On the Schwarzschild type metric in nonlocal de Sitter gravity

Zoran Rakić

Faculty of Mathematics, University of Belgrade, Serbia

(joint work with I. Dimitrijević, B. Dragovich, and J. Stanković)

THE INTERNATIONAL WORKSHOP Problems of the Modern Mathematical Physics – PMMP February 19–23, 2024

イロト イポト イヨト イヨト

General theory of relativity

- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold *M* with metric $(g_{\mu\nu})$ of signature (1, 3).
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
 - sphere S³ (of constant positive sectional curvature).
 - flat space R³ (of curvature equal 0),
 - hyperbolic space III^o (of constant negative sectional culvature).
- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \ k \in \{-1, 0, 1\},$$
(1)

where a(t) is a cosmic scale factor which describes the evolution (in time) of Universe and parameter k which describes the curvature of the space.

- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold *M* with metric $(g_{\mu\nu})$ of signature (1, 3).
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
 - sphere S³ (of constant positive sectional curvature).
 - flat space \mathbb{R}^3 (of curvature equal 0),
 - hyperbolic space \mathbb{H}^3 (of constant negative sectional cutvature).
- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold *M* with metric $(g_{\mu\nu})$ of signature (1, 3).

There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:

- sphere S³ (of constant positive sectional curvature).
- flat space \mathbb{R}^3 (of curvature equal 0),
- hyperbolic space \mathbb{H}^3 (of constant negative sectional cutvature).

Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

where a(t) is a cosmic scale factor which describes the evolution (in time) of Universe and parameter k which describes the curvature of the space. FRW metric

(日)

- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold *M* with metric $(g_{\mu\nu})$ of signature (1, 3).
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
 - sphere \mathbb{S}^3 (of constant positive sectional curvature),
 - \circ flat space \mathbb{R}^3 (of curvature equal 0),
 - hyperbolic space \mathbb{H}^3 (of constant negative sectional cutvature).
- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold *M* with metric $(g_{\mu\nu})$ of signature (1, 3).
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
 - sphere \mathbb{S}^3 (of constant positive sectional curvature),
 - flat space \mathbb{R}^3 (of curvature equal 0),
 - hyperbolic space \mathbb{H}^3 (of constant negative sectional cutvature).
 - Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold *M* with metric $(g_{\mu\nu})$ of signature (1, 3).
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
 - sphere \mathbb{S}^3 (of constant positive sectional curvature),
 - flat space \mathbb{R}^3 (of curvature equal 0),
 - hyperbolic space \mathbb{H}^3 (of constant negative sectional cutvature).
 - Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold *M* with metric $(g_{\mu\nu})$ of signature (1, 3).
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
 - sphere \mathbb{S}^3 (of constant positive sectional curvature),
 - flat space \mathbb{R}^3 (of curvature equal 0),
 - hyperbolic space \mathbb{H}^3 (of constant negative sectional cutvature).
- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold *M* with metric $(g_{\mu\nu})$ of signature (1, 3).
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
 - sphere \mathbb{S}^3 (of constant positive sectional curvature),
 - flat space \mathbb{R}^3 (of curvature equal 0),
 - hyperbolic space \mathbb{H}^3 (of constant negative sectional cutvature).
- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

$$S = \int \left(\frac{R-2\Lambda}{16\pi\,G\,c^4} + \mathcal{L}_m\right) \sqrt{-g} \,d^4x$$

where *R* is scalar curvature, $g = det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and \mathcal{L}_m is Lagrangian of matter.

The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1$$
(2)

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

The energy momentum tensor for ideal fluid (matter in cosmology) is

$$\Gamma = diag(-\rho \, g_{00}, g_{11}\rho, g_{22}\rho, g_{33}\rho), \tag{3}$$

where ρ is energy density and p is pressure.

- コン (雪) (ヨ) (ヨ)

э.

$$S = \int \left(\frac{R-2\Lambda}{16\,\pi\,G\,c^4} + \mathcal{L}_m\right) \sqrt{-g} \,d^4x$$

where *R* is scalar curvature, $g = det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and \mathcal{L}_m is Lagrangian of matter.

The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1$$
 (2)

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and *R* is scalar curvature.

The energy momentum tensor for ideal fluid (matter in cosmology) is

$$T = \text{diag}(-\rho \, g_{00}, g_{11} \rho, g_{22} \rho, g_{33} \rho), \tag{3}$$

where ρ is energy density and p is pressure.

・ 同 ト ・ ヨ ト ・ ヨ

$$S = \int \left(\frac{R-2\Lambda}{16\pi\,G\,c^4} + \mathcal{L}_m\right) \sqrt{-g} \, d^4x$$

where *R* is scalar curvature, $g = det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and \mathcal{L}_m is Lagrangian of matter.

The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1$$
 (2)

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

The energy momentum tensor for ideal fluid (matter in cosmology) is

$$T = \text{diag}(-\rho g_{00}, g_{11}p, g_{22}p, g_{33}p), \tag{3}$$

where ρ is energy density and p is pressure.

A (1) > A (2) > A (2)

$$S = \int \left(\frac{R-2\Lambda}{16\pi\,G\,c^4} + \mathcal{L}_m\right) \sqrt{-g} \, d^4x$$

where *R* is scalar curvature, $g = det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and \mathcal{L}_m is Lagrangian of matter.

The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1$$
 (2)

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

The energy momentum tensor for ideal fluid (matter in cosmology) is

 $T = \text{diag}(-\rho \, g_{00}, g_{11} \rho, g_{22} \rho, g_{33} \rho), \tag{3}$

where ρ is energy density and p is pressure.

$$S = \int \left(\frac{R-2\Lambda}{16\pi\,G\,c^4} + \mathcal{L}_m\right) \sqrt{-g} \, d^4x$$

where *R* is scalar curvature, $g = det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and \mathcal{L}_m is Lagrangian of matter.

The variation of the action *S* we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1$$
 (2)

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and *R* is scalar curvature.

The energy momentum tensor for ideal fluid (matter in cosmology) is

 $\Gamma = \text{diag}(-\rho \, g_{00}, g_{11} \rho, g_{22} \rho, g_{33} \rho), \tag{3}$

where ρ is energy density and p is pressure.

• □ • • □ • • □ • • □ • • □ •

$$S = \int \left(\frac{R-2\Lambda}{16 \pi G c^4} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

where *R* is scalar curvature, $g = det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and \mathcal{L}_m is Lagrangian of matter.

The variation of the action *S* we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1$$
 (2)

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and *R* is scalar curvature.

The energy momentum tensor for ideal fluid (matter in cosmology) is

$$T = \text{diag}(-\rho \, g_{00}, g_{11}\rho, g_{22}\rho, g_{33}\rho), \tag{3}$$

where ρ is energy density and p is pressure.

$$0 = \nabla_{\mu} T_{0}^{\mu} = -\dot{\rho} - 3\frac{\dot{a}}{\dot{a}}(\rho + \rho).$$
(4)

- Since in the cosmology equation of state is $\rho = w\rho$, where w is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
- Televine of matter in the Universe are:
- ϕ cosmic dust w = 0, and $\rho_m = 0 a^{-1}$.
- w = 1/3, and $p_r = 0.a^{-4}$
- In this moment the ratio $\frac{Pm}{2}pprox 10^6$
- From the expression for FRW metric it follows

$$R(t) = rac{6(a(t)\ddot{a}(t) + \dot{a}(t)^2 + k)}{a(t)^2}$$

・ロ・・(型・・目・・(目・)

э.

$$0 = \nabla_{\mu} T_{0}^{\mu} = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p).$$
(4)

- Since in the cosmology equation of state is $p = w\rho$, where *w* is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
 - I The basic types of matter in the Universe are:
- cosmic dust w = 0, and $\rho_m = C a^{-3}$
- radiation - w = 1/3, and $\rho_r = C a^{-4}$
- In this moment the ratio $\frac{
 ho_m}{r} pprox$ 10 6 .
- From the expression for FRW metric it follows

$$R(t) = \frac{6(a(t)\ddot{a}(t) + \dot{a}(t)^{2} + k)}{a(t)^{2}}$$

< D > < P > < E > < E</p>

$$0 = \nabla_{\mu} T_{0}^{\mu} = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p).$$
(4)

- Since in the cosmology equation of state is $p = w\rho$, where *w* is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
 - I The basic types of matter in the Universe are:
- cosmic dust w = 0, and $\rho_m = C a^{-3}$
- radiation - w = 1/3, and $\rho_r = C a^{-4}$
- In this moment the ratio $\frac{
 ho_m}{r}pprox$ 10 6 .
- From the expression for FRW metric it follows

$$R(t) = \frac{6(a(t)\ddot{a}(t) + \dot{a}(t)^{2} + k)}{a(t)^{2}}$$

< ロ > < 同 > < 三 > < 三 >

$$0 = \nabla_{\mu} T_{0}^{\mu} = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p).$$
(4)

- Since in the cosmology equation of state is $p = w\rho$, where w is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
 - I I The basic types of matter in the Universe are:
- cosmic dust w = 0, and $\rho_m = C a^{-3}$
- radiation - w = 1/3, and $\rho_r = C a^{-4}$
- In this moment the ratio $\frac{\rho_m}{r} \approx 10^6$.
- From the expression for FRW metric it follows

$$R(t) = \frac{6(a(t)\ddot{a}(t) + \dot{a}(t)^{2} + k)}{a(t)^{2}}$$

$$0 = \nabla_{\mu} T_{0}^{\mu} = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p).$$
(4)

- Since in the cosmology equation of state is $\rho = w\rho$, where *w* is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
 - The basic types of matter in the Universe are:
- cosmic dust w = 0, and $\rho_m = C a^{-3}$.
- radiation - w = 1/3, and $\rho_r = C a^{-4}$.
- In this moment the ratio $\frac{\rho_m}{\rho_r} \approx 10^6$.
- From the expression for FRW metric it follows

$$R(t) = \frac{6(a(t)\ddot{a}(t) + \dot{a}(t)^{2} + k)}{a(t)^{2}}$$

$$0 = \nabla_{\mu} T_{0}^{\mu} = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p).$$
(4)

- Since in the cosmology equation of state is $\rho = w\rho$, where *w* is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
 - The basic types of matter in the Universe are:
- cosmic dust w = 0, and $\rho_m = C a^{-3}$.
- radiation - w = 1/3, and $\rho_r = C a^{-4}$.
- In this moment the ratio $\frac{\rho_m}{\rho_r} \approx 10^6$.
- From the expression for FRW metric it follows

$$R(t) = \frac{6(a(t)\ddot{a}(t) + \dot{a}(t)^{2} + k)}{a(t)^{2}}$$

$$0 = \nabla_{\mu} T_{0}^{\mu} = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p).$$
(4)

- Since in the cosmology equation of state is $\rho = w\rho$, where *w* is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
 - The basic types of matter in the Universe are:
- cosmic dust w = 0, and $\rho_m = C a^{-3}$.
- radiation - w = 1/3, and $\rho_r = C a^{-4}$.
- In this moment the ratio $\frac{\rho_m}{\rho_r} \approx 10^6$.

From the expression for FRW metric it follows

$$R(t) = \frac{6(a(t)\ddot{a}(t) + \dot{a}(t)^{2} + k)}{a(t)^{2}}$$

$$0 = \nabla_{\mu} T_{0}^{\mu} = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p).$$
(4)

- Since in the cosmology equation of state is $\rho = w\rho$, where *w* is a constant, we have that equation (4) has solution $\rho = Ca^{-3(1+w)}$.
 - The basic types of matter in the Universe are:
- cosmic dust w = 0, and $\rho_m = C a^{-3}$.
- radiation - w = 1/3, and $\rho_r = C a^{-4}$.
- In this moment the ratio $\frac{\rho_m}{\rho_r} \approx 10^6$.

From the expression for FRW metric it follows

$$R(t) = \frac{6\left(a(t)\ddot{a}(t) + \dot{a}(t)^2 + k\right)}{a(t)^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3\rho) + \frac{\Lambda}{3}, \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3},$$

 Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{a}{a} .$$
 (5)

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields.
- the gravitational redshift of light
- the gravitational lensing.
- and other ...

GTR has certain deficiencies

・ロ・・ (日・・ (日・・)

э

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{a}{a} . \tag{5}$$

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields.
- the gravitational redshift of light
- the gravitational lensing
- and other ...

GTR has certain deficiencies.

A = A = A = A = A = A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3},\qquad \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}+\frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{\dot{a}}{a} .$$
 (5)

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields,
- the gravitational redshift of light
- the gravitational lensing,
- and other ...

GTR has certain deficiencies.

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3},\qquad \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}+\frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{\dot{a}}{a} .$$
 (5)

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields.
- the gravitational redshift of light
- the gravitational lensing
- and other ...

GTR has certain deficiencies.

< D > < P > < E > < E</p>

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3},\qquad \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}+\frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{a}{a} .$$
 (5)

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields
- the gravitational redshift of light
- the gravitational lensing
- and other ...

GTR has certain deficiencies.

< D > < P > < E > < E</p>

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3},\qquad \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}+\frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{\dot{a}}{a} .$$
 (5)

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields,
- the gravitational redshift of light
- the gravitational lensing,
- and other ..

GTR has certain deficiencies.

< 同 > < 三 > < 三 >

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3},\qquad \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}+\frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{a}{a} .$$
 (5)

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields,
- the gravitational redshift of light
- the gravitational lensing.
- and other ..

GTR has certain deficiencies.

A (1) > A (2) > A (2)

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3},\qquad \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}+\frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{\dot{a}}{a} .$$
 (5)

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields,
- the gravitational redshift of light
- the gravitational lensing
- and other ..

GTR has certain deficiencies.

A (1) > A (2) > A (2)

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3},\qquad \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}+\frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{a}{a} .$$
 (5)

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields,
- the gravitational redshift of light
- the gravitational lensing,
- and other ..

GTR has certain deficiencies.

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3},\qquad \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}+\frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{\dot{a}}{a} .$$
 (5)

Despite to the great success of GRT in describing:

- the precession of Merkur perihelion,
- the bending of light in gravitational fields,
- the gravitational redshift of light
- the gravitational lensing,
- and other ...

GTR has certain deficiencies.

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3},\qquad \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}+\frac{\Lambda}{3}.$$

Hubble parameter is a measure used to describe the expansion of the Universe

$$H = \frac{\dot{a}}{a} .$$
 (5)

Despite to the great success of GRT in describing:

Zoran Rakić

- the precession of Merkur perihelion,
- the bending of light in gravitational fields,
- the gravitational redshift of light
- the gravitational lensing,
- and other ...

GTR has certain deficiencies.

Motivation

Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933)
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

< ロ > < 同 > < 回 > < 回 > < 回 > <

э

Motivation

Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

< ロ > < 同 > < 三 > < 三 >
Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

< ロ > < 同 > < 三 > < 三 >

Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

< ロ > < 同 > < 三 > < 三 >

Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).

Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

Great cosmological observational discoveries of 20th century, which could not be explained by GTR without additional matter

- High orbital speeds of galaxies in clusters (Fritz Zwicky, 1933).
- High orbital speeds of stars in spiral galaxies (Vera Rubin, at the end of 1960es).
- Accelerated expansion of the Universe (1998).

Big Bang

- Another cosmological problem is related to the Big Bang singularity. General relativity yields cosmological solutions with zero size of the Universe at its beginning, and what means an infinite matter density.
- When physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

< ロ > < 同 > < 三 > < 三 > -

- Dark matter and energy
- Modification of Einstein theory of gravity, i.e. modification of its Lagrangian L

$$\mathcal{L} = \frac{R - 2\Lambda}{16 \pi G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then Grant and about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

- コン (雪) (ヨ) (ヨ)

э.

Dark matter and energy

Modification of Einstein theory of gravity, i.e. modification of its Lagrangian L

$$\mathcal{L} = rac{R-2\Lambda}{16 \pi G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then
 the Universe contains) about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

Dark matter and energy

Modification of Einstein theory of gravity, i.e. modification of its Lagrangian L

$$\mathcal{L} = rac{R-2\Lambda}{16 \pi G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then
 the Universe contains) about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

Zoran Rakić

Dark matter and energy

Modification of Einstein theory of gravity, i.e. modification of its Lagrangian \mathcal{L}

$$\mathcal{L} = rac{R-2\Lambda}{16 \pi G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then
 the Universe contains) about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

Dark matter and energy

Modification of Einstein theory of gravity, i.e. modification of its Lagrangian L

$$\mathcal{L} = \frac{R-2\Lambda}{16 \pi G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then
 the Universe contains) about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

- Dark matter and energy
- Modification of Einstein theory of gravity, i.e. modification of its Lagrangian L

$$\mathcal{L} = \frac{R-2\Lambda}{16 \pi G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then
 the Universe contains) about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

- Dark matter and energy
- Modification of Einstein theory of gravity, i.e. modification of its Lagrangian L

$$\mathcal{L} = \frac{R-2\Lambda}{16 \pi G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then
 the Universe contains) about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

- Dark matter and energy
- Modification of Einstein theory of gravity, i.e. modification of its Lagrangian L

$$\mathcal{L} = \frac{R-2\Lambda}{16 \pi G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then
 the Universe contains) about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

- Dark matter and energy
- Modification of Einstein theory of gravity, i.e. modification of its Lagrangian L

$$\mathcal{L} = \frac{R-2\Lambda}{16 \pi G} + \mathcal{L}_m, \qquad c = 1.$$

Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then
 the Universe contains) about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

Modification of Einstein theory of gravity

Motivation for modification of Einstein theory of gravity

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity Einstein General Theory of Relativity

From action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} \, d^4x$$

using variational methods we get field equations

$$R_{\mu
u}-rac{1}{2}\,R\,g_{\mu
u}+\Lambda g_{\mu
u}=8\pi G T_{\mu
u},\qquad c=1.$$

where $\mathcal{T}_{\mu
u}$ is stress-energy tensor, $g_{\mu
u}$ is the metric tensor, $R_{\mu
u}$ is Ricci tensor and R

- コン (雪) (ヨ) (ヨ)

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity
Einstein General Theory of Relativity
From action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

using variational methods we get field equations

$$R_{\mu\nu} - rac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

where $\mathcal{T}_{\mu
u}$ is stress-energy tensor, $g_{\mu
u}$ is the metric tensor, $R_{\mu
u}$ is Ricci tensor and R

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity
 Einstein General Theory of Relativity
 From action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

using variational methods we get field equations

$$R_{\mu\nu} - rac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

where $\mathcal{T}_{\mu
u}$ is stress-energy tensor, $g_{\mu
u}$ is the metric tensor, $\mathcal{R}_{\mu
u}$ is Ricci tensor and \mathcal{R}

(日)

Modification of Einstein theory of gravity

Motivation for modification of Einstein theory of gravity

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity
 Einstein General Theory of Relativity
 From action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

using variational methods we get field equations

$$R_{\mu\nu} - rac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

where $T_{\mu\nu}$ is stress-energy tensor, $g_{\mu\nu}$ is the metric tensor, $R_{\mu\nu}$ is Ricci tensor and R

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity
 Einstein General Theory of Relativity
 From action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

using variational methods we get field equations

$$R_{\mu\nu} - rac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

where $\mathcal{T}_{\mu
u}$ is stress-energy tensor, $g_{\mu
u}$ is the metric tensor, $\mathcal{R}_{\mu
u}$ is Ricci tensor and \mathcal{R}

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity

Einstein General Theory of Relativity From action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

using variational methods we get field equations

$$R_{\mu
u} - rac{1}{2} R g_{\mu
u} + \Lambda g_{\mu
u} = 8\pi G T_{\mu
u}, \qquad c = 1.$$

where $\mathcal{T}_{\mu
u}$ is stress-energy tensor, $g_{\mu
u}$ is the metric tensor, $\mathcal{R}_{\mu
u}$ is Ricci tensor and \mathcal{R}

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity

Einstein General Theory of Relativity

From action

$$S = \int \left(rac{R-2\Lambda}{16\pi G} + \mathcal{L}_m
ight)\sqrt{-g} d^4x$$

using variational methods we get field equations

$$R_{\mu
u} - rac{1}{2} R g_{\mu
u} + \Lambda g_{\mu
u} = 8\pi G T_{\mu
u}, \qquad c = 1.$$

where $\mathcal{T}_{\mu
u}$ is stress-energy tensor, $g_{\mu
u}$ is the metric tensor, $\mathcal{R}_{\mu
u}$ is Ricci tensor and \mathcal{R}

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity

Einstein General Theory of Relativity
 From action

$$S = \int \left(rac{R-2\Lambda}{16\pi G} + \mathcal{L}_m
ight)\sqrt{-g} d^4x$$

using variational methods we get field equations

$$R_{\mu\nu} - rac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

where $T_{\mu
u}$ is stress-energy tensor, $g_{\mu
u}$ is the metric tensor, $R_{\mu
u}$ is Ricci tensor and R

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Different approaches to modification of Einstein theory of gravity

Einstein General Theory of Relativity
 From action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} \ d^4x$$

using variational methods we get field equations

$$R_{\mu\nu} - rac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

where $T_{\mu\nu}$ is stress-energy tensor, $g_{\mu\nu}$ is the metric tensor, $R_{\mu\nu}$ is Ricci tensor and R

• □ ▶ • • □ ▶ • • □ ▶ • • □ ▶ •

Einstein-Hilbert action

$$S = \int \left(rac{R-2\Lambda}{16\pi G} + \mathcal{L}_m
ight) \sqrt{-g} d^4 x$$

modification

$$R \longrightarrow f(R, \Lambda, R_{\mu
u}, R^lpha_{\mueta
u}, \Box, \ldots), \quad \Box =
abla_\mu
abla^\mu = rac{1}{\sqrt{-g}}\, \partial_\mu \sqrt{-g}\, g^{\mu
u}\, \partial_
u$$

Gauss-Bonnet invariant

$$\mathcal{G}=R^2-4\,R^{\mu
u}R_{\mu
u}+R^{lphaeta\mu
u}\,R_{lphaeta\mu
u}$$

<ロ> <同> <同> < 同> < 同> < 三> < 三> <

Einstein-Hilbert action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4 \lambda$$

modification

$$R \longrightarrow f(R, \Lambda, R_{\mu
u}, R^lpha_{\mueta
u}, \Box, \ldots), \quad \Box =
abla_\mu
abla^\mu = rac{1}{\sqrt{-g}}\,\partial_\mu \sqrt{-g}\,g^{\mu
u}\,\partial_
u$$

Gauss-Bonnet invariant

$$\mathcal{G}=\textit{R}^2-4\,\textit{R}^{\mu\nu}\textit{R}_{\mu\nu}+\textit{R}^{\alpha\beta\mu\nu}\,\textit{R}_{\alpha\beta\mu\nu}$$

Einstein-Hilbert action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4 x$$

modification

$$R \longrightarrow f(R, \Lambda, R_{\mu
u}, R^lpha_{\mueta
u}, \Box, \ldots), \quad \Box =
abla_\mu
abla^\mu = rac{1}{\sqrt{-g}}\,\partial_\mu \sqrt{-g}\,g^{\mu
u}\,\partial_
u$$

Gauss-Bonnet invariant

$$\mathcal{G}=\textit{R}^2-4\,\textit{R}^{\mu\nu}\textit{R}_{\mu\nu}+\textit{R}^{\alpha\beta\mu\nu}\,\textit{R}_{\alpha\beta\mu\nu}$$

<ロ> < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

Einstein-Hilbert action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4 x$$

modification

$$R \longrightarrow f(R, \Lambda, R_{\mu
u}, R^{lpha}_{\mueta
u}, \Box, \ldots), \quad \Box =
abla_{\mu}
abla^{\mu} = rac{1}{\sqrt{-g}} \, \partial_{\mu} \sqrt{-g} \, g^{\mu
u} \, \partial_{
u}$$

Gauss-Bonnet invariant

$$\mathcal{G}=\textit{R}^2-4\,\textit{R}^{\mu\nu}\textit{R}_{\mu\nu}+\textit{R}^{\alpha\beta\mu\nu}\,\textit{R}_{\alpha\beta\mu\nu}$$

Einstein-Hilbert action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4 x$$

modification

$$R \longrightarrow f(R,\Lambda,R_{\mu
u},R^lpha_{\mueta
u},\square,\dots), \quad \square =
abla_\mu
abla^\mu = rac{1}{\sqrt{-g}}\,\partial_\mu\sqrt{-g}\,g^{\mu
u}\,\partial_
u$$

Gauss-Bonnet invariant

$$\mathcal{G}=\textit{R}^2-4\,\textit{R}^{\mu\nu}\textit{R}_{\mu\nu}+\textit{R}^{\alpha\beta\mu\nu}\,\textit{R}_{\alpha\beta\mu\nu}$$

Einstein-Hilbert action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4 x$$

modification

$$R \longrightarrow f(R, \Lambda, R_{\mu
u}, R^{lpha}_{\mueta
u}, \Box, \ldots), \quad \Box =
abla_{\mu}
abla^{\mu} = rac{1}{\sqrt{-g}} \, \partial_{\mu} \sqrt{-g} \, g^{\mu
u} \, \partial_{
u}$$

Gauss-Bonnet invariant

$$\mathcal{G} = R^2 - 4 \, R^{\mu\nu} R_{\mu\nu} + R^{\alpha\beta\mu\nu} \, R_{\alpha\beta\mu\nu}$$

Einstein-Hilbert action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4 x$$

modification

$$R \longrightarrow f(R, \Lambda, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...), \quad \Box = \nabla_{\mu} \nabla^{\mu} = \frac{1}{\sqrt{-g}} \partial_{\mu} \sqrt{-g} g^{\mu\nu} \partial_{\nu}$$

Gauss-Bonnet invariant

$$\mathcal{G}=\textit{R}^2-4\,\textit{R}^{\mu\nu}\textit{R}_{\mu\nu}+\textit{R}^{\alpha\beta\mu\nu}\,\textit{R}_{\alpha\beta\mu\nu}$$

< ロ > < 同 > < 三 > < 三 >

Einstein-Hilbert action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4 x$$

modification

$$R \longrightarrow f(R, \Lambda, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...), \quad \Box = \nabla_{\mu} \nabla^{\mu} = \frac{1}{\sqrt{-g}} \partial_{\mu} \sqrt{-g} g^{\mu\nu} \partial_{\nu}$$

Gauss-Bonnet invariant

$$\mathcal{G} = \mathbf{R}^2 - 4 \, \mathbf{R}^{\mu\nu} \mathbf{R}_{\mu\nu} + \mathbf{R}^{\alpha\beta\mu\nu} \, \mathbf{R}_{\alpha\beta\mu\nu}$$

Einstein-Hilbert action

$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4 x$$

modification

$$R \longrightarrow f(R, \Lambda, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...), \quad \Box = \nabla_{\mu} \nabla^{\mu} = \frac{1}{\sqrt{-g}} \partial_{\mu} \sqrt{-g} g^{\mu\nu} \partial_{\nu}$$

Gauss-Bonnet invariant

$$\mathcal{G} = \textit{R}^2 - 4 \,\textit{R}^{\mu\nu}\textit{R}_{\mu\nu} + \textit{R}^{\alpha\beta\mu\nu}\textit{R}_{\alpha\beta\mu\nu}$$

< D > < P > < E > < E</p>

\blacksquare f(R) modified gravity

$$S = \int \Big(rac{f(R)}{16\pi G} + \mathcal{L}_m\Big)\sqrt{-g} d^4x$$

Gauss-Bonnet modified gravity

$$S = \int \left(rac{R+lpha \mathcal{G}}{16\pi G} + \mathcal{L}_{m}
ight) \sqrt{-g} d^{4}x$$

nonlocal modified gravity

$$S=\int \Big(rac{F(R,R_{\mu
u},R^{lpha}_{\mueta
u},\Box,...)}{16\pi G}+\mathcal{L}_{m}\Big)\sqrt{-g}\;d^{4}x$$

< □ > < □ > < □ > < □ > < □ > <

æ.

 \blacksquare *f*(*R*) modified gravity

$$S = \int \left(\frac{f(R)}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

Gauss-Bonnet modified gravity

$$S = \int \left(\frac{R+\alpha \mathcal{G}}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

nonlocal modified gravity

$$S = \int \Big(\frac{F(R, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...)}{16\pi G} + \mathcal{L}_m\Big)\sqrt{-g} d^4x$$

< D > < P > < E > < E</p>
■ f(R) modified gravity

$$S = \int \left(\frac{f(R)}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

Gauss-Bonnet modified gravity

$$S = \int \left(\frac{R + lpha \mathcal{G}}{16\pi G} + \mathcal{L}_m \right) \sqrt{-g} d^4x$$

nonlocal modified gravity

$$S = \int \Big(\frac{F(R, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...)}{16\pi G} + \mathcal{L}_m\Big)\sqrt{-g} d^4x$$

< D > < P > < E > < E</p>

f(R) modified gravity

$$S = \int \left(\frac{f(R)}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

Gauss-Bonnet modified gravity

$$S = \int \left(\frac{R+\alpha \mathcal{G}}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

nonlocal modified gravity

$$S = \int \Big(\frac{F(R, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...)}{16\pi G} + \mathcal{L}_m\Big)\sqrt{-g} d^4x$$

< D > < P > < E > < E</p>

■ f(R) modified gravity

$$S = \int \left(\frac{f(R)}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

Gauss-Bonnet modified gravity

$$S = \int \left(\frac{R+\alpha \mathcal{G}}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

nonlocal modified gravity

$$S = \int \Big(\frac{F(R, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...)}{16\pi G} + \mathcal{L}_m\Big)\sqrt{-g} d^4x$$

< 同 > < 三 > < 三 >

f(R) modified gravity

$$S = \int \left(\frac{f(R)}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

Gauss-Bonnet modified gravity

$$S = \int \left(\frac{R+\alpha \mathcal{G}}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

nonlocal modified gravity

$$S = \int \Big(\frac{F(R, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...)}{16\pi G} + \mathcal{L}_m\Big)\sqrt{-g} d^4x$$

< 同 > < 三 > < 三 >

■ f(R) modified gravity

$$S = \int \left(\frac{f(R)}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

Gauss-Bonnet modified gravity

$$S = \int \left(\frac{R + lpha \mathcal{G}}{16\pi G} + \mathcal{L}_m \right) \sqrt{-g} d^4x$$

nonlocal modified gravity

$$S = \int \left(\frac{F(R, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...)}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

同 > < 三 > < 三 >

f(R) modified gravity

$$S = \int \left(\frac{f(R)}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

Gauss-Bonnet modified gravity

$$S = \int \left(\frac{R+\alpha \mathcal{G}}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

nonlocal modified gravity

$$S = \int \Big(\frac{F(R, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...)}{16\pi G} + \mathcal{L}_m\Big)\sqrt{-g} d^4x$$

★ ∃ → ★ ∃

- Under nonlocal modification of gravity we understand replacement of the scalar curvature *R* in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_{\mu} \nabla^{\mu}$ is d'Alembert operator and ∇_{μ} denotes the covariant derivative
- Let *M* be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4} x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of \Box , and Λ is cosmolo-

gical constant.

In the case of FRW metric the scalar curvature and d'Alambert operator are given by

$$R = \frac{6(a\ddot{a} + \dot{a}^2 + k)}{a^2} , \quad \Box R = -\ddot{R} - 3H\dot{R}, \quad H = \frac{\dot{a}}{a} .$$

- Under nonlocal modification of gravity we understand replacement of the scalar curvature *R* in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_{\mu} \nabla^{\mu}$ is d'Alembert operator and ∇_{μ} denotes the covariant derivative
- Let *M* be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_{M} \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of \Box , and Λ is cosmolo-

gical constant.

In the case of FRW metric the scalar curvature and d'Alambert operator are given by

$$R = \frac{6(a\ddot{a} + \dot{a}^2 + k)}{a^2}, \quad \Box R = -\ddot{R} - 3H\dot{R}, \quad H = \frac{\dot{a}}{a}.$$

< ロ > < 同 > < 三 > < 三 >

- Under nonlocal modification of gravity we understand replacement of the scalar curvature *R* in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_{\mu} \nabla^{\mu}$ is d'Alembert operator and ∇_{μ} denotes the covariant derivative
- Let *M* be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of \Box , and Λ is cosmolo-

gical constant.

In the case of FRW metric the scalar curvature and d'Alambert operator are given by

$$R = \frac{6(a\ddot{a} + \dot{a}^2 + k)}{a^2}, \quad \Box R = -\ddot{R} - 3H\dot{R}, \quad H = \frac{\dot{a}}{a}.$$

< ロ > < 同 > < 三 > < 三 >

- Under nonlocal modification of gravity we understand replacement of the scalar curvature *R* in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_{\mu} \nabla^{\mu}$ is d'Alembert operator and ∇_{μ} denotes the covariant derivative
- Let *M* be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \,\mathrm{d}^{4}x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of \Box , and Λ is cosmological constant.

In the case of FRW metric the scalar curvature and d'Alambert operato are given by

$$R = \frac{6(a\ddot{a} + \dot{a}^2 + k)}{a^2} , \quad \Box R = -\ddot{R} - 3H\dot{R}, \quad H = \frac{\dot{a}}{a} .$$

→ < 同 > < 回 > < 回 >

- Under nonlocal modification of gravity we understand replacement of the scalar curvature *R* in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_{\mu} \nabla^{\mu}$ is d'Alembert operator and ∇_{μ} denotes the covariant derivative
- Let *M* be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of \Box , and Λ is cosmolo-

gical constant.

In the case of FRW metric the scalar curvature and d'Alambert operator are given by

$$R = \frac{6\left(a\ddot{a} + \dot{a}^2 + k\right)}{a^2}, \quad \Box R = -\ddot{R} - 3H\dot{R}, \quad H = \frac{\dot{a}}{a}.$$

- Under nonlocal modification of gravity we understand replacement of the scalar curvature *R* in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_{\mu} \nabla^{\mu}$ is d'Alembert operator and ∇_{μ} denotes the covariant derivative
- Let *M* be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \,\mathrm{d}^{4}x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of \Box , and Λ is cosmolo-

gical constant.

In the case of FRW metric the scalar curvature and d'Alambert operator are given by

$$R = \frac{6(a\ddot{a} + \dot{a}^2 + k)}{a^2}, \quad \Box R = -\ddot{R} - 3H\dot{R}, \quad H = \frac{\dot{a}}{a}.$$

- Under nonlocal modification of gravity we understand replacement of the scalar curvature *R* in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_{\mu} \nabla^{\mu}$ is d'Alembert operator and ∇_{μ} denotes the covariant derivative
- Let *M* be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of \Box , and Λ is cosmolo-

gical constant.

In the case of FRW metric the scalar curvature and d'Alambert operator are given by

$$R = \frac{6(a\ddot{a} + \dot{a}^2 + k)}{a^2}, \quad \Box R = -\ddot{R} - 3H\dot{R}, \quad H = \frac{\dot{a}}{a}.$$

For calculating variation of the action, $\delta S = \frac{1}{16\pi G} \delta S_0 + \delta S_1$, we need the following

Lemma 1. For any two scalar functions G and H hold

$$\begin{split} \int_{M} \mathcal{H}\delta(\sqrt{-g}) \, \mathrm{d}^{4}x &= -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}\delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta \mathcal{H}\sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(\mathcal{R}_{\mu\nu} \mathcal{H} - \mathcal{K}_{\mu\nu} \mathcal{H}\right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta(\mathcal{F}(\Box)\mathcal{G})\sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(\mathcal{R}_{\mu\nu} - \mathcal{K}_{\mu\nu}\right) \left(\mathcal{G}'\mathcal{F}(\Box)\mathcal{H}\right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x \\ &+ \sum_{n=1}^{\infty} \frac{f_{n}}{2} \sum_{k=0}^{n-1} \int_{M} \mathcal{S}_{\mu\nu} (\Box'\mathcal{H}, \Box^{n-1-l}\mathcal{G}) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x. \end{split}$$

where

$$\begin{split} & K_{\mu\nu} = \nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box, \\ & S_{\mu\nu}(A, B) = g_{\mu\nu} \nabla^{\alpha} A \nabla_{\alpha} B - 2 \nabla_{\mu} A \nabla_{\nu} B + g_{\mu\nu} A \Box B, \end{split}$$

For calculating variation of the action, $\delta S = \frac{1}{16\pi G} \delta S_0 + \delta S_1$, we need the following

Lemma 1. For any two scalar functions \mathcal{G} and \mathcal{H} hold

$$\begin{split} \int_{M} \mathcal{H}\delta(\sqrt{-g}) \, \mathrm{d}^{4}x &= -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}\delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta R \sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(R_{\mu\nu} \mathcal{H} - K_{\mu\nu} \mathcal{H} \right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta(\mathcal{F}(\Box)\mathcal{G}) \sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(R_{\mu\nu} - K_{\mu\nu} \right) \left(\mathcal{G}' \mathcal{F}(\Box) \mathcal{H} \right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x \\ &+ \sum_{n=1}^{\infty} \frac{f_{n}}{2} \sum_{l=0}^{n-1} \int_{M} S_{\mu\nu} (\Box^{l} \mathcal{H}, \Box^{n-1-l} \mathcal{G}) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x. \end{split}$$

where

$$egin{aligned} & \mathcal{K}_{\mu
u} =
abla_{\mu}
abla_{
u} - g_{\mu
u} \Box, \ & S_{\mu
u}(\mathcal{A},\mathcal{B}) = g_{\mu
u}
abla^{lpha} \mathcal{A}
abla_{lpha} \mathcal{B} - 2
abla_{\mu} \mathcal{A}
abla_{
u} \mathcal{B} + g_{\mu
u} \mathcal{A} \Box \mathcal{B}, \end{aligned}$$

Equations of motion

For calculating variation of the action, $\delta S = \frac{1}{16\pi G} \delta S_0 + \delta S_1$, we need the following

Lemma 1. For any two scalar functions \mathcal{G} and \mathcal{H} hold

$$\begin{split} \int_{M} \mathcal{H}\delta(\sqrt{-g}) \, \mathrm{d}^{4}x &= -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}\delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta R \sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(R_{\mu\nu} \mathcal{H} - K_{\mu\nu} \mathcal{H} \right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta(\mathcal{F}(\Box)\mathcal{G}) \sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(R_{\mu\nu} - K_{\mu\nu} \right) \left(\mathcal{G}' \mathcal{F}(\Box) \mathcal{H} \right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x \\ &+ \sum_{n=1}^{\infty} \frac{f_{n}}{2} \sum_{l=0}^{n-1} \int_{M} S_{\mu\nu} (\Box^{l} \mathcal{H}, \Box^{n-1-l} \mathcal{G}) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x. \end{split}$$

where

$$egin{aligned} & \mathcal{K}_{\mu
u} =
abla_{\mu}
abla_{
u} - g_{\mu
u} \square, \ & S_{\mu
u}(\mathcal{A}, \mathcal{B}) = g_{\mu
u}
abla^{lpha} \mathcal{A}
abla_{lpha} \mathcal{B} - 2
abla_{\mu} \mathcal{A}
abla_{
u} \mathcal{B} + g_{\mu
u} \mathcal{A} \square \mathcal{B}, \end{aligned}$$

For calculating variation of the action, $\delta S = \frac{1}{16\pi G} \delta S_0 + \delta S_1$, we need the following

Lemma 1. For any two scalar functions \mathcal{G} and \mathcal{H} hold

$$\begin{split} \int_{M} \mathcal{H}\delta(\sqrt{-g}) \, \mathrm{d}^{4}x &= -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}\delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta R \sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(R_{\mu\nu} \mathcal{H} - K_{\mu\nu} \mathcal{H} \right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta(\mathcal{F}(\Box)\mathcal{G}) \sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(R_{\mu\nu} - K_{\mu\nu} \right) \left(\mathcal{G}' \mathcal{F}(\Box) \mathcal{H} \right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x \\ &+ \sum_{n=1}^{\infty} \frac{f_{n}}{2} \sum_{l=0}^{n-1} \int_{M} S_{\mu\nu} (\Box^{l} \mathcal{H}, \Box^{n-1-l} \mathcal{G}) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x. \end{split}$$

where

$$egin{aligned} & \mathcal{K}_{\mu
u} =
abla_{\mu}
abla_{
u} - g_{\mu
u}
abla, \ & \mathcal{S}_{\mu
u}(\mathcal{A},\mathcal{B}) = g_{\mu
u}
abla^{lpha} \mathcal{A}
abla_{lpha} \mathcal{B} - 2
abla_{\mu} \mathcal{A}
abla_{
u} \mathcal{B} + g_{\mu
u} \mathcal{A}
abla \mathcal{B}, \end{aligned}$$

The action S_0 is Einstein-Hilbert action without matter its variation is

$$\delta S_0 = \int_M G_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x + \Lambda \int_M g_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x, \tag{6}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor.

Using previous theorem we find the variation of S₁.

$$\delta S_{1} = -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \delta g^{\mu\nu} \sqrt{-g} d^{4}x + \int_{M} \left(R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{-g} d^{4}x.$$
(7)

Since, $S = \frac{1}{16\pi G} S_0 + S_1$, finally we get equations of motion (EOM).

(日)
 (日)

The action S_0 is Einstein-Hilbert action without matter its variation is

$$\delta S_0 = \int_M G_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x + \Lambda \int_M g_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x, \tag{6}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor.

Using previous theorem we find the variation of S₁,

$$\delta S_{1} = -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \delta g^{\mu\nu} \sqrt{-g} d^{4}x + \int_{M} \left(R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{-g} d^{4}x.$$
(7)

Since, $S = \frac{1}{16\pi G} S_0 + S_1$, finally we get equations of motion (EOM).

・ロト ・四ト ・ヨト ・ヨト

The action S_0 is Einstein-Hilbert action without matter its variation is

$$\delta S_0 = \int_M G_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x + \Lambda \int_M g_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x, \qquad (6)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor.

Using previous theorem we find the variation of S₁,

$$\delta S_{1} = -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \delta g^{\mu\nu} \sqrt{-g} d^{4}x + \int_{M} \left(R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{-g} d^{4}x.$$
(7)

Since, $S = \frac{1}{16\pi G} S_0 + S_1$, finally we get equations of motion (EOM).

・ロ・・(型・・目・・(目・)

э.

The action S_0 is Einstein-Hilbert action without matter its variation is

$$\delta S_0 = \int_M G_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x + \Lambda \int_M g_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x, \qquad (6)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor.

Using previous theorem we find the variation of S_1 ,

$$\delta S_{1} = -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \delta g^{\mu\nu} \sqrt{-g} d^{4}x + \int_{M} \left(R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{-g} d^{4}x.$$
(7)

Since, $S = \frac{1}{16\pi G} S_0 + S_1$, finally we get equations of motion (EOM).

<ロ> <部> < E> < E> < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > < E > <

The action S_0 is Einstein-Hilbert action without matter its variation is

$$\delta S_0 = \int_M G_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x + \Lambda \int_M g_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x, \qquad (6)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor.

Using previous theorem we find the variation of S₁,

$$\delta S_{1} = -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \delta g^{\mu\nu} \sqrt{-g} d^{4}x + \int_{M} \left(R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{-g} d^{4}x.$$
(7)

Since, $S = \frac{1}{16\pi G} S_0 + S_1$, finally we get equations of motion (EOM).

◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ◆ ○ ● ◆ ○ ●

Equations of motion

Let us note that $abla^{\mu} ilde{G}_{\mu
u} = \mathbf{0}.$

EOM are invariant on the replacement of functions $\mathcal G$ and $\mathcal H$ in S.

<ロ> <同> < 回> < 回> < 回> < => <</p>

Ъ.

Theorem 2 (EOM) The equations of motion for system given by S erection $\tilde{G}_{\mu\nu}=0,$ (8)

Let us note that $\nabla^{\mu} \widehat{G}_{\mu\nu} = 0$.

EOM are invariant on the replacement of functions ${\cal G}$ and ${\cal H}$ in ${\cal S}$.

< ロ > < 同 > < 回 > < 回 > .

Theorem 2 (EOM) The equations of motion for system given by S vare:

$$\tilde{G}_{\mu\nu} = 0,$$
 (8)

where

$$\begin{split} \tilde{G}_{\mu\nu} &= \frac{G_{\mu\nu} + \Lambda g_{\mu\nu}}{16\pi G} - \frac{1}{2} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) + R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu}, \\ \Omega_{\mu\nu} &= \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} S_{\mu\nu} \big(\Box^l \mathcal{H}(R), \Box^{n-1-l} \mathcal{G}(R) \big), \\ K_{\mu\nu} &= \nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box, \\ S_{\mu\nu}(A, B) &= g_{\mu\nu} \nabla^{\alpha} A \nabla_{\alpha} B - 2 \nabla_{\mu} A \nabla_{\nu} B + g_{\mu\nu} A \Box B, \\ W &= \mathcal{H}'(R) \mathcal{F}(\Box) \mathcal{G}(R) + \mathcal{G}'(R) \mathcal{F}(\Box) \mathcal{H}(R). \end{split}$$

Let us note that $abla^\mu \widehat{G}_{\mu u}=0.$

FEOM are invariant on the replacement of functions ${\cal G}$ and ${\cal H}$ in S.

Theorem 2 (EOM) The equations of motion for system given by S vare:

$$\tilde{G}_{\mu\nu} = \mathbf{0},\tag{8}$$

where

$$\begin{split} \tilde{G}_{\mu\nu} &= \frac{G_{\mu\nu} + \Lambda g_{\mu\nu}}{16\pi G} - \frac{1}{2} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) + R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu}, \\ \Omega_{\mu\nu} &= \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} S_{\mu\nu} \big(\Box^l \mathcal{H}(R), \Box^{n-1-l} \mathcal{G}(R) \big), \\ K_{\mu\nu} &= \nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box, \\ S_{\mu\nu}(A, B) &= g_{\mu\nu} \nabla^{\alpha} A \nabla_{\alpha} B - 2 \nabla_{\mu} A \nabla_{\nu} B + g_{\mu\nu} A \Box B, \\ W &= \mathcal{H}'(R) \mathcal{F}(\Box) \mathcal{G}(R) + \mathcal{G}'(R) \mathcal{F}(\Box) \mathcal{H}(R). \end{split}$$

Let us note that $\nabla^{\mu} \tilde{G}_{\mu\nu} = 0$.

EOM are invariant on the replacement of functions \mathcal{G} and \mathcal{H} in S.

Theorem 2 (EOM) The equations of motion for system given by S vare:

$$\tilde{G}_{\mu\nu} = 0,$$
 (8)

where

$$\begin{split} \tilde{G}_{\mu\nu} &= \frac{G_{\mu\nu} + \Lambda g_{\mu\nu}}{16\pi G} - \frac{1}{2} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) + R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu}, \\ \Omega_{\mu\nu} &= \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} S_{\mu\nu} \big(\Box^l \mathcal{H}(R), \Box^{n-1-l} \mathcal{G}(R) \big), \\ K_{\mu\nu} &= \nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box, \\ S_{\mu\nu}(A, B) &= g_{\mu\nu} \nabla^{\alpha} A \nabla_{\alpha} B - 2 \nabla_{\mu} A \nabla_{\nu} B + g_{\mu\nu} A \Box B, \\ W &= \mathcal{H}'(R) \mathcal{F}(\Box) \mathcal{G}(R) + \mathcal{G}'(R) \mathcal{F}(\Box) \mathcal{H}(R). \end{split}$$

Let us note that $\nabla^{\mu} \tilde{G}_{\mu\nu} = 0$.

EOM are invariant on the replacement of functions \mathcal{G} and \mathcal{H} in S.

• • • • • • • •

Q(R) = P(R) and

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) P^2 + 2\mathcal{F}(q) (R_{\mu\nu} - K_{\mu\nu}) PP' \qquad (9)$$
$$+ \frac{1}{2} \mathcal{F}'(q) S_{\mu\nu}(P, P) = 0.$$

If we suppose that the manifold M is endowed with FRW metric, then we have just suppose linearly independent equations: trace and 00-equation.

.

- Q(R) = P(R) and
- = P(R) be an eigenfunction of the corresponding d'Alembert-Beltrami C operator: GP(R) = q P(R), and consequently P(G) P(R) = P(q) P(R).

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) P^2 + 2\mathcal{F}(q) (R_{\mu\nu} - K_{\mu\nu}) PP' \qquad (9)$$
$$+ \frac{1}{2} \mathcal{F}'(q) S_{\mu\nu}(P, P) = 0.$$

If we suppose that the manifold *M* is endowed with FRW metric, then we have just integral independent equations: trace and 00-equation.

• Q(R) = P(R) and

• P(R) be an eigenfunction of the corresponding d'Alembert-Beltrami operator: $\Box P(R) = q P(R)$, and consequently $\mathcal{F}(\Box)P(R) = \mathcal{F}(q)P(R)$,

we obtain

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) P^2 + 2\mathcal{F}(q) (R_{\mu\nu} - K_{\mu\nu}) PP' \qquad (9)$$
$$+ \frac{1}{2} \mathcal{F}'(q) S_{\mu\nu}(P, P) = 0.$$

If we suppose that the manifold *M* is endowed with FRW metric, then we have just integral independent equations: trace and 00-equation.

- Q(R) = P(R) and
- *P*(*R*) be an eigenfunction of the corresponding d'Alembert-Beltrami □ operator: □*P*(*R*) = *qP*(*R*), and consequently *F*(□)*P*(*R*) = *F*(*q*)*P*(*R*),

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) P^2 + 2\mathcal{F}(q) (R_{\mu\nu} - K_{\mu\nu}) PP' \qquad (9)$$

+ $\frac{1}{2} \mathcal{F}'(q) S_{\mu\nu}(P, P) = 0.$

If we suppose that the manifold *M* is endowed with FRW metric, then we have just <u>be</u> linearly independent equations: trace and 00-equation.

< D > < P > < E > < E > <</p>

- Q(R) = P(R) and
- P(R) be an eigenfunction of the corresponding d'Alembert-Beltrami □ operator: □P(R) = q P(R), and consequently F(□)P(R) = F(q)P(R),

we obtain

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) P^2 + 2\mathcal{F}(q) (R_{\mu\nu} - K_{\mu\nu}) PP' \qquad (9)$$
$$+ \frac{1}{2} \mathcal{F}'(q) S_{\mu\nu}(P, P) = 0.$$

If we suppose that the manifold *M* is endowed with FRW metric, then we have just <u>be</u> linearly independent equations: trace and 00-equation.

- Q(R) = P(R) and
- P(R) be an eigenfunction of the corresponding d'Alembert-Beltrami □ operator: □P(R) = q P(R), and consequently F(□)P(R) = F(q)P(R),

we obtain

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) P^2 + 2\mathcal{F}(q) (R_{\mu\nu} - K_{\mu\nu}) PP' \qquad (9)$$
$$+ \frac{1}{2} \mathcal{F}'(q) S_{\mu\nu}(P, P) = 0.$$

If we suppose that the manifold *M* is endowed with FRW metric, then we have just by linearly independent equations: trace and 00-equation.

э.

Earlier, we considered models of nonlocal gravity without matter which are described by the action,

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(\mathbf{R}) = \mathbf{R}, \mathcal{G}(\mathbf{R}) = \mathbf{R},$
- 2. $\mathcal{H}(R) = R^{-1}, \mathcal{G}(R) = R,$
- 3. $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q},$
- 4. $\mathcal{H}(\boldsymbol{R}) = (\boldsymbol{R} + \boldsymbol{R}_0)^m, \, \mathcal{G}(\boldsymbol{R}) = (\boldsymbol{R} + \boldsymbol{R}_0)^m,$

5. R = const.

<ロ> <同> <同> < 回> < 回> < 三</p>

Earlier, we considered models of nonlocal gravity without matter which are described by the action,

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(R) = R, \mathcal{G}(R) = R,$
- 2. $\mathcal{H}(\mathbf{R}) = \mathbf{R}^{-1}, \mathcal{G}(\mathbf{R}) = \mathbf{R},$
- 3. $\mathcal{H}(\mathbf{R}) = \mathbf{R}^{p}, \mathcal{G}(\mathbf{R}) = \mathbf{R}^{q},$
- 4. $\mathcal{H}(R) = (R + R_0)^m, \, \mathcal{G}(R) = (R + R_0)^m,$

5. R = const.

< D > < P > < E > < E</p>

Earlier, we considered models of nonlocal gravity without matter which are described by the action,

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(R) = R, \mathcal{G}(R) = R,$
- 2. $\mathcal{H}(\mathbf{R}) = \mathbf{R}^{-1}, \mathcal{G}(\mathbf{R}) = \mathbf{R},$
- 3. $\mathcal{H}(\mathbf{R}) = \mathbf{R}^p, \, \mathcal{G}(\mathbf{R}) = \mathbf{R}^q,$
- 4. $\mathcal{H}(R) = (R + R_0)^m, \, \mathcal{G}(R) = (R + R_0)^m,$

5. R = const.

< ロ > < 同 > < 回 > < 回 > .
$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(R) = R, \mathcal{G}(R) = R,$
- 2. $\mathcal{H}(\mathbf{R}) = \mathbf{R}^{-1}, \mathcal{G}(\mathbf{R}) = \mathbf{R},$
- 3. $\mathcal{H}(\mathbf{R}) = \mathbf{R}^{p}, \mathcal{G}(\mathbf{R}) = \mathbf{R}^{q},$
- 4. $\mathcal{H}(R) = (R + R_0)^m, \, \mathcal{G}(R) = (R + R_0)^m,$
- 5. R = const.

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(R) = R, \mathcal{G}(R) = R$,
- 2. $\mathcal{H}(R) = R^{-1}, \mathcal{G}(R) = R,$
- 3. $\mathcal{H}(R) = R^p, \mathcal{G}(R) = R^q,$
- 4. $\mathcal{H}(R) = (R + R_0)^m, \, \mathcal{G}(R) = (R + R_0)^m,$

5. R = const.

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(R) = R, \mathcal{G}(R) = R$,
- 2. $\mathcal{H}(R) = R^{-1}, \mathcal{G}(R) = R$,
- 3. $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q},$
- 4. $\mathcal{H}(R) = (R + R_0)^m, \, \mathcal{G}(R) = (R + R_0)^m,$

5. R = const.

< ロ > < 同 > < 三 > < 三 > -

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(R) = R, \mathcal{G}(R) = R$,
- 2. $\mathcal{H}(R) = R^{-1}, \mathcal{G}(R) = R$,
- 3. $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q},$
- 4. $\mathcal{H}(R) = (R + R_0)^m, \, \mathcal{G}(R) = (R + R_0)^m,$

5. R = const.

< ロ > < 同 > < 回 > < 回 > .

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(R) = R, \mathcal{G}(R) = R$,
- 2. $\mathcal{H}(R) = R^{-1}, \mathcal{G}(R) = R$,
- 3. $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q},$
- 4. $\mathcal{H}(R) = (R + R_0)^m, \, \mathcal{G}(R) = (R + R_0)^m,$

5. R = const.

・ 同 ト ・ ヨ ト ・ ヨ ト

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(R) = R, \mathcal{G}(R) = R$,
- 2. $\mathcal{H}(R) = R^{-1}, \mathcal{G}(R) = R$,
- 3. $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q},$
- 4. $\mathcal{H}(R) = (R + R_0)^m, \, \mathcal{G}(R) = (R + R_0)^m,$
- 5. R = const.

・ 同 ト ・ ヨ ト ・ ヨ ト

1. model $\mathcal{H}(R) = R, \mathcal{G}(R) = R$.

- Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
- Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
- Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh(\sqrt{\frac{\Lambda}{3}}t)$, in the second and third case for k = 0 we obtain de Sitter solution.
- All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.

2. model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.

- Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR, \ c \in \mathbb{R}^*$.
- there are cosmological solutions of form $a(t) = a_0|t t_0|^{\alpha}$, in the case k = 0, for $\alpha \neq 0, 1/2$ and $3\alpha \in 1 + 2\mathbb{N}$, in cases $k \neq 0$, for $\alpha = 1$.
- Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

(日)

1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.

- Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
- Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
- Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh\left(\sqrt{\frac{\Lambda}{3}}t\right)$, in the second and third case for k = 0 we obtain de Sitter solution.
- All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.

2. model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.

- Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR, \ c \in \mathbb{R}^*$.
- there are cosmological solutions of form a(t) = a₀|t − t₀|^α, in the case k = 0, for α ≠ 0, 1/2 and 3 α ∈ 1 + 2 N, in cases k ≠ 0, for α = 1.
- Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

э.

- 1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.
 - Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
 - Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
 - Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh\left(\sqrt{\frac{\Lambda}{3}t}\right)$, in the second and third case for k = 0 we obtain de Sitter solution.
 - All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.
- **2.** model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.
 - Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR, \ c \in \mathbb{R}^*$.
 - there are cosmological solutions of form a(t) = a₀|t − t₀|^α, in the case k = 0, for α ≠ 0, 1/2 and 3 α ∈ 1 + 2 N, in cases k ≠ 0, for α = 1.
 - Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

(日)

- 1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.
 - Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
 - Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
 - Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh(\sqrt{\frac{\Lambda}{3}t})$, in the second and third case for k = 0 we obtain de Sitter solution.
 - All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.
- **2.** model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.
 - Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR$, $c \in \mathbb{R}^*$.
 - there are cosmological solutions of form a(t) = a₀|t − t₀|^α, in the case k = 0, for α ≠ 0, 1/2 and 3 α ∈ 1 + 2 N, in cases k ≠ 0, for α = 1.
 - Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

- 1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.
 - Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
 - Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
 - Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh(\sqrt{\frac{\Lambda}{3}t})$, in the second and third case for k = 0 we obtain de Sitter solution.
 - All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.
- **2.** model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.
 - Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR$, $c \in \mathbb{R}^*$.
 - there are cosmological solutions of form $a(t) = a_0|t t_0|^{\alpha}$, in the case k = 0, for $\alpha \neq 0, 1/2$ and $3\alpha \in 1 + 2\mathbb{N}$, in cases $k \neq 0$, for $\alpha = 1$.
 - Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

< ロ > < 同 > < 回 > < 回 > < 回 > <

- 1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.
 - Using ansatz $\Box R = r R + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
 - Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
 - Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh(\sqrt{\frac{\Lambda}{3}t})$, in the second and third case for k = 0 we obtain de Sitter solution.
 - All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.

2. model $\mathcal{H}(R) = R^{-1}, \mathcal{G}(R) = R$.

- Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR, \ c \in \mathbb{R}^*$.
- there are cosmological solutions of form a(t) = a₀|t − t₀|^α, in the case k = 0, for α ≠ 0, 1/2 and 3 α ∈ 1 + 2 N, in cases k ≠ 0, for α = 1.
- Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

< ロ > < 同 > < 回 > < 回 > .

- 1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.
 - Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
 - Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
 - Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh(\sqrt{\frac{\Lambda}{3}t})$, in the second and third case for k = 0 we obtain de Sitter solution.
 - All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.
- **2.** model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.
 - Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR$, $c \in \mathbb{R}^*$.
 - there are cosmological solutions of form a(t) = a₀|t − t₀|^α, in the case k = 0, for α ≠ 0, 1/2 and 3 α ∈ 1 + 2 N, in cases k ≠ 0, for α = 1.
 - Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

- 1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.
 - Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
 - Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
 - Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh(\sqrt{\frac{\Lambda}{3}t})$, in the second and third case for k = 0 we obtain de Sitter solution.
 - All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.
- **2.** model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.
 - Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR$, $c \in \mathbb{R}^*$.
 - there are cosmological solutions of form $a(t) = a_0|t t_0|^{\alpha}$, in the case k = 0, for $\alpha \neq 0, 1/2$ and $3 \alpha \in 1 + 2 \mathbb{N}$, in cases $k \neq 0$, for $\alpha = 1$.
 - Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

- 1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.
 - Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
 - Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
 - Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh(\sqrt{\frac{\Lambda}{3}t})$, in the second and third case for k = 0 we obtain de Sitter solution.
 - All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.
- **2.** model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.
 - Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR$, $c \in \mathbb{R}^*$.
 - there are cosmological solutions of form $a(t) = a_0|t t_0|^{\alpha}$, in the case k = 0, for $\alpha \neq 0, 1/2$ and $3\alpha \in 1 + 2\mathbb{N}$, in cases $k \neq 0$, for $\alpha = 1$.
 - Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

- 1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.
 - Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
 - Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
 - Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh(\sqrt{\frac{\Lambda}{3}t})$, in the second and third case for k = 0 we obtain de Sitter solution.
 - All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.
- **2.** model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.
 - Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR$, $c \in \mathbb{R}^*$.
 - there are cosmological solutions of form $a(t) = a_0|t t_0|^{\alpha}$, in the case k = 0, for $\alpha \neq 0, 1/2$ and $3\alpha \in 1 + 2\mathbb{N}$, in cases $k \neq 0$, for $\alpha = 1$.
 - Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

- 1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.
 - Using ansatz $\Box R = rR + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
 - Solutions exist for all three values of parameter k = 0, ±1, under certain conditions on function F(□), and parameters σ, τ, λ, Λ, k.
 - Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh(\sqrt{\frac{\Lambda}{3}t})$, in the second and third case for k = 0 we obtain de Sitter solution.
 - All obtained solutions satisfy ä(t) = λ²a(t) > 0, i.e. are consistent with observational data.
- **2.** model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.
 - Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR$, $c \in \mathbb{R}^*$.
 - there are cosmological solutions of form $a(t) = a_0 |t t_0|^{\alpha}$, in the case k = 0, for $\alpha \neq 0, 1/2$ and $3\alpha \in 1 + 2\mathbb{N}$, in cases $k \neq 0$, for $\alpha = 1$.
 - Case $a(t) = |t t_0|$ for k = -1 corresponds to Milne's model.

э.

3. model $\mathcal{H}(R) = R^p, \mathcal{G}(R) = R^q, p \ge q$.

- We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
- For ρ = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy γ = −12Λ.
- In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4. model** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp\left(-\frac{\gamma}{12}t^2\right)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

- Using this ansatz we obtined the followinf five solutions:
 - $r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$
 - $t = m\gamma, \ n = 0, \ B_0 = \frac{2}{3}, \ m = 1$
 - $t = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 1$
 - $T = III \gamma$, $II = \frac{1}{2}$, $III = \frac{3}{2} \gamma$, $III = \frac{1}{2}$

3. model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.

- We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
- For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
- In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4. model** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

- Using this ansatz we obtined the followinf five solutions:
 - $r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$
 - $a_{1} = m \gamma_{1}, n = 0, R_{0} = \frac{2}{2}, m = 1$
 - $t = m\gamma, \ n = \frac{1}{2}, \ B_0 = \frac{1}{2}\gamma, \ m = 1$
 - $r = m\gamma$, $n = \frac{1}{2}$, $R = 3\gamma$, m = -1

・ロッ ・ 一 ・ ・ 日 ・ ・ 日 ・

3. model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.

- We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
- For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
- In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4. model** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

- Using this ansatz we obtined the followinf five solutions:
 - $r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$
 - $t = m\gamma, \ n = 0, \ B_0 = \frac{2}{3}, \ m = 1$
 - $\mathbf{p} = \mathbf{r} = \mathbf{m} \gamma, \ \mathbf{n} = \frac{1}{2}, \ \mathbf{R}_{\mathbf{0}} = \frac{1}{2} \gamma, \ \mathbf{m} = 1$
 - $T = m\gamma$, $n = \frac{1}{2}$, $R_0 = 3\gamma$, m = -1

(0)

э.

3. model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.

- We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
- For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.

In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.

- **4. model** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

• Using this ansatz we obtined the followinf five solutions:

- $r = m\gamma, \ n = 0, \ P_0 = \gamma, \ m = \frac{1}{2}$
- $0, \quad I = m\gamma, \ n = 0, \ R_0 = \frac{2}{3}, \ m = 1$
- $t = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 1$
- \bullet $T = m\gamma$, $n = \frac{1}{2}$, $P_0 = 3\gamma$, $m = -\frac{1}{2}$

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4. model** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form
 - $a(t) = At^{n} \exp(-\frac{\gamma}{12}t^{2})$ and $\Box (R + R_{0})^{m} = r(R + R_{0})^{m}$.
 - Using this ansatz we obtined the followinf five solutions:
 - $\mathbf{e} = \mathbf{r} = \mathbf{m} \gamma, \ \mathbf{n} = \mathbf{0}, \ \mathbf{R}_0 = \gamma, \ \mathbf{m} = \frac{1}{2}$
 - $n = m \gamma_1, n = 0, R_0 = \frac{2}{2}, m = 1$
 - $t = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 1$
 - $r = m\gamma, n = \frac{1}{2}, P_0 = 3\gamma, m = -\frac{1}{2}$

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4. model** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form
 - $a(t) = At^{n} \exp(-\frac{\gamma}{12}t^{2})$ and $\Box (R + R_{0})^{m} = r(R + R_{0})^{m}$.
 - Using this ansatz we obtined the followinf five solutions:
 - $\bullet \quad T = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$
 - **a** $I = m\gamma_1 \ n = 0, \ R_0 = \frac{1}{2}, \ m = 1$
 - $t = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 1$
 - $r = m\gamma, n = \frac{1}{2}, R_{\rm b} = 3\gamma, m = -$

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.

• We considered scale factor and ansatz of the form $a(t) = At^{a} \exp(-\frac{\gamma}{12}t^{2})$ and $\Box (R + R_{0})^{m} = r(R + R_{0})^{a}$

- \bullet $T = m\gamma, n = 0, R_0 = \gamma, m = \frac{1}{2}$
- $t = m\gamma, \ n = 0, \ R_0 = \frac{2}{2}, \ m = 1$
- $t = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 1$
- **e** $r = m\gamma, n = \frac{1}{2}, R_0 = 3\gamma, m = -$

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R + R_0)^m = r(R + R_0)^m$. • Using this ansatz we obtined the followinf five solutions:

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R + R_0)^m = r(R + R_0)^m$. • Using this ansatz we obtined the followinf five solutions:

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

• Using this ansatz we obtined the followinf five solutions:

< ロ > < 同 > < 三 > < 三 > < 三 > < ○

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

• Using this ansatz we obtined the followinf five solutions:

•
$$r = m \gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$$

•
$$r = m\gamma, \ n = 0, \ R_0 = \frac{\gamma}{3}, \ m = 1$$

•
$$r = m \gamma, \ n = \frac{1}{2}, R_0 = \frac{4}{3} \gamma, \ m = 1$$

•
$$r = m\gamma, \ n = \frac{1}{2}, \ R_0 = 3\gamma, \ m = -\frac{1}{4}$$

•
$$r = m\gamma, \ n = \frac{2m+1}{3}, \ R_0 = \frac{7}{3}\gamma, \ m = \frac{1}{2}$$

◆□▶ ◆帰▶ ◆≧▶ ◆≧▶ ─ ≧ ∽��や

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

• Using this ansatz we obtined the followinf five solutions:

•
$$r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$$

•
$$r = m\gamma, \ n = 0, \ R_0 = \frac{\gamma}{3}, \ m = 1$$

•
$$r = m \gamma, \ n = \frac{1}{2}, R_0 = \frac{4}{3} \gamma, \ m = 1$$

•
$$r = m\gamma, \ n = \frac{1}{2}, \ R_0 = 3\gamma, \ m = -\frac{1}{4}$$

•
$$r = m\gamma, \ n = \frac{2m+1}{3}, \ R_0 = \frac{7}{3}\gamma, \ m = \frac{1}{2}$$

◆□▶ ◆帰▶ ◆≧▶ ◆≧▶ ─ ≧ ∽��や

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

- Using this ansatz we obtined the followinf five solutions:
 - $r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$
 - $r = m\gamma, \ n = 0, \ R_0 = \frac{\gamma}{3}, \ m = 1$
 - $r = m \gamma, \ n = \frac{1}{2}, R_0 = \frac{4}{3} \gamma, \ m = 1$
 - $r = m\gamma, \ n = \frac{1}{2}, \ R_0 = 3\gamma, \ m = -\frac{1}{4}$
 - $r = m\gamma, \ n = \frac{2m+1}{3}, \ R_0 = \frac{7}{3}\gamma, \ m = \frac{1}{2}.$

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

• Using this ansatz we obtined the followinf five solutions:

•
$$r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$$

•
$$r = m\gamma, \ n = 0, \ R_0 = \frac{\gamma}{3}, \ m = 1$$

•
$$r = m\gamma, \ n = \frac{1}{2}, R_0 = \frac{4}{3}\gamma, \ m = 1$$

- $r = m\gamma, \ n = \frac{1}{2}, \ R_0 = 3\gamma, \ m = -\frac{1}{4}$
- $r = m\gamma, \ n = \frac{2m+1}{3}, \ R_0 = \frac{7}{3}\gamma, \ m = \frac{1}{2}.$

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

• Using this ansatz we obtined the followinf five solutions:

•
$$r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$$

•
$$r = m\gamma, \ n = 0, \ R_0 = \frac{\gamma}{3}, \ m = 1$$

•
$$r = m\gamma, \ n = \frac{1}{2}, R_0 = \frac{4}{3}\gamma, \ m = 1$$

•
$$r = m\gamma, \ n = \frac{1}{2}, \ R_0 = 3\gamma, \ m = -\frac{1}{4}$$

•
$$r = m\gamma, \ n = \frac{2m+1}{3}, \ R_0 = \frac{7}{3}\gamma, \ m = \frac{1}{2}$$

◆□▶ ◆帰▶ ◆≧▶ ◆≧▶ ─ ≧ ∽��や

- **3.** model $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q}, p \geq q$.
 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
 - For p = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
 - In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

Using this ansatz we obtined the followinf five solutions:

•
$$r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$$

•
$$r = m\gamma, \ n = 0, \ R_0 = \frac{\gamma}{3}, \ m = 1$$

•
$$r = m\gamma, \ n = \frac{1}{2}, R_0 = \frac{4}{3}\gamma, \ m = 1$$

•
$$r = m\gamma, \ n = \frac{1}{2}, \ R_0 = 3\gamma, \ m = -\frac{1}{4}$$

•
$$r = m\gamma, \ n = \frac{2m+1}{3}, \ R_0 = \frac{7}{3}\gamma, \ m = \frac{1}{2}$$

- **4. model** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, m = ¹/₂ we found unique solution for arbitrary F(^γ/₂) and F'(^γ/₂).
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{1}{2})$ and $\mathcal{F}'(\frac{1}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.

5. model R = const.

- If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If R = R₀ < 0, then there exists non-trivial singular cyclic solution a(t) = √ ⁻¹²/_{R₀} | cos ¹/₂ (√ ^{R₀}/₃ t φ)| za k = −1.
- Case R₀ = 0 is considered as an limit case when R₀ → 0, and in both cases R₀ < 0 and R₀ > 0, we obtain Minkowski space.

- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, m = ¹/₂ we found unique solution for arbitrary F(²/₂) and F'(²/₂).
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{1}{2})$ and $\mathcal{F}'(\frac{1}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.

5. model R = const.

- If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If $R = R_0 < 0$, then there exists non-trivial singular cyclic solution $a(t) = \sqrt{\frac{-12}{R_0}} |\cos \frac{1}{2}(\sqrt{-\frac{R_0}{3}}t - \varphi)|$ za k = -1.
- Case R₀ = 0 is considered as an limit case when R₀ → 0, and in both cases R₀ < 0 and R₀ > 0, we obtain Minkowski space.

- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, $m = \frac{1}{2}$ we found unique solution for arbitrary $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$.
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.

5. model R =const.

- If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If $R = R_0 < 0$, then there exists non-trivial singular cyclic solution $a(t) = \sqrt{\frac{-12}{R_0}} |\cos \frac{1}{2}(\sqrt{-\frac{R_0}{3}}t \varphi)| \text{ za } k = -1.$
- Case R₀ = 0 is considered as an limit case when R₀ → 0, and in both cases R₀ < 0 and R₀ > 0, we obtain Minkowski space.
- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, $m = \frac{1}{2}$ we found unique solution for arbitrary $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$.
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.
- **5.** model R =const.
 - If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
 - If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
 - If $R = R_0 < 0$, then there exists non-trivial singular cyclic solution $a(t) = \sqrt{\frac{-12}{R_0}} |\cos \frac{1}{2}(\sqrt{-\frac{R_0}{3}}t \varphi)| \operatorname{za} k = -1.$
 - Case R₀ = 0 is considered as an limit case when R₀ → 0, and in both cases R₀ < 0 and R₀ > 0, we obtain Minkowski space.

- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, $m = \frac{1}{2}$ we found unique solution for arbitrary $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$.
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.

5. model R = const.

- If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If R = R₀ < 0, then there exists non-trivial singular cyclic solution a(t) = √ -12/R₀ | cos 1/2 (√ R₀/3 t − φ) | za k = −1.
- Case R₀ = 0 is considered as an limit case when R₀ → 0, and in both cases R₀ < 0 and R₀ > 0, we obtain Minkowski space.

- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, $m = \frac{1}{2}$ we found unique solution for arbitrary $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$.
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.

- If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If $R = R_0 < 0$, then there exists non-trivial singular cyclic

solution $a(t) = \sqrt{\frac{-12}{R_0}} |\cos \frac{1}{2}(\sqrt{-\frac{R_0}{3}}t - \varphi)|$ za k = -1.

• Case $R_0 = 0$ is considered as an limit case when $R_0 \rightarrow 0$, and in both cases $R_0 < 0$ and $R_0 > 0$, we obtain Minkowski space.

- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, $m = \frac{1}{2}$ we found unique solution for arbitrary $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$.
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.

- If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If $R = R_0 < 0$, then there exists non-trivial singular cyclic

solution $a(t) = \sqrt{\frac{-12}{R_0}} |\cos \frac{1}{2}(\sqrt{-\frac{R_0}{3}}t - \varphi)|$ za k = -1.

• Case $R_0 = 0$ is considered as an limit case when $R_0 \rightarrow 0$, and in both cases $R_0 < 0$ and $R_0 > 0$, we obtain Minkowski space.

- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, $m = \frac{1}{2}$ we found unique solution for arbitrary $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$.
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.

- If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If $R = R_0 < 0$, then there exists non-trivial singular cyclic solution $a(t) = \sqrt{\frac{-12}{R_0}} |\cos \frac{1}{2}(\sqrt{-\frac{R_0}{3}}t \varphi)|$ za k = -1.
- Case $R_0 = 0$ is considered as an limit case when $R_0 \rightarrow 0$, and in both cases $R_0 < 0$ and $R_0 > 0$, we obtain Minkowski space.

- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, $m = \frac{1}{2}$ we found unique solution for arbitrary $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$.
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.

- If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If $R = R_0 < 0$, then there exists non-trivial singular cyclic solution $a(t) = \sqrt{\frac{-12}{R_0}} |\cos \frac{1}{2}(\sqrt{-\frac{R_0}{3}}t \varphi)|$ za k = -1.
- Case $R_0 = 0$ is considered as an limit case when $R_0 \rightarrow 0$, and in both cases $R_0 < 0$ and $R_0 > 0$, we obtain Minkowski space.

- **4.** model $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - In the case n = 0, $m = \frac{1}{2}$ we found unique solution for arbitrary $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$.
 - In the case $n = \frac{2}{3}$, $m = \frac{1}{2}$ we found unique solution for $\mathcal{F}(\frac{\gamma}{2})$ and $\mathcal{F}'(\frac{\gamma}{2})$ which satisfy $\Lambda = -\frac{7}{6}\gamma$.
 - In the case $n = \frac{1}{2}$, $m = -\frac{1}{4}$ there is no solutions of EOM.

- If R = R₀ > 0, then there exist non-singlar solutions for all three values of parameter k = 0, ±1, which are bounced in the cases k = 0, 1.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If $R = R_0 < 0$, then there exists non-trivial singular cyclic solution $a(t) = \sqrt{\frac{-12}{R_0}} |\cos \frac{1}{2}(\sqrt{-\frac{R_0}{3}}t \varphi)|$ za k = -1.
- Case $R_0 = 0$ is considered as an limit case when $R_0 \rightarrow 0$, and in both cases $R_0 < 0$ and $R_0 > 0$, we obtain Minkowski space.

• • • • • • • • •

э.

Recently, we have considered the nonlocal gravity model with cosmological constant A and without matter, given by

(MS)
$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R} - 2\Lambda \mathcal{F}(\Box) \sqrt{R} - 2\Lambda \right) \sqrt{-g} \, \mathrm{d}^4 x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

It is a **Constant of** since the EOM (9), for $P(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(\sqrt{R - 2\Lambda}, \sqrt{R - 2\Lambda}) = 0,$$
 (10)

where we take $q = \zeta \Lambda$.

- It is evident that EOM (10) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.
- One such nonlocal operator $\mathcal{F}(\Box)$ is

$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} \tilde{t}_n \Big[\Big(rac{\Box}{q} \Big)^n + \Big(rac{q}{\Box} \Big)^n \Big] = 1 - rac{1}{2e} \Big(rac{\Box}{q} e^{rac{\Box}{q}} + rac{q}{\Box} e^{rac{B}{\Box}} \Big), \quad q eq 0.$

<ロ> <同> <同> < 回> < 回> < 三</p>

(MS)
$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda} \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \right) \sqrt{-g} \, \mathrm{d}^4 x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

It is a very special case since the EOM (9), for $P(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(\sqrt{R - 2\Lambda}, \sqrt{R - 2\Lambda}) = 0,$$
 (10)

where we take $q = \zeta \Lambda$.

It is evident that EOM (10) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.

One such nonlocal operator $\mathcal{F}(\Box)$ is

$$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} ilde{f}_n \Big[\Big(rac{\Box}{q} \Big)^n + \Big(rac{q}{\Box} \Big)^n \Big] = 1 - rac{1}{2e} \Big(rac{\Box}{q} e^{rac{\Box}{q}} + rac{q}{\Box} e^{rac{q}{\Box}} \Big), \quad q
eq 0.$$

< ロ > < 同 > < 回 > < 回 > < 回 > <

(MS)
$$S = \frac{1}{16\pi G} \int_{M} \left(R - 2\Lambda + \sqrt{R - 2\Lambda} \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

It is a very special case since the EOM (9), for $P(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$\left(G_{\mu\nu}+\Lambda g_{\mu\nu}\right)\left(1+\mathcal{F}(q)\right)+\frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(\sqrt{R-2\Lambda},\sqrt{R-2\Lambda})=0, \ (10)$$

where we take $q = \zeta \Lambda$.

- It is evident that EOM (10) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.
- One such nonlocal operator $\mathcal{F}(\Box)$ is

$$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} \widetilde{f}_n \Big[\Big(rac{\Box}{q} \Big)^n + \Big(rac{q}{\Box} \Big)^n \Big] = 1 - rac{1}{2e} \Big(rac{\Box}{q} e^{rac{\Box}{q}} + rac{q}{\Box} e^{rac{q}{\Box}} \Big), \quad q
eq 0.$$

< ロ > < 同 > < 三 > < 三 > -

(MS)
$$S = \frac{1}{16\pi G} \int_{M} \left(R - 2\Lambda + \sqrt{R - 2\Lambda} \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

It is a very special case since the EOM (9), for $P(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(\sqrt{R-2\Lambda}, \sqrt{R-2\Lambda}) = 0,$$
 (10)

where we take $q = \zeta \Lambda$.

It is evident that EOM (10) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.

One such nonlocal operator $\mathcal{F}(\Box)$ is

$$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} ilde{f}_n \Big[\Big(rac{\Box}{q} \Big)^n + \Big(rac{q}{\Box} \Big)^n \Big] = 1 - rac{1}{2e} \Big(rac{\Box}{q} e^{rac{\Box}{q}} + rac{q}{\Box} e^{rac{q}{\Box}} \Big), \quad q
eq 0.$$

< ロ > < 同 > < 三 > < 三 > -

(MS)
$$S = \frac{1}{16\pi G} \int_{M} \left(R - 2\Lambda + \sqrt{R - 2\Lambda} \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

It is a very special case since the EOM (9), for $P(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(\sqrt{R - 2\Lambda}, \sqrt{R - 2\Lambda}) = 0,$$
 (10)

where we take $q = \zeta \Lambda$.

It is evident that EOM (10) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.

One such nonlocal operator $\mathcal{F}(\Box)$ is

$$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} \widetilde{f}_n \Big[\Big(rac{\Box}{q} \Big)^n + \Big(rac{q}{\Box} \Big)^n \Big] = 1 - rac{1}{2e} \Big(rac{\Box}{q} e^{rac{\Box}{q}} + rac{q}{\Box} e^{rac{q}{\Box}} \Big), \quad q
eq 0.$$

Ъ.

(MS)
$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda} \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \right) \sqrt{-g} \, \mathrm{d}^4 x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

It is a very special case since the EOM (9), for $P(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(\sqrt{R - 2\Lambda}, \sqrt{R - 2\Lambda}) = 0,$$
 (10)

where we take $q = \zeta \Lambda$.

It is evident that EOM (10) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.

One such nonlocal operator $\mathcal{F}(\Box)$ is

$$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} \tilde{f}_n \Big[\Big(\frac{\Box}{q} \Big)^n + \Big(\frac{q}{\Box} \Big)^n \Big] = 1 - \frac{1}{2e} \Big(\frac{\Box}{q} e^{\frac{\Box}{q}} + \frac{q}{\Box} e^{\frac{q}{\Box}} \Big), \quad q \neq 0.$$

1. Cosmological solution in the flat Universe (k = 0)

1.1. Solutions of the form $a(t) = A t^{\alpha} e^{\gamma t}$

There are two solutions:

 $a(t) = A_1 b a^{\frac{1}{2} t^2}, \qquad \mathcal{F}(-\frac{3}{2}h) = -1, \quad \mathcal{F}(-\frac{3}{2}h) = 0,$ $a(t) = A_1 b a^{\frac{1}{2} t^2}, \qquad \mathcal{F}(-A) = -1, \quad \mathcal{F}'(-A) = 0.$

1.2. Now solutions of the form $z(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\alpha}$ (w) In this case for $\alpha\beta \neq 0$, $\beta \neq 2\lambda$ and $q \neq 0$ we have solutions if

 $\gamma = \frac{2}{3}, \quad \eta = \frac{2}{3}\Lambda, \quad \lambda = \pm \sqrt{\frac{2}{3}\Lambda}.$

III When $\alpha\beta \neq 0$, we have the following two special solutions:

au () = A could () (2A (), _____ Z (2A) = -1, Z (2A) = au () = A could () (2A (), _____ Z (2A) = -1, Z (2A) = -

1. Cosmological solution in the flat Universe (k = 0)

- 1.1. Solutions of the form $a(t) = A t^n e^{\gamma t}$
 - There are two solutions:

$$a_{1}(t) = A t^{\frac{3}{2}} e^{\frac{A}{4}t^{2}}, \qquad \qquad \mathcal{F}(-\frac{3}{7}\Lambda) = -1, \quad \mathcal{F}'(-\frac{3}{7}\Lambda) = 0,$$

$$a_{2}(t) = A e^{\frac{A}{4}t^{2}}, \qquad \qquad \mathcal{F}(-\Lambda) = -1, \quad \mathcal{F}'(-\Lambda) = 0.$$

1.2. New solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$ In this case for $\alpha\beta \neq 0$, $\beta \neq 2\Lambda$ and $\sigma \neq 0$ we have solution

$$\gamma=rac{2}{3}, \qquad q=rac{3}{8}\Lambda, \qquad \lambda=\pm\sqrt{rac{3}{8}\Lambda}.$$

When $\alpha\beta \neq 0$, we have the following two special solutions:

$$a_{3}(t) = A \cosh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0,$$
$$a_{4}(t) = A \sinh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0.$$

- 1.1. Solutions of the form $a(t) = A t^n e^{\gamma t}$
 - There are two solutions:

$$a_{1}(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^{2}}, \qquad \mathcal{F}(-\frac{3}{7}\Lambda) = -1, \ \mathcal{F}'(-\frac{3}{7}\Lambda) = 0,$$

$$a_{2}(t) = A e^{\frac{\Lambda}{6}t^{2}}, \qquad \mathcal{F}(-\Lambda) = -1, \ \mathcal{F}'(-\Lambda) = 0.$$

1.2. New solutions of the form a(t) = (α e^{λt} + β e^{-λt})^γ
In this case for αβ ≠ 0, R ≠ 2Λ and q ≠ 0 we have solutions if

$$\gamma = \frac{2}{3}, \qquad q = \frac{3}{8}\Lambda, \qquad \lambda = \pm \sqrt{\frac{3}{8}\Lambda}.$$

When $\alpha\beta \neq 0$, we have the following two special solutions:

$$\begin{aligned} a_3(t) &= A \cosh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0, \\ a_4(t) &= A \sinh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0. \end{aligned}$$

- 1. Cosmological solution in the flat Universe (k = 0)
 - 1.1. Solutions of the form $a(t) = A t^n e^{\gamma t^2}$
 - There are two solutions:

$$a_{1}(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^{2}}, \qquad \mathcal{F}(-\frac{3}{7}\Lambda) = -1, \ \mathcal{F}'(-\frac{3}{7}\Lambda) = 0,$$

$$a_{2}(t) = A e^{\frac{\Lambda}{6}t^{2}}, \qquad \mathcal{F}(-\Lambda) = -1, \ \mathcal{F}'(-\Lambda) = 0.$$

1.2. New solutions of the form a(t) = (α e^{λt} + β e^{-λt})^γ
In this case for αβ ≠ 0, R ≠ 2Λ and q ≠ 0 we have solutions if

$$\gamma = \frac{2}{3}, \qquad q = \frac{3}{8}\Lambda, \qquad \lambda = \pm \sqrt{\frac{3}{8}\Lambda}.$$

When $\alpha\beta \neq 0$, we have the following two special solutions:

$$\begin{aligned} a_3(t) &= A \,\cosh^{\frac{2}{3}}\left(\sqrt{\frac{3}{8}}\Lambda \,t\right), \qquad \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0, \\ a_4(t) &= A \,\sinh^{\frac{2}{3}}\left(\sqrt{\frac{3}{8}}\Lambda \,t\right), \qquad \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0. \end{aligned}$$

- 1.1. Solutions of the form $a(t) = A t^n e^{\gamma t^2}$
 - There are two solutions:

$$a_{1}(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^{2}}, \qquad \mathcal{F}(-\frac{3}{7}\Lambda) = -1, \ \mathcal{F}'(-\frac{3}{7}\Lambda) = 0,$$

$$a_{2}(t) = A e^{\frac{\Lambda}{6}t^{2}}, \qquad \mathcal{F}(-\Lambda) = -1, \ \mathcal{F}'(-\Lambda) = 0.$$

1.2. New solutions of the form a(t) = (α e^{λt} + β e^{-λt})^γ
In this case for αβ ≠ 0, R ≠ 2Λ and q ≠ 0 we have solutions if

$$\gamma = \frac{2}{3}, \qquad q = \frac{3}{8}\Lambda, \qquad \lambda = \pm \sqrt{\frac{3}{8}\Lambda}.$$

When $\alpha\beta \neq 0$, we have the following two special solutions:

$$\begin{aligned} a_3(t) &= A \cosh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0, \\ a_4(t) &= A \sinh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0. \end{aligned}$$

- 1.1. Solutions of the form $a(t) = A t^n e^{\gamma t^2}$
 - There are two solutions:

$$a_{1}(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^{2}}, \qquad \mathcal{F}(-\frac{3}{7}\Lambda) = -1, \ \mathcal{F}'(-\frac{3}{7}\Lambda) = 0,$$

$$a_{2}(t) = A e^{\frac{\Lambda}{6}t^{2}}, \qquad \mathcal{F}(-\Lambda) = -1, \ \mathcal{F}'(-\Lambda) = 0.$$

1.2. New solutions of the form $a(t) = (\alpha \ e^{\lambda t} + \beta \ e^{-\lambda t})^{\gamma}$ In this case for $\alpha \beta \neq 0$, $R \neq 2\Lambda$ and $q \neq 0$ we have solutions if

$$\gamma = \frac{2}{3}, \qquad q = \frac{3}{8}\Lambda, \qquad \lambda = \pm \sqrt{\frac{3}{8}\Lambda}.$$

When $\alpha\beta \neq 0$, we have the following two special solutions:

$$\begin{aligned} a_3(t) &= A \cosh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0, \\ a_4(t) &= A \sinh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0. \end{aligned}$$

< ロ > < 同 > < 回 > < 回 > < 回 > <

- 1.1. Solutions of the form $a(t) = A t^n e^{\gamma t^2}$
 - There are two solutions:

$$a_{1}(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^{2}}, \qquad \mathcal{F}(-\frac{3}{7}\Lambda) = -1, \ \mathcal{F}'(-\frac{3}{7}\Lambda) = 0,$$

$$a_{2}(t) = A e^{\frac{\Lambda}{6}t^{2}}, \qquad \mathcal{F}(-\Lambda) = -1, \ \mathcal{F}'(-\Lambda) = 0.$$

1.2. New solutions of the form a(t) = (α e^{λt} + β e^{-λt})^γ
In this case for αβ ≠ 0, R ≠ 2Λ and q ≠ 0 we have solutions if

$$\gamma = \frac{2}{3}, \qquad q = \frac{3}{8}\Lambda, \qquad \lambda = \pm \sqrt{\frac{3}{8}\Lambda}.$$

When $\alpha\beta \neq 0$, we have the following two special solutions:

$$\begin{aligned} a_3(t) &= A \,\cosh^{\frac{2}{3}}\left(\sqrt{\frac{3}{8}}\Lambda \,t\right), \qquad \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0, \\ a_4(t) &= A \,\sinh^{\frac{2}{3}}\left(\sqrt{\frac{3}{8}}\Lambda \,t\right), \qquad \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0. \end{aligned}$$

< ロ > < 同 > < 回 > < 回 > .

- 1.1. Solutions of the form $a(t) = A t^n e^{\gamma t^2}$
 - There are two solutions:

$$a_{1}(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^{2}}, \qquad \mathcal{F}(-\frac{3}{7}\Lambda) = -1, \ \mathcal{F}'(-\frac{3}{7}\Lambda) = 0,$$

$$a_{2}(t) = A e^{\frac{\Lambda}{6}t^{2}}, \qquad \mathcal{F}(-\Lambda) = -1, \ \mathcal{F}'(-\Lambda) = 0.$$

1.2. New solutions of the form a(t) = (α e^{λt} + β e^{-λt})^γ
In this case for αβ ≠ 0, R ≠ 2Λ and q ≠ 0 we have solutions if

$$\gamma = \frac{2}{3}, \qquad q = \frac{3}{8}\Lambda, \qquad \lambda = \pm \sqrt{\frac{3}{8}\Lambda}.$$

When $\alpha\beta \neq 0$, we have the following two special solutions:

$$a_{3}(t) = A \cosh^{\frac{2}{3}}\left(\sqrt{\frac{3}{8}\Lambda} t\right), \qquad \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0,$$

$$a_{4}(t) = A \sinh^{\frac{2}{3}}\left(\sqrt{\frac{3}{8}\Lambda} t\right), \qquad \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0.$$

< ロ > < 同 > < 三 > < 三 > -

Ъ.

- 1. Cosmological solution in the flat Universe (k = 0)
 - 1.3. New solutions of the form $a(t) = (\alpha \sin \lambda t + \beta \cos \lambda t)^{\gamma}$
 - If For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for $\gamma, \gamma = \frac{1}{2}$. Taking $\beta = \pm \alpha$, and $A = \alpha^{\frac{1}{2}}$, we have the following two solutions:



 $0 = A \cos^2 (q - \frac{2}{3} A) = 0 = A \sin^2 (q - \frac{2}{3} A) = 0.$

<ロ> <同> <同> < 同> < 同> < 三> < 三> <

.

- 1. Cosmological solution in the flat Universe (k = 0)
 - 1.3. New solutions of the form $a(t) = (\alpha \sin \lambda t + \beta \cos \lambda t)^{\gamma}$
 - For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ , $\gamma = \frac{2}{3}$. Taking $\beta = \pm \alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

$$\begin{aligned} \mathbf{a}_{5}(t) &= A\left(1 + \sin\left(2\sqrt{-\frac{3}{8}}\Lambda t\right)\right)^{\frac{1}{3}}, \qquad \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0, \\ \mathbf{a}_{6}(t) &= A\left(1 - \sin\left(2\sqrt{-\frac{3}{8}}\Lambda t\right)\right)^{\frac{1}{3}}, \qquad \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0. \end{aligned}$$

For $\alpha = 0$ or $\beta = 0$, we have also two cosmological solutions with $\gamma = \frac{2}{3}$:

$$a_{7}(t) = A \sin^{\frac{3}{4}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0,$$
$$a_{8}(t) = A \cos^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0.$$

1. Cosmological solution in the flat Universe (k = 0)

- 1.3. New solutions of the form $a(t) = (\alpha \sin \lambda t + \beta \cos \lambda t)^{\gamma}$
 - For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ , $\gamma = \frac{2}{3}$. Taking $\beta = \pm \alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

$$a_{5}(t) = A \left(1 + \sin\left(2\sqrt{-\frac{3}{8}}\Lambda t\right) \right)^{\frac{1}{3}}, \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0,$$

$$a_{6}(t) = A \left(1 - \sin\left(2\sqrt{-\frac{3}{8}}\Lambda t\right) \right)^{\frac{1}{3}}, \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0.$$

For $\alpha = 0$ or $\beta = 0$, we have also two cosmological solutions with $\gamma = \frac{2}{3}$:

$$a_{7}(t) = A \sin^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0,$$
$$a_{8}(t) = A \cos^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0.$$

A = A = A = A = A = A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

- 1. Cosmological solution in the flat Universe (k = 0)
 - 1.3. New solutions of the form $a(t) = (\alpha \sin \lambda t + \beta \cos \lambda t)^{\gamma}$

For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ , $\gamma = \frac{2}{3}$. Taking $\beta = \pm \alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

$$a_{5}(t) = A \left(1 + \sin\left(2\sqrt{-\frac{3}{8}\Lambda t}\right)\right)^{\frac{1}{3}}, \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0,$$

$$a_{6}(t) = A \left(1 - \sin\left(2\sqrt{-\frac{3}{8}\Lambda t}\right)\right)^{\frac{1}{3}}, \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0.$$

For $\alpha = 0$ or $\beta = 0$, we have also two cosmological solutions with $\gamma = \frac{2}{3}$:

$$a_{7}(t) = A \sin^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0,$$
$$a_{8}(t) = A \cos^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0.$$

(日)

1. Cosmological solution in the flat Universe (k = 0)

- 1.3. New solutions of the form $a(t) = (\alpha \sin \lambda t + \beta \cos \lambda t)^{\gamma}$
 - For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ , $\gamma = \frac{2}{3}$. Taking $\beta = \pm \alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

$$a_{5}(t) = A \left(1 + \sin\left(2\sqrt{-\frac{3}{8}\Lambda} t\right) \right)^{\frac{1}{3}}, \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0,$$

$$a_{6}(t) = A \left(1 - \sin\left(2\sqrt{-\frac{3}{8}\Lambda} t\right) \right)^{\frac{1}{3}}, \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0.$$

For $\alpha = 0$ or $\beta = 0$, we have also two cosmological solutions with $\gamma = \frac{2}{3}$:

$$a_{7}(t) = A \sin^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0,$$
$$a_{8}(t) = A \cos^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0.$$

・ 同 ト ・ ヨ ト ・ ヨ ト

1. Cosmological solution in the flat Universe (k = 0)

- 1.3. New solutions of the form $a(t) = (\alpha \sin \lambda t + \beta \cos \lambda t)^{\gamma}$
 - For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ , $\gamma = \frac{2}{3}$. Taking $\beta = \pm \alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

$$a_{5}(t) = A \left(1 + \sin\left(2\sqrt{-\frac{3}{8}\Lambda} t\right) \right)^{\frac{1}{3}}, \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0,$$

$$a_{6}(t) = A \left(1 - \sin\left(2\sqrt{-\frac{3}{8}\Lambda} t\right) \right)^{\frac{1}{3}}, \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0.$$

For $\alpha = 0$ or $\beta = 0$, we have also two cosmological solutions with $\gamma = \frac{2}{3}$:

$$a_{7}(t) = A \sin^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0,$$
$$a_{8}(t) = A \cos^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0.$$

A = > < = >
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

- 2. Cosmological solution in the open and closed Universe $(k=\pm 1)$
 - 2.1. Solutions of the form $a(t) = A e^{\pm \sqrt{b}t}$, $(k = \pm 1)$
 - For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution: $a_{0}(t) = A e^{2\sqrt{2}t}, \qquad k = \pm 1, \qquad \pi(\frac{1}{2}\Lambda) = -1, \quad \pi'(\frac{1}{2}\Lambda) = 0, \quad \Lambda > 0$
 - New solutions of the form $a(t) := (a \cdot a^{\lambda} + b \cdot a^{\lambda}) 2p(k := \pm 1)$
 - Integration and the second seco
 - $$\begin{split} & a_{11}(0) = A \ \text{subst} \left\{ \sqrt{\frac{2}{3}} A \left(0 \right), \quad A = A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) =$$

훈.

- 2. Cosmological solution in the open and closed Universe $(k = \pm 1)$
 - 2.1. Solutions of the form $a(t) = A e^{\pm \sqrt{6}t}$, $(k = \pm 1)$
 - For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution: $a_0(t) = A \ e^{\pm \sqrt{\frac{2}{8}}t}, \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{2}\Lambda) = -1, \quad \mathcal{F}'(\frac{1}{2}\Lambda) = 0, \quad \Lambda > 0$
 - 2.2. New solutions of the form $a(t) = (\alpha \ e^{\lambda t} + \beta \ e^{-\lambda t})^{\gamma}$, $(k = \pm 1)$
 - For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $q \neq 0$ there are two following cosmological solutions:

$$\begin{aligned} \mathbf{a}_{10}(t) &= A \, \cosh^{\frac{1}{2}}\left(\sqrt{\frac{2}{3}}\Lambda \, t\right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0, \\ \mathbf{a}_{11}(t) &= A \, \sinh^{\frac{1}{2}}\left(\sqrt{\frac{2}{3}}\Lambda \, t\right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0. \end{aligned}$$

- 2. Cosmological solution in the open and closed Universe $(k = \pm 1)$
 - 2.1. Solutions of the form $a(t) = A \ e^{\pm \sqrt{\frac{1}{6}t}}, \ (k=\pm 1)$
 - For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution:

$$a_9(t) = A e^{\pm \sqrt{\frac{\Lambda}{6}}t}, \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3}\Lambda) = -1, \quad \mathcal{F}'(\frac{1}{3}\Lambda) = 0, \quad \Lambda > 0.$$

- 2.2. New solutions of the form $a(t) = (\alpha \ e^{\lambda t} + \beta \ e^{-\lambda t})^{\gamma}$, $(k = \pm 1)$
 - For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $q \neq 0$ there are two following cosmological solutions:

$$\begin{aligned} a_{10}(t) &= A \cosh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3}\Lambda) = -1, \ \mathcal{F}'(\frac{1}{3}\Lambda) = 0, \\ a_{11}(t) &= A \sinh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3}\Lambda) = -1, \ \mathcal{F}'(\frac{1}{3}\Lambda) = 0. \end{aligned}$$

・ロト ・同 ・ ・ ヨ ・ ・ ヨ ・

Ъ.

- 2. Cosmological solution in the open and closed Universe $(k = \pm 1)$
 - 2.1. Solutions of the form $a(t) = A e^{\pm \sqrt{\frac{\Lambda}{6}}t}$, $(k = \pm 1)$

For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution: $a_{g}(t) = A \ e^{\pm \sqrt{\frac{\Lambda}{6}}t}, \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3}\Lambda) = -1, \quad \mathcal{F}'(\frac{1}{3}\Lambda) = 0, \quad \Lambda > 0$

- 2.2. New solutions of the form $a(t) = (\alpha \ e^{\lambda t} + \beta \ e^{-\lambda t})^{\gamma}$, $(k = \pm 1)$
 - For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $q \neq 0$ there are two following cosmological solutions:

 $a_{10}(t) = A \cosh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0,$ $a_{11}(t) = A \sinh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0.$

- 2. Cosmological solution in the open and closed Universe $(k = \pm 1)$
 - 2.1. Solutions of the form $a(t) = A e^{\pm \sqrt{\frac{\Lambda}{6}}t}$, $(k = \pm 1)$
 - For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution:

 $a_9(t) = A e^{\pm \sqrt{\frac{\Lambda}{6}}t}, \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3}\Lambda) = -1, \quad \mathcal{F}'(\frac{1}{3}\Lambda) = 0, \quad \Lambda > 0.$

- 2.2. New solutions of the form $a(t) = (\alpha \ e^{\lambda t} + \beta \ e^{-\lambda t})^{\gamma}$, $(k = \pm 1)$
 - For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $q \neq 0$ there are two following cosmological solutions:

 $\begin{aligned} \mathbf{a}_{10}(t) &= \mathbf{A} \, \cosh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda \, t \right), \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3} \Lambda) = -1, \ \mathcal{F}'(\frac{1}{3} \Lambda) = 0, \\ \mathbf{a}_{11}(t) &= \mathbf{A} \, \sinh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda \, t \right), \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3} \Lambda) = -1, \ \mathcal{F}'(\frac{1}{3} \Lambda) = 0. \end{aligned}$

- 2. Cosmological solution in the open and closed Universe $(k = \pm 1)$
 - 2.1. Solutions of the form $a(t) = A e^{\pm \sqrt{\frac{\Lambda}{6}}t}$, $(k = \pm 1)$
 - For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution:

$$a_9(t) = A e^{\pm \sqrt{rac{\Lambda}{6}}t}, \qquad k = \pm 1, \quad \mathcal{F}(rac{1}{3}\Lambda) = -1, \quad \mathcal{F}'(rac{1}{3}\Lambda) = 0, \quad \Lambda > 0.$$

- 2.2. New solutions of the form $a(t) = (\alpha \ e^{\lambda t} + \beta \ e^{-\lambda t})^{\gamma}$, $(k = \pm 1)$
 - For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $q \neq 0$ there are two following cosmological solutions:

$$a_{10}(t) = A \cosh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0,$$
$$a_{11}(t) = A \sinh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0.$$

< ロ > < 同 > < 三 > < 三 > -

Ъ.

- 2. Cosmological solution in the open and closed Universe $(k = \pm 1)$
 - 2.1. Solutions of the form $a(t) = A e^{\pm \sqrt{\frac{\Lambda}{6}}t}$, $(k = \pm 1)$
 - For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution:

$$a_9(t)=A\ e^{\pm\sqrt{rac{\Lambda}{6}}t},\qquad k=\pm1,\quad \mathcal{F}(rac{1}{3}\Lambda)=-1,\ \mathcal{F}'(rac{1}{3}\Lambda)=0,\quad \Lambda>0.$$

- 2.2. New solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$, $(k = \pm 1)$
 - For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $q \neq 0$ there are two following cosmological solutions:

$$a_{10}(t) = A \cosh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0,$$

$$a_{11}(t) = A \sinh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0.$$

- 2. Cosmological solution in the open and closed Universe $(k = \pm 1)$
 - 2.1. Solutions of the form $a(t) = A e^{\pm \sqrt{\frac{\Lambda}{6}}t}$, $(k = \pm 1)$
 - For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution:

$$a_9(t)=A\ e^{\pm\sqrt{\Lambda\over 6}t},\qquad k=\pm1,\quad \mathcal{F}({1\over 3}\Lambda)=-1,\ \mathcal{F}'({1\over 3}\Lambda)=0,\quad \Lambda>0.$$

- 2.2. New solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$, $(k = \pm 1)$
 - For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $q \neq 0$ there are two following cosmological solutions:

$$\begin{aligned} a_{10}(t) &= A \, \cosh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda \, t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \; \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0, \\ a_{11}(t) &= A \, \sinh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda \, t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \; \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0. \end{aligned}$$

- 1. Cosmological solution for $a_1(t) = A t^{\frac{5}{3}} e^{\frac{h}{14}t^2}$, k = 0
- The corresponding **Constant and the scalar** 2 curvature are:

$$H_{1}(t) = \frac{\dot{a}_{1}}{a_{1}} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

$$\ddot{a}_{1}(t) = \left(-\frac{2}{9}\frac{1}{t^{2}} + \frac{1}{3}\Lambda + \frac{1}{49}\Lambda^{2}t^{2}\right)a_{1}(t)$$

$$R_{1}(t) = \frac{4}{3}\frac{1}{t^{2}} + \frac{22}{7}\Lambda + \frac{12}{49}\Lambda^{2}t^{2},$$

Friedman equations gives

$$\bar{\rho}(t) = \frac{2t^{-2} + \frac{9}{96}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}, \quad \bar{\rho}(t) = -\frac{\Lambda}{56\pi G} (\frac{3}{7}\Lambda t^2 - 1), \quad (11)$$

where $\bar{\rho}$ and $\bar{\rho}$ are analogs of the energy density and pressure of the dark side of the universe, respectively. The corresponding equation of state is $\bar{\rho}(t) = \bar{w}(t) \bar{\rho}(t)$.
- 1. Cosmological solution for $a_1(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$, k = 0
- The corresponding Hubble parameter, acceleration and the scalar 2 curvature are:

$$H_{1}(t) = \frac{\dot{a}_{1}}{a_{1}} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

$$\ddot{a}_{1}(t) = \left(-\frac{2}{9}\frac{1}{t^{2}} + \frac{1}{3}\Lambda + \frac{1}{49}\Lambda^{2}t^{2}\right)a_{1}(t),$$

$$R_{1}(t) = \frac{4}{3}\frac{1}{t^{2}} + \frac{22}{7}\Lambda + \frac{12}{49}\Lambda^{2}t^{2},$$

• Friedman equations gives

$$\bar{\rho}(t) = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}, \quad \bar{p}(t) = -\frac{\Lambda}{56\pi G} (\frac{3}{7}\Lambda t^2 - 1), \quad (11)$$

where $\bar{\rho}$ and \bar{p} are analogs of the energy density and pressure of the dark side of the universe, respectively. The corresponding equation of state is $\bar{\rho}(t) = \bar{w}(t) \bar{\rho}(t)$.

Model $\mathcal{H}(R) = \mathcal{G}(R) = \sqrt{R - 2\Lambda}$

- 1. Cosmological solution for $a_1(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$, k = 0
- The corresponding Hubble parameter, acceleration and the scalar 2 curvature are:

$$H_{1}(t) = \frac{\dot{a}_{1}}{a_{1}} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

$$\ddot{a}_{1}(t) = \left(-\frac{2}{9}\frac{1}{t^{2}} + \frac{1}{3}\Lambda + \frac{1}{49}\Lambda^{2}t^{2}\right)a_{1}(t),$$

$$R_{1}(t) = \frac{4}{3}\frac{1}{t^{2}} + \frac{22}{7}\Lambda + \frac{12}{49}\Lambda^{2}t^{2},$$

Friedman equations gives

$$\bar{\rho}(t) = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}, \quad \bar{p}(t) = -\frac{\Lambda}{56\pi G} (\frac{3}{7}\Lambda t^2 - 1), \quad (11)$$

where $\bar{\rho}$ and \bar{p} are analogs of the energy density and pressure of the dark side of the universe, respectively. The corresponding equation of state is $\bar{p}(t) = \bar{w}(t) \bar{\rho}(t)$.

(日)

Model $\mathcal{H}(R) = \mathcal{G}(R) = \sqrt{R - 2\Lambda}$

- 1. Cosmological solution for $a_1(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$, k = 0
- The corresponding Hubble parameter, acceleration and the scalar 2 curvature are:

$$H_{1}(t) = \frac{\dot{a}_{1}}{a_{1}} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

$$\ddot{a}_{1}(t) = \left(-\frac{2}{9}\frac{1}{t^{2}} + \frac{1}{3}\Lambda + \frac{1}{49}\Lambda^{2}t^{2}\right)a_{1}(t),$$

$$R_{1}(t) = \frac{4}{3}\frac{1}{t^{2}} + \frac{22}{7}\Lambda + \frac{12}{49}\Lambda^{2}t^{2},$$

Friedman equations gives

$$\bar{\rho}(t) = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}, \quad \bar{p}(t) = -\frac{\Lambda}{56\pi G} (\frac{3}{7}\Lambda t^2 - 1), \quad (11)$$

where $\bar{\rho}$ and \bar{p} are analogs of the energy density and pressure of the dark side of the universe, respectively. The corresponding equation of state is $\bar{p}(t) = \bar{w}(t) \bar{\rho}(t)$.

- 1. Cosmological solution for $a_1(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$, k = 0
- The corresponding Hubble parameter, acceleration and the scalar 2 curvature are:

$$\begin{aligned} H_1(t) &= \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t, \\ \ddot{a}_1(t) &= \left(-\frac{2}{9}\frac{1}{t^2} + \frac{1}{3}\Lambda + \frac{1}{49}\Lambda^2 t^2\right) a_1(t), \\ R_1(t) &= \frac{4}{3}\frac{1}{t^2} + \frac{22}{7}\Lambda + \frac{12}{49}\Lambda^2 t^2, \end{aligned}$$

Friedman equations gives

$$\bar{\rho}(t) = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}, \quad \bar{\rho}(t) = -\frac{\Lambda}{56\pi G} (\frac{3}{7}\Lambda t^2 - 1), \quad (11)$$

where $\bar{\rho}$ and \bar{p} are analogs of the energy density and pressure of the dark side of the universe, respectively. The corresponding equation of state is $\bar{\rho}(t) = \bar{w}(t) \bar{\rho}(t)$.

< ロ > < 同 > < 三 > < 三 > -

- Control (11) implies that w
 (t) → -1 when t → ∞, what corresponds to an analog of ∧ dark energy dominance in the standard cosmological model.
- It means that this nonlocal gravity model with cosmological solution
 a(t) = A t³ e^{At²} describes some effects usually attributed to the dark
 matter and dark energy.
- This solution is invariant under transformation t → −t and singular at cosmic time t = 0.
- Let us recall, the second Friedman equation

$$H^{2} = \frac{\ddot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho - \frac{k}{a^{2}} + \frac{\Lambda}{3},$$
 (12)

where 🔝 is energy density in the standard model of cosmology.

- The expressions (11) implies that $\overline{w}(t) \rightarrow -1$ when $t \rightarrow \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model.
- It means that this nonlocal gravity model with cosmological solution $a(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$ describes some effects usually attributed to the dark matter and dark energy.
- This solution is invariant under transformation t → −t and singular at cosmic time t = 0.
- Let us recall, the second Friedman equation

$$H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho - \frac{k}{a^{2}} + \frac{\Lambda}{3},$$
 (12)

where \bigcirc is energy density in the standard model of cosmology.

- The expressions (11) implies that $\overline{w}(t) \rightarrow -1$ when $t \rightarrow \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model.
- It means that this nonlocal gravity model with cosmological solution $a(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$ describes some effects usually attributed to the dark matter and dark energy.
- This solution is invariant under transformation t → −t and singular at cosmic time t = 0.
- Let us recall, the second Friedman equation

$$H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho - \frac{k}{a^{2}} + \frac{\Lambda}{3},$$
 (12)

where $\mathbf{P}_{\mathcal{P}}$ is energy density in the standard model of cosmology.

- The expressions (11) implies that $\overline{w}(t) \rightarrow -1$ when $t \rightarrow \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model.
- It means that this nonlocal gravity model with cosmological solution $a(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$ describes some effects usually attributed to the dark matter and dark energy.
- This solution is invariant under transformation $t \rightarrow -t$ and singular at cosmic time t = 0.
- Let us recall, the second Friedman equation

$$H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho - \frac{k}{a^{2}} + \frac{\Lambda}{3},$$
 (12)

where \bigcirc is energy density in the standard model of cosmology.

< ロ > < 同 > < 回 > < 回 > < 回 > <

- The expressions (11) implies that $\overline{w}(t) \rightarrow -1$ when $t \rightarrow \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model.
- It means that this nonlocal gravity model with cosmological solution $a(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$ describes some effects usually attributed to the dark matter and dark energy.
- This solution is invariant under transformation t → −t and singular at cosmic time t = 0.
- Let us recall, the second Friedman equation

$$H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho - \frac{k}{a^{2}} + \frac{\Lambda}{3},$$
 (12)

where \bigcirc is energy density in the standard model of cosmology.

< 口 > < 同 > < 三 > < 三 > 、

- The expressions (11) implies that $\overline{w}(t) \rightarrow -1$ when $t \rightarrow \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model.
- It means that this nonlocal gravity model with cosmological solution $a(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$ describes some effects usually attributed to the dark matter and dark energy.
- This solution is invariant under transformation t → −t and singular at cosmic time t = 0.
- Let us recall, the second Friedman equation

$$H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho - \frac{k}{a^{2}} + \frac{\Lambda}{3},$$
 (12)

where $\bigcirc p$ is energy density in the standard model of cosmology.

< 口 > < 同 > < 三 > < 三 > 、

$$\begin{aligned} H^2 &= -\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_r + \frac{8\pi G}{3}\rho_m - \frac{k}{a^2} + \frac{\Lambda}{3} \\ &= -\frac{8 C_r \pi G}{a^4} + \frac{8 C_m \pi G}{a^3} - \frac{k}{a^2} + \frac{\Lambda}{3} \end{aligned}$$

It follows

$$rac{H^2}{H_0^2}=rac{\Omega_r}{a^4}+rac{\Omega_m}{a^3}+rac{\Omega_k}{a^2}+\Omega_\Lambda$$

Observational data obtained by Planck-2018 for the ACDM model:

 $t_0 = (13.801 \pm 0.024) \times 10^9$ yr – age of the universe,

 $H(t_0) = (67.40 \pm 0.50) \text{ km/s/Mpc} - \text{Hubble parameter}$

 $\Omega_m = 0.315 \pm 0.007$ – matter density parameter,

 $\Omega_{\Lambda} = 0.685 - \Lambda$ density parameter.

 $w_0 = -1.03 \pm 0.03$ ratio of pressure to energy density.

- コン (雪) (ヨ) (ヨ)

$$H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho_{r} + \frac{8\pi G}{3}\rho_{m} - \frac{k}{a^{2}} + \frac{\Lambda}{3}$$
$$= \frac{8C_{r}\pi G}{a^{4}} + \frac{8C_{m}\pi G}{a^{3}} - \frac{k}{a^{2}} + \frac{\Lambda}{3}$$

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

Observational data obtained by Planck-2018 for the ACDM model:

 $t_0 = (13.801 \pm 0.024) \times 10^9$ yr – age of the universe,

 $H(t_0) = (67.40 \pm 0.50) \text{ km/s/Mpc} - \text{Hubble parameter},$

 $\Omega_m = 0.315 \pm 0.007$ – matter density parameter,

 $\Omega_{\Lambda} = 0.685 - \Lambda$ density parameter,

Zoran Rakić

 $w_0 = -1.03 \pm 0.03$ - ratio of pressure to energy density.

$$H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho_{r} + \frac{8\pi G}{3}\rho_{m} - \frac{k}{a^{2}} + \frac{\Lambda}{3}$$
$$= \frac{8 C_{r}\pi G}{a^{4}} + \frac{8 C_{m}\pi G}{a^{3}} - \frac{k}{a^{2}} + \frac{\Lambda}{3}$$

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

Observational data obtained by Planck-2018 for the ACDM model:

 $t_0 = (13.801 \pm 0.024) \times 10^9$ yr – age of the universe,

 $H(t_0) = (67.40 \pm 0.50) \text{ km/s/Mpc} - \text{Hubble parameter},$

 $\Omega_m = 0.315 \pm 0.007$ – matter density parameter,

 $\Omega_{\Lambda} = 0.685 - \Lambda$ density parameter,

 $w_0 = -1.03 \pm 0.03$ - ratio of pressure to energy density.

< D > < P > < E > < E</p>

$$H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho_{r} + \frac{8\pi G}{3}\rho_{m} - \frac{k}{a^{2}} + \frac{\Lambda}{3}$$
$$= \frac{8 C_{r}\pi G}{a^{4}} + \frac{8 C_{m}\pi G}{a^{3}} - \frac{k}{a^{2}} + \frac{\Lambda}{3}$$

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

Observational data obtained by Planck-2018 for the ACDM model:

 $t_0 = (13.801 \pm 0.024) \times 10^9$ yr – age of the universe,

 $H(t_0) = (67.40 \pm 0.50) \text{ km/s/Mpc} - \text{Hubble parameter},$

 $\Omega_m = 0.315 \pm 0.007$ – matter density parameter,

 $\Omega_{\Lambda} = 0.685 - \Lambda$ density parameter,

 $w_0 = -1.03 \pm 0.03$ - ratio of pressure to energy density.

< 同 > < 三 > <

$$H^{2} = \frac{\dot{a}^{2}}{a^{2}} = \frac{8\pi G}{3}\rho_{r} + \frac{8\pi G}{3}\rho_{m} - \frac{k}{a^{2}} + \frac{\Lambda}{3}$$
$$= \frac{8 C_{r}\pi G}{a^{4}} + \frac{8 C_{m}\pi G}{a^{3}} - \frac{k}{a^{2}} + \frac{\Lambda}{3}$$

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

Observational data obtained by Planck-2018 for the ACDM model:

 $t_0 = (13.801 \pm 0.024) \times 10^9$ yr – age of the universe,

 $H(t_0) = (67.40 \pm 0.50) \text{ km/s/Mpc} - \text{Hubble parameter},$

 $\Omega_m = 0.315 \pm 0.007$ – matter density parameter,

 $\Omega_{\Lambda} = 0.685 - \Lambda$ density parameter,

 $w_0 = -1.03 \pm 0.03$ - ratio of pressure to energy density.

From, $H_1(t) = \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t$

taking $H_1(t_0) = H(t_0)$ we calculate $\Lambda_1 = 1.05 \times 10^{-35} \text{s}^{-2}$ that differs from $\Lambda = 3H^2(t_0) \Omega_{\Lambda} = 0.98 \times 10^{-35} \text{s}^{-2}$ (by ΛCDM model).

We also computed

 $\ddot{a}_1(t_0)/a_1(t_0) = 2.7 \times 10^{-36} s^{-2}$ $R(t_0) = 4.5 \times 10^{-35} s^{-2}$ and consequently $R(t_0) - 2\Lambda = 2.4 \times 10^{-35} s^{-2}$.

Replacing solution a₁(t) with k = 0, Friedman equations give

$$\bar{p}_{1}(t) = \frac{3}{8\pi G} \Big(H_{1}^{2}(t) - \frac{\Lambda_{1}}{3} \Big) = \frac{3}{8\pi G} \Big(\frac{4}{9} t^{-2} - \frac{1}{7} \Lambda_{1} + \frac{1}{49} \Lambda_{1}^{2} t^{2} \Big),$$
$$\bar{p}_{1}(t) = \frac{\Lambda_{1}}{56\pi G} \Big(1 - \frac{3}{7} \Lambda_{1} t^{2} \Big).$$

• From,

$$H_1(t) = \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

taking $H_1(t_0) = H(t_0)$ we calculate $\Lambda_1 = 1.05 \times 10^{-35} \text{s}^{-2}$ that differs from $\Lambda = 3H^2(t_0) \Omega_{\Lambda} = 0.98 \times 10^{-35} \text{s}^{-2}$ (by ΛCDM model).

We also computed

$$\ddot{a}_1(t_0)/a_1(t_0) = 2.7 \times 10^{-36} s^{-2}$$

 $R(t_0) = 4.5 \times 10^{-35} s^{-2}$ and consequently
 $R(t_0) - 2\Lambda = 2.4 \times 10^{-35} s^{-2}$.

• Replacing solution $a_1(t)$ with k = 0, Friedman equations give

$$\bar{\rho}_{1}(t) = \frac{3}{8\pi G} \Big(H_{1}^{2}(t) - \frac{\Lambda_{1}}{3} \Big) = \frac{3}{8\pi G} \Big(\frac{4}{9} t^{-2} - \frac{1}{7} \Lambda_{1} + \frac{1}{49} \Lambda_{1}^{2} t^{2} \Big),$$
$$\bar{\rho}_{1}(t) = \frac{\Lambda_{1}}{56\pi G} \Big(1 - \frac{3}{7} \Lambda_{1} t^{2} \Big).$$

< ロ > < 同 > < 回 > < 回 > < 回 > <

• From,

$$H_1(t) = \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

taking $H_1(t_0) = H(t_0)$ we calculate $\Lambda_1 = 1.05 \times 10^{-35} s^{-2}$ that differs from $\Lambda = 3H^2(t_0) \Omega_{\Lambda} = 0.98 \times 10^{-35} s^{-2}$ (by ΛCDM model).

We also computed

 $\ddot{a}_1(t_0)/a_1(t_0) = 2.7 \times 10^{-36} s^{-2}$ $R(t_0) = 4.5 \times 10^{-35} s^{-2}$ and consequently $R(t_0) - 2\Lambda = 2.4 \times 10^{-35} s^{-2}$.

• Replacing solution $a_1(t)$ with k = 0, Friedman equations give

$$\bar{\rho}_1(t) = \frac{3}{8\pi G} \Big(H_1^2(t) - \frac{\Lambda_1}{3} \Big) = \frac{3}{8\pi G} \Big(\frac{4}{9} t^{-2} - \frac{1}{7} \Lambda_1 + \frac{1}{49} \Lambda_1^2 t^2 \Big),$$

$$\bar{\rho}_1(t) = \frac{\Lambda_1}{56\pi G} \Big(1 - \frac{3}{7} \Lambda_1 t^2 \Big).$$

< ロ > < 同 > < 回 > < 回 > .

From,

$$H_1(t) = \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

taking $H_1(t_0) = H(t_0)$ we calculate $\Lambda_1 = 1.05 \times 10^{-35} \text{s}^{-2}$ that differs from $\Lambda = 3H^2(t_0) \Omega_{\Lambda} = 0.98 \times 10^{-35} \text{s}^{-2}$ (by ΛCDM model).

We also computed

$$\begin{split} \ddot{a}_1(t_0)/a_1(t_0) &= 2.7 \times 10^{-36} s^{-2} \\ R(t_0) &= 4.5 \times 10^{-35} s^{-2} \\ R(t_0) - 2\Lambda &= 2.4 \times 10^{-35} s^{-2}. \end{split}$$

• Replacing solution $a_1(t)$ with k = 0, Friedman equations give

$$\bar{\rho}_1(t) = \frac{3}{8\pi G} \Big(H_1^2(t) - \frac{\Lambda_1}{3} \Big) = \frac{3}{8\pi G} \Big(\frac{4}{9} t^{-2} - \frac{1}{7} \Lambda_1 + \frac{1}{49} \Lambda_1^2 t^2 \Big),$$
$$\bar{\rho}_1(t) = \frac{\Lambda_1}{56\pi G} \Big(1 - \frac{3}{7} \Lambda_1 t^2 \Big).$$

< ロ > < 同 > < 回 > < 回 > .

• From,

$$H_1(t) = \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

taking $H_1(t_0) = H(t_0)$ we calculate $\Lambda_1 = 1.05 \times 10^{-35} s^{-2}$ that differs from $\Lambda = 3H^2(t_0) \Omega_{\Lambda} = 0.98 \times 10^{-35} s^{-2}$ (by ΛCDM model).

We also computed

$$\begin{split} \ddot{a}_1(t_0)/a_1(t_0) &= 2.7 \times 10^{-36} s^{-2} \\ R(t_0) &= 4.5 \times 10^{-35} s^{-2} \\ R(t_0) - 2\Lambda &= 2.4 \times 10^{-35} s^{-2}. \end{split}$$

• Replacing solution $a_1(t)$ with k = 0, Friedman equations give

$$\bar{\rho}_{1}(t) = \frac{3}{8\pi G} \Big(H_{1}^{2}(t) - \frac{\Lambda_{1}}{3} \Big) = \frac{3}{8\pi G} \Big(\frac{4}{9} t^{-2} - \frac{1}{7} \Lambda_{1} + \frac{1}{49} \Lambda_{1}^{2} t^{2} \Big),$$
$$\bar{\rho}_{1}(t) = \frac{\Lambda_{1}}{56\pi G} \Big(1 - \frac{3}{7} \Lambda_{1} t^{2} \Big).$$

< 同 > < 三 > < 三 > -

• For $t = t_0$, from previous formula, and from ACDM model we have $\bar{\rho}_1(t_0) = 2.26 \times 10^{-30} \frac{g}{cm^3}$, $\rho(t_0) = \frac{3}{8\pi G} \left(H_0^2 - \frac{\Lambda}{3}\right) = 2.68 \times 10^{-30} \frac{g}{cm^3}$.

 Then, for vacuum energy density of background solution a₁(t) and ACDM model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_{\Lambda} = 0.42 \times 10^{-30} \frac{g}{cm^3},$$

• Critical energy density: $ho_c = rac{3 H_0^2}{8 \pi G} = 8.51 imes 10^{-30} rac{g}{cm^3}$

• and consequently,

$$\Omega_{\Lambda_{1}} = \frac{\rho_{\Lambda_{1}}}{\rho_{c}} = 0.734, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{c}} = 0.685, \quad \Delta\Omega_{\Lambda} = \Omega_{\Lambda_{1}} - \Omega_{\Lambda} = 0.049, \quad (13)$$
$$\Omega_{m_{1}} = \frac{\bar{\rho}_{1}(t_{0})}{\rho_{c}} = 0.266, \quad \Omega_{m} = \frac{\bar{\rho}(t_{0})}{\rho_{c}} = 0.315, \quad \Delta\Omega_{m} = \Omega_{m} - \Omega_{m_{1}} = 0.049. \quad (14)$$

<ロ> <四> <四> <四> <四> <四> <四> <四> <四> <四</p>

• For $t = t_0$, from previous formula, and from Λ CDM model we have

$$ar{
ho}_1(t_0) = 2.26 imes 10^{-30} \; rac{g}{cm^3},$$
 $ho(t_0) = rac{3}{8\pi G} \Big(H_0^2 - rac{\Lambda}{3} \Big) = 2.68 imes 10^{-30} \; rac{g}{cm^3}.$

• Then, for vacuum energy density of background solution $a_1(t)$ and Λ CDM model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_{\Lambda} = 0.42 \times 10^{-30} \frac{g}{cm^3},$$

• Critical energy density: $\rho_c = \frac{3 H_0^2}{8 \pi G} = 8.51 \times 10^{-30} \frac{g}{cm^3}$

• and consequently,

$$\Omega_{\Lambda_{1}} = \frac{\rho_{\Lambda_{1}}}{\rho_{c}} = 0.734, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{c}} = 0.685, \quad \Delta\Omega_{\Lambda} = \Omega_{\Lambda_{1}} - \Omega_{\Lambda} = 0.049, \quad (13)$$
$$\Omega_{m_{1}} = \frac{\bar{\rho}_{1}(t_{0})}{\rho_{c}} = 0.266, \quad \Omega_{m} = \frac{\rho(t_{0})}{\rho_{c}} = 0.315, \quad \Delta\Omega_{m} = \Omega_{m} - \Omega_{m_{1}} = 0.049. \quad (14)$$

э

• For $t = t_0$, from previous formula, and from ΛCDM model we have

$$ar{
ho}_1(t_0) = 2.26 imes 10^{-30} \ rac{g}{cm^3},$$
 $ho(t_0) = rac{3}{8\pi G} \Big(H_0^2 - rac{\Lambda}{3} \Big) = 2.68 imes 10^{-30} \ rac{g}{cm^3}$

• Then, for vacuum energy density of background solution $a_1(t)$ and Λ CDM model, we have

$$ho(t_0) - \bar{
ho}_1(t_0) = rac{\Lambda_1 - \Lambda}{8\pi G} =
ho_{\Lambda_1} -
ho_{\Lambda} = 0.42 imes 10^{-30} \ rac{g}{cm^3},$$

• Critical energy density: $\rho_c = \frac{3 H_0^2}{8 \pi G} = 8.51 \times 10^{-30} \frac{g}{cm^3}$

• and consequently,

$$\Omega_{\Lambda_{1}} = \frac{\rho_{\Lambda_{1}}}{\rho_{c}} = 0.734, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{c}} = 0.685, \quad \Delta\Omega_{\Lambda} = \Omega_{\Lambda_{1}} - \Omega_{\Lambda} = 0.049, \quad (13)$$
$$\Omega_{m_{1}} = \frac{\bar{\rho}_{1}(t_{0})}{\rho_{c}} = 0.266, \quad \Omega_{m} = \frac{\rho(t_{0})}{\rho_{c}} = 0.315, \quad \Delta\Omega_{m} = \Omega_{m} - \Omega_{m_{1}} = 0.049. \quad (14)$$

<ロ> <同> <同> < 同> < 同>

э

• For $t = t_0$, from previous formula, and from ΛCDM model we have

$$ar{
ho}_1(t_0) = 2.26 imes 10^{-30} \ rac{g}{cm^3},$$
 $ho(t_0) = rac{3}{8\pi G} \Big(H_0^2 - rac{\Lambda}{3} \Big) = 2.68 imes 10^{-30} \ rac{g}{cm^3}$

• Then, for vacuum energy density of background solution $a_1(t)$ and Λ CDM model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_{\Lambda} = 0.42 \times 10^{-30} \frac{g}{cm^3},$$

• Critical energy density:
$$\rho_c = \frac{3 H_0^2}{8 \pi G} = 8.51 \times 10^{-30} \frac{g}{cm^3}$$

and consequently,

$$\Omega_{\Lambda_{1}} = \frac{\rho_{\Lambda_{1}}}{\rho_{c}} = 0.734, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{c}} = 0.685, \quad \Delta\Omega_{\Lambda} = \Omega_{\Lambda_{1}} - \Omega_{\Lambda} = 0.049, \quad (13)$$
$$\Omega_{m_{1}} = \frac{\bar{\rho}_{1}(t_{0})}{\rho_{c}} = 0.266, \quad \Omega_{m} = \frac{\rho(t_{0})}{\rho_{c}} = 0.315, \quad \Delta\Omega_{m} = \Omega_{m} - \Omega_{m_{1}} = 0.049. \quad (14)$$

0 2

• For $t = t_0$, from previous formula, and from ΛCDM model we have

$$ar{
ho}_1(t_0) = 2.26 imes 10^{-30} \ rac{g}{cm^3},$$
 $ho(t_0) = rac{3}{8\pi G} \Big(H_0^2 - rac{\Lambda}{3} \Big) = 2.68 imes 10^{-30} \ rac{g}{cm^3}.$

 Then, for vacuum energy density of background solution a₁(t) and **ACDM** model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_{\Lambda} = 0.42 \times 10^{-30} \frac{g}{cm^3},$$

• Critical energy density: $\rho_c = \frac{3H_0^2}{8\pi G} = 8.51 \times 10^{-30} \frac{g}{cm^3}$
• and consequently,

$$\Omega_{\Lambda_{1}} = \frac{\rho_{\Lambda_{1}}}{\rho_{c}} = 0.734, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{c}} = 0.685, \quad \Delta\Omega_{\Lambda} = \Omega_{\Lambda_{1}} - \Omega_{\Lambda} = 0.049, \quad (13)$$
$$\Omega_{m_{1}} = \frac{\bar{\rho}_{1}(t_{0})}{\rho_{c}} = 0.266, \quad \Omega_{m} = \frac{\rho(t_{0})}{\rho_{c}} = 0.315, \quad \Delta\Omega_{m} = \Omega_{m} - \Omega_{m_{1}} = 0.049. \quad (14)$$

• For $t = t_0$, from previous formula, and from ACDM model we have

$$ar{
ho}_1(t_0) = 2.26 imes 10^{-30} \ rac{g}{cm^3},$$
 $ho(t_0) = rac{3}{8\pi G} \Big(H_0^2 - rac{\Lambda}{3} \Big) = 2.68 imes 10^{-30} \ rac{g}{cm^3}.$

• Then, for vacuum energy density of background solution $a_1(t)$ and Λ CDM model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_{\Lambda} = 0.42 \times 10^{-30} \frac{g}{cm^3},$$

• Critical energy density:
$$\rho_c = \frac{3 H_0^2}{8 \pi G} = 8.51 \times 10^{-30} \frac{g}{cm^3}$$

and consequently,

$$\Omega_{\Lambda_{1}} = \frac{\rho_{\Lambda_{1}}}{\rho_{c}} = 0.734, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{c}} = 0.685, \quad \Delta\Omega_{\Lambda} = \Omega_{\Lambda_{1}} - \Omega_{\Lambda} = 0.049, \quad (13)$$
$$\Omega_{m_{1}} = \frac{\bar{\rho}_{1}(t_{0})}{\rho_{c}} = 0.266, \quad \Omega_{m} = \frac{\rho(t_{0})}{\rho_{c}} = 0.315, \quad \Delta\Omega_{m} = \Omega_{m} - \Omega_{m_{1}} = 0.049. \quad (14)$$

- According to (13) and (14), we obtain that Ω_{m1} = 26, 6% corresponds to dark matter and ΔΩ_m = ΔΩ_Λ = 4.9% is related to visible matter, what is in a very good agreement with the standard model of cosmology.
- Effective presure. At the beginning, $\bar{p}_1(0) = \frac{\Lambda_1}{56\pi d} > 0$, then decreases and equals zero at $t = \sqrt{\frac{7}{3\Lambda_1}} = 4.71 \times 10^{17} s = 14,917 \times 10^9 yr$.

we have parameter $\bar{w}_1(t) = \frac{p_1(t)}{\bar{p}_1(t)}$ which has future behavior in agreement with standard model of cosmology, i.e. $\bar{w}_1(t \to \infty) \to -1$.

Note that **Constant of the second se**

According to (13) and (14), we obtain that Ω_{m1} = 26, 6% corresponds to dark matter and ΔΩ_m = ΔΩ_Λ = 4.9% is related to visible matter, what is in a very good agreement with the standard model of cosmology.
 Efective presure. At the beginning, p
₁(0) = Λ₁/_{56πG} > 0, then decreases

and equals zero at $t = \sqrt{\frac{7}{3\Lambda_1}} = 4.71 \times 10^{17} s = 14,917 \times 10^9 yr.$

• According to (11). we have parameter $\bar{w}_1(t) = \frac{\bar{p}_1(t)}{\bar{p}_1(t)}$ which has future behavior in agreement with standard model of cosmology, i.e. $\bar{w}_1(t \to \infty) \to -1$.

Note that the Hubble parameter has minimum at $t_{min} = 21.1 \times 10^9 yr$ and it is $H_1(t_{min}) = 61.72 km/s/Mpc$. It also, follows that the Universe experiences decelerated expansion during matter dominance, that turns to acceleration at time $t_{acc} = 7.84 \times 10^9 yr$ when, $\ddot{a} = 0$.

< ロ > < 同 > < 回 > < 回 > < 回 > <

- According to (13) and (14), we obtain that $\Omega_{m_1} = 26,6\%$ corresponds to dark matter and $\Delta \Omega_m = \Delta \Omega_{\Lambda} = 4.9\%$ is related to visible matter, what is in a very good agreement with the standard model of cosmology.
- Effective presure. At the beginning, $\bar{p}_1(0) = \frac{\Lambda_1}{56\pi G} > 0$, then decreases and equals zero at $t = \sqrt{\frac{7}{3\Lambda_1}} = 4.71 \times 10^{17} s = 14,917 \times 10^9 yr$.

• According to (11). we have parameter $\bar{w}_1(t) = \frac{\bar{p}_1(t)}{\bar{p}_1(t)}$ which has future behavior in agreement with standard model of cosmology, i.e. $\bar{w}_1(t \to \infty) \to -1$.

Note that the Huddle parameter has minimum at $t_{min} = 21.1 \times 10^9 yr$ and it is $H_1(t_{min}) = 61.72 km/s/Mpc$. It also, follows that the Universe experiences decelerated expansion during matter dominance, that turns to acceleration at time $t_{acc} = 7.84 \times 10^9 yr$ when, $\ddot{a} = 0$.

< ロ > < 同 > < 回 > < 回 > .

According to (13) and (14), we obtain that Ω_{m1} = 26, 6% corresponds to dark matter and ΔΩ_m = ΔΩ_Λ = 4.9% is related to visible matter, what is in a very good agreement with the standard model of cosmology.

Efective presure. At the beginning, $\bar{p}_1(0) = \frac{\Lambda_1}{56\pi G} > 0$, then decreases and equals zero at $t = \sqrt{\frac{7}{3\Lambda_1}} = 4.71 \times 10^{17} s = 14,917 \times 10^9 yr$.

• According to (11). we have parameter $\bar{w}_1(t) = \frac{\bar{p}_1(t)}{\bar{p}_1(t)}$ which has future behavior in agreement with standard model of cosmology, i.e. $\bar{w}_1(t \to \infty) \to -1$.

Note that the Hubble parameter has minimum at $t_{min} = 21.1 \times 10^9 yr$ and it is $H_1(t_{min}) = 61.72 km/s/Mpc$. It also, follows that the Universe experiences decelerated expansion during matter dominance, that turns to acceleration at time $t_{acc} = 7.84 \times 10^9 yr$ when, $\ddot{a} = 0$.

< ロ > < 同 > < 回 > < 回 > .

• According to (13) and (14), we obtain that $\Omega_{m_1} = 26,6\%$ corresponds to dark matter and $\Delta\Omega_m = \Delta\Omega_{\Lambda} = 4.9\%$ is related to visible matter, what is in a very good agreement with the standard model of cosmology.

Efective presure. At the beginning, $\bar{p}_1(0) = \frac{\Lambda_1}{56\pi G} > 0$, then decreases and equals zero at $t = \sqrt{\frac{7}{3\Lambda_1}} = 4.71 \times 10^{17} s = 14,917 \times 10^9 yr$.

According to (11). we have parameter $\bar{w}_1(t) = \frac{\bar{p}_1(t)}{\bar{\rho}_1(t)}$ which has future behavior in agreement with standard model of cosmology, i.e. $\bar{w}_1(t \to \infty) \to -1$.

Note that the Hubble parameter has minimum at $t_{min} = 21.1 \times 10^9 yr$ and it is $H_1(t_{min}) = 61.72 km/s/Mpc$. It also, follows that the Universe experiences decelerated expansion during matter dominance, that turns to acceleration at time $t_{acc} = 7.84 \times 10^9 yr$ when, $\ddot{a} = 0$.

• According to (13) and (14), we obtain that $\Omega_{m_1} = 26,6\%$ corresponds to dark matter and $\Delta \Omega_m = \Delta \Omega_{\Lambda} = 4.9\%$ is related to visible matter, what is in a very good agreement with the standard model of cosmology.

Efective presure. At the beginning, $\bar{p}_1(0) = \frac{\Lambda_1}{56\pi G} > 0$, then decreases and equals zero at $t = \sqrt{\frac{7}{3\Lambda_1}} = 4.71 \times 10^{17} s = 14,917 \times 10^9 yr$.

According to (11). we have parameter $\bar{w}_1(t) = \frac{\bar{p}_1(t)}{\bar{p}_1(t)}$ which has future behavior in agreement with standard model of cosmology, i.e. $\bar{w}_1(t \to \infty) \to -1$.

Note that • the Hubble parameter has minimum at $t_{min} = 21.1 \times 10^9 yr$ and it is $H_1(t_{min}) = 61.72 km/s/Mpc$. It also, follows that the Universe experiences decelerated expansion during matter dominance, that turns to acceleration at time $t_{acc} = 7.84 \times 10^9 yr$ when, $\ddot{a} = 0$.

$$ds^{2} = -A(r)dt^{2} + B(r)dr^{2} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta \,d\varphi^{2}.$$
 (15)

The corresponding scalar curvature *R* of above metric (15) $R = \frac{2}{r^2} - \frac{2}{r^2 B(r)} - \frac{2A'(r)}{rA(r)B(r)} + \frac{A'(r)^2}{2A(r)^2 B(r)} + \frac{2B'(r)}{rB^2(r)} + \frac{A'(r)B'(r)}{2A(r)B(r)^2} - \frac{A''(r)}{A(r)B(r)} \quad (16)$ We will consider **1** $B(r) = A(r)^{-1}$, and formula (16) becomes $R = \frac{2 - 2A(r) - 4rA'(r) - r^2A''(r)}{r^2} = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} [r^2(1 - A(r))]. \quad (17)$

(0)

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2. \bullet \mathrm{SdS\,\,metric\,\,GC}$$
(15)

The corresponding scalar curvature *R* of above metric (15) $R = \frac{2}{r^2} - \frac{2}{r^2 B(r)} - \frac{2A'(r)}{rA(r)B(r)} + \frac{A'(r)^2}{2A(r)^2 B(r)} + \frac{2B'(r)}{rB^2(r)} + \frac{A'(r)B'(r)}{2A(r)B(r)^2} - \frac{A''(r)}{A(r)B(r)}$ (16)

We will consider \bullet the case $B(r) = A(r)^{-1}$, and formula (16) becomes

$$R = \frac{2 - 2A(r) - 4rA'(r) - r^2A''(r)}{r^2} = \frac{1}{r^2}\frac{\partial^2}{\partial r^2} [r^2(1 - A(r))].$$
(17)

・ 同 ト ・ ヨ ト ・ ヨ ト

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2. \quad \text{SdS metric-GC}$$
(15)

The corresponding scalar curvature *R* of above metric (15) $R = \frac{2}{r^2} - \frac{2}{r^2 B(r)} - \frac{2A'(r)}{rA(r)B(r)} + \frac{A'(r)^2}{2A(r)^2 B(r)} + \frac{2B'(r)}{rB^2(r)} + \frac{A'(r)B'(r)}{2A(r)B(r)^2} - \frac{A''(r)}{A(r)B(r)}$ (16) We will consider r the case $B(r) = A(r)^{-1}$, and formula (16) becomes $R = \frac{2 - 2A(r) - 4rA'(r) - r^2A''(r)}{r^2} = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} [r^2(1 - A(r))].$ (17)

<<p>(日)

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2. \qquad (15)$$

The corresponding scalar curvature *R* of above metric (15) $R = \frac{2}{r^2} - \frac{2}{r^2 B(r)} - \frac{2A'(r)}{rA(r)B(r)} + \frac{A'(r)^2}{2A(r)^2 B(r)} + \frac{2B'(r)}{rB^2(r)} + \frac{A'(r)B'(r)}{2A(r)B(r)^2} - \frac{A''(r)}{A(r)B(r)}$ (16) We will consider **Constant** $B(r) = A(r)^{-1}$, and formula (16) becomes $R = \frac{2 - 2A(r) - 4rA'(r) - r^2A''(r)}{r^2} = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} [r^2(1 - A(r))].$ (17)

<<p>(日)
We want to investigate our model outside the spherically symmetric massive body - it is natural to consider a generalization of the Schwarzschild-de Sitter (SdS) metric starting from the standard Schwarzschild expression,

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2. \, \bullet \, \mathrm{SdS \,\,metric-GC}$$
(15)

The corresponding scalar curvature *R* of above metric (15) $R = \frac{2}{r^2} - \frac{2}{r^2 B(r)} - \frac{2A'(r)}{rA(r)B(r)} + \frac{A'(r)^2}{2A(r)^2 B(r)} + \frac{2B'(r)}{rB^2(r)} + \frac{A'(r)B'(r)}{2A(r)B(r)^2} - \frac{A''(r)}{A(r)B(r)}$ (16)

We will consider \triangleright the case $B(r) = A(r)^{-1}$, and formula (16) becomes

$$R = \frac{2 - 2A(r) - 4rA'(r) - r^2A''(r)}{r^2} = \frac{1}{r^2}\frac{\partial^2}{\partial r^2}[r^2(1 - A(r))].$$
 (17)

> < 同 > < 三 > < 三 > < 三 > < ○</p>

Firstly, we have to solve an eigenvalue problem,

 $\Box \sqrt{R} - 2\Lambda = q \sqrt{R} - 2\Lambda,$

where d'Alembertian \Box acts in the following way:

$$\Box u(r) = A(r) u''(r) + (A'(r) + \frac{2}{r} A(r)) u'(r) = \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 A(r) \frac{\partial u}{\partial r}], \quad (18)$$

where u(r) is any differentiable scalar function.

We consider function A(r) in the form

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda}{3}r^2 - f(r),$$
(19)

where μ and u are some parameters



$$R(r) = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} [r^2 (1 - A(r))] = 4\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} [r^2 f(r)].$$
(20)

・ロ・・(型・・目・・(目・)

Firstly, we have to solve an eigenvalue problem,

 $\Box\sqrt{R-2\Lambda}=q\sqrt{R-2\Lambda}$

where d'Alembertian \Box acts in the following way:

$$\Box u(r) = A(r) u''(r) + (A'(r) + \frac{2}{r} A(r)) u'(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 A(r) \frac{\partial u}{\partial r} \right], \quad (18)$$

where u(r) is any differentiable scalar function.

We consider function A(r) in the form

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda}{3}r^2 - f(r),$$
(19)

where μ and u are some parameters.

Then one can show that for A(r) given by (19) holds

$$R(r) = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left[r^2 \left(1 - A(r) \right) \right] = 4\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left[r^2 f(r) \right].$$
(20)

I Firstly, we have to solve an eigenvalue problem,

 $\Box\sqrt{R-2\Lambda}=q\sqrt{R-2\Lambda},$

where d'Alembertian \Box acts in the following way:

$$\Box u(r) = A(r) u''(r) + (A'(r) + \frac{2}{r} A(r)) u'(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 A(r) \frac{\partial u}{\partial r} \right], \quad (18)$$

where u(r) is any differentiable scalar function.

We consider function A(r) in the form

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda}{3}r^2 - f(r),$$
(19)

where μ and u are some parameters.

Then one can show that for A(r) given by (19) holds

$$R(r) = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} [r^2 (1 - A(r))] = 4\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} [r^2 f(r)].$$
(20)

< ロ > < 同 > < 回 > < 回 > < 回 > <

I Firstly, we have to solve an eigenvalue problem,

 $\Box\sqrt{R-2\Lambda}=q\sqrt{R-2\Lambda},$

where d'Alembertian \Box acts in the following way:

$$\Box u(r) = A(r) u''(r) + (A'(r) + \frac{2}{r} A(r)) u'(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 A(r) \frac{\partial u}{\partial r} \right], \quad (18)$$

where u(r) is any differentiable scalar function.

We consider function A(r) in the form

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda}{3}r^2 - f(r),$$
(19)

where μ and ν are some parameters.

Then one can show that for A(r) given by (19) holds

$$R(r) = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left[r^2 (1 - A(r)) \right] = 4\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left[r^2 f(r) \right].$$
(20)

A = A = A = A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A
 A = A

I Firstly, we have to solve an eigenvalue problem,

 $\Box\sqrt{R-2\Lambda}=q\sqrt{R-2\Lambda},$

where d'Alembertian \Box acts in the following way:

$$\Box u(r) = A(r) u''(r) + (A'(r) + \frac{2}{r} A(r)) u'(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 A(r) \frac{\partial u}{\partial r} \right], \quad (18)$$

where u(r) is any differentiable scalar function.

We consider function A(r) in the form

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda}{3}r^2 - f(r),$$
(19)

where μ and ν are some parameters.

Then one can show that for A(r) given by (19) holds

$$R(r) = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left[r^2 (1 - A(r)) \right] = 4\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left[r^2 f(r) \right].$$
(20)

< ロ > < 同 > < 回 > < 回 > < 回 > <

$$\Box \sqrt{R-2\Lambda} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 A(r) \frac{\partial}{\partial r} \sqrt{R-2\Lambda} \right] = q \sqrt{R-2\Lambda}.$$
 (21)

 Since equation (21) is very complicated and it is very hard to find exact solution, we search for an approximative solution, and we take A(r) ≈ 1 in (21), what is applicable when

$$\left|\frac{\mu}{r}\right| \ll 1, \quad \left|\frac{\nu}{r^2}\right| \ll 1, \quad |\Lambda r^2| \ll 1, \quad |f(r)| \ll 1.$$
 (22)

Under conditions (22), equation (21) becomes

$$\Delta\sqrt{R-2\Lambda} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial}{\partial r} \sqrt{R-2\Lambda} \right] = q \sqrt{R-2\Lambda}, \tag{23}$$

where \triangle is the Laplace operator (Laplacian) in spherical coordinates.

Thee general solution of equation

$$\Delta u(t) = \frac{\partial^2 u(t)}{\partial t^2} + \frac{2}{r} \frac{\partial u(t)}{\partial r} = q u(t) \quad \text{is}$$
(24)

・ロ・・(型・・目・・(目・)

$$\Box \sqrt{R-2\Lambda} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 A(r) \frac{\partial}{\partial r} \sqrt{R-2\Lambda} \right] = q \sqrt{R-2\Lambda}.$$
 (21)

• Since equation (21) is very complicated and it is very hard to find exact solution, we search for an approximative solution, and we take $A(r) \approx 1$ in (21), what is applicable when

$$\frac{\mu}{r} | \ll 1, \quad |\frac{\nu}{r^2} | \ll 1, \quad |\Lambda r^2| \ll 1, \quad |f(r)| \ll 1.$$
 (22)

Under conditions (22), equation (21) becomes

where \triangle is the Laplace operator (Laplacian) in spherical coordinates.

Thee general solution of equation

$$\Delta u(r) = \frac{\partial^2 u(r)}{\partial r^2} + \frac{2}{r} \frac{\partial u(r)}{\partial r} = q u(r) \quad \text{is} \quad (24)$$

$$\Box \sqrt{R-2\Lambda} = \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 A(r) \frac{\partial}{\partial r} \sqrt{R-2\Lambda}] = q \sqrt{R-2\Lambda}.$$
 (21)

• Since equation (21) is very complicated and it is very hard to find exact solution, we search for an approximative solution, and we take $A(r) \approx 1$ in (21), what is applicable when

$$\left|\frac{\mu}{r}\right| \ll 1, \quad \left|\frac{\nu}{r^2}\right| \ll 1, \quad |\Lambda r^2| \ll 1, \quad |f(r)| \ll 1.$$
 (22)

Under conditions (22), equation (21) becomes

where \triangle is the Laplace operator (Laplacian) in spherical coordinates.

Thee general solution of equation

$$\Delta u(r) = \frac{\partial^2 u(r)}{\partial r^2} + \frac{2}{r} \frac{\partial u(r)}{\partial r} = q u(r) \quad \text{is} \quad (24)$$

・ロン ・ 日 ・ ・ 日 ・ ・ 日 ・

$$\Box \sqrt{R-2\Lambda} = \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 A(r) \frac{\partial}{\partial r} \sqrt{R-2\Lambda}] = q \sqrt{R-2\Lambda}.$$
 (21)

Since equation (21) is very complicated and it is very hard to find exact solution, we search for an approximative solution, and we take A(r) ≈ 1 in (21), what is applicable when

$$\left|\frac{\mu}{r}\right| \ll 1, \quad \left|\frac{\nu}{r^2}\right| \ll 1, \quad |\Lambda r^2| \ll 1, \quad |f(r)| \ll 1.$$
 (22)

Under conditions (22), equation (21) becomes

where \triangle is the Laplace operator (Laplacian) in spherical coordinates.

Thee general solution of equation

$$\Delta u(r) = \frac{\partial^2 u(r)}{\partial r^2} + \frac{2}{r} \frac{\partial u(r)}{\partial r} = q u(r) \quad \text{is} \quad (24)$$

< ロ > < 同 > < 回 > < 回 > < 回 > <

$$\Box \sqrt{R-2\Lambda} = \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 A(r) \frac{\partial}{\partial r} \sqrt{R-2\Lambda}] = q \sqrt{R-2\Lambda}.$$
 (21)

Since equation (21) is very complicated and it is very hard to find exact solution, we search for an approximative solution, and we take A(r) ≈ 1 in (21), what is applicable when

$$\frac{\mu}{r} | \ll 1, \quad |\frac{\nu}{r^2} | \ll 1, \quad |\Lambda r^2| \ll 1, \quad |f(r)| \ll 1.$$
 (22)

Under conditions (22), equation (21) becomes

where \bigtriangleup is the Laplace operator (Laplacian) in spherical coordinates.

Thee general solution of equation

$$\Delta u(r) = \frac{\partial^2 u(r)}{\partial r^2} + \frac{2}{r} \frac{\partial u(r)}{\partial r} = q u(r) \quad \text{is} \quad (24)$$

< ロ > < 同 > < 回 > < 回 > .

$$\Box \sqrt{R-2\Lambda} = \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 A(r) \frac{\partial}{\partial r} \sqrt{R-2\Lambda}] = q \sqrt{R-2\Lambda}.$$
 (21)

• Since equation (21) is very complicated and it is very hard to find exact solution, we search for an approximative solution, and we take $A(r) \approx 1$ in (21), what is applicable when

$$\frac{\mu}{r} | \ll 1, \quad |\frac{\nu}{r^2} | \ll 1, \quad |\Lambda r^2| \ll 1, \quad |f(r)| \ll 1.$$
 (22)

Under conditions (22), equation (21) becomes

where \triangle is the Laplace operator (Laplacian) in spherical coordinates.

Thee general solution of equation

$$\Delta u(r) = \frac{\partial^2 u(r)}{\partial r^2} + \frac{2}{r} \frac{\partial u(r)}{\partial r} = q u(r) \quad \text{is} \quad (24)$$

< ロ > < 同 > < 回 > < 回 > < 回 > <

$$u(r) = \frac{C_1}{r} e^{\sqrt{q} r} + \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (25)

$$u(r) = \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (26)

Then we can rewrite equation (20) in the form

$$R(r) - 2\Lambda = 2\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} [r^2 f(r)] = u^2(r).$$
 (27)

or in more details the equation (27) is equivalent to

$$(28)^{2}f''(r) + 4rf'(r) + 2f(r) = -2\Lambda r^{2} + C_{2}^{2} e^{-2\sqrt{q} r}.$$

General solution of equation (28) is

$$(r) = -\frac{1}{6}r^{2} + \frac{1}{4q}r^{2}e^{-2\sqrt{q}r} + \frac{1}{7}e^{-2\sqrt{q}r} + \frac{1}{7}e^{-2\sqrt{q}r}$$
(29)
$$(-2) + (-$$

$$u(r) = \frac{C_1}{r} e^{\sqrt{q} r} + \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (25)

$$u(r) = \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (26)

Then we can rewrite equation (20) in the form

$$R(r) - 2\Lambda = 2\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left[r^2 f(r) \right] = u^2(r).$$
(27)

or in more details the equation (27) is equivalent to

$$r^{2}f''(r) + 4rf'(r) + 2f(r) = -2\Lambda r^{2} + C_{2}^{2} e^{-2\sqrt{q} r}.$$
 (28)

General solution of equation (28) is

$$f(r) = -\frac{\Lambda}{6}r^2 + \frac{C_2^2}{4q} \frac{1}{r^2} e^{-2\sqrt{q}r} + \frac{C_3}{r} + \frac{C_4}{r^2}.$$
 (29)

$$u(r) = \frac{C_1}{r} e^{\sqrt{q} r} + \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (25)

$$u(r) = \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (26)

Then we can rewrite equation (20) in the form

$$R(r) - 2\Lambda = 2\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} [r^2 f(r)] = u^2(r).$$
(27)

or in more details the equation (27) is equivalent to

$$f^{2}f^{\prime\prime}(r) + 4rf^{\prime}(r) + 2f(r) = -2\Lambda r^{2} + C_{2}^{2} e^{-2\sqrt{q} r}.$$
 (28)

General solution of equation (28) is

$$f(r) = -\frac{\Lambda}{6}r^2 + \frac{C_2^2}{4q} \frac{1}{r^2} e^{-2\sqrt{q}r} + \frac{C_3}{r} + \frac{C_4}{r^2}.$$
 (29)

<ロ> <同> <同> < 同> < 同> < 同>

$$u(r) = \frac{C_1}{r} e^{\sqrt{q} r} + \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (25)

$$u(r) = \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (26)

32

Then we can rewrite equation (20) in the form

$$\mathbf{R}(r) - 2\Lambda = 2\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left[r^2 f(r) \right] = u^2(r).$$
(27)

or in more details the equation (27) is equivalent to

$$r^{2}f^{\prime\prime}(r) + 4rf^{\prime}(r) + 2f(r) = -2\Lambda r^{2} + C_{2}^{2} e^{-2\sqrt{q} r}.$$
(28)

General solution of equation (28) is

$$f(r) = -\frac{\Lambda}{6}r^2 + \frac{C_2^2}{4q} \frac{1}{r^2} e^{-2\sqrt{q}r} + \frac{C_3}{r} + \frac{C_4}{r^2}.$$
 (29)

$$u(r) = \frac{C_1}{r} e^{\sqrt{q} r} + \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (25)

$$u(r) = \frac{C_2}{r} e^{-\sqrt{q} r}.$$
 (26)

Then we can rewrite equation (20) in the form

$$\mathbf{R}(r) - 2\Lambda = 2\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \left[r^2 f(r) \right] = u^2(r).$$
(27)

or in more details the equation (27) is equivalent to

$$r^{2}f^{\prime\prime}(r) + 4rf^{\prime}(r) + 2f(r) = -2\Lambda r^{2} + C_{2}^{2} e^{-2\sqrt{q} r}.$$
(28)

General solution of equation (28) is

$$f(r) = -\frac{\Lambda}{6}r^2 + \frac{C_2^2}{4q} \frac{1}{r^2} e^{-2\sqrt{q}r} + \frac{C_3}{r} + \frac{C_4}{r^2}.$$
 (29)

< ロ > < 同 > < 回 > < 回 > < 回 > <

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda r^2}{6} - \frac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}.$$
 (30)

Then the corresponding scalar curvature becomes

$$R(r) = 2\Lambda + \frac{G_2^2}{r^2} e^{-2\sqrt{q} r}.$$
(31)

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda r^2}{6} - \frac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}.$$
 (30)

Then the corresponding scalar curvature becomes

$$R(r) = 2\Lambda + \frac{C_2^2}{r^2} e^{-2\sqrt{q} r}.$$
(31)

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda r^2}{6} - \frac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}.$$
 (30)

Then the corresponding scalar curvature becomes

$$R(r) = 2\Lambda + \frac{C_2^2}{r^2} e^{-2\sqrt{q} r}.$$
(31)

- コン (雪) (ヨ) (ヨ)

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda r^2}{6} - \frac{C_2^2}{4gr^2} e^{-2\sqrt{q} r}.$$
 (30)

Then the corresponding scalar curvature becomes

$$R(r) = 2\Lambda + \frac{C_2^2}{r^2} e^{-2\sqrt{q} r}.$$
 (31)

It is well known that A(r) of the standard Schwarzschild-de Sitter metric,
 i.e. in the case of local de Sitter gravity, is

$$A_{\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3c^2}, \qquad r \ge r_0,$$
(32)

where r_0 is the radius of spherically symmetric massive body (*M*–mass) The nonlocal version of A(r) can be rewritten as

$$A_{n\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{6c^2} + \frac{\delta^2}{qr^2} \left(1 - e^{-2\sqrt{q} r}\right), \qquad q = \frac{\eta \Lambda}{c^2}, \qquad (33)$$

where we add terms $rac{\nu_2}{r^2}$ and $rac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}$ taking $\delta^2 = rac{C_2^2}{4},
u = -rac{\delta^2}{q}$

- Here δ and η are dimensionless parameters, and their values should be determined by experiments (astronomical observations).
- Testing formula (33) in the Solar and other astronomical systems is one of our main next tasks.

(0)

It is well known that A(r) of the standard Schwarzschild-de Sitter metric,
 i.e. in the case of local de Sitter gravity, is

$$A_{\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3c^2}, \qquad r \ge r_0,$$
 (32)

where r_0 is the radius of spherically symmetric massive body (*M*-mass) • The nonlocal version of A(r) can be rewritten as

$$A_{n\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{6c^2} + \frac{\delta^2}{qr^2} \left(1 - e^{-2\sqrt{q} r}\right), \qquad q = \frac{\eta \Lambda}{c^2}, \qquad (33)$$

where we add terms $\frac{\nu}{r^2}$ and $\frac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}$ taking $\delta^2 = \frac{C_2^2}{4}, \nu = -\frac{\delta^2}{q}$.

- Here δ and η are dimensionless parameters, and their values should be determined by experiments (astronomical observations).
- Testing formula (33) in the Solar and other astronomical systems is one of our main next tasks.

It is well known that A(r) of the standard Schwarzschild-de Sitter metric,
 i.e. in the case of local de Sitter gravity, is

$$A_{\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3c^2}, \qquad r \ge r_0,$$
(32)

where r_0 is the radius of spherically symmetric massive body (*M*-mass) • The nonlocal version of A(r) can be rewritten as

$$A_{n\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{6c^2} + \frac{\delta^2}{qr^2} \left(1 - e^{-2\sqrt{q} r}\right), \qquad q = \frac{\eta \Lambda}{c^2}, \qquad (33)$$

where we add terms $\frac{\nu}{r^2}$ and $\frac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}$ taking $\delta^2 = \frac{C_2^2}{4}, \nu = -\frac{\delta^2}{q}$.

- Here δ and η are dimensionless parameters, and their values should be determined by experiments (astronomical observations).
- Testing formula (33) in the Solar and other astronomical systems is one of our main next tasks.

It is well known that A(r) of the standard Schwarzschild-de Sitter metric,
 i.e. in the case of local de Sitter gravity, is

$$A_{\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3c^2}, \qquad r \ge r_0,$$
 (32)

where r₀ is the radius of spherically symmetric massive body (*M*-mass)
The nonlocal version of A(r) can be rewritten as

$$A_{n\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{6c^2} + \frac{\delta^2}{qr^2} \left(1 - e^{-2\sqrt{q} r}\right), \qquad q = \frac{\eta \Lambda}{c^2}, \qquad (33)$$

where we add terms $\frac{\nu}{r^2}$ and $\frac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}$ taking $\delta^2 = \frac{C_2^2}{4}, \nu = -\frac{\delta^2}{q}$.

- Here δ and η are dimensionless parameters, and their values should be determined by experiments (astronomical observations).
- Testing formula (33) in the Solar and other astronomical systems is one of our main next tasks.

< ロ > < 同 > < 回 > < 回 > < 回 > <

It is well known that A(r) of the standard Schwarzschild-de Sitter metric,
 i.e. in the case of local de Sitter gravity, is

$$A_{\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3c^2}, \qquad r \ge r_0,$$
 (32)

where r_0 is the radius of spherically symmetric massive body (*M*-mass) • The nonlocal version of A(r) can be rewritten as

$$A_{n\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{6c^2} + \frac{\delta^2}{qr^2} (1 - e^{-2\sqrt{q} r}), \qquad q = \frac{\eta \Lambda}{c^2}, \qquad (33)$$

where we add terms $\frac{\nu}{r^2}$ and $\frac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}$ taking $\delta^2 = \frac{C_2^2}{4}, \nu = -\frac{\delta^2}{q}$.

- Here δ and η are dimensionless parameters, and their values should be determined by experiments (astronomical observations).
- Testing formula (33) in the Solar and other astronomical systems is one of our main next tasks.

< ロ > < 同 > < 回 > < 回 > .

It is well known that A(r) of the standard Schwarzschild-de Sitter metric,
 i.e. in the case of local de Sitter gravity, is

$$A_{\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3c^2}, \qquad r \ge r_0,$$
 (32)

where r_0 is the radius of spherically symmetric massive body (*M*-mass) • The nonlocal version of A(r) can be rewritten as

$$A_{n\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{6c^2} + \frac{\delta^2}{qr^2} (1 - e^{-2\sqrt{q} r}), \qquad q = \frac{\eta \Lambda}{c^2}, \qquad (33)$$

where we add terms $\frac{\nu}{r^2}$ and $\frac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}$ taking $\delta^2 = \frac{C_2^2}{4}, \nu = -\frac{\delta^2}{q}$.

- Here δ and η are dimensionless parameters, and their values should be determined by experiments (astronomical observations).
- Testing formula (33) in the Solar and other astronomical systems is one of our main next tasks.

< ロ > < 同 > < 三 > < 三 > -

It is well known that A(r) of the standard Schwarzschild-de Sitter metric,
 i.e. in the case of local de Sitter gravity, is

$$A_{\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3c^2}, \qquad r \ge r_0,$$
 (32)

where r_0 is the radius of spherically symmetric massive body (*M*-mass) • The nonlocal version of A(r) can be rewritten as

$$A_{n\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{6c^2} + \frac{\delta^2}{qr^2} (1 - e^{-2\sqrt{q} r}), \qquad q = \frac{\eta \Lambda}{c^2}, \qquad (33)$$

where we add terms $\frac{\nu}{r^2}$ and $\frac{C_2^2}{4qr^2} e^{-2\sqrt{q} r}$ taking $\delta^2 = \frac{C_2^2}{4}, \nu = -\frac{\delta^2}{q}$.

- Here δ and η are dimensionless parameters, and their values should be determined by experiments (astronomical observations).
- Testing formula (33) in the Solar and other astronomical systems is one of our main next tasks.

- I. Dimitrijevic, B. Dragovich, Z. Rakic, J. Stankovic, *Nonlocal de Sitter gravity and its exact cosmological solutions*, Journal of High Energy Physics 2022 (12), 1-28.
- I. Dimitrijevic, B. Dragovich, A. S. Koshelev, Z. Rakic, J. Stankovic, Cosmological solutions of a nonlocal square root gravity, Phys. Lett. B 797 (2019) 134848, arXiv:1906.07560 [gr-qc].
- I. Dimitrijevic, B. Dragovich, Z. Rakic, J. Stankovic, On the Schwarzschild-de Sitter metric of nonlocal de Sitter gravity, Filomat, 2023, Vol. 37 (25), 8641-8650.
- S. Nojiri, S.D. Odintsov, V. K. Oikonomou, Modified Gravity Theories on a Nutshell: inflation, bounce, and late-time evolution, Phys. Rep. 692 (2017), 1–104.
- I. Dimitrijevic, B. Dragovich, A. S. Koshelev, Z. Rakic, J. Stankovic, Some cosmological solutions of a new nonlocal gravity model, Symmetry 12, 917 (2020), arXiv:2006.16041 [gr-qc].
- I.Dimitrijevic, B.Dragovich, Z.Rakic, and J.Stankovic, New Cosmological Solutions of a Nonlocal Gravity Model. Symmetry (2022) Volume 14 (1), 3.
- I. Dimitrijevic, B. Dragovich, J. Grujic, A.S. Koshelev, Z. Rakic, *Cosmology of modified gravity with a nonlocal f(R),* Filomat 33 (2019) 1163–1178, arXiv:1509.04254[hep=th].
- T. Biswas, T. Koivisto, A. Mazumdar, *Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity*, JCAP **1011** (2010) 008 [arXiv:1005.0590v2 [hep-th]].
- A. S. Koshelev, S. Yu. Vernov, On bouncing solutions in non-local gravity, Phys. Part. Nuclei 43, 666–668 (2012) [arXiv:1202.1289v1 [hep-th]].

THANK YOU FOR

YOUR ATTENTION !!!

Zoran Rakić On the Schwarzschild type metric in nonlocal de Sitter gravity

◆ロ ▶ ◆屈 ▶ ◆ 臣 ▶ ◆ 臣 ● の Q ()

Non-trivial Christoffel symbols of Friedman – Robertson – Walker metric

$$\Gamma_{01}^{1} = \frac{\dot{a}}{a} \qquad \Gamma_{02}^{2} = \frac{\dot{a}}{a} \qquad \Gamma_{03}^{3} = \frac{\dot{a}}{a}$$

$$\Gamma_{11}^{0} = \frac{a\dot{a}}{1 - kr^{2}} \qquad \Gamma_{11}^{1} = \frac{kr}{1 - kr^{2}} \qquad \Gamma_{12}^{2} = \frac{1}{r}$$

$$\Gamma_{13}^{3} = \frac{1}{r}$$

$$\Gamma_{22}^{0} = r^{2}a\dot{a} \qquad \Gamma_{22}^{1} = r(kr^{2} - 1) \qquad \Gamma_{23}^{3} = \cot\theta$$

$$\Gamma_{33}^{0} = r^{2}a\dot{a}\sin^{2}\theta \qquad \Gamma_{33}^{1} = r(kr^{2} - 1)\sin^{2}\theta \qquad \Gamma_{33}^{2} = -\sin\theta\cos\theta$$

・ロト ・四ト ・ヨト ・ヨト

Ъ.

Non-trivial Christoffel symbols of Friedman - Robertson - Walker metric

 $\Gamma_{01}^{1} = \frac{\dot{a}}{a} \qquad \Gamma_{02}^{2} = \frac{\dot{a}}{a} \qquad \Gamma_{03}^{3} = \frac{\dot{a}}{a}$ $\Gamma_{01}^{0} = \frac{a\dot{a}}{1 - kr^{2}} \qquad \Gamma_{11}^{1} = \frac{kr}{1 - kr^{2}} \qquad \Gamma_{12}^{2} = \frac{1}{r}$ $\Gamma_{13}^{3} = \frac{1}{r}$ $\Gamma_{22}^{0} = r^{2}a\dot{a} \qquad \Gamma_{22}^{1} = r(kr^{2} - 1) \qquad \Gamma_{23}^{3} = \cot\theta$ $\Gamma_{33}^{0} = r^{2}a\dot{a}\sin^{2}\theta \qquad \Gamma_{33}^{1} = r(kr^{2} - 1)\sin^{2}\theta \qquad \Gamma_{33}^{2} = -\sin\theta\cos\theta$

Non-trivial Christoffel symbols of Friedman-Robertson-Walker metric

$$\Gamma_{01}^{1} = \frac{\dot{a}}{a} \qquad \Gamma_{02}^{2} = \frac{\dot{a}}{a} \qquad \Gamma_{03}^{3} = \frac{\dot{a}}{a}$$

$$\Gamma_{11}^{0} = \frac{a\dot{a}}{1-kr^{2}} \qquad \Gamma_{11}^{1} = \frac{kr}{1-kr^{2}} \qquad \Gamma_{12}^{2} = \frac{1}{r}$$

$$\Gamma_{13}^{3} = \frac{1}{r}$$

$$\Gamma_{22}^{0} = r^{2}a\dot{a} \qquad \Gamma_{22}^{1} = r(kr^{2}-1) \qquad \Gamma_{23}^{3} = \cot\theta$$

$$\Gamma_{33}^{0} = r^{2}a\dot{a}\sin^{2}\theta \qquad \Gamma_{33}^{1} = r(kr^{2}-1)\sin^{2}\theta \qquad \Gamma_{33}^{2} = -\sin\theta\cos\theta$$

・ロ・・ (日・・ (日・・)

Ъ.

Non-trivial components of curvature tensor

$$R_{0110} = \frac{a\ddot{a}}{1-kr^2} \qquad R_{1221} = -\frac{r^2a^2(\dot{a}^2+k)}{1-kr^2}$$
$$R_{0220} = r^2a\ddot{a} \qquad R_{1331} = -\frac{r^2a^2\sin^2\theta(\dot{a}^2+k)}{1-kr^2}$$
$$R_{0330} = r^2a\ddot{a}\sin^2\theta \qquad R_{2332} = -r^4a^2\sin^2\theta(\dot{a}^2+k)$$

Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3a}{a} & 0 & 0 & 0\\ 0 & u g_{11} & 0 & 0\\ 0 & 0 & u g_{22} & 0\\ 0 & 0 & 0 & u g_{33} \end{pmatrix}, \qquad u = \frac{a\ddot{a} + 2(\dot{a}^2 + k)}{a^2}$$

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

훈.

Non-trivial components of curvature tensor

$$R_{0110} = \frac{\ddot{a}\ddot{a}}{1 - k r^2} \qquad R_{1221} = -\frac{r^2 a^2 (\ddot{a}^2 + k)}{1 - k r^2}$$
$$R_{0220} = r^2 \ddot{a}\ddot{a} \qquad R_{1331} = -\frac{r^2 a^2 \sin^2 \theta (\dot{a}^2 + k)}{1 - k r^2}$$
$$R_{0330} = r^2 \ddot{a}\ddot{a} \sin^2 \theta \qquad R_{2332} = -r^4 a^2 \sin^2 \theta (\dot{a}^2 + k)$$

Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3\ddot{a}}{a} & 0 & 0 & 0\\ 0 & u g_{11} & 0 & 0\\ 0 & 0 & u g_{22} & 0\\ 0 & 0 & 0 & u g_{33} \end{pmatrix}, \qquad u = \frac{\ddot{a}\ddot{a} + 2(\dot{a}^2 + k)}{a^2}$$

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Non-trivial components of curvature tensor

$$R_{0110} = \frac{\ddot{a}\ddot{a}}{1 - k r^2} \qquad R_{1221} = -\frac{r^2 a^2 (\ddot{a}^2 + k)}{1 - k r^2}$$
$$R_{0220} = r^2 \ddot{a}\ddot{a} \qquad R_{1331} = -\frac{r^2 a^2 \sin^2 \theta (\dot{a}^2 + k)}{1 - k r^2}$$
$$R_{0330} = r^2 \ddot{a}\ddot{a} \sin^2 \theta \qquad R_{2332} = -r^4 a^2 \sin^2 \theta (\dot{a}^2 + k)$$

Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3a}{a} & 0 & 0 & 0\\ 0 & u g_{11} & 0 & 0\\ 0 & 0 & u g_{22} & 0\\ 0 & 0 & 0 & u g_{33} \end{pmatrix}, \qquad u = \frac{a\ddot{a} + 2(\dot{a}^2 + k)}{a^2}$$

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <
Non-trivial components of curvature tensor

$$R_{0110} = \frac{\ddot{a}\ddot{a}}{1 - k r^2} \qquad R_{1221} = -\frac{r^2 a^2 (\dot{a}^2 + k)}{1 - k r^2}$$
$$R_{0220} = r^2 \ddot{a}\ddot{a} \qquad R_{1331} = -\frac{r^2 a^2 \sin^2 \theta (\dot{a}^2 + k)}{1 - k r^2}$$
$$R_{0330} = r^2 \ddot{a}\ddot{a} \sin^2 \theta \qquad R_{2332} = -r^4 a^2 \sin^2 \theta (\dot{a}^2 + k)$$

Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3a}{a} & 0 & 0 & 0\\ 0 & u g_{11} & 0 & 0\\ 0 & 0 & u g_{22} & 0\\ 0 & 0 & 0 & u g_{33} \end{pmatrix}, \qquad u = \frac{a\ddot{a} + 2(\dot{a}^2 + k)}{a^2}$$

・ロ・・(型・・目・・(目・)

э.

Non-trivial components of curvature tensor

$$R_{0110} = \frac{\ddot{a}\ddot{a}}{1 - k r^2} \qquad R_{1221} = -\frac{r^2 a^2 (\dot{a}^2 + k)}{1 - k r^2}$$
$$R_{0220} = r^2 \ddot{a}\ddot{a} \qquad R_{1331} = -\frac{r^2 a^2 \sin^2 \theta (\dot{a}^2 + k)}{1 - k r^2}$$
$$R_{0330} = r^2 \ddot{a}\ddot{a} \sin^2 \theta \qquad R_{2332} = -r^4 a^2 \sin^2 \theta (\dot{a}^2 + k)$$

Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3a}{a} & 0 & 0 & 0\\ 0 & u g_{11} & 0 & 0\\ 0 & 0 & u g_{22} & 0\\ 0 & 0 & 0 & u g_{33} \end{pmatrix}, \qquad u = \frac{a\ddot{a} + 2(\dot{a}^2 + k)}{a^2}$$

< □ > < □ > < □ > < □ > < □ > <

э.

$$R = \frac{6\left(a\ddot{a} + \dot{a}^2 + k\right)}{a^2}$$

Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3(\dot{a}^2 + k)}{a^2} & 0 & 0 & 0\\ 0 & -v g_{11} & 0 & 0\\ 0 & 0 & -v g_{22} & 0\\ 0 & 0 & 0 & -v g_{33} \end{pmatrix}, \qquad v = \frac{2 \, a \ddot{a} + \dot{a}^2 + k}{a^2}$$

<ロ> <同> <同> < 同> < 同> < 三> < 三> <

÷.

$$R = \frac{6(a\ddot{a} + \dot{a}^2 + k)}{a^2}$$

Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3(\dot{a}^2+k)}{a^2} & 0 & 0 & 0\\ 0 & -v g_{11} & 0 & 0\\ 0 & 0 & -v g_{22} & 0\\ 0 & 0 & 0 & -v g_{33} \end{pmatrix}, \qquad v = \frac{2 \, a \ddot{a} + \ddot{a}^2 + k}{a^2}$$

<ロ> <同> <同> < 同> < 同> < 三> < 三> <

÷.

$$R = \frac{6\left(a\ddot{a} + \dot{a}^2 + k\right)}{a^2}$$

Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3(\dot{a}^2+k)}{a^2} & 0 & 0 & 0\\ 0 & -v g_{11} & 0 & 0\\ 0 & 0 & -v g_{22} & 0\\ 0 & 0 & 0 & -v g_{33} \end{pmatrix}, \qquad v = \frac{2 \, a \ddot{a} + \dot{a}^2 + k}{a^2}$$

$$R = \frac{6\left(a\ddot{a} + \dot{a}^2 + k\right)}{a^2}$$

Einstein tensor



$$R = \frac{6\left(a\ddot{a} + \dot{a}^2 + k\right)}{a^2}$$

Einstein tensor



Non-trivial Christoffel symbols of Schwarzshield-de Sitter type metric

$$\Gamma_{01}^{0} = \frac{1}{2} \frac{A'}{A}, \qquad \Gamma_{00}^{1} = \frac{1}{2} \frac{A'}{B}, \qquad \Gamma_{11}^{1} = \frac{1}{2} \frac{B'}{B},$$

$$\Gamma_{22}^{1} = -\frac{r}{B}, \qquad \Gamma_{33}^{1} = -\frac{r \sin^{2} \theta}{B}, \qquad \Gamma_{12}^{2} = \frac{1}{r},$$

$$\Gamma_{33}^{2} = -\sin \theta \cos \theta, \qquad \Gamma_{13}^{3} = \frac{1}{r}, \qquad \Gamma_{23}^{3} = \cot \theta.$$

Non-trivial components of curvature tensor

$$R_{0101} = \frac{A}{4} \left(-\left(\frac{A'}{A}\right)^2 - \frac{A'}{A}\frac{B'}{B} + 2\frac{A''}{A} \right), \qquad R_{0202} = \frac{r}{2}\frac{A'}{B},$$

$$R_{0303} = \frac{r}{2}\frac{A'}{B}\sin^2\theta, \qquad R_{1212} = \frac{r}{2}\frac{B'}{B},$$

$$R_{1313} = \frac{r}{2}\frac{B'}{B}\sin^2\theta, \qquad R_{2323} = r^2\frac{B-1}{B}\sin^2\theta$$

< 同 > < 三 > < 三 >

Non-trivial Christoffel symbols of Schwarzshield-de Sitter type metric

$$\Gamma_{01}^{0} = \frac{1}{2} \frac{A'}{A}, \qquad \Gamma_{00}^{1} = \frac{1}{2} \frac{A'}{B}, \qquad \Gamma_{11}^{1} = \frac{1}{2} \frac{B'}{B},$$

$$\Gamma_{22}^{1} = -\frac{r}{B}, \qquad \Gamma_{33}^{1} = -\frac{r \sin^{2} \theta}{B}, \qquad \Gamma_{12}^{2} = \frac{1}{r},$$

$$\Gamma_{33}^{2} = -\sin \theta \cos \theta, \qquad \Gamma_{13}^{3} = \frac{1}{r}, \qquad \Gamma_{23}^{3} = \cot \theta.$$

Non-trivial components of curvature tensor

$$R_{0101} = \frac{A}{4} \left(-\left(\frac{A'}{A}\right)^2 - \frac{A'}{A}\frac{B'}{B} + 2\frac{A''}{A} \right), \qquad R_{0202} = \frac{r}{2}\frac{A'}{B},$$

$$R_{0303} = \frac{r}{2}\frac{A'}{B}\sin^2\theta, \qquad R_{1212} = \frac{r}{2}\frac{B'}{B},$$

$$R_{1313} = \frac{r}{2}\frac{B'}{B}\sin^2\theta, \qquad R_{2323} = r^2\frac{B-1}{B}\sin^2\theta$$

< 同 > < 三 > < 三 >

Non-trivial Christoffel symbols of Schwarzshield-de Sitter type metric

$$\begin{split} \Gamma^{0}_{01} &= \frac{1}{2} \frac{A'}{A}, & \Gamma^{1}_{00} &= \frac{1}{2} \frac{A'}{B}, & \Gamma^{1}_{11} &= \frac{1}{2} \frac{B'}{B}, \\ \Gamma^{1}_{22} &= -\frac{r}{B}, & \Gamma^{1}_{33} &= -\frac{r \sin^{2} \theta}{B}, & \Gamma^{2}_{12} &= \frac{1}{r}, \\ \Gamma^{2}_{33} &= -\sin \theta \cos \theta, & \Gamma^{3}_{13} &= \frac{1}{r}, & \Gamma^{3}_{23} &= \cot \theta. \end{split}$$

Non-trivial components of curvature tensor

$$R_{0101} = \frac{A}{4} \left(-\left(\frac{A'}{A}\right)^2 - \frac{A'}{A}\frac{B'}{B} + 2\frac{A''}{A} \right), \qquad R_{0202} = \frac{r}{2}\frac{A'}{B},$$

$$R_{0303} = \frac{r}{2}\frac{A'}{B}\sin^2\theta, \qquad R_{1212} = \frac{r}{2}\frac{B'}{B},$$

$$R_{1313} = \frac{r}{2}\frac{B'}{B}\sin^2\theta, \qquad R_{2323} = r^2\frac{B-1}{B}\sin^2\theta.$$

A (1) > A (2) > A (2)

Non-trivial Christoffel symbols of Schwarzshield-de Sitter type metric

$$\begin{split} \Gamma_{01}^{0} &= \frac{1}{2} \frac{A'}{A}, & \Gamma_{00}^{1} &= \frac{1}{2} \frac{A'}{B}, & \Gamma_{11}^{1} &= \frac{1}{2} \frac{B'}{B}, \\ \Gamma_{22}^{1} &= -\frac{r}{B}, & \Gamma_{33}^{1} &= -\frac{r \sin^{2} \theta}{B}, & \Gamma_{12}^{2} &= \frac{1}{r}, \\ \Gamma_{33}^{2} &= -\sin \theta \cos \theta, & \Gamma_{13}^{3} &= \frac{1}{r}, & \Gamma_{23}^{3} &= \cot \theta. \end{split}$$

Non-trivial components of curvature tensor

$$\begin{aligned} R_{0101} &= \frac{A}{4} \left(-\left(\frac{A'}{A}\right)^2 - \frac{A'}{A} \frac{B'}{B} + 2\frac{A''}{A} \right), \qquad R_{0202} &= \frac{r}{2} \frac{A'}{B}, \\ R_{0303} &= \frac{r}{2} \frac{A'}{B} \sin^2 \theta, \qquad \qquad R_{1212} &= \frac{r}{2} \frac{B'}{B}, \\ R_{1313} &= \frac{r}{2} \frac{B'}{B} \sin^2 \theta, \qquad \qquad R_{2323} &= r^2 \frac{B-1}{B} \sin^2 \theta. \end{aligned}$$

The Ricci tensor is diagonal and its components are:

$$\begin{aligned} R_{00} &= \frac{A''}{2B} - \frac{A'B'}{4B^2} - \frac{A'^2}{4AB} + \frac{A'}{rB}, \qquad R_{11} = -\frac{A''}{2A} + \frac{A'B'}{4A(r)B(r)} + \frac{A'^2}{4A^2} + \frac{B'}{r}, \\ R_{22} &= -\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1, \qquad R_{33} = \left(-\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1\right)\sin^2\theta. \end{aligned}$$

The scalar curvature is

$$R = -\frac{A''}{AB} + \frac{A'B'}{2AB^2} + \frac{A'^2}{2A^2B} - \frac{2A'}{rAB} + \frac{2B'}{rB^2} - \frac{2}{r^2B} + \frac{2}{r^2}$$

The Einstein tensor is diagonal and its components are

$$G_{00} = \frac{AB'}{rB^2} - \frac{A}{r^2B} + \frac{A}{r^2}, \qquad G_{22} = \frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2},$$
$$G_{11} = \frac{A'}{rA} - \frac{B}{r^2} + \frac{1}{r^2}, \qquad G_{33} = \left(\frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2}\right)\sin^2\theta.$$

SdS metric-GC

< D > < P > < E > < E</p>

$$\begin{aligned} R_{00} &= \frac{A''}{2B} - \frac{A'B'}{4B^2} - \frac{A'^2}{4AB} + \frac{A'}{rB}, \qquad R_{11} = -\frac{A''}{2A} + \frac{A'B'}{4A(r)B(r)} + \frac{A'^2}{4A^2} + \frac{B'}{r}, \\ R_{22} &= -\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1, \qquad R_{33} = \left(-\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1\right)\sin^2\theta. \end{aligned}$$

The scalar curvature is

$$R = -\frac{A''}{AB} + \frac{A'B'}{2AB^2} + \frac{A'^2}{2A^2B} - \frac{2A'}{rAB} + \frac{2B'}{rB^2} - \frac{2}{r^2B} + \frac{2}{r^2}$$

The Einstein tensor is diagonal and its components are

$$G_{00} = \frac{AB'}{rB^2} - \frac{A}{r^2B} + \frac{A}{r^2}, \qquad G_{22} = \frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2},$$
$$G_{11} = \frac{A'}{rA} - \frac{B}{r^2} + \frac{1}{r^2}, \qquad G_{33} = \left(\frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2}\right)\sin^2\theta.$$

SdS metric-GC

< ロ > < 同 > < 三 > < 三 >

The Ricci tensor is diagonal and its components are:

$$\begin{aligned} R_{00} &= \frac{A''}{2B} - \frac{A'B'}{4B^2} - \frac{A'^2}{4AB} + \frac{A'}{rB}, \qquad R_{11} = -\frac{A''}{2A} + \frac{A'B'}{4A(r)B(r)} + \frac{A'^2}{4A^2} + \frac{B'}{r}, \\ R_{22} &= -\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1, \qquad R_{33} = \left(-\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1\right)\sin^2\theta. \end{aligned}$$

The scalar curvature is

$$R = -\frac{A''}{AB} + \frac{A'B'}{2AB^2} + \frac{A'^2}{2A^2B} - \frac{2A'}{rAB} + \frac{2B'}{rB^2} - \frac{2}{r^2B} + \frac{2}{r^2}$$

The Einstein tensor is diagonal and its components are

$$G_{00} = \frac{AB'}{rB^2} - \frac{A}{r^2B} + \frac{A}{r^2}, \qquad G_{22} = \frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2},$$
$$G_{11} = \frac{A'}{rA} - \frac{B}{r^2} + \frac{1}{r^2}, \qquad G_{33} = \left(\frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2}\right)\sin^2\theta.$$

SdS metric-GC

< < >> < <</p>

2a

The Ricci tensor is diagonal and its components are:

$$R_{00} = \frac{A''}{2B} - \frac{A'B'}{4B^2} - \frac{A'^2}{4AB} + \frac{A'}{rB}, \qquad R_{11} = -\frac{A''}{2A} + \frac{A'B'}{4A(r)B(r)} + \frac{A'^2}{4A^2} + \frac{B'}{r},$$
$$R_{22} = -\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1, \qquad R_{33} = \left(-\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1\right)\sin^2\theta.$$

The scalar curvature is

$$R = -\frac{A''}{AB} + \frac{A'B'}{2AB^2} + \frac{A'^2}{2A^2B} - \frac{2A'}{rAB} + \frac{2B'}{rB^2} - \frac{2}{r^2B} + \frac{2}{r^2}$$

The Einstein tensor is diagonal and its components are

$$G_{00} = \frac{AB'}{rB^2} - \frac{A}{r^2B} + \frac{A}{r^2}, \qquad G_{22} = \frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2},$$
$$G_{11} = \frac{A'}{rA} - \frac{B}{r^2} + \frac{1}{r^2}, \qquad G_{33} = \left(\frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2}\right)\sin^2\theta.$$

SdS metric-GC

< ロ > < 同 > < 三 > < 三 >

The Ricci tensor is diagonal and its components are:

$$R_{00} = \frac{A''}{2B} - \frac{A'B'}{4B^2} - \frac{A'^2}{4AB} + \frac{A'}{rB}, \qquad R_{11} = -\frac{A''}{2A} + \frac{A'B'}{4A(r)B(r)} + \frac{A'^2}{4A^2} + \frac{B'}{r},$$
$$R_{22} = -\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1, \qquad R_{33} = \left(-\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1\right)\sin^2\theta.$$

The scalar curvature is

$$R = -\frac{A''}{AB} + \frac{A'B'}{2AB^2} + \frac{A'^2}{2A^2B} - \frac{2A'}{rAB} + \frac{2B'}{rB^2} - \frac{2}{r^2B} + \frac{2}{r^2}$$

The Einstein tensor is diagonal and its components are

 $G_{00} = \frac{AB'}{rB^2} - \frac{A}{r^2B} + \frac{A}{r^2}, \qquad G_{22} = \frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2},$ $G_{11} = \frac{A'}{rA} - \frac{B}{r^2} + \frac{1}{r^2}, \qquad G_{33} = \left(\frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2}\right)\sin^2\theta.$

The Ricci tensor is diagonal and its components are:

$$R_{00} = \frac{A''}{2B} - \frac{A'B'}{4B^2} - \frac{A'^2}{4AB} + \frac{A'}{rB}, \qquad R_{11} = -\frac{A''}{2A} + \frac{A'B'}{4A(r)B(r)} + \frac{A'^2}{4A^2} + \frac{B'}{r},$$
$$R_{22} = -\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1, \qquad R_{33} = \left(-\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1\right)\sin^2\theta.$$

The scalar curvature is

$$R = -\frac{A''}{AB} + \frac{A'B'}{2AB^2} + \frac{A'^2}{2A^2B} - \frac{2A'}{rAB} + \frac{2B'}{rB^2} - \frac{2}{r^2B} + \frac{2}{r^2}$$

The Einstein tensor is diagonal and its components are

$$G_{00} = \frac{AB'}{rB^2} - \frac{A}{r^2B} + \frac{A}{r^2}, \qquad G_{22} = \frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2},$$
$$G_{11} = \frac{A'}{rA} - \frac{B}{r^2} + \frac{1}{r^2}, \qquad G_{33} = \left(\frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2}\right)\sin^2\theta.$$

SdS metric-G0

A (1) > A (2) > A (2)

The Ricci tensor is diagonal and its components are:

$$R_{00} = \frac{A''}{2B} - \frac{A'B'}{4B^2} - \frac{A'^2}{4AB} + \frac{A'}{rB}, \qquad R_{11} = -\frac{A''}{2A} + \frac{A'B'}{4A(r)B(r)} + \frac{A'^2}{4A^2} + \frac{B'}{r},$$
$$R_{22} = -\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1, \qquad R_{33} = \left(-\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1\right)\sin^2\theta.$$

The scalar curvature is

$$R = -\frac{A''}{AB} + \frac{A'B'}{2AB^2} + \frac{A'^2}{2A^2B} - \frac{2A'}{rAB} + \frac{2B'}{rB^2} - \frac{2}{r^2B} + \frac{2}{r^2}$$

The Einstein tensor is diagonal and its components are

$$G_{00} = \frac{AB'}{rB^2} - \frac{A}{r^2B} + \frac{A}{r^2}, \qquad G_{22} = \frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2},$$

$$G_{11} = \frac{A'}{rA} - \frac{B}{r^2} + \frac{1}{r^2}, \qquad G_{33} = \left(\frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2}\right)\sin^2\theta.$$

SdS metric-GC

< 同 > < 三 > < 三 >

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{A(r)}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2.$$

The Christoffel symbols are:

$$\Gamma_{01}^{0} = \frac{1}{2} \frac{A'}{A}, \quad \Gamma_{00}^{1} = \frac{1}{2} A A', \qquad \Gamma_{11}^{1} = -\frac{1}{2} \frac{A'}{A}, \quad \Gamma_{22}^{1} = -rA, \quad \Gamma_{33}^{1} = -rA \sin^{2} \theta,$$

$$\Gamma_{12}^{2} = \frac{1}{r}, \qquad \Gamma_{33}^{2} = -\sin \theta \cos \theta, \quad \Gamma_{13}^{3} = \frac{1}{r}, \qquad \Gamma_{23}^{3} = \cot \theta.$$

Non-trivial components of curvature tensor are:

$$\begin{aligned} R_{0101} &= \frac{1}{2}A'', \qquad R_{0202} &= \frac{r}{2}AA', \qquad R_{0303} &= \frac{r}{2}AA'\sin^2\theta, \\ R_{1212} &= -\frac{r}{2}\frac{A'}{A}, \quad R_{1313} &= -\frac{r}{2}\frac{A'}{A}\sin^2\theta, \quad R_{2323} &= r^2(1-A)\sin^2\theta. \end{aligned}$$

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{A(r)}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2.$$

The Christoffel symbols are:

$$\Gamma_{01}^{0} = \frac{1}{2} \frac{A'}{A}, \quad \Gamma_{00}^{1} = \frac{1}{2} A A', \qquad \Gamma_{11}^{1} = -\frac{1}{2} \frac{A'}{A}, \quad \Gamma_{22}^{1} = -rA, \quad \Gamma_{33}^{1} = -rA \sin^{2} \theta,$$

$$\Gamma_{12}^{2} = \frac{1}{r}, \qquad \Gamma_{33}^{2} = -\sin \theta \cos \theta, \quad \Gamma_{13}^{3} = \frac{1}{r}, \qquad \Gamma_{23}^{3} = \cot \theta.$$

Non-trivial components of curvature tensor are:

$$\begin{aligned} R_{0101} &= \frac{1}{2}A'', \qquad R_{0202} &= \frac{r}{2}AA', \qquad R_{0303} &= \frac{r}{2}AA'\sin^2\theta, \\ R_{1212} &= -\frac{r}{2}\frac{A'}{A}, \quad R_{1313} &= -\frac{r}{2}\frac{A'}{A}\sin^2\theta, \quad R_{2323} &= r^2(1-A)\sin^2\theta. \end{aligned}$$

$$ds^{2} = -A(r)dt^{2} + \frac{1}{A(r)}dr^{2} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\varphi^{2}.$$

The Christoffel symbols are:

$$\Gamma_{01}^{0} = \frac{1}{2} \frac{A'}{A}, \quad \Gamma_{00}^{1} = \frac{1}{2} A A', \qquad \Gamma_{11}^{1} = -\frac{1}{2} \frac{A'}{A}, \quad \Gamma_{22}^{1} = -rA, \quad \Gamma_{33}^{1} = -rA \sin^{2} \theta,$$

$$\Gamma_{12}^{2} = \frac{1}{r}, \qquad \Gamma_{33}^{2} = -\sin \theta \cos \theta, \quad \Gamma_{13}^{3} = \frac{1}{r}, \qquad \Gamma_{23}^{3} = \cot \theta.$$

Non-trivial components of curvature tensor are:

$$\begin{aligned} &R_{0101} = \frac{1}{2}A'', \qquad R_{0202} = \frac{r}{2}AA', \qquad R_{0303} = \frac{r}{2}AA'\sin^2\theta, \\ &R_{1212} = -\frac{r}{2}\frac{A'}{A}, \quad R_{1313} = -\frac{r}{2}\frac{A'}{A}\sin^2\theta, \quad R_{2323} = r^2(1-A)\sin^2\theta. \end{aligned}$$

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{A(r)}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2.$$

The Christoffel symbols are:

$$\Gamma_{01}^{0} = \frac{1}{2} \frac{A'}{A}, \quad \Gamma_{00}^{1} = \frac{1}{2} A A', \qquad \Gamma_{11}^{1} = -\frac{1}{2} \frac{A'}{A}, \quad \Gamma_{22}^{1} = -rA, \quad \Gamma_{33}^{1} = -rA \sin^{2} \theta,$$

$$\Gamma_{12}^{2} = \frac{1}{r}, \qquad \Gamma_{33}^{2} = -\sin \theta \cos \theta, \quad \Gamma_{13}^{3} = \frac{1}{r}, \qquad \Gamma_{23}^{3} = \cot \theta.$$

Non-trivial components of curvature tensor are:

$$\begin{aligned} R_{0101} &= \frac{1}{2}A'', \qquad R_{0202} &= \frac{r}{2}AA', \qquad R_{0303} &= \frac{r}{2}AA'\sin^2\theta, \\ R_{1212} &= -\frac{r}{2}\frac{A'}{A}, \quad R_{1313} &= -\frac{r}{2}\frac{A'}{A}\sin^2\theta, \quad R_{2323} &= r^2(1-A)\sin^2\theta. \end{aligned}$$

<ロ> <同> <同> < 同> < 同> < 同>

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{A(r)}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2.$$

The Christoffel symbols are:

$$\Gamma_{01}^{0} = \frac{1}{2} \frac{A'}{A}, \quad \Gamma_{00}^{1} = \frac{1}{2} A A', \qquad \Gamma_{11}^{1} = -\frac{1}{2} \frac{A'}{A}, \quad \Gamma_{22}^{1} = -rA, \quad \Gamma_{33}^{1} = -rA \sin^{2} \theta,$$

$$\Gamma_{12}^{2} = \frac{1}{r}, \qquad \Gamma_{33}^{2} = -\sin \theta \cos \theta, \quad \Gamma_{13}^{3} = \frac{1}{r}, \qquad \Gamma_{23}^{3} = \cot \theta.$$

Non-trivial components of curvature tensor are:

$$\begin{aligned} R_{0101} &= \frac{1}{2}A'', \qquad R_{0202} &= \frac{r}{2}AA', \qquad R_{0303} &= \frac{r}{2}AA'\sin^2\theta, \\ R_{1212} &= -\frac{r}{2}\frac{A'}{A}, \quad R_{1313} &= -\frac{r}{2}\frac{A'}{A}\sin^2\theta, \quad R_{2323} &= r^2(1-A)\sin^2\theta. \end{aligned}$$

• □ ▶ • • □ ▶ • □ ▶ • • □ ▶ •

$$R_{00} = \frac{1}{2}AA'' + \frac{1}{r}AA', \qquad R_{11} = -\frac{1}{2}\frac{A''}{A} - \frac{1}{r}\frac{A'}{A},$$
$$R_{22} = 1 - A - rA', \qquad R_{33} = (1 - A - rA')\sin^2\theta.$$

The scalar curvature is

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}A + \frac{2}{r^2}$$

The Einstein tensor is presented as follows:

$$G_{00} = -\frac{A(r)A'(r)}{r} - \frac{A(r)^2}{r^2} + \frac{A(r)}{r^2}, \qquad G_{11} = \frac{A'(r)}{rA(r)} - \frac{1}{r^2A(r)} + \frac{1}{r^2},$$
$$G_{22} = \frac{1}{2}r^2A''(r) + rA'(r), \qquad \qquad G_{33} = \left(\frac{1}{2}r^2A''(r) + rA'(r)\right)\sin^2\theta.$$

SdS metric-C:B=1/A

$$R_{00} = \frac{1}{2}AA'' + \frac{1}{r}AA', \qquad R_{11} = -\frac{1}{2}\frac{A''}{A} - \frac{1}{r}\frac{A'}{A},$$
$$R_{22} = 1 - A - rA', \qquad R_{33} = (1 - A - rA')\sin^2\theta.$$

The scalar curvature is

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}A + \frac{2}{r^2}$$

The Einstein tensor is presented as follows:

$$G_{00} = -\frac{A(r)A'(r)}{r} - \frac{A(r)^2}{r^2} + \frac{A(r)}{r^2}, \qquad G_{11} = \frac{A'(r)}{rA(r)} - \frac{1}{r^2A(r)} + \frac{1}{r^2},$$
$$G_{22} = \frac{1}{2}r^2A''(r) + rA'(r), \qquad \qquad G_{33} = \left(\frac{1}{2}r^2A''(r) + rA'(r)\right)\sin^2\theta.$$

SdS metric-C:B=1/A

$$R_{00} = \frac{1}{2}AA'' + \frac{1}{r}AA', \qquad R_{11} = -\frac{1}{2}\frac{A''}{A} - \frac{1}{r}\frac{A'}{A},$$
$$R_{22} = 1 - A - rA', \qquad R_{33} = (1 - A - rA')\sin^2\theta.$$

The scalar curvature is

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}A + \frac{2}{r^2}$$

The Einstein tensor is presented as follows:

$$G_{00} = -\frac{A(r)A'(r)}{r} - \frac{A(r)^2}{r^2} + \frac{A(r)}{r^2}, \qquad G_{11} = \frac{A'(r)}{rA(r)} - \frac{1}{r^2A(r)} + \frac{1}{r^2},$$
$$G_{22} = \frac{1}{2}r^2A''(r) + rA'(r), \qquad \qquad G_{33} = \left(\frac{1}{2}r^2A''(r) + rA'(r)\right)\sin^2\theta.$$

SdS metric-C:B=1/A

$$\begin{aligned} R_{00} &= \frac{1}{2}AA'' + \frac{1}{r}AA', \qquad R_{11} = -\frac{1}{2}\frac{A''}{A} - \frac{1}{r}\frac{A'}{A}, \\ R_{22} &= 1 - A - rA', \qquad R_{33} = (1 - A - rA')\sin^2\theta. \end{aligned}$$

The scalar curvature is

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}A + \frac{2}{r^2}$$

The Einstein tensor is presented as follows:

$$G_{00} = -\frac{A(r)A'(r)}{r} - \frac{A(r)^2}{r^2} + \frac{A(r)}{r^2}, \qquad G_{11} = \frac{A'(r)}{rA(r)} - \frac{1}{r^2A(r)} + \frac{1}{r^2},$$
$$G_{22} = \frac{1}{2}r^2A''(r) + rA'(r), \qquad \qquad G_{33} = \left(\frac{1}{2}r^2A''(r) + rA'(r)\right)\sin^2\theta.$$

SdS metric-C:B=1/A

$$R_{00} = \frac{1}{2}AA'' + \frac{1}{r}AA', \qquad R_{11} = -\frac{1}{2}\frac{A''}{A} - \frac{1}{r}\frac{A'}{A},$$
$$R_{22} = 1 - A - rA', \qquad R_{33} = (1 - A - rA')\sin^2\theta.$$

The scalar curvature is

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}A + \frac{2}{r^2}$$

The Einstein tensor is presented as follows:

$$G_{00} = -\frac{A(r)A'(r)}{r} - \frac{A(r)^2}{r^2} + \frac{A(r)}{r^2}, \qquad G_{11} = \frac{A'(r)}{rA(r)} - \frac{1}{r^2A(r)} + \frac{1}{r^2},$$
$$G_{22} = \frac{1}{2}r^2A''(r) + rA'(r), \qquad \qquad G_{33} = \left(\frac{1}{2}r^2A''(r) + rA'(r)\right)\sin^2\theta.$$

SdS metric-C:B=1/A

$$\begin{aligned} R_{00} &= \frac{1}{2}AA'' + \frac{1}{r}AA', \qquad R_{11} = -\frac{1}{2}\frac{A''}{A} - \frac{1}{r}\frac{A'}{A}, \\ R_{22} &= 1 - A - rA', \qquad R_{33} = (1 - A - rA')\sin^2\theta. \end{aligned}$$

The scalar curvature is

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}A + \frac{2}{r^2}$$

The Einstein tensor is presented as follows:

$$G_{00} = -\frac{A(r)A'(r)}{r} - \frac{A(r)^2}{r^2} + \frac{A(r)}{r^2}, \qquad G_{11} = \frac{A'(r)}{rA(r)} - \frac{1}{r^2A(r)} + \frac{1}{r^2},$$

$$G_{22} = \frac{1}{2}r^2A''(r) + rA'(r), \qquad \qquad G_{33} = \left(\frac{1}{2}r^2A''(r) + rA'(r)\right)\sin^2\theta.$$

SdS metric-C:B=1/A

• □ ▶ • • □ ▶ • □ ▶ • • □ ▶ •