS gauge theory?

HS Multiplets

Infinite set of spins $s = 0, 1/2, 1, 3/2, 2 \ldots$

$\omega_{\alpha_1 \ldots \alpha_n, \dot\beta_1 \ldots \dot\beta_m}$ **and** $C_{\alpha_1 \ldots \alpha_n, \dot\beta_1 \ldots \dot\beta_m}$ **with all** $n \geq 0$ **and** $m \geq 0$.

Generating functions $\omega(Y|x)$ **and** $C(Y|x)$: **unrestricted functions of commuting spinor variables** $Y = (y_\alpha, \bar y_{\dot\alpha})$

$$A(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} A_{\alpha_1 \ldots \alpha_n, \dot\alpha_1 \ldots \dot\alpha_m}(x) y^{\alpha_1} \ldots y^{\alpha_n} \bar y^{\dot\alpha_1} \ldots \bar y^{\dot\alpha_m}$$

Gauge one-forms $\omega_{\alpha_1 \ldots \alpha_n, \dot\beta_1 \ldots \dot\beta_m}, \qquad n + m = 2(s-1)$

$s = 1: \quad \omega(x) = dx^\nu \omega_\nu(x)$

$s = 2: \quad \omega_{\alpha\dot\beta}(x), \qquad \omega_{\alpha\beta}(x), \qquad \bar\omega_{\dot\alpha\dot\beta}(x)$

$s = 3/2: \quad \omega_\alpha(x), \qquad \bar\omega_{\dot\alpha}(x)$

Frame-like fields: $|n-m| = 0$ **(bosons) or** $|n-m| = 1$ **fermions**

Auxiliary Lorentz-like fields: $|n-m| = 2$ **(bosons)**

Extra fields: $|n-m| > 2$

Zero-forms $C(Y|x)$: **matter fields and higher derivatives of massless fields**

Free Field Unfolded Massless Equations

The full unfolded system for free massless bosonic fields is 1989

$$\star \qquad R_1(y,\bar{y}\mid x) = \frac{i}{4}\Big(\eta \overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\bar{y}^{\dot\alpha}\partial\bar{y}^{\dot\beta}}\, C(0,\bar{y}\mid x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\, C(y,0\mid x)\Big)$$

$$\star\star \qquad \tilde{\mathrm{D}}_0 C(y,\bar{y}\mid x) = 0$$

$$R_1(y,\bar{y}\mid x) := D_0^{ad}\omega(y,\bar{y}\mid x) \qquad D_0^{ad} := D^L - e^{\alpha\dot\beta}\Big(y_\alpha\frac{\partial}{\partial\bar{y}^{\dot\beta}} + \frac{\partial}{\partial y^\alpha}\bar{y}_{\dot\beta}\Big)$$

$$\tilde{\mathrm{D}}_0 = D^L + e^{\alpha\dot\beta}\Big(y_\alpha\bar{y}_{\dot\beta} + \frac{\partial^2}{\partial \mathbf{y}^\alpha\partial\bar{\mathbf{y}}^{\dot\beta}}\Big) \qquad D^L := \mathrm{d}_x - \Big(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta} + \bar\omega^{\dot\alpha\dot\beta}\bar{y}_{\dot\alpha}\frac{\partial}{\partial\bar{y}^{\dot\beta}}\Big)$$

$$H^{\alpha\beta} := e^{\alpha}{}_{\dot\alpha}e^{\beta\dot\alpha}\,, \qquad \overline{H}^{\dot\alpha\dot\beta} := e_{\alpha}{}^{\dot\alpha}e^{\alpha\dot\beta}$$

$\star\star$ **implies that higher-order terms in y and $\bar{y}$ describe higher-derivative descendants of the primary HS fields**

Zero-Form Sector

Equations on the gauge invariant zero-forms C

$$C(Y;K|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} C_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}(x) y^{\alpha_1}\ldots y^{\alpha_n}\bar y^{\dot\alpha_1}\ldots \bar y^{\dot\alpha_m}$$

decompose into independent subsystems associated with different spins

Spin-s zero-forms are $C_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}(x)$ with

$$n - m = \pm 2s$$

Perturbative unfolded equations

$$\mathrm{d}_x C = \sigma_- C + \text{lower-derivative and nonlinear terms}$$

$$\sigma_- := \mathbf{e}^{\alpha\dot\beta}\frac{\partial^2}{\partial \mathbf{y}^\alpha \partial \bar{\mathbf{y}}^{\dot\beta}}\,, \qquad \sigma_-^2 = 0$$

$C_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}(x)$ contain $\frac{n+m}{2} - \{s\}$ space-time derivatives of the spin-s dynamical fields. Presence of zero-forms C in the HS vertices may induce infinite towers of derivatives and, hence, non-locality.

HS Vertices

Diffeomorphisms without Riemannian geometry: Cartan formalism of differential forms with field equations in the unfolded form

$$\mathrm{d}_x\omega = -\omega * \omega + \Upsilon(\omega,\omega,C) + \Upsilon(\omega,\omega,C,C) + \dots\,,$$

$$\mathrm{d}_x C = -[\omega, C]_* + \Upsilon(\omega,C,C) + \dots$$

The problem: consistent non-linear corrections 1988 in the local frame

The vertices can be put into the form

$$\Upsilon(\Phi,\Phi,\dots) = F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})\Phi(Y_1)\dots\Phi(Y_n)|_{Y_i=0}$$

with $\Phi = \omega$, C and some non-polynomial functions $F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})$ of the Lorentz-covariant combinations

$$Q^i := y^\alpha \frac{\partial}{\partial {y_i}^\alpha}\,, \qquad P^{ij} := \frac{\partial}{\partial y_i^\alpha}\frac{\partial}{\partial y_{j\alpha}}\,, \qquad \bar{Q}^i := \bar{y}^{\dot\alpha}\frac{\partial}{\partial {\bar{y}_i}^{\dot\alpha}}\,, \qquad \bar{P}^{ij} := \frac{\partial}{\partial {\bar{y}_i}^{\dot\alpha}}\frac{\partial}{\partial \bar{y}_{j\dot\alpha}}$$

The fundamental problem: find a proper class of functions $F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}$ guaranteeing spin-locality (minimal non-locality) of the HS theory

Locality and Non-Locality

arXiv: 2208.02004

Equations of motion in perturbatively local field theory $E_{A_0,s_0}(\partial,\phi)=0$

$$E_{A_0,s_0}(\partial,\phi)=\sum_{k=0,l=1}^{\infty} a^{n_1\dots n_k}_{A_0\,A_1\dots A_l}(s_0,s_1,\dots,s_l)\partial_{n_1}\dots\partial_{n_k}\phi^{A_1}_{s_1}\dots\phi^{A_l}_{s_l}$$

have a finite # of non-zero coefficients $a^{n_1\dots n_k}_{A_0\dots A_l}$ at any order l.

s_0 is the spin of the field on which the linearized equation is imposed

HS theory involves infinite towers of Fronsdal fields of all spins.

$a^{n_1\dots n_k}_{A_0\dots A_l}$ may take an infinite # of values.

It makes sense to distinguish between — Gelfond, MV 2018

local: finite number of derivatives at any order

$$a^{n_1\dots n_k}_{A_0\dots A_l}(s_0,s_1,\dots s_l)=0 \quad \text{at } k>k_{max}(l)$$

spin-local: finite number of derivatives for any finite subset of fields

$$a^{n_1\dots n_k}_{A_0\dots A_l}(s_0,s_1,s_2,\dots s_l)=0 \quad \text{at } k>k_{max}(s_0,s_1,s_2,\dots s_l)$$

non-local: infinite number of derivatives for a finite subset of fields at some order.

Compact Spin-Locality

The simplest option: replacement of the class of local field theories with the finite # of fields by spin-local models with infinite # of fields.

Spin-local-compact vertices in addition obey

$$a^{n_1\dots n_k}_{A_0\,A_1\dots A_l}(s_0, s_1, \dots, s_k + t_k, \dots, s_l) = 0 \quad t_k > t^0_k \quad \forall k$$

non-compact otherwise.

Compactness is in the space of spins, not in space-time

Both types of vertices in HS theory:

Cubic HS vertices $\omega * \omega$ built from HS gauge potentials are spin-local-compact: spins s_0, s_1, s_2 obey the triangle inequalities $s_0 \leq s_1 + s_2$ etc.

Vertices associated with the conserved currents built from gauge invariant field strength are spin-local non-compact. These include conserved currents of any integer s_0 built from two spin-zero fields $(s_1 = s_2 = 0)$.

Field Redefinitions

A local theory remains local under perturbatively local field redefinitions

$$\phi^B_{s_0} \to \phi^B_{s_0} + \delta\phi^B_{s_0}\,, \qquad \delta\phi^B_{s_0} = \sum_{k=0,l=1}^{\infty} b^{Bn_1\ldots n_k}{}_{A_1\ldots A_l}(s_0, s_1, \ldots, s_l)\partial_{n_1}\ldots\partial_{n_k}\phi^{A_1}_{s_1}\ldots\phi^{A_l}_{s_l}$$

with a finite # of non-zero coefficients at any order.

Which field redefinitions leave vertices spin-local?

General spin-local field redefinitions do not work since contributions of all spin s_p redefined fields may develop non-locality

$$\delta E_{A_0,s_0}(\partial,\phi) = \sum_{s_p=0}^{\infty}\ \sum_{p,k,k'=0,l,l'=1}^{\infty} a^{n_1\ldots n_k}_{A_0\,A_1\ldots A_l}(s_0, s_1, s_2, \ldots, s_p, \ldots, s_l)$$
$$\partial_{n_1}\ldots\partial_{n_k}\phi^{A_1}_{s_1}\ldots\phi^{A_{p-1}}_{s_{p-1}}\phi^{A_{p+1}}_{s_{p+1}}\ldots\phi^{A_l}_{s_l} b^{A_p\,m_1\ldots m_{k'}}{}_{B_1\ldots B_{l'}}(s_p, t_1, \ldots, t_{l'})\partial_{m_1}\ldots\partial_{m_k}\phi^{B_1}_{t_1}\ldots\phi^{B_{l'}}_{t_{l'}}$$

Spin-local-compact field redefinitions in spin-local theories: proper substitute since summation over s_p is finite.

One of the central problems in HS theory is to find a field frame making it (spin-)local. Given non-locally looking field theory, the essential question is whether or not it is spin-local in some other variables.

Spinor Spin-Locality

Polynomiality of $F(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl})$ in either P^{ij} or $\bar{P}^{ij}$ $\forall i, j$ associated with C

Restriction to the fixed spin relates the degrees in P^{ij} and $\bar{P}^{kl}$ since

$$n - m = \pm 2s$$

Non-linear corrections can affect the relation between spinor and space-time spin-locality making obscure the space-time interpretation of the locality analysis in the spinor space.

This does not happen for projectively-compact spin-local vertices with

$$F(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl}) = Q_\omega G(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl}) + \bar{Q}_\omega \bar{G}(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl})$$

Q_ω and $\bar{Q}_\omega$ being associated with the one-forms ω among Φ.

arXiv: 2208.02004

Projectiely-Compact Spin-Local Vertices

Using background frame $e^{\alpha\dot\beta}$ HS equations can be represented as

$$D^L C(y,\bar y) = e^{\alpha\dot\alpha}\Big(\partial_\alpha\bar\partial_{\dot\alpha}F^{++}(y,\bar y)+y_\alpha\bar\partial_{\dot\alpha}F^{-+}(y,\bar y)+\bar y_{\dot\alpha}\partial_\alpha F^{+-}(y,\bar y)+y_\alpha\bar y_{\dot\alpha}F^{--}(y,\bar y$$

Generally, nonlinear corrections can contribute to any of F^{ab}.

The contribution to F^{++} **can be singled out by the projector**

$$\Pi^{des} := N_y^{-1}\bar N_{\bar y}^{-1} y^\alpha \bar y^{\dot\alpha}\frac{\partial}{\partial e^{\alpha\dot\alpha}}\,, \qquad N_y := y^\alpha\partial_\alpha\,, \quad N_{\bar y} := \bar y^{\dot\alpha}\bar\partial_{\dot\alpha}$$

A spin-local vertex Υ is called projectively compact if $\Pi^{des}\Upsilon$ is spin-local-compact. In particular, if $\Pi^{des}\Upsilon = 0$.

The contribution of the projectively-compact spin-local vertices can affect the expressions of the descendants in terms of derivatives of the ground fields only by spin-local-compact terms that preserve space-time locality of the vertex associated with the spin-local spinor vertex.

Nonlinear System via Doubling of Spinors

How to find nonlinear corrections to HS equations? The efficient trick MV 1992 reduces the problem to De Rham cohomology with respect to additional spinor variables $Z^A = (z^\alpha, \bar{z}^{\dot\alpha})$ in presence of Klein operators K

$$\omega(Y;K|x) \longrightarrow W(Z;Y;K|x)\,, \qquad C(Y;K|x) \longrightarrow B(Z;Y;K|x)\,, \quad Y^A = (y^\alpha, \bar{y}^{\dot\alpha})$$

Some of the nonlinear HS equations

$$\begin{cases} \mathsf{d}_x W + W \star W = 0 \\ \mathsf{d}_x B + W \star B - B \star W = 0 \\ \mathsf{d}_x S + W \star S + S \star W = 0 \\ \mathbf{S \star B - B \star S = 0} \\ \mathbf{S \star S = i(\theta^A \theta_A + \eta \theta^\alpha \theta_\alpha B \star k \star \kappa + \bar\eta \bar\theta^{\dot\alpha} \bar\theta_{\dot\alpha} B \star k \star \bar\kappa)} \end{cases} \qquad \mathbf{1992}$$

determine Z_A-dependence in terms of "initial data" $\omega(Y;K|x)$ and $C(Y;K|x)$

$S(Z;Y;K|x) = \theta^A S_A(Z;Y;K|x)$ is a connection along Z^A $(\theta^A \equiv dZ^A)$

Klein operators $K = (k, \bar{k})$ generate chirality automorphisms

$$kf(A) = f(\tilde{A})k\,, \qquad A = (a_\alpha\,, \bar{a}_{\dot\alpha}): \qquad \tilde{A} = (-a_\alpha\,, \bar{a}_{\dot\alpha})$$

Inner Klein operators: $\kappa = \exp iz_\alpha y^\alpha\,, \bar\kappa = \exp i\bar{z}_{\dot\alpha}\bar{y}^{\dot\alpha}\,, \kappa \star f = \tilde{f} \star \kappa\,, \qquad \kappa \star \kappa = 1$

Dynamical content is in the d-independent twistor sector

Perturbative Analysis

Vacuum solution

$$B_0 = 0\,, \qquad S_0 = \theta^A Z_A\,, \qquad W_0 = \frac{1}{2} w^{AB}(x) Y_A Y_B$$

$$\mathsf{d}_x W_0 + W_0 \star W_0 = 0\,, \qquad w^{AB} : AdS_4$$

$$[\mathbf{S_0}\,,\mathbf{f}]_\star = -\mathbf{2i}\mathsf{d}_\mathbf{Z}\mathbf{f}\,, \qquad \mathsf{d}_\mathbf{Z} = \theta^\mathbf{A} \frac{\partial}{\partial \mathbf{Z}^\mathbf{A}}$$

First-order fluctuations

$$B_1 = C(Y)\,, \qquad S = S_0 + S_1\,, \qquad W = W_0(Y) + W_1(Y) + W_0(Y)C(Y)$$

Order-n equations containing S have the form

$$\mathsf{d}_Z U_n(Z;Y|dZ) = V[U_{<n}](Z;Y|\theta) \qquad \mathsf{d}_Z V[U_{<n}](Z;Y|\theta) = 0$$

can be solved by shifted homotopy with shift parameters Q

$$U_n(Z;Y|\theta) = \mathsf{d}^*_Z V[U_{<n}](Z;Y|\theta) + \mathbf{h(Y)} + \mathsf{d}_Z \epsilon(Z;Y|\theta)$$

$$\mathsf{d}^*_Z V(Z;Y|\theta) = (Z^A - Q^A)\frac{\partial}{\partial \theta^A}\int_0^1 \frac{dt}{t} V(tZ + (1-t)Q; Y|t\theta)$$

Interpretation

The contracting homotopy freedom encodes:

All possible gauge choices in d_z-exact forms $\mathsf{d}_z\epsilon(Z;Y|dZ)$

All possible choices of field variables in d_z cohomology $\mathbf{h(Y)}$

Any unfolded HS system is associated with one or another solution to the nonlinear HS system.

How to single out the proper (e.g., minimally nonlocal) frames?

Spin-local limit: $\beta \to -\infty$ with $Q_A = \beta \frac{\partial}{\partial Y^A}$

Didenko, Gelfond, Korybut, MV 1909.04876

Projectively compact vertex was obtained by hand 1605.02662 but so far has not been reached by a systematic homotopy method

Differential Homotopy

Homotopy and shift parameters t^a are treated as coordinates of some manifold $\mathcal{M}$ with the total differential

$$\mathsf{d} := \mathsf{d}_Z + \mathsf{d}_t\,, \qquad \mathsf{d}_Z := \theta^A \frac{\partial}{\partial Z^A}\,, \qquad \mathsf{d}_t := dt^a \frac{\partial}{\partial t^a}\,,$$

Equations to be solved at every perturbation order still have the form

$$\mathsf{d} f(Z,t,\theta,dt) = g(Z,t,\theta,dt)\,, \qquad \mathsf{d} g(Z,t,\theta,dt) = 0\,.$$

Fundamental Ansatz

Lower-order computation yields expressions of the remarkable form

$$f_\mu = \int_{p_i^2 r_i^2 u^2 v^2 \tau\sigma\beta\rho} \mu(\tau,\sigma,\beta,\rho,u,v,p,r)\,\mathrm{d}\Omega^2 \mathcal{E}(\Omega) G(g(r))\,,$$

where $\mu(\tau,\sigma,\beta,\rho,\ldots)$ **is demanded to have compact support in** τ,σ,β,ρ

$$\mathrm{d}\Omega^2 := \mathrm{d}\Omega^\alpha \mathrm{d}\Omega_\alpha$$

$$\Omega_\alpha(\tau,t) := \tau z_\alpha - (1-\tau)(p_\alpha(\sigma) - \beta v_\alpha + \rho(y_\alpha + p_{+\alpha} + u_\alpha))$$

$$p_{+\alpha} := \sum_{i=1}^{l} p_{i\alpha}\,, \qquad p_\alpha(\sigma) = \sum_{i=1}^{k} p_{i\alpha}\sigma_i$$

$$\mathcal{E}(\Omega) := \exp i\Big(\Omega_\beta(y^\beta + p_+^\beta + u^\beta) + u_\alpha v^\alpha - \sum_{i=1}^{l} p_{i\alpha} r_i^\alpha - \sum_{k\geq j>i\geq 1} p_{i\beta} p_j^\beta\Big)\,,$$

$$G_l(g) := g_1(r_1)\ldots g_l(r_l)k\,,$$

$g_i(y)$ **are some functions of** y_α **(e.g.,** $C(y)$ **or** $\omega(y)$**).**

Homology map

Fundamental Ansatz has the remarkable property that

$$\mathsf{d}\Big(\mathsf{d}\Omega^2 \exp i\Big(\Omega_\beta(y^\beta + p_+^\beta + u^\beta) + u_\alpha v^\alpha - \sum_{i=1}^{l} p_{i\alpha} r_i^\alpha - \sum_{k\ge j>i\ge 1} p_{i\beta} p_j^\beta\Big)\Big) = 0$$

$(\mathsf{d}\Omega)^3 = 0$ since the one-forms $\mathsf{d}\Omega^\alpha$ are anticommuting and $\alpha = 1, 2$.

As a result d effectively acts only on μ

$$\mathsf{d} f_\mu = f_{\mathsf{d}\mu}\,,$$

mapping homological problem in terms of spinors Z^A to that on $\mu(t_i)$.

Great advantage of this formalism is that there is no more need to use the Schouten identity: the only formula where it manifests itself is the homology map.

The problem takes purely geometric form on $\mathcal{M}$

$$\mathsf{d}\mu_f \cong \mu_g$$

That $\mathsf{d}\mu_g \cong 0$ implies $\mu_g \cong \mathsf{d}h_g$ allowing to set

$$\mu_f = h_g$$

Projectively-compact spin-local vertex

Final result

$$\mathcal{D}C = J^{\eta}_{pc} + J^{\bar\eta}_{pc}$$

$$J^{\eta}_{pc} = \frac{i\eta}{4}\int_{u^2v^2\tau\rho\beta\sigma_1\sigma_2} d^2u d^2v \mathbb{D}(\tau)\mathbb{D}(\sigma_1+\rho)\mathbb{D}(\sigma_2-(1-\beta-\rho))\vartheta(-\sigma_1)\vartheta(\sigma_2)\mathrm{d}\Omega^2\tilde{\mu}(\beta)\mathcal{E}(\Omega)$$
$$\Big[P(\beta-1,\sigma_\omega,\sigma_1,\sigma_2,1-\beta)\omega(r_\omega,\bar y;K)\bar{*}C(r_1,\bar y;K)\bar{*}C(r_2,\bar y;K)$$
$$+P(\beta-1,\sigma_1,\sigma_\omega,\sigma_2,1-\beta)C(r_1,\bar y;K)\bar{*}\omega(r_\omega,\bar y;K)\bar{*}C(r_2,\bar y;K)$$
$$+P(\beta-1,\sigma_1,\sigma_2,\sigma_\omega,1-\beta)C(r_1,\bar y;K)\bar{*}C(r_2,\bar y;K)\bar{*}\omega(r_\omega,\bar y;K)\Big]\Big|_{r_{\omega,C_i}=0} k\,,$$

$$P_k(\sigma_l,\sigma_{l+1},\dots,\sigma_{l+k}) := \vartheta(\sigma_{l+k}-\sigma_{l+k-1})\dots\vartheta(\sigma_{l+1}-\sigma_l)\,,\qquad \mathbb{D}(a) := da\delta(a)$$

J^{η}_{pc} **is both spin-local and projectively-compact containing an overall factor of $y^\alpha p_{\omega\alpha}$. To reach projective compactness at higher orders the prexponent factors have to be of the form**

$$\prod_i \mathbb{D}(\sigma_{C_i}+\rho+\dots)$$

Conclusion

HS gauge theories contain gravity along with infinite towers of other fields with various spins including ordinary matter fields: singlet scalar!

Infinite-dimensional HS symmetry

HS theories exist in various dimensions.

Unbroken HS symmetries demand AdS background

HS vertices contain higher derivatives.

Customary concepts of Riemann geometry are not applicable: study of exact solutions is very instructive:

BH-like solutions Didenko, MV 2009, Iazeola, Sundell 2010

One of the central problems is the mechanism of spontaneous breakdown of HS symmetries

HS holography is closely related with the locality properties of HS theory

Concepts of projectively-compact vertices are introduced for which spin-locality in the spinor space and space-time are equivalent.

The new differential homotopy approach is designed to figure out the actual level of non-locality of the HS theory.

It is both far more general and far simpler than other approaches, avoiding the necessity to use Schouten identity. In particular it allowed us to evaluate the projectively-compact spin-local vertex in HS theory.

The differential homotopy approach is geometric: cohomology problem on polyhedra in the space $\mathcal{M}$ of homotopy parameters.

To do: higher orders in HS theories of various kinds