On the backreaction issue for the black hole in de Sitter space-time

Akhmedov Emil, Kirill Bazarov

21.02/2024

PHD student at MIPT and ITEP of NRC “Kurchatov Institute”

Introduction - Main Statement///

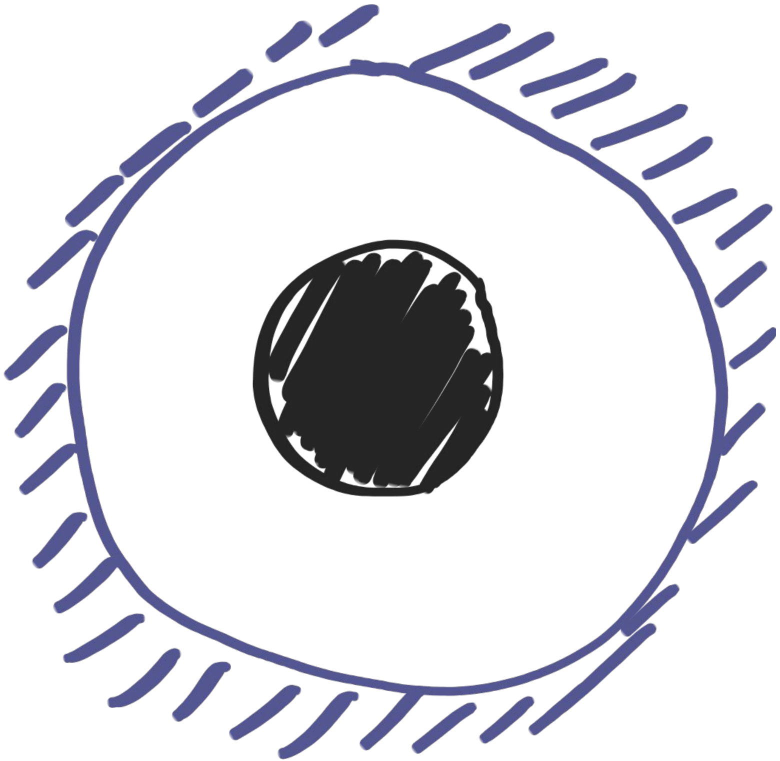
Quantum fluctuations may completely destroy event horizon:

- e.g. Black hole evaporation

Semiclassical approximation:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \langle : \hat{T}_{\mu\nu} : \rangle$$

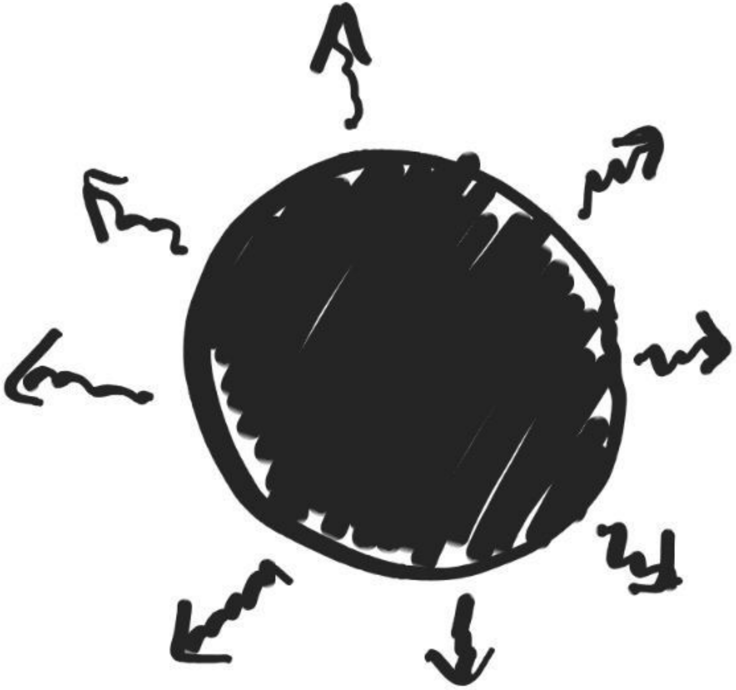
Introduction - Main Statement///



Quantum fluctuations completely destroy event horizon in multi horizon scena
(black hole in expanding universe)

Introduction and setup

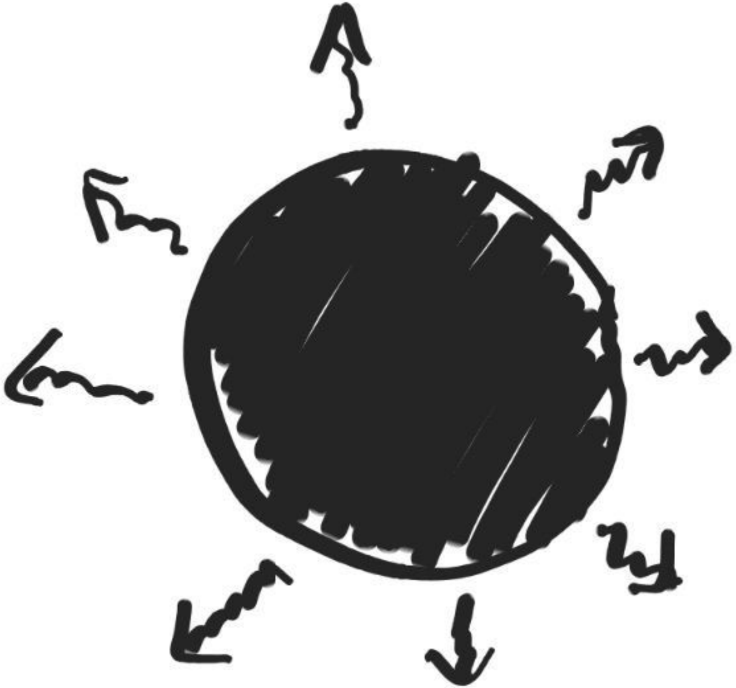
Black hole set up///



$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2 d\Omega^2$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0$$

Black hole set up///



$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2 d\Omega^2$$

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = 8\pi G\langle : \hat{T}_{\mu\nu} : \rangle$$

Is the right hand side neglectable?

Black hole set up///

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G\langle : \hat{T}_{\mu\nu} : \rangle$$

G is small

$$g_{00} = g_{00}^{(0)} + G\delta g_{00}^{(1)}$$

$$g_{00}^{(0)} = \left(1 - \frac{2M}{r}\right)$$

Metric is sensitive near r=2M!

Can we neglect RHS? ///

$$ds^2 = (1 - \frac{2M}{r})dt^2 - (1 - \frac{2M}{r})^{-1}dr^2 - r^2 d\Omega^2$$

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(\partial_\mu\varphi\partial_\mu\varphi - m^2\varphi^2\right)$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G\langle : \hat{T}_{\mu\nu} : \rangle$$

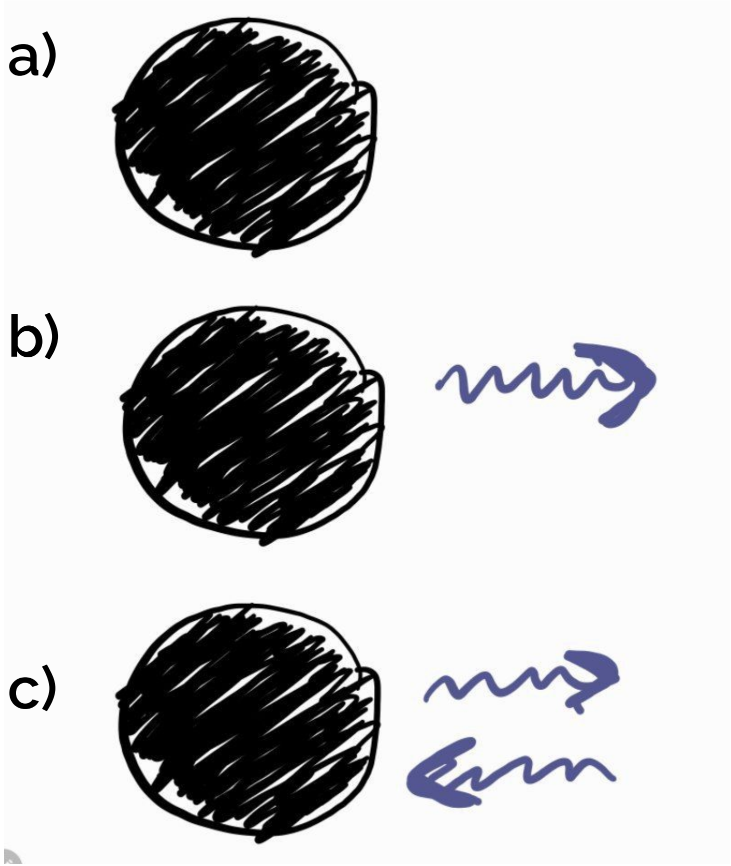
Quantum state///

$$\langle : \hat{T}_{\mu\nu} : \rangle = ???$$

Quantum state?

95% of literature is about:

a) Boulware state
b) Unruh state
c) Hartle-Hawking (HH) state



Quantum state///

$$\langle : \hat{T}_{\mu\nu} : \rangle = ???$$

$$\hat{\rho} = e^{-\beta \hat{H}}, \quad \beta = \frac{1}{T}$$

Quantum state?

95% of literature is about:

a) Boulware state
b) Unruh state
c) Hartle-Hawking (HH) state

$$\beta = \infty \to a)$$

$$\beta = 8\pi M \to c)$$

Known results///

Covariant Point Splitting Regularization for a Sc
Walker Universe with Spatial Curvature

T.S. Bunch (King's Coll. London), P.C.W. Davies (King's Coll. London)
1977

14 pages
Published in: *Proc.Roy.Soc.Lond.A* 357 (1977) 381-394
DOI: 10.1098/rspa.1977.0174

Notes on black hole evaporation

W.G
197

Pub
DOI

Covariant Point Splitting Regularization for a Sc

CONFORMAL ANOMALIES AND MASSL
FRIEDMANN UNIVERSE

I.L. Buchbinder (Tomsk Pedagogical Inst.), S.D. Odintsov (Toms
1984

4 pages
Published in: *Sov.Phys.J.* 27 (1984) 674-677

Vacuum Polarization in Schwarzschi

P. Candelas (Texas U.)

Energy Momentum Tensor Near an

P.C.W. Davies (King's Coll. London), S.A. Fulling (King's

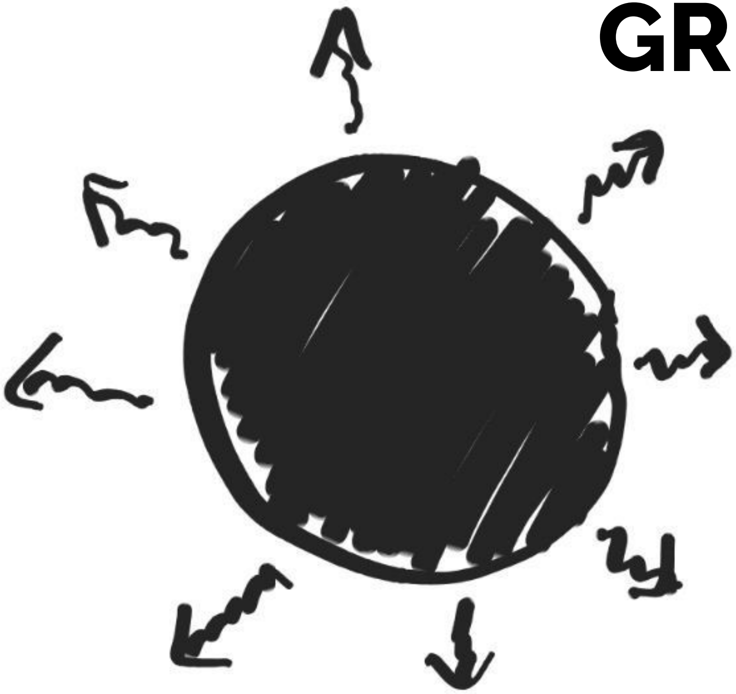
Regularization, Renormalization, a

S.M. Christensen (Utah U. and Harvard U.)
1978

18 pages
Published in: *Phys.Rev.D* 17 (1978) 946-963

Massive field? Arbitrary state?
Arbitrary metric?

Hawking temperature////



GR ⇔ Thermodynamics

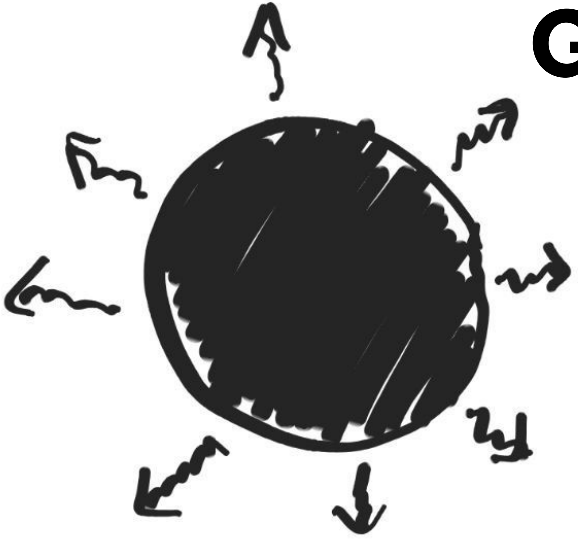
$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2 d\Omega^2$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0$$

$$T_H = \frac{1}{8\pi M}$$

Hawking temperature

Hawking temperature////



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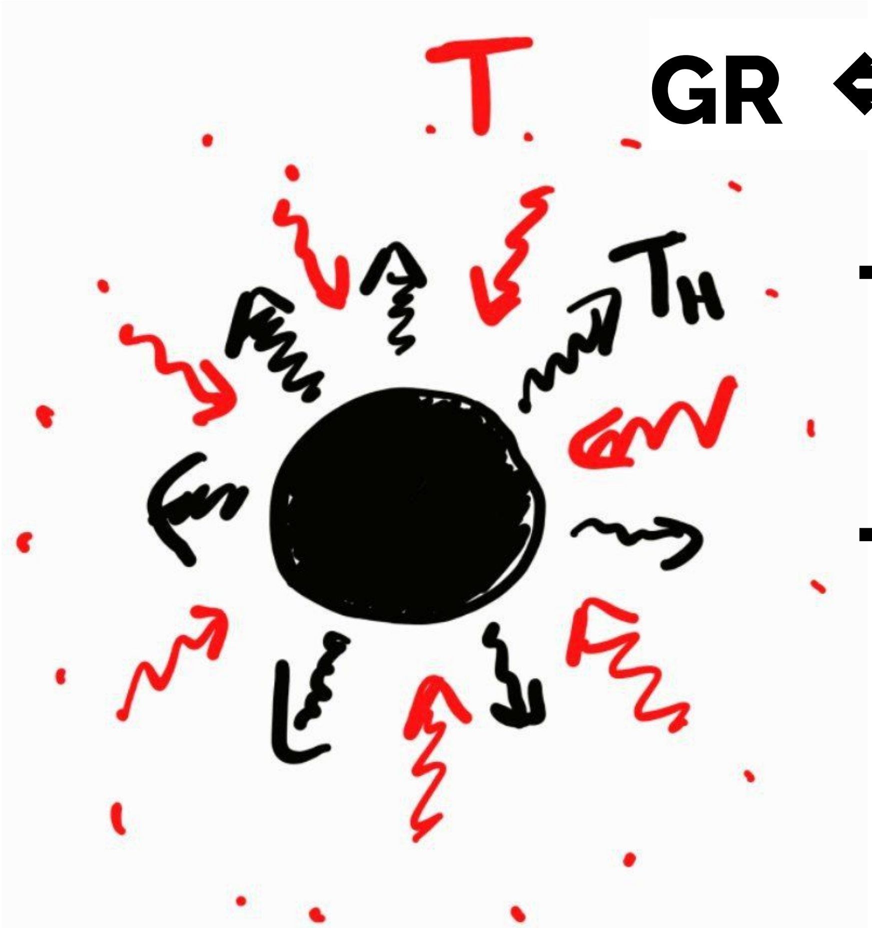


GR ⇔ Thermodynamics

$T=T_H$ stability

$T \neq T_H$ instability

Stress-energy tensor (leading terms)//

$$M = \frac{1}{4}, \quad T_H = \frac{1}{2\pi}$$

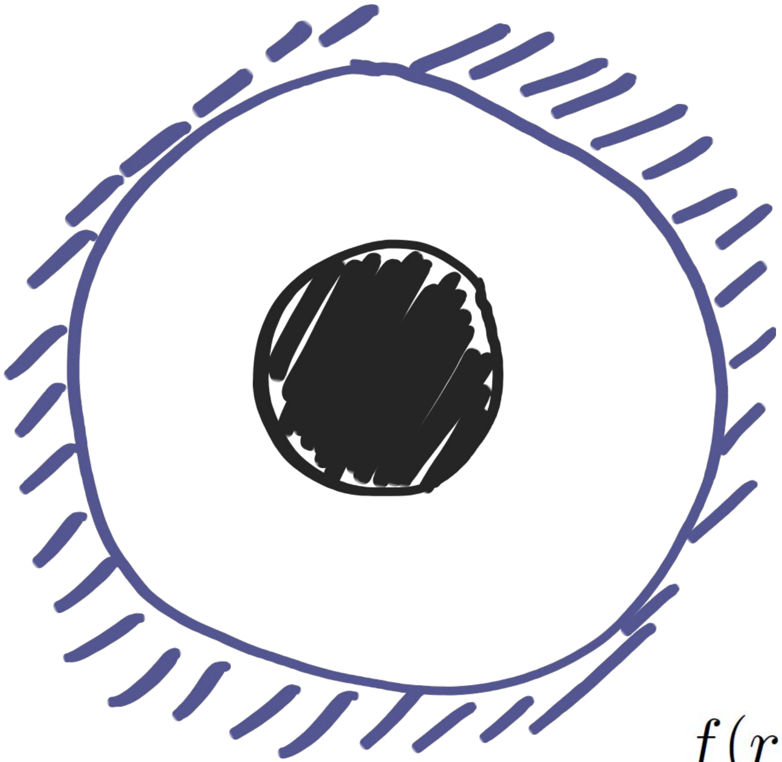
Singular near the horizon! (r=½)

$$T \neq T_H$$ **"Broken" state**

$$\langle : \hat{T}^{\mu}_{\nu} : \rangle_{\beta = \frac{1}{T}} \approx \frac{1}{480\pi^2}\left(1 - \frac{1}{2r}\right)^{-2}\left((2\pi T)^4 - 1\right)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} \end{pmatrix} +$$

$$+ \frac{1}{48\pi^2}\left(1 - \frac{1}{2r}\right)^{-2}\left((2\pi T)^2 - 1\right)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & \frac{2}{3} \end{pmatrix}$$

Multihorizon situation///



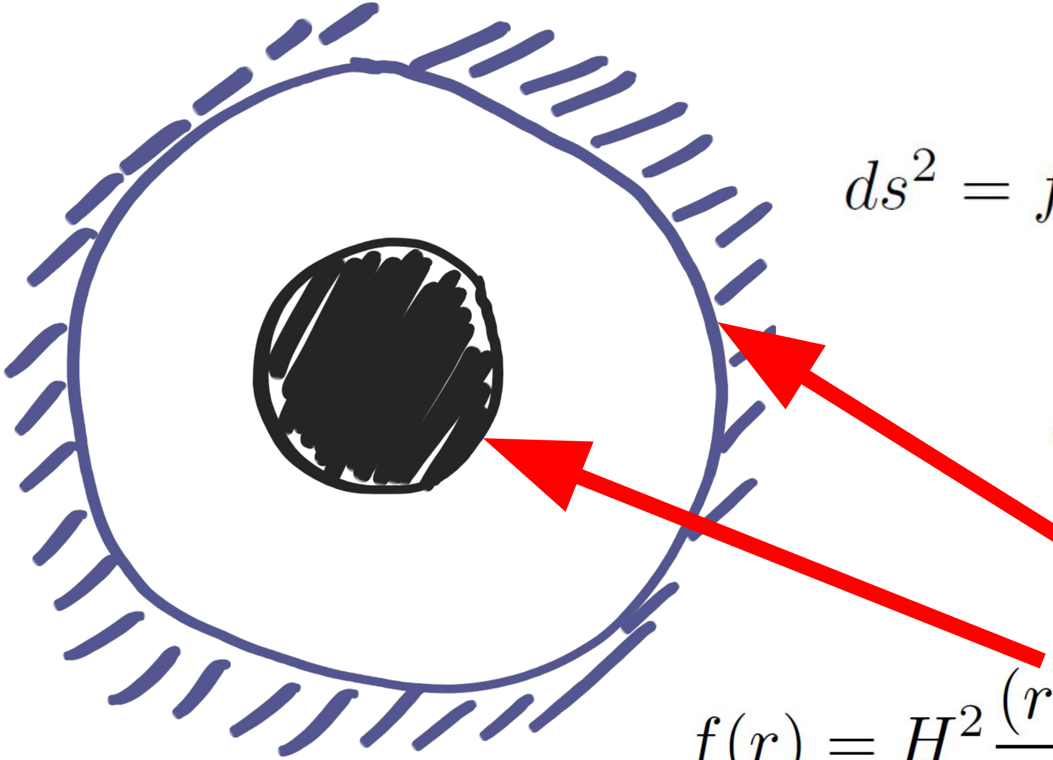
De Sitter–Schwarzschild space-time:

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 d\Omega_2$$

$$f(r) = 1 - \frac{2M}{r} - H^2 r^2$$

$$f(r) = H^2 \frac{(r - r_b)(r_c - r)(r + r_c + r_b)}{r}$$

Multihorizon situation///



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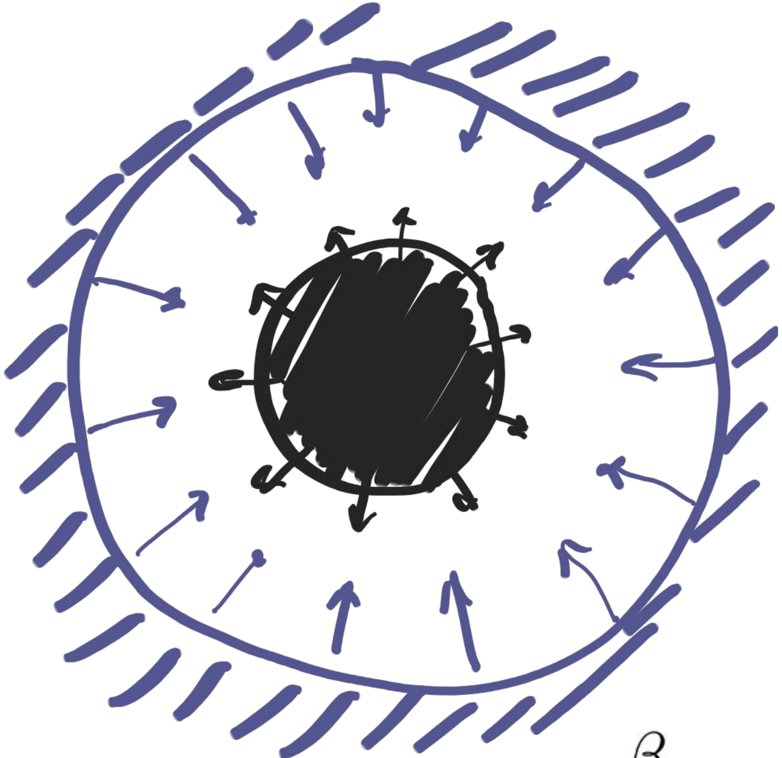
Multihorizon situation///



De Sitter–Schwarzschild space-time:

$$\beta_b = \frac{4\pi r_b}{H^2(r_c - r_b)(2r_b + r_c)} = \frac{2\pi}{\kappa_b}$$

$$\beta_c = \frac{4\pi r_c}{H^2(r_c - r_b)(r_b + 2r_c)} = \frac{2\pi}{\kappa_c}$$

$$\beta_c - \beta_b = \frac{2\pi(r_b + r_c)}{H^2(2r_b + r_c)(r_c + 2r_b)} > 0$$

From 4d
to 2d

Technical problems///

Two dimensional analog:

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 d\Omega_2 \longrightarrow ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)}$$

Rindler example:

2D:

$$\langle : T_{\mu\nu} : \rangle_\beta = \frac{1}{24}\left(\left(\frac{2\pi}{\beta}\right)^2 - 1\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4D:

$$\langle : \hat{T}^\mu_\nu : \rangle_{\beta=\frac{1}{T}} \approx \frac{1}{480\pi^2} e^{-4\xi}\left((2\pi T)^4 - 1\right)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

SET in 2d (GENERAL CASE)///

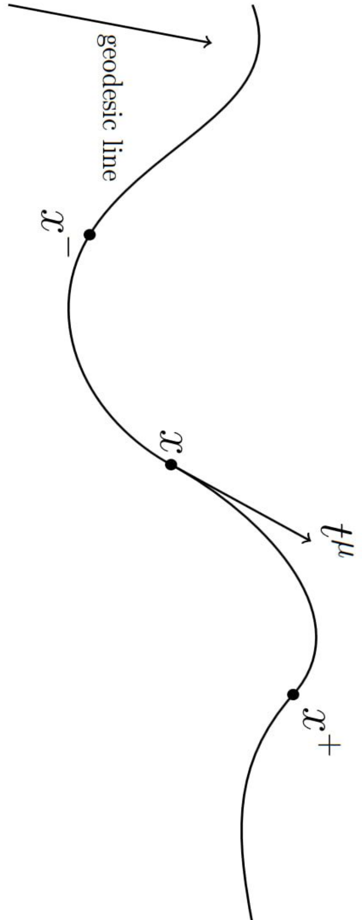
$$\langle : \hat{T}_{\alpha\beta} : \rangle = \Theta_{\alpha\beta} + \frac{R}{48\pi} g_{\alpha\beta}$$

$$ds^2 = C(u,v)dudv$$

$$\Theta_{uu} = \frac{1}{48\pi}\left(\frac{2\pi}{\beta}\right)^2 - \frac{1}{12\pi} C^{1/2} \partial_u^2 C^{-1/2}$$

$$\Theta_{vv} = \frac{1}{48\pi}\left(\frac{2\pi}{\beta}\right)^2 - \frac{1}{12\pi} C^{1/2} \partial_v^2 C^{-1/2}$$

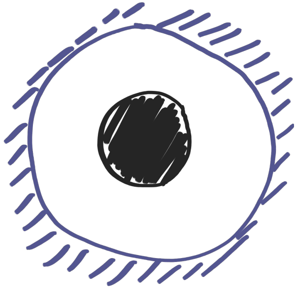
$$\Theta_{uv} = \Theta_{vu} = ($$



Black hole in expanding universe ///

General formula:

$$\langle :T_{00}: \rangle \approx \frac{\pi}{6}\frac{1}{\beta^2} - \frac{1}{6\pi} f(r^*)^{1/2} \frac{\partial^2}{\partial^2 r^*} f(r^*)^{-1/2}$$



Black hole horizon $\langle :T_{00}: \rangle \approx \frac{\pi}{6}\left[\frac{1}{\beta^2} - \frac{1}{\beta_b^2}\right]$

Cosmological horizon $\langle :T_{00}: \rangle \approx \frac{\pi}{6}\left[\frac{1}{\beta^2} - \frac{1}{\beta_c^2}\right].$

Dilaton Gravity

2d dilaton gravity////

$$S^{\mathrm{grav}} = \frac{1}{16\pi G}\int d^2x\sqrt{-g}e^{-2\phi}\left[R - 4\omega(\partial_\mu\phi)^2 + 4\lambda^2\right]$$

- $\omega = 0$ case is the Jackiw-Teitelboim theory;
- if $\omega = -\frac{1}{2}$ one obtains planar general relativity;
- $\omega = -1$ one has the first-order string theory.

2d dilaton gravity////

$$S^{\text{grav}} = \frac{1}{16\pi G}\int d^2x\sqrt{-g}e^{-2\phi}\left[R - 4\omega(\partial_\mu\phi)^2 + 4\lambda^2\right]$$

$$S = S^{\text{grav}} + S^{\text{matter}} = S^{\text{grav}} - \frac{1}{2}\int d^2x\,\partial_\mu\varphi\,\partial^\mu\varphi$$

2d dilaton gravity///

Only gravity:

$$e^{2\phi}T_{\mu\nu}^{\text{grav}} \equiv -2(\omega+1)D_\mu\phi D_\nu\phi + D_\mu D_\nu\phi - g_{\mu\nu}D_\mu D^\nu\phi + (\omega+2)g_{\mu\nu}D_\mu\phi D^\mu\phi - g_{\mu\nu}\lambda^2 = 0,$$

$$R - 4\omega D_\mu D^\mu\phi + 4\omega D_\mu\phi D^\mu\phi + 4\lambda^2 = 0.$$

Back reaction can be considered

2d dilaton gravity////

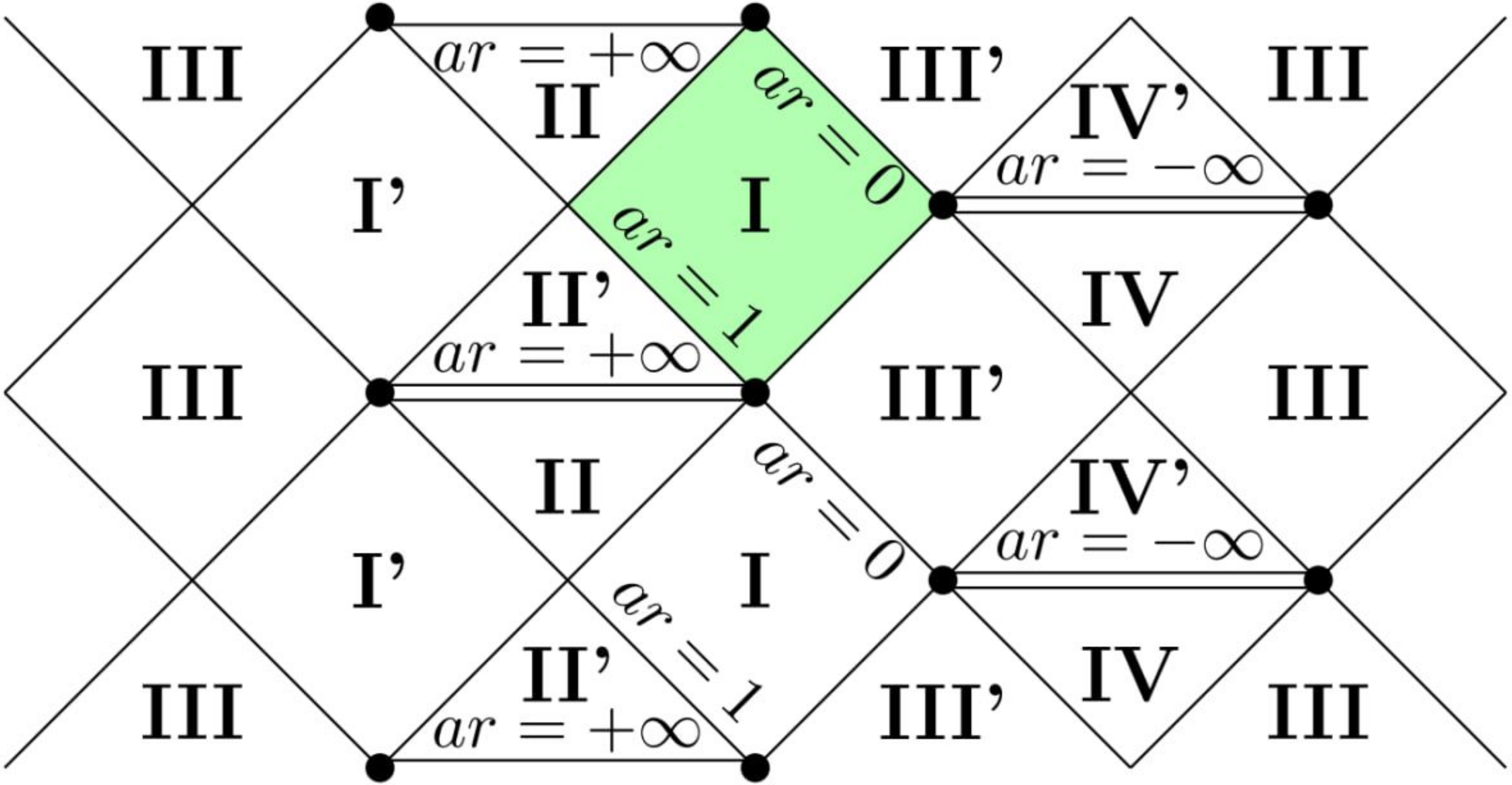
$$\omega = -\frac{4}{3} \longrightarrow ds^2 = -[a^2r^2 - a^4r^4]dt^2 + \frac{dr^2}{a^2r^2 - a^4r^4},$$

$$\phi = \frac{3}{2}\log(ar),$$

Multi Horizon

$$T = \begin{cases} \frac{a}{2\pi}, & r \to 1/a, \\ 0, & r \to 0. \end{cases}$$

2d dilaton gravity



2d dilaton gravity

$$ds^2 = -e^{2\nu(X)}dt^2 + dX^2,$$

$T = 0 \quad \rightarrow \quad$ at $r = 0$ all is ok

$r \rightarrow 0$ equals $X \rightarrow \infty$

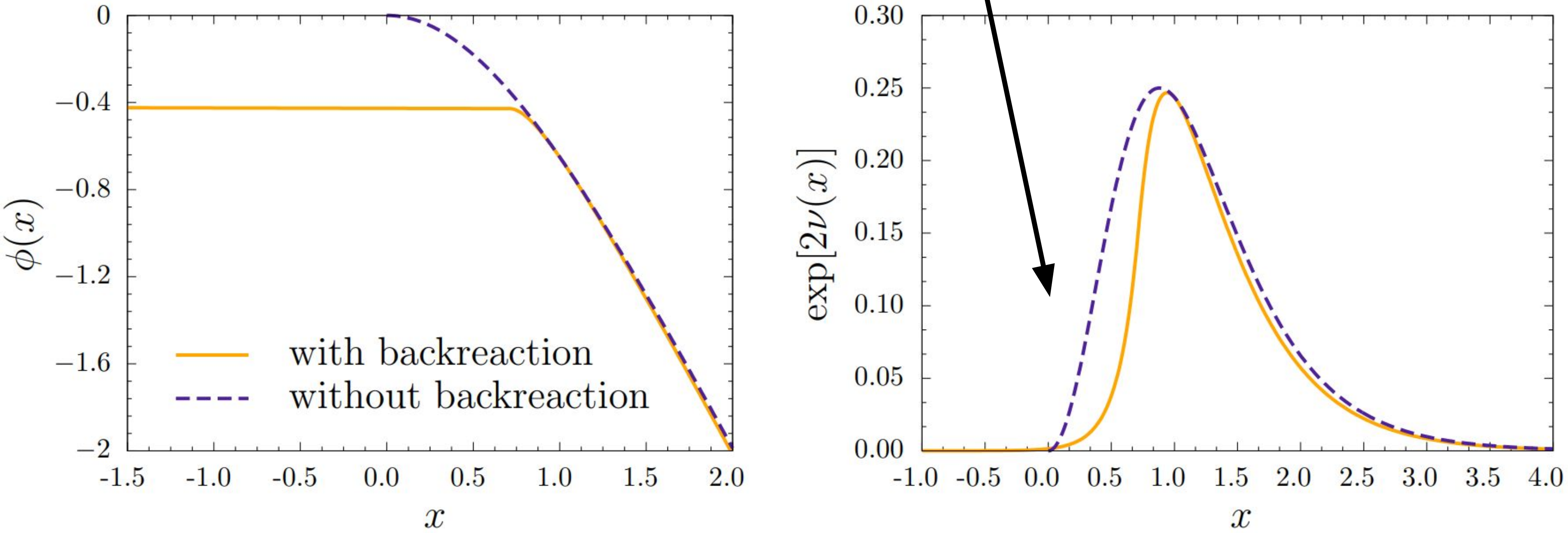
$$\langle : \hat{T}_{\alpha\beta} : \rangle$$

$$\lambda^2 - \frac{2}{3}\phi'(X)^2 + \phi''(X) = 8\pi G e^{2\phi(X)}\left(\frac{e^{-2\nu(X)}\pi}{6\beta^2} + \frac{1}{24\pi}\left[\nu'(X)^2 + 2\nu''(X)\right]\right),$$

$$-\lambda^2 - \phi'(X)\nu'(X) + \frac{4}{3}\phi'(X)^2 = 8\pi G e^{2\phi(X)}\left(\frac{e^{-2\nu(X)}\pi}{6\beta^2} - \frac{\nu'(X)^2}{24\pi}\right),$$

2d dilaton gravity (numerical solution)

Horizon is changed!



Based on///

inspire-hep:

- **Notes on peculiarities of quantum fields in space–times with horizons**

  K.V. Bazarov

  2112.02188 Class.Quant.Grav. 39 (2022) 21, 217001

- **Backreaction issue for the black hole in de Sitter spacetime**

  E.T. Akhmedov, K.V. Bazarov

  2212.06433 Phys.Rev.D 107 (2023) 10, 105012

- **On a non trivial self-consistent backreaction of quantum fields in 2D dilaton gravity**

  E.T. Akhmedov, P.A. Anempodistov, K.V. Bazarov

  2401.07645

Future///

4D case?

More space-times?

inspire-hep:

More states?

Non-thermal states?

Thank you for your attention!

Any comments?

THE END

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)}, \qquad f(r) = 1 - \frac{2M}{r} - H^2 r^2.$$

$$f(r) = H^2 \frac{(r - r_b)(r_c - r)(r + r_c + r_b)}{r}.$$

$$H^2 = \frac{1}{r_c^2 + r_b r_c + r_b^2}, \qquad M = \frac{r_b r_c (r_b + r_c)}{2(r_c^2 + r_b r_c + r_b^2)} = H^2 \frac{r_b r_c (r_b + r_c)}{2}.$$

Black hole temperature

Cosmological horizon temperature

$$\langle \hat{a}^\dagger_\omega, \hat{a}_{\omega'} \rangle = n_{out}(\omega)\delta(\omega - \omega'), \qquad \langle \hat{b}^\dagger_\omega, \hat{b}_{\omega'} \rangle = n_{in}(\omega)\delta(\omega - \omega'), \qquad \langle \hat{b}^\dagger_\omega, \hat{a}_{\omega'} \rangle = 0.$$

$$n_{out}(\omega) = \frac{1}{e^{\beta_0 \omega} - 1} + \delta n(\omega), \quad n_{in}(\omega) = \frac{1}{e^{\beta_0 \omega} - 1} - \delta n(\omega), \quad \text{where} \quad \frac{2}{\beta_0^2} = \frac{1}{\beta_b^2} + \frac{1}{\beta_c^2},$$

$$\int_0^\infty d\omega \omega \delta n(\omega) |R_\omega|^2 = \frac{\pi^2}{12}\Big[\frac{1}{\beta_b^2} - \frac{1}{\beta_c^2}\Big], \qquad \int_0^\infty d\omega \omega \delta n(\omega) |T_\omega|^2 = 0.$$

Thermalization?

State

Stress-energy tensor:

$$T_{\mu\nu}(x)_\beta = \left(\frac{\partial}{\partial x_1^\mu} \frac{\partial}{\partial x_2^\nu} - \frac{1}{2} g_{\mu\nu} \left[g^{\alpha\beta} \frac{\partial}{\partial x_1^\alpha} \frac{\partial}{\partial x_2^\beta} - m^2 \right] \right) W_\beta(x_1|x_2) \Big|_{x_1 = x_2 = x}$$

Wightman function:

$$W_\beta(x_1|x_2) = \langle \hat{\varphi}(x_1) \hat{\varphi}(x_2) \rangle \qquad \langle \hat{O} \rangle = \frac{\mathrm{Tr} \hat{O} \hat{\rho}}{\mathrm{Tr} \hat{\rho}}$$

Thermal density matrix:

$$\hat{\rho} = e^{-\beta \hat{H}}$$

Technical problems

Two dimensional analog:

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - \cancel{r^2 d\Omega_2} \longrightarrow ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)}$$

Rindler example:

2D:

$$\langle : T_{\mu\nu} : \rangle_\beta = \frac{1}{24}\left(\left(\frac{2\pi}{\beta}\right)^2 - 1\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

4D:

$$\langle : \hat{T}^\mu_\nu : \rangle_{\beta=\frac{1}{T}} \approx \frac{1}{480\pi^2} e^{-4\xi}\left((2\pi T)^4 - 1\right)\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -\frac{1}{3} & 0 & 0\\ 0 & 0 & -\frac{1}{3} & 0\\ 0 & 0 & 0 & -\frac{1}{3}\end{pmatrix}$$

Technical problems

Field operator:

$$\hat{\varphi}(t,r^*) = \int_0^\infty \frac{d\omega}{\sqrt{2\pi}}\frac{1}{\sqrt{2\omega}}\left[e^{i\omega t}\left(\hat{a}^\dagger_\omega \overrightarrow{\varphi}_\omega(r^*) + \hat{b}^\dagger_\omega \overleftarrow{\varphi}_\omega(r^*)\right) + h.c.\right]$$

EOM:

$$\left[-\partial^2_{r^*} + m^2 f(r^*)\right]\varphi_\omega(r^*) = \omega^2\varphi_\omega(r^*),$$

$$r^* = \int \frac{dr}{f(r)}$$

$$r \to r_b,\ r^* \to -\infty$$

$$r \to r_c,\ r^* \to +\infty$$

Boundary conditions:

	$r^* \to -\infty$	$r^* \to +\infty$
$\overrightarrow{\varphi}_\omega(r^*)$	$e^{i\omega r^*} + R_\omega e^{-i\omega r^*}$	$T_\omega e^{i\omega r^*}$
$\overleftarrow{\varphi}_\omega(r^*)$	$T_\omega e^{-i\omega r^*}$	$e^{-i\omega r^*} + R_\omega e^{i\omega r^*}$

Technical problems

Field operator:

$$\hat{\varphi}(t,r^*) = \int_0^\infty \frac{d\omega}{\sqrt{2\pi}} \frac{1}{\sqrt{2\omega}} \left[e^{i\omega t} \left(\hat{a}^\dagger_\omega \overrightarrow{\varphi}_\omega(r^*) + \hat{b}^\dagger_\omega \overleftarrow{\varphi}_\omega(r^*) \right) + h.c. \right]$$

Thermal averaging:

$$\hat{\rho} = e^{-\beta \hat{H}}, \quad \text{then} \quad \langle \hat{a}^\dagger_\omega, \hat{a}_{\omega'} \rangle = \frac{1}{e^{\beta\omega}-1} \delta(\omega - \omega'), \quad \langle \hat{b}^\dagger_\omega, \hat{b}_{\omega'} \rangle = \frac{1}{e^{\beta\omega}-1} \delta(\omega - \omega')$$

Quantum average of Wightman function:

$$W(t_2, r_2^* | t_1, r_1^*) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{e^{i\omega(t_2 - t_1)}}{e^{\beta\omega}-1} \frac{1}{2\omega} \left[\overrightarrow{\varphi}_\omega(r_1^*) \overrightarrow{\varphi^*}_\omega(r_2^*) + \overleftarrow{\varphi}_\omega(r_1^*) \overleftarrow{\varphi^*}_\omega(r_2^*) \right]$$

Energy density

Thermal averaging:

$$W(t_2, r_2^*|t_1, r_1^*) \approx \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{e^{i\omega(t_2-t_1)}}{e^{\beta\omega}-1} \frac{1}{2\omega} \Big[e^{i\omega(r_2^*-r_1^*)} + |R_\omega|^2 e^{-i\omega(r_2^*-r_1^*)} + \\ + R_\omega e^{i\omega(r_1^*+r_2^*)} + R_\omega^* e^{-i\omega(r_1^*+r_2^*)} + |T_\omega|^2 e^{-i\omega(r_2^*-r_1^*)} \Big]$$

Quantum average of stress-energy tensor:

$$\langle T_{00} \rangle \approx \int_0^\infty \frac{\omega d\omega}{4\pi} \left[1 + \frac{2}{e^{\beta\omega}-1} \right] \left(1 + |R_\omega|^2 + |T_\omega|^2 \right) =$$

$$= \int_0^\infty \frac{\omega d\omega}{2\pi} \left[1 + \frac{2}{e^{\beta\omega}-1} \right]$$

Energy density

Thermal averaging:

$$W(t_2, r_2^*|t_1, r_1^*) \approx \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{e^{i\omega(t_2-t_1)}}{e^{\beta\omega}-1} \frac{1}{2\omega} \Big[e^{i\omega(r_2^*-r_1^*)} + |R_\omega|^2 e^{-i\omega(r_2^*-r_1^*)} + \\ + R_\omega e^{i\omega(r_1^*+r_2^*)} + R_\omega^* e^{-i\omega(r_1^*+r_2^*)} + |T_\omega|^2 e^{-i\omega(r_2^*-r_1^*)} \Big]$$

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$$= \int_0^\infty \frac{\omega d\omega}{2\pi} \left[1 + \frac{2}{e^{\beta\omega}-1} \right]$$

Divergence

Regularization

$$\langle \hat{T}_{\mu\nu}(x) \rangle = D_{\mu\nu} \langle \hat{\varphi}(x^+) \hat{\varphi}(x^-) \rangle \Big|_{x^+ = x^- = x}$$

Geodesic point-splitting: $x^\mu(\tau) = x^\mu + \tau t^\mu + \frac{1}{2}\tau^2 a^\mu + \frac{1}{6}\tau^3 b^\mu + ...,$

$$\langle T_{00} \rangle = \int_{-\infty}^{\infty} \frac{\omega d\omega}{2\pi} \left[1 + \frac{2}{e^{\beta\omega} - 1} \right]$$

$$\langle : T_{00} : \rangle \approx \frac{\pi}{6} \frac{1}{\beta^2} - \frac{1}{6\pi} f(r^*)^{1/2} \frac{\partial^2}{\partial^2 r^*} f(r^*)^{-1/2}$$

Regularization

$$\langle \hat{T}_{\mu\nu}(x) \rangle = D_{\mu\nu} \langle \hat{\varphi}(x^+) \hat{\varphi}(x^-) \rangle \big|_{x^+ = x^- = x}$$

Geodesic point-splitting: $$x^\mu(\tau) = x^\mu + \tau t^\mu + \frac{1}{2}\tau^2 a^\mu + \frac{1}{6}\tau^3 b^\mu + \dots,$$

$$\langle T_{00} \rangle = \int_{-\infty}^{\infty} \frac{\omega d\omega}{2\pi} \left[1 + \frac{2}{e^{\beta\omega} - 1} \right]$$

$$\langle : T_{00} : \rangle \approx \frac{\pi}{6} \frac{1}{\beta^2} - \frac{1}{6\pi} f(r^*)^{1/2} \frac{\partial^2}{\partial^2 r^*} f(r^*)^{-1/2}$$

Sums

$$W_\beta(x,x') \equiv \langle \hat{\varphi}(t,r,\theta,\phi)\varphi(t',r',\theta',\phi')\rangle_\beta =$$

$$= \int_{-\infty}^{+\infty} d\omega \frac{1}{e^{\beta\omega}-1} \frac{e^{-i\omega(t'-t)}}{rr'4\pi\omega} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Y_l^m(\theta,\phi)\left(Y_l^m(\theta',\phi')\right)^* \times$$

$$\times \left[R^*_{\omega,l}(r')R_{\omega,l}(r) + L^*_{\omega,l}(r')L_{\omega,l}(r)\right] =$$

$$= \int_{-\infty}^{+\infty} d\omega \frac{1}{e^{\beta\omega}-1} \frac{e^{-i\omega(t'-t)}}{rr'4\pi\omega} \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} P_l(\vec{x}\cdot\vec{x}')\left[R^*_{\omega,l}(r')R_{\omega,l}(r) + L^*_{\omega,l}(r')L_{\omega,l}(r)\right].$$

Sums

$$\sum_{l=0}^{\infty}(2l+1)\left[\left|L_{\omega,l}(r)\right|^2+\left|R_{\omega,l}(r)\right|^2\right]\approx\sum_{l=0}^{\infty}(2l+1)\left[\left|R_{\omega,l}(r)\right|^2\right]\approx\omega^2\left(1-\frac{1}{2r}\right)^{-1},$$

$$\sum_{l=0}^{\infty}(2l+1)l(l+1)\left[\left|L_{\omega,l}(r)\right|^2+\left|R_{\omega,l}(r)\right|^2\right]\approx\frac{1}{6}\left(\omega^2+\omega^4\right)\left(1-\frac{1}{2r}\right)^{-2},$$

$$\sum_{l=0}^{\infty}(2l+1)\left[\left|\partial_{r^*}L_{\omega,l}(r)\right|^2+\left|\partial_{r^*}R_{\omega,l}(r)\right|^2\right]\approx\frac{1}{3}\left(4\omega^2+\omega^4\right)\left(1-\frac{1}{2r}\right)^{-1}.$$

Sums