On the Schwarzschild type metric in nonlocal de Sitter gravity

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- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold M with metric $(g_{\mu\nu})$ of signature $(1, 3)$.
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
  - sphere S^3 (of constant positive sectional curvature),
  - flat space $\mathbb{R}^3$ (of curvature equal 0),
  - hyperbolic space $\mathbb{H}^3$ (of constant negative sectional curvature).
- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

where $a(t)$ is a cosmic scale factor which describes the evolution (in time) of Universe and parameter k which describes the curvature of the space.

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$$S = \int \left(\frac{R - 2\Lambda}{16\,\pi\, G\, c^4} + \mathcal{L}_m \right) \sqrt{-g}\; d^4x$$

where R is scalar curvature, $g = \det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and $\mathcal{L}_m$ is Lagrangian of matter.

- The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi\, G\, T_{\mu\nu}, \quad c = 1 \tag{2}$$

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

- The energy momentum tensor for ideal fluid (matter in cosmology) is

$$T = \mathrm{diag}(-\rho\, g_{00}, g_{11}p, g_{22}p, g_{33}p), \tag{3}$$

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- Using the conservation law we get

$$0 = \nabla_\mu T^\mu_0 = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p). \qquad (4)$$

- Since in the cosmology equation of state is $p = w\rho$, where w is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
- ▶ The basic types of matter in the Universe are:
  - cosmic dust - $w = 0$, and $\rho_m = Ca^{-3}$
  - radiation – $w = 1/3$, and $\rho_r = Ca^{-4}$
  - In this moment the ratio $\frac{\rho_m}{\rho_r} \approx 10^6$.
- From the expression for FRW metric it follows

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- Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{\dot{a}}{a}\,. \tag{5}$$

- Despite to the great success of GRT in describing:
  - the precession of Merkur perihelion,
  - the bending of light in gravitational fields,
  - the gravitational redshift of light
  - the gravitational lensing,
  - and other ...

GTR has certain deficiencies.

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Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

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There are two natural approaches:

- Dark matter and energy
- Modification of Einstein theory of gravity, i.e. modification of its Lagrangian $\mathcal{L}$

$$\mathcal{L} = \frac{R - 2\Lambda}{16\,\pi\,G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then the Universe contains about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

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Motivation for modification of Einstein theory of gravity

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity

- Einstein General Theory of Relativity

From action

$$S = \int \Big(\frac{R - 2\Lambda}{16\pi G} + \mathcal{L}_m \Big) \sqrt{-g}\, d^4x$$

using variational methods we get field equations

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

where $T_{\mu\nu}$ is stress-energy tensor, $g_{\mu\nu}$ is the metric tensor, $R_{\mu\nu}$ is Ricci tensor and R

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$$R \longrightarrow f(R, \Lambda, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, \dots), \quad \Box = \nabla_\mu \nabla^\mu = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g}\, g^{\mu\nu} \partial_\nu$$

Gauss-Bonnet invariant

$$\mathcal{G} = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}$$

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- Under nonlocal modification of gravity we understand replacement of the scalar curvature R in the Einstein-Hilbert action by a suitable function $F(R,\Box)$, where $\Box = \nabla_\mu \nabla^\mu$ is d'Alembert operator and ∇_μ denotes the covariant derivative
- Let M be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_M \Big(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R)\, \mathcal{F}(\Box)\, \mathcal{G}(R) \Big) \sqrt{-g}\, \mathrm{d}^4 x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of $\Box$, and Λ is cosmological constant.

- In the case of FRW metric the scalar curvature and d'Alambert operator are given by

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where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is Einstein tensor.

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Theorem 2 (EOM) *The equations of motion for system given by S are:*

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where

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$$S = \int_M \Big(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R)\,\mathcal{F}(\Box)\,\mathcal{G}(R) \Big) \sqrt{-g}\, d^4x,$$

for the following cases:

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- Using ansatz $\Box R = r R + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
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3. model $\mathcal{H}(R) = R^p, \mathcal{G}(R) = R^q, p \geq q.$

- We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12} t^2)$
- For $p = q = 1$ there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
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- If $R = R_0 > 0$, then there exist non-singlar solutions for all three values of parameter $k = 0, \pm 1$, which are bounced in the cases $k = 0, 1$.
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where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

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1.1 Solutions of the form $a(t) = A\, t^n e^{\gamma t^2}$

- There are two solutions:

1.2 New solutions of the form $a(t) = (\alpha\, e^{\lambda t} + \beta\, e^{-\lambda t})^{\gamma}$

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$$\gamma = \frac{2}{3}, \qquad q = \frac{3}{8}\Lambda, \qquad \lambda = \pm\sqrt{\frac{3}{8}\Lambda}\,.$$

- When $\alpha\beta \neq 0$, we have the following two special solutions:

$$a_3(t) = A\, \cosh^{\frac{2}{3}}\left(\sqrt{\frac{3}{8}\Lambda}\; t\right), \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \;\; \mathcal{F}'(\frac{3}{8}\Lambda) = 0,$$

$$a_4(t) = A\, \sinh^{\frac{2}{3}}\left(\sqrt{\frac{3}{8}\Lambda}\; t\right), \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \;\; \mathcal{F}'(\frac{3}{8}\Lambda) = 0.$$

1. **Cosmological solution in the flat Universe** $(k = 0)$

1.3. New solutions of the form $a(t) = (\alpha \sin \lambda t + \beta \cos \lambda t)^{\gamma}$?

- For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for [illegible]. Taking [illegible] and $A = $ [illegible], we have the following two solutions:

$$a_5(t) = A\Big(1 + \sin\big(2\sqrt{[illegible]\Lambda\, t}\big)\Big)^{[illegible]}, \qquad \mathcal{F}([illegible]\Lambda) = -1, \quad \mathcal{F}'([illegible]\Lambda) = 0,$$

$$a_6(t) = A\Big(1 - \sin\big(2\sqrt{[illegible]\Lambda\, t}\big)\Big)^{[illegible]}, \qquad \mathcal{F}([illegible]\Lambda) = -1, \quad \mathcal{F}'([illegible]\Lambda) = 0.$$

- For $\alpha = 0$ or $\beta = 0$, we have also two cosmological solutions with $\gamma = \frac{2}{3}$:

$$a_7(t) = A \sin^{\frac{2}{3}}\Big(\sqrt{-\frac{3}{8}\Lambda}\; t\Big), \qquad \mathcal{F}\big(\frac{3}{8}\Lambda\big) = -1, \quad \mathcal{F}'\big(\frac{3}{8}\Lambda\big) = 0,$$

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1. **Cosmological solution in the flat Universe** $(k = 0)$

1.3. New solutions of the form $a(t) = (\alpha \ \sin \lambda t + \beta \ \cos \lambda t)^{\gamma}$

- For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ, $\gamma = \frac{2}{3}$. Taking $\beta = \pm\alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

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2. **Cosmological solution in the open and closed Universe** $(k = \pm 1)$

2.2. New solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$, $(k = \pm 1)$

- For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $\varphi \neq 0$ there are two following cosmological solutions:

$$a_{10}(t) = A \cosh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}\Lambda}\, t\right), \quad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \; \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0,$$

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2. **Cosmological solution in the open and closed Universe** $(k = \pm 1)$

2.1. Solutions of the form $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}t}$, $(k = \pm 1)$

- For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution:

$$a_9(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}t}, \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3}\Lambda) = -1, \quad \mathcal{F}'(\frac{1}{3}\Lambda) = 0, \quad \Lambda > 0.$$

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- 1. Cosmological solution for $a_1(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14}t^2}, \ k = 0$

  - The corresponding Hubble parameter, acceleration and the scalar 2 curvature are:

$$H_1(t) = \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

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  - Friedman equations gives

$$\bar{\rho}(t) = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}, \quad \bar{p}(t) = -\frac{\Lambda}{56\pi G}\Big(\frac{3}{7}\Lambda t^2 - 1\Big), \qquad (11)$$

where $\bar{\rho}$ and $\bar{p}$ are analogs of the energy density and pressure of the dark side of the universe, respectively. The corresponding equation of state is $\bar{p}(t) = \bar{w}(t)\,\bar{\rho}(t)$.

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$$\bar{\rho}(t) = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}\,, \quad \bar{p}(t) = -\frac{\Lambda}{56\pi G}\big(\frac{3}{7}\Lambda t^2 - 1\big), \qquad (11)$$

where $\bar{\rho}$ and $\bar{p}$ are analogs of the energy density and pressure of the dark side of the universe, respectively. The corresponding equation of state is $\bar{p}(t) = \bar{w}(t)\, \bar{\rho}(t)$.

- The expression (11) implies that $\bar{w}(t) \to -1$ when $t \to \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model.
- It means that this nonlocal gravity model with cosmological solution $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}$ describes some effects usually attributed to the dark matter and dark energy.
- This solution is invariant under transformation $t \to -t$ and singular at cosmic time $t = 0$.

- Let us recall, the second Friedman equation

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \qquad (12)$$

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- Then we can rewrite the previous equation as,

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_r + \frac{8\pi G}{3}\rho_m - \frac{k}{a^2} + \frac{\Lambda}{3}$$
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$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

- Observational data obtained by Planck-2018 for the ΛCDM model:

$t_0 = (13.801 \pm 0.024) \times 10^9$yr – age of the universe,

$H(t_0) = (67.40 \pm 0.50)$ km/s/Mpc – Hubble parameter,

$\Omega_m = 0.315 \pm 0.007$– matter density parameter,

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taking $H_1(t_0) = H(t_0)$ we calculate $\Lambda_1 = 1.05 \times 10^{-35}s^{-2}$ that differs from $\Lambda = 3H^2(t_0)\,\Omega_\Lambda = 0.98 \times 10^{-35}s^{-2}$ (by ΛCDM model).

- We also computed

$$\ddot{a}_1(t_0)/a_1(t_0) = 2.7 \times 10^{-36}s^{-2}$$

$$R(t_0) = 4.5 \times 10^{-35}s^{-2} \quad \text{and consequently}$$

$$R(t_0) - 2\Lambda = 2.4 \times 10^{-35}s^{-2}.$$

- Replacing solution $a_1(t)$ with $k = 0$, Friedman equations give

$$\bar{\rho}_1(t) = \frac{3}{8\pi G}\Big(H_1^2(t) - \frac{\Lambda_1}{3}\Big) = \frac{3}{8\pi G}\Big(\frac{4}{9}t^{-2} - \frac{1}{7}\Lambda_1 + \frac{1}{49}\Lambda_1^2 t^2\Big),$$

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- For $t = t_0$, from previous formula, and from ΛCDM model we have

$$\bar{\rho}_1(t_0) = 2.26 \times 10^{-30} \ \frac{g}{cm^3},$$

$$\rho(t_0) = \frac{3}{8\pi G}\Big(H_0^2 - \frac{\Lambda}{3}\Big) = 2.68 \times 10^{-30} \ \frac{g}{cm^3}.$$

- Then, for vacuum energy density of background solution $a_1(t)$ and ΛCDM model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_\Lambda = 0.42 \times 10^{-30} \ \frac{g}{cm^3},$$

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$$\Omega_{\Lambda_1} = \frac{\rho_{\Lambda_1}}{\rho_c} = 0.734, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = 0.685, \quad \Delta\Omega_\Lambda = \Omega_{\Lambda_1} - \Omega_\Lambda = 0.049, \qquad (13)$$

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- According to (13) and (14), we obtain that $\Omega_{m_1} = 26,6\%$ corresponds to dark matter and $\Delta\Omega_m = \Delta\Omega_\Lambda = 4.9\%$ is related to visible matter, what is in a very good agreement with the standard model of cosmology.
- Efective presure. At the beginning, $\bar{p}_1(0) = \frac{\Lambda_1}{56\pi G} > 0$, then decreases and equals zero at $t = \sqrt{\frac{7}{3\Lambda_1}} = 4.71 \times 10^{17}\, s = 14,917 \times 10^9\, yr$.
- According to (11), we have parameter $\bar{w}_1(t) = \frac{\bar{p}_1(t)}{\bar{\rho}_1(t)}$ which has future behavior in agreement with standard model of cosmology, i.e. $\bar{w}_1(t \to \infty) \to -1$.
- Note that the Hubble parameter has minimum at $t_{min} = 21.1 \times 10^9 yr$ and it is $H_1(t_{min}) = 61.72 km/s/Mpc$. It also, follows that the Universe experiences decelerated expansion during matter dominance, that turns to acceleration at time $t_{acc} = 7.84 \times 10^9 yr$ when, $\ddot{a} = 0$.

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$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2. \tag{15}$$

- The corresponding scalar curvature R of above metric (15)

$$R = \frac{2}{r^2} - \frac{2}{r^2 B(r)} - \frac{2A'(r)}{rA(r)B(r)} + \frac{A'(r)^2}{2A(r)^2 B(r)} + \frac{2B'(r)}{rB^2(r)} + \frac{A'(r)B'(r)}{2A(r)B(r)^2} - \frac{A''(r)}{A(r)B(r)} \tag{16}$$

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- Then we can rewrite equation (20) in the form

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  or in more details the equation (27) is equivalent to

  $$r^2 f''(r) + 4rf'(r) + 2f(r) = -2\Lambda r^2 + C_2^2\, e^{-2\sqrt{q}\, r}. \tag{28}$$

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  $$f(r) = -\frac{\Lambda}{6} r^2 + \frac{C_2^2}{4q}\,\frac{1}{r^2}\, e^{-2\sqrt{q}\, r} + \frac{C_3}{r} + \frac{C_4}{r^2}. \tag{29}$$

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$$A_{\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3c^2}, \qquad r \geq r_0, \tag{32}$$

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THANK YOU FOR

YOUR ATTENTION !!!

Non-trivial Christoffel symbols of Friedman – Robertson – Walker metric

$$\Gamma^1_{01} = \frac{\dot a}{a} \qquad \Gamma^2_{02} = \frac{\dot a}{a} \qquad \Gamma^3_{03} = \frac{\dot a}{a}$$

$$\Gamma^0_{11} = \frac{a\,\dot a}{1 - k\,r^2} \qquad \Gamma^1_{11} = \frac{k\,r}{1 - k\,r^2} \qquad \Gamma^2_{12} = \frac{1}{r}$$

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Non-trivial components of curvature tensor

$$R_{0110} = \frac{a\,\ddot{a}}{1 - k\,r^2} \qquad R_{1221} = -\frac{r^2\,a^2\,(\dot{a}^2 + k)}{1 - k\,r^2}$$

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Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3\ddot{a}}{a} & 0 & 0 & 0 \\ 0 & u\,g_{11} & 0 & 0 \\ 0 & 0 & u\,g_{22} & 0 \\ 0 & 0 & 0 & u\,g_{33} \end{pmatrix}, \qquad u = \frac{a\,\ddot{a} + 2\,(\dot{a}^2 + k)}{a^2}$$

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Scalar curvature

$$R = \frac{6\,(a\,\ddot{a} + \dot{a}^2 + k)}{a^2}$$

Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3\,(\dot{a}^2+k)}{a^2} & 0 & 0 & 0 \\ 0 & -v\,g_{11} & 0 & 0 \\ 0 & 0 & -v\,g_{22} & 0 \\ 0 & 0 & 0 & -v\,g_{33} \end{pmatrix}, \qquad v = \frac{2\,a\,\ddot{a} + \dot{a}^2 + k}{a^2}$$

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$$\Gamma^1_{22} = -\frac{r}{B}, \qquad \Gamma^1_{33} = -\frac{r\sin^2\theta}{B}, \qquad \Gamma^2_{12} = \frac{1}{r},$$

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Non-trivial components of curvature tensor

$$R_{0101} = \frac{A}{4}\left(-\left(\frac{A'}{A}\right)^2 - \frac{A'}{A}\frac{B'}{B} + 2\frac{A''}{A}\right), \qquad R_{0202} = \frac{r}{2}\frac{A'}{B},$$

$$R_{0303} = \frac{r}{2}\frac{A'}{B}\sin^2\theta, \qquad R_{1212} = \frac{r}{2}\frac{B'}{B},$$

$$R_{1313} = \frac{r}{2}\frac{B'}{B}\sin^2\theta, \qquad R_{2323} = r^2\frac{B-1}{B}\sin^2\theta.$$

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$$\Gamma^0_{01} = \frac{1}{2}\frac{A'}{A}, \qquad \Gamma^1_{00} = \frac{1}{2}\frac{A'}{B}, \qquad \Gamma^1_{11} = \frac{1}{2}\frac{B'}{B},$$

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SdS metric-GC

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SdS metric-GC

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▸ SdS metric-C:B=1/A

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$$R_{00} = \frac{1}{2}AA'' + \frac{1}{r}AA', \qquad R_{11} = -\frac{1}{2}\frac{A''}{A} - \frac{1}{r}\frac{A'}{A},$$

$$R_{22} = 1 - A - rA', \qquad R_{33} = \left(1 - A - rA'\right)\sin^2\theta.$$

The scalar curvature is

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}A + \frac{2}{r^2}.$$

The Einstein tensor is presented as follows:

$$G_{00} = -\frac{A(r)A'(r)}{r} - \frac{A(r)^2}{r^2} + \frac{A(r)}{r^2}, \qquad G_{11} = \frac{A'(r)}{rA(r)} - \frac{1}{r^2A(r)} + \frac{1}{r^2},$$

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