Tachyon condensation in a chromomagnetic center-vortex background

M. Bordag

This lecture is aimed at an introduction to chromomagnetic fields in non-Abelian gauge theory.

Topics are
-magnetic fields, Landau levels and magnetic moment
-Heisenberg-Euler Lagrangian in QED
-generalization to SU(N) and the unstable mode
-chromomagnetic vacuum and approaches to its stabilization
-tachyon condensation

based on

M. Bordag. Tachyon condensation in a chromomagnetic background field and the groundstate of QCD. *Eur. Phys. J. A*, 59:55, 2023

M. Bordag. Tachyon condensation in a chromomagnetic center-vortex background. *Universe*, 10:38, 2024

Introduction

- QCD is the part of the Standard Model which describes the strong interaction
- It is well understood for high energies where the perturbative approach works well thanks to the asymptotic freedom
- In contrast, at low energies we have a strong coupling regime and no perturbation theory. Also, due to the masslessness of the gluons, one observes not yet resolved infrared problems
- The goundstate (vacuum) of QCD cannot be the perturbative one, the asymptotic states are not gluons and quarks, in opposite one expects the picture of a many-particle state.
- Quarks and gluons are not observed as free particles; one expects their confinement, formulated e.g. in terms of the Wilson criterion
- a further, not yet resolved problem is the behavior of the theory at high temperature and density (quark-gluon plasma)

Approaches

- numerical approaches like lattice calculations, Schwinger-Dyson equations, functional renormalization group (flow equations)
many expected features confirmed, e.g. dual superconductor picture, phase transitions
also, one expects gluon condensates
- phenomenological models
- bag models (baryon spectroscopy)
(there are much more attempts and approaches)

- classical solutions (background fields) like selfdual background (chromoelectric and chromomagnetic) fields, monopoles and instantons

chromomagnetic background field

relevant for this talk: chromomagnetic model for the ground state (Savvidy vacuum)
it rests on the *Euler-Heisenberg Lagrangian* [1]
that is QED in a homogeneous magnetic background field,

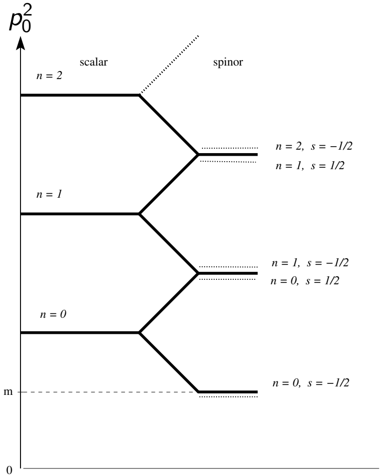
Return, for a moment, to the Landau levels ($n = 0, 1, ...$) for a particle with charge, magnetic moment and the spin projection σ

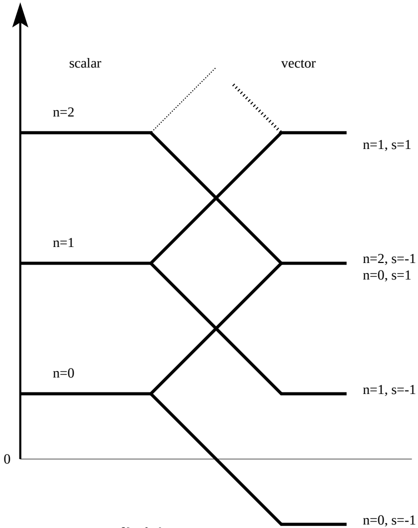
$$E_n = \sqrt{m^2 + p_z^2 + gB(2n + 1 + 2\sigma)}, \quad \sigma = \begin{cases} 0 & (scalar) \\ \pm\frac{1}{2} & (spinor) \\ \pm 1 & (vector) \end{cases}$$

[1] W. Heisenberg and H. Euler. Consequences of Dirac's theory of positrons.
Z. Phys., 98:714–732, 1936

Level scheme

p_0^2

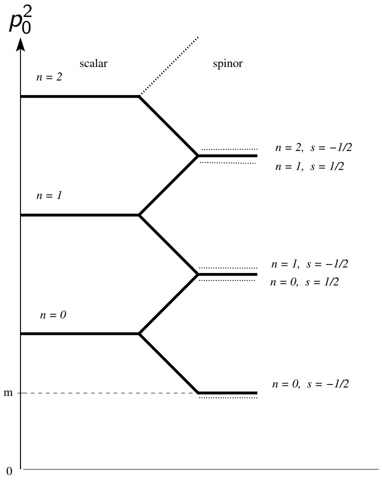


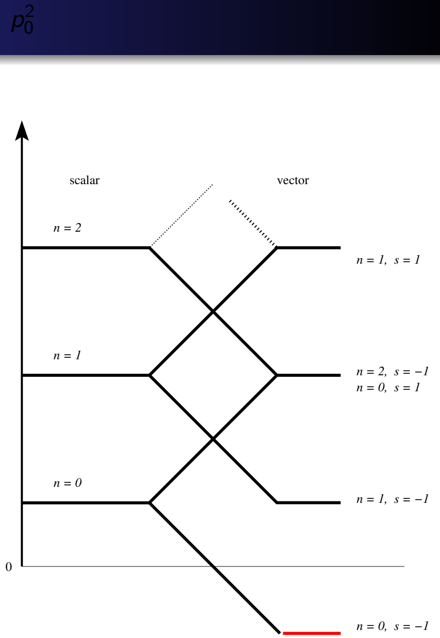


The lowest energy levels in a homogeneous magnetic field.

Comment on anomalous magnetic moment

The tachyonic mode





the tachyonic mode has $n = 0,\ \sigma = -1$
and $E_{ta}^2 = p_z^2 - gB$
divide the modes into unstable (tachyonic) and stable (all other) modes

The effective Lagrangian

at one loop, the first quantum corrections to the classical ground state, results in (Euler-Heisenberg Lagrangian)

$$\mathcal{L}_{eff}^{QED} = -\frac{B^2}{2} + \frac{e^2(eB)^2}{48\pi^2} \ln \frac{(2eB)^2}{m^4}, \qquad \beta = \frac{e^2}{12\pi^2}$$
$$\mathcal{L}_{eff}^{QCD} = -\frac{B^2}{2} - \frac{11(gB)^2}{192\pi^2} \ln \frac{(gB)^2}{\mu^4} + i\frac{(gB)^2}{8\pi}, \qquad \beta = -\frac{11N_c - 2N_f}{3}\frac{g^2}{16\pi^2}$$

where μ is a normalization constant.
effective potential $V_{ell} = -\mathcal{L}_{eff}$
$\mathcal{L}_{eff}^{QCD}$ found in 1977 (Savvidy) [1],
its imaginary part (instability) found in 1978 (Nielesen, Olesen) [2]

[1] G.K. Savvidy. Infrared instability of vacuum state of gauge theories and asymptotic freedom.
Phys. Lett. B, 71(1):133–134, 1977

[2] N.K Nielsen and P. Olesen. Unstable Yang-Mills Field Mode.
Nucl. Phys. B, 144(2-3):376–396, 1978

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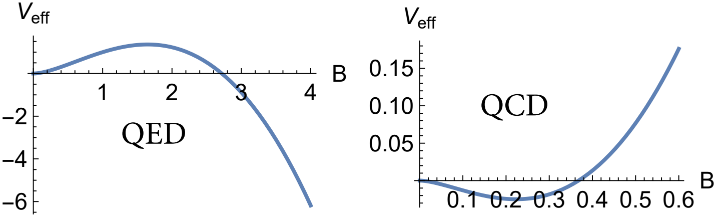

responsible is the sign of the beta-function, that is, the asymptotic freedom in case of QCD

the minimum is

$$gB_{|_{min}} = \mu^2 \exp\left(-\frac{24\pi^2}{11g^2}\right), \qquad V_{eff\,|_{min}} = -\frac{11\mu^4}{96\pi^2} \exp\left(-\frac{48\pi^2}{11g^2}\right),$$

the minimum and spontaneous creation of a magnetic field

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Since the energy is below zero, the system tries to enter this state and the magnetic field will be created spontaneously. This state is a new goundstate (sometimes called Savvidy vacuum), which is below the perturbative one.
However, there are objections (beyond its instability)
- it is shallow (exponentially small for small coupling),
- it is not stable (due to the imaginary part)
- the symmetry is not restored at high temperature [1]
- using the renormalization group invariant field strength, the energy density reads
$gB = \mu^2 \exp\left(-\frac{48\pi^2}{\beta g^2(\mu)}\right) = \Lambda^2_{QCD}$,
as pointed out in [2]. This is by far too much as candidate for the dark matter

[1] Walter Dittrich and Volker Schanbacher. The effective QCD lagrangian at finite temperature.
Physics Letters B, 100(5):415–419, 1981

[2] H.B. Nielsen. Approximate qcd lower bound for the bag constant b.
Physics Letters B, 80(1):133–137, 1978

Attempts to overcome the instability

- *Copenhagen vacuum* [1]
  It rests on the observation that the instability for its formation needs a certain spatial region of a slowly varying background field, for instance to have $gB < p_z^2$. One expected a certain domain structure to be formed.
- In [2] (and successors) the idea was spelled out that the self-interaction of the tachyonic mode, which is a consequence of the non-Abelian structure of the theory, should remove the imaginary part like it happens with the quartic oscillator in quantum mechanics.
- In [3], an attempt was undertaken to sum ring (*daisy*) diagrams using the gluon polarization tensor in some tractable approximation.

[1] J. Ambjrn and P. Olesen. On the Formation of a Random Color Magnetic Quantum Liquid in QCD. *Nucl. Phys. B*, 170(1):60–78, 1980
[2] Curt A. Flory. Covariant Constant Chromomagnetic Fields and Elimination of the One Loop Instabilities. 1983.
Preprint, SLAC-PUB3244, 1983
[3] Vladimir Skalozub and Michael Bordag. Color ferromagnetic vacuum state at finite temperature. *Nucl. Phys. B*, 576:430–44, 2000

Selfdual background

Another approach starts from a selfdual background. In such background, which necessarily involves also a chromoelectric field, the effective potential has also a minimum, but without imaginary part. In place, one has an infinite number of zero modes [1]. Also, the formulation is in Euclidean space and returning to Minkowski space, the electric field becomes imaginary.
Recently, [2] was able to sum up these zero modes. Further, there it was shown, that the electric field may be switched off keeping the imaginary part away. The previous result without imaginary part was re-obtained, which is the common result for all these approaches.
However, as shown in [3], as soon as one includes some A_0-background, the tachyonic instability reappears.

[1] H. Leutwyler. Vacuum Fluctuations Surrounding Soft Gluon Fields.
Phys. Lett., 96B:154–158, 1980

[2] George Savvidy. Stability of Yang Mills vacuum state.
Nuclear Physics B, 990:116187, 2023

[3] M. Bordag and V. Skalozub. Effective potential of gluodynamics in background of Polyakov loop and colormagnetic field.
Eur. Phys. J. C, 82:390, 2022.
arXiv 2112.01043

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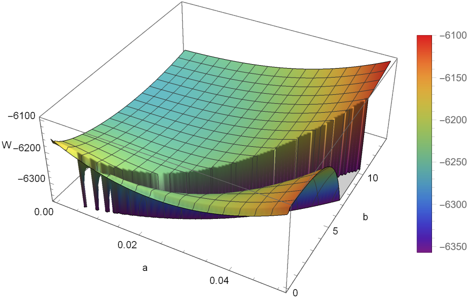


The effective potential in the background of both, A_0 and B

there is a minimum for $B = 0$ and a minimum for $A_0 = 0$

with both, the imaginary part reappears and the energy is not bounded from below

A field theory for the tachyonic mode

Consider SU(2) gluodynamics

Lagrangian $\mathcal{L} = -\frac{1}{4}\left(F^a_{\mu\nu}[A]\right)^2, \quad (a = 1, 2, 3 - \text{color})$

introduce background field $A^a_\mu = B^a_\mu + Q^a_\mu$

field strength

$$F^a_{\mu\nu}[B+Q] = F^a_{\mu\nu}[B] + D^{ab}_\mu Q^b_\nu - D^{ab}_\nu Q^b_\mu + g\varepsilon^{abc} Q^b_\mu Q^c_\nu$$

with covariant derivative $D^{ab}_\mu = \partial_\mu \delta^{ab} + g\varepsilon^{acb} B^c_\mu$.

split Lagrangian $\mathcal{L}_{\rm YM} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4$

classical background $\mathcal{L}_0 = -\frac{1}{4}\left(F^a_{\mu\nu}[B]\right)^2$

source term $\mathcal{L}_1 = Q^a_\nu D^{ab}_\mu B^b_{\mu\nu}$

quadratic part $\mathcal{L}_2 = -\frac{1}{2} Q^a_\mu \left(-(D^a_\lambda)^2 \delta_{\mu\nu} - 2g\varepsilon^{acb} B^c_{\mu\nu} \right) Q^b_\nu$

$$\mathcal{L}_3 = -g\varepsilon^{abc}(D^{ad}_{\mu\nu} Q^d_\nu) Q^b_\mu Q^c_\nu, \quad \mathcal{L}_4 = -\frac{g^2}{4}(Q^a_\mu Q^a_\mu Q^b_\nu Q^b_\nu - Q^a_\mu Q^a_\nu Q^b_\mu Q^b_\nu)$$

The background field shows up in the covariant derivatives (besides $\mathcal{L}_0$) and in the spin-term.

Abelian background and charged basis

$$B_\mu^a = \delta^{a3} B_\mu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

turn the fields

$$Q_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu + W_\mu^*), \qquad Q_\mu^3 = Q_\mu,$$
$$Q_\mu^2 = \frac{1}{\sqrt{2}i}(W_\mu - W_\mu^*), \qquad W_\mu = \frac{1}{\sqrt{2}}\left(Q_\mu^1 + iQ_\mu^2\right).$$

interpretation: Q_μ - color neutral vector field (Abelian)
W_μ - color charged vector field (non Abelian)

Lagrangian:

$$\mathcal{L}_2 = -\tfrac{1}{2} Q_\mu \Big(-\partial^2 \delta_{\mu\nu} \Big) Q_\nu - W_\mu^* \Big(-(D_\lambda)^2 \delta_{\mu\nu} - 2igB_{\mu\nu} \Big) W_\nu \ ,$$
$$\mathcal{L}_3 = -ig(Q_\mu W_{\mu\nu}^* W_\nu - Q_\mu W_{\mu\nu} W_\nu^* - Q_{\mu\nu} W_\mu^* W_\nu)$$
$$\mathcal{L}_4 = -g^2(Q_\mu Q_\mu W_\nu^* W_\nu - Q_\mu Q_\nu W_\mu^* W_\nu + W_\mu^* W_\mu W_\nu^* W_\nu - W_\mu^* W_\nu W_\mu^* W_\nu)$$

where

$$Q_{\mu\nu} = \partial_\mu Q_\nu - \partial_\nu Q_\mu, \quad W_{\mu\nu} = D_\mu W_\nu - D_\nu W_\mu, \qquad D_\mu = \partial_\mu - iB_\mu,$$

Cylindrical symmetric background

$$\text{potential:} \quad B_\mu = \begin{pmatrix} 0 \\ B_1 \\ B_2 \\ 0 \end{pmatrix}_\mu , \quad \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \vec{B} = \vec{e}_\varphi \frac{\mu(r)}{r}$$

($\mu(r)$ – profile function)

homog. field $\quad \mu(r) = \frac{Br^2}{2}, \qquad \frac{\mu'(r)}{r} \equiv B(r) = B$

center-vortex $\quad \mu(r) = \frac{Br^2}{2}\Theta(R-r) + \frac{BR^2}{2}\Theta(r-R), \quad \frac{\mu'(r)}{r} \equiv B(r) = B\,\Theta(R-r)$

center-vortex was earlier considered in [1] and [2], for effective potential obtained similar results as in homogeneous background

[1] Dmitri Diakonov and Martin Maul. Center-vortex solutions of the Yang-Mills effective action in three and four dimensions.
Phys. Rev. D, 66:096004, 2002

[2] M. Bordag. Vacuum energy of a color magnetic vortex.
Phys. Rev., D67:065001, 2003

Center-vortex background

magnetic field homogeneous inside a cylinder of radius R and zero outside:

$$\mu(r) = \frac{Br^2}{2}\Theta(R-r) + \frac{BR^2}{2}\Theta(r-R), \; \frac{\mu'(r)}{r} \equiv B(r) = B\,\Theta(R-r)$$

flux: $\Phi = \int d^2x_\perp B(r) = \pi R^2 B = 2\pi\delta,$

$$\delta = \frac{BR^2}{2}$$

energy: $E_{bg} = \frac{1}{2}\int d^2x_\perp B(r)^2 = \frac{\pi}{2}B^2R^2 = \pi\frac{\delta^2}{R^2}$

and consider the following expansion of the tachyonic mode

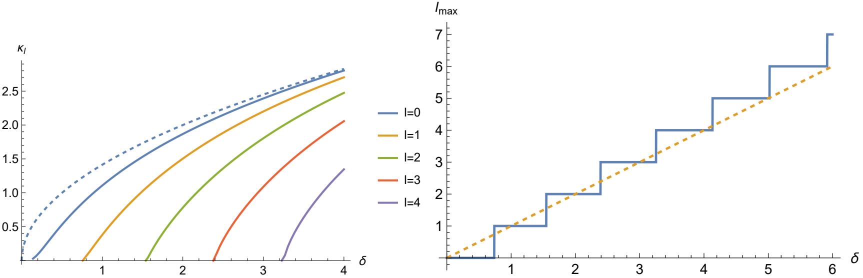
$$W_\mu^{\rm ta}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}_\mu \sum_{l=0}^{l_{max}} \frac{e^{il\varphi}}{\sqrt{2\pi}}\phi_l(r)\psi_l(x_\alpha), \qquad (\alpha = 0,3) \quad x_\perp = (r,\varphi),$$

allowing for several orbital momenta,

$\phi_l(x_\perp)$ are the eigenfunctions of the spatial part of the operator,

$$\left(\partial_r^2 + \frac{1}{r}\partial_r - \frac{(l-\mu(r))^2}{r^2} + 2\frac{\mu'(r)}{r}\right)\phi_l(r) = \kappa_l^2\phi_l(r).$$

with eigenvalues κ_l (bound state solutions) need normalizable solutions: $\int_0^\infty dr\, r\ \phi_l(r)\phi_{l'}(r) = \delta_{ll'}$,



Indeed, for $l > 0$, there are tachyonic levels, although less strong coupled, there is one state per unite flux

for the coefficients we get a 2d theory: $\tilde{\mathcal{L}} = \int dx_\perp\ \mathcal{L} \equiv \tilde{\mathcal{L}}_2 + \tilde{\mathcal{L}}_4$ with

$$\tilde{\mathcal{L}}_2 = -\sum_{l=0}^{l_{max}} \psi_l^*(x_\alpha)\left(-\partial_\alpha^2 + m_l^2\right)\psi_l(x_\alpha), \qquad (\alpha = 0, 3)$$

$$\tilde{\mathcal{L}}_4 = -\lambda\delta \sum_{l_1,\ldots,l_4 \le l_{max}} \delta_{l_1-l_2,l_3-l_4} N_4(l_i)\ \psi_{l_1}^*(x_\alpha)\psi_{l_2}(x_\alpha)\psi_{l_3}^*(x_\alpha)\psi_{l_4}(x_\alpha)$$

2d theory for the tachyonic modes

$$\tilde{\mathcal{L}}_2 = -\sum_{l=0}^{l_{max}} \psi_l^*(x_\alpha)\left(-\partial_\alpha^2 + m_l^2\right)\psi_l(x_\alpha), \qquad (\alpha = 0, 3)$$

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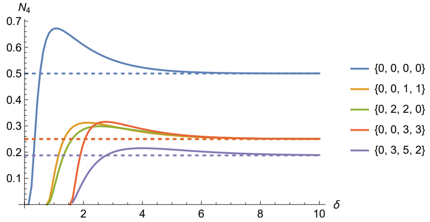
and the coefficients are

$$m_l^2 = -\kappa_l^2, \qquad \lambda = \frac{g^2}{\pi},$$

$$N_4(l_i) = \int_0^\infty dr\, r\, \phi_{l_1}(r)\phi_{l_2}(r)\phi_{l_3}(r)\phi_{l_4}(r).$$



$$N_4^{hom}(l_i) = \frac{\Gamma(l_1+l_3+1)}{2^{l_1+l_3}\sqrt{\Gamma(l_1+1)\Gamma(l_2+1)\Gamma(l_3+1)\Gamma(l_4+1)}}$$

A stable tachyon condensate

Due to the selfrepulsion of the tachyonic modes, represented by $\mathcal{L}_4$, the energy has a stable minimum

represent as module-phase: $\psi_l(x_\alpha) = \frac{1}{\sqrt{2}}\varphi_l(x_\alpha)\, e^{i\Theta_l(x_\alpha)}$

consider condensates: $\varphi_l(x_\alpha) \to v_l + \varphi_l(x_\alpha), \quad \Theta_l(x_\alpha) \to \vartheta_l + \Theta_l(x_\alpha)$

insert into the lagrangian and expand: $\hat{\mathcal{L}} = \hat{\mathcal{L}}_0 + \hat{\mathcal{L}}_1 + \hat{\mathcal{L}}_2 + \dots$ with

$\hat{\mathcal{L}}_0 = \frac{1}{2}\sum_{l=0}^{l_{max}} \kappa_l^2 v_l^2 - \frac{g^2}{2\pi}\delta \sum_{l_1,\dots,l_4 \le l_{max}} \delta_{l_1-l_2,l_3-l_4} N_4(l_i)\; v_{l_1} v_{l_2} v_{l_3} v_{l_4}\; e^{i\vartheta_{l1}-i\vartheta_{l2}+i\vartheta_{l3}-i\vartheta_{l4}}$

$\hat{\mathcal{L}}_1 = \sum_{l=0}^{l_{max}} \left[\kappa_l^2 v_l - 4\frac{g^2}{2\pi}\delta \sum_{l_1,\dots,l_4 \le l_{max}} \delta_{l_1-l_2,l_3-l_4} N_4(l_i)\; \delta_{l,l_1} v_{l_2} v_{l_3} v_{l_4}\right] \varphi_l(x_\alpha),$

$\hat{\mathcal{L}}_2 = -\sum_{l=0}^{l_{max}} \left[\frac{1}{2}\varphi_l(x_\alpha)\left(-\partial_\alpha^2 \delta_{ll'} + m_{ll'}^2\right)\varphi_l(x_\alpha) + v_l^2 \Theta_l(x_\alpha)(-\partial_\alpha^2)\Theta_l(x_\alpha)\right]$

compare with single field:

$\frac{1}{2}(p^2-m^2)(v+\varphi)^2+\lambda(v+\varphi)^4 = -\frac{m^2}{2}v^2+\lambda v^4+v\varphi(-m^2+\lambda v^2)+\frac{1}{2}(p^2-m^2+3\lambda v^2)\varphi^2$

The mass is now a matrix with entries

$$m_{ll'}^2 = -\kappa_l^2 \delta_{ll'} + 3\frac{g^2}{2\pi}\delta \sum_{l_1,\dots,l_4 \le l_{max}} \delta_{l_1-l_2,l_3-l_4} N_4(l_i)\delta_{l,l_1}\delta_{l,l_2}\; v_{l_3} v_{l_4}.$$

The effective potential on tree level and numerical work

$$V_{eff}^{tree} \equiv -\hat{\mathcal{L}}_0\Big|_{v_l^{tree}} \,,$$

further goes the 'tr ln' contribution
The minimum of $\hat{\mathcal{L}}_0$ was found numerically:

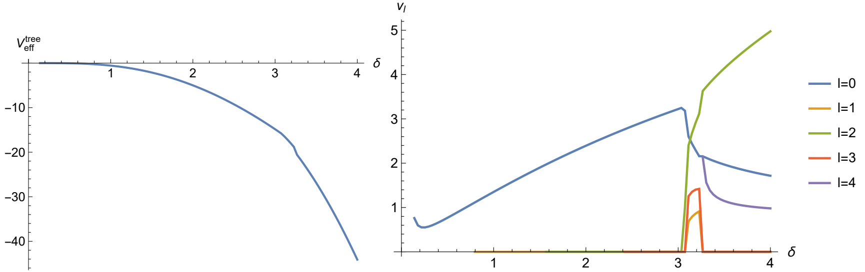


Figure: Left panel: The value of $V_{eff}^{tree} = -\mathcal{L}$ in the minimum. Right panel: The tree level condensates v_l^{tree} as function of the flux δ. The mesh for these plots is $\Delta\delta = 0.039$.

It was checked that the first variation in the minimum is zero within the given precision (up to 100 digits of numerical precision).
see non-regular behavior after $\delta = 3.03$

Diagonalization of the mass matrix

The mass matrix $m_{l'l}$ can be diagonalized, the eigenvalues are in the plot:

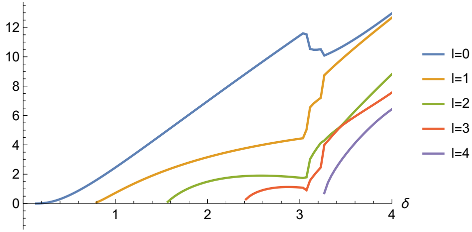


Figure: The mass eigenvalues on tree level, i.e., after diagonalization.

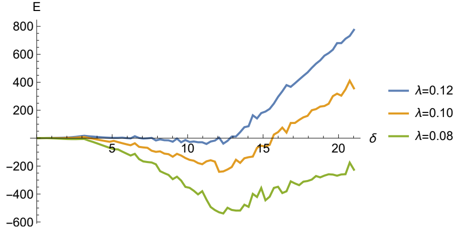
As can be seen, each eigenvalue is non-zero (even when only one condensate is non-zero)
For $\delta > 3.03$, the behavior is also non-regular.
Interpretation: classical chaos ?
(known, for example for a hydrogen atom in a magnetic field)

The minimum of the total energy

We add the energy of the background field:

$$E = \frac{1}{2}B^2 + V_{eff}^{tree}$$

and consider larger magnetic field



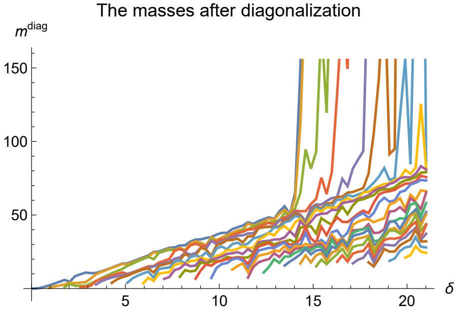


Figure: Left panel: The energy E of the system. Right panel: The mass eigenvalues on tree level, i.e., after diagonalization, for $\lambda = 0.1$. The mesh for these plots is $\Delta\delta = 0.28$.

note: the uneveness of the plots is NOT due to numerics, but it is intrinsic

Compare with the homogeneous background field

The energy is proportional to the volume of the plane perpendicular to the magnetic field, hence infinite. The finite radius R can be viewed as a regularization parameter. However, the number l_{max} of the involved orbital momenta can serve also as a regularization parameter. Now, if taking for $N_4(l_l)$ the values from the homogeneous space (i.e., the expression in terms of Gamma functions), we get a different picture

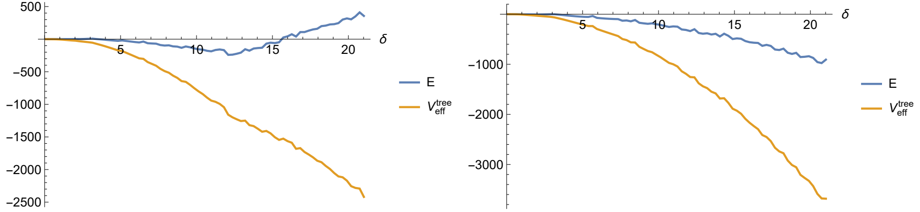


Figure: Left panel: The effective potential and the energy for $\lambda = 0.1$. Right panel: The effective potential and the energy for $\lambda = 0.1$ calculated with N_4^{hom}, in place of $N_4(R)$.

Thus, taking the infinite space expressions for the N_4^{hom} or l_{max} as regularization parameter, is not a good approximation.

Conclusions

In the background of a finite radius chromomagnetic flux tube, we considered the tachyonic modes. There is, roughly speaking, one per flux quantum. These modes produce the instability of the Savvidy vacuum.

On the classical level, accounting for their self-repulsion (ϕ^4-like term in the action), these form a condensate. Making a shift of the tachyonic modes (like in a Higgs model) we get modes with nonzero masses; thus a stable model. Also, there are Goldstone modes.

The energy of the background and the condensate (together) have a minimum as function of the magnetic field (or, of the flux). The depth increases with decreasing coupling constant (in distinction from the Savvidy vacuum).

We investigate the condensate in some detail. The behavior of the effective potential and the condensates as function of the flux show an unexpected complex behavior, which is, possible, related to classical chaos.

For further work, since all masses are real, the path is open for calculating quantum corrections, including at finite temperature.

An open problem in the investigation of the phase transition in this model (the tachyonic modes 'live' in two dimensions) is the relation to the Mermin-Wagner theorem.

Further, one needs to include the 'remaining' (stable) components of the gluon field.

open problem: phase transition in 2d

consider homogeneous chromomagnetic background field, the tachyonic modes is

$$W_\mu^{ta}(x) = u_0(x_\perp)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\\ 0\\ 0\end{pmatrix}_\mu \psi(x_\alpha), \qquad u_0(x_\perp) \quad = \left(\frac{B}{2\pi}\right)^{1/2} \exp\left(-\frac{B}{4}x_\perp^2\right)$$

take $\psi(x_\alpha) = v + \eta(x_\alpha) + i\phi(x_\alpha)$, get the effective potential in the form

$$W = \frac{m^2}{2}v^2 - \frac{\lambda}{8}v^4 + \frac{1}{2}\mathrm{tr}\ln\beta_\eta + \frac{1}{2}\mathrm{tr}\ln\beta_\phi - \frac{1}{2}\mathrm{tr}\Delta_\eta^{-1}\beta_\eta - \frac{1}{2}\mathrm{tr}\Delta_\pi^{-1}\beta_\pi + W^{2\mathrm{PI}}[\beta_\eta, \beta_\phi]$$

The inverse free propagators read
$\Delta_\eta^{-1} = k_\alpha^2 + \mu_\eta^2$, $\Delta_\phi^{-1} = k_\alpha^2 + \mu_\phi^2$,
with

$$\mu_\eta^2 = -gB + \frac{3}{2}\lambda v^2, \qquad \mu_\phi^2 = -gB + \frac{1}{2}\lambda v^2$$

this is after applying the formalism of the second Legendre transform (CJT formalism)

Phase transitions and symmetry restoration

With the above formulas, and after solving the corresponding gap equations in Hartree approximation, in this $O(2)$-model one comes to a first-order phase transition; at some T_c, the condensate v disappears and the symmetry is restored.

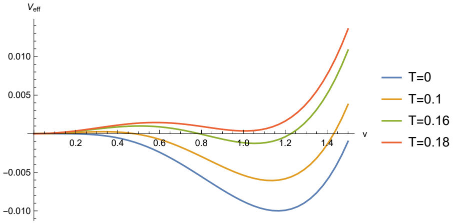


Figure: V_{eff} as function of v for $b = 0.1$ and $g = 1$.

Comment From general grounds one would expect a second order transition. That we see a first order is a known defect of the Hartree approximation. However, the symmetry breaking at low temperature, and its restoration at higher temperature are correct also in the given approximation.

Minimum of the effective potential and symmetry restoration

The complete effective potential consists of the classical energy and the quantum correction, $\tilde{V}_{eff}(v, B) = \frac{B^2}{2} + V_{eff}$, and, in fact, we have to look for a minimum of <u>this</u> expression. This is, what comes in place of the old Heisenberg-Euler formula. A numerical evaluation shows a minimum in the plane of two parameters, v and B. The symmetry is restored at high temperature in opposite to the Savvidy vacuum

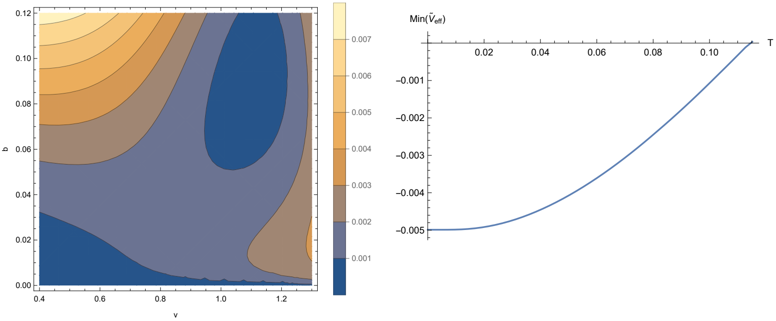


Figure: The vacuum energy $\tilde{V}_{eff}(v, B)$, for $T = 0.12$ (left panel) with two minima. The right panel shows the depth $Min(\tilde{V}_{eff})$ of the minimum as function of the temperature.