On Z-dominance, shift symmetry and spin-locality in HS theory

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based on arXiv:2212.05006 (with V.Didenko)

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Short bibliography on HS locality

Locality problem in HS theory within the framework of Vasiliev generating system was studied in

- 1605.02662 (M.Vasiliev)
- 1805.11941 (O.Gelfond, M.Vasiliev)
- 1807.00001 (V.Didenko, O.Gelfond, AK, M.Vasiliev)
- 1909.04876 (V.Didenko, O.Gelfond, AK, M.Vasiliev)
- 1910.00487 (O.Gelfond, M.Vasiliev)
- 2009.02811 (V.Didenko, O.Gelfond, AK, M.Vasiliev)
- 2101.01683 (O.Gelfond, AK)
- 2212.05006 (V.Didenko, AK)
- 2307.09331 (M.Vasiliev)
- 2308.16281 (O.Gelfond)

$$d_x \omega + \omega * \omega = \Upsilon(\omega, \omega, C) + \dots,$$
$$d_x C + [\omega, C]_* = \Upsilon(\omega, C, C) + \dots$$

Spin-locality and Z-dominance

Solving equations of the Vasiliev generating system one can systematically obtain higher order corrections to interaction vertices which acquire the following form (schematically)

$$\Upsilon(\omega, C, \ldots, C) = \sum \int_0^1 d\mathcal{T} \ldots e^{i\mathcal{T} z_\alpha y^\alpha + \cdots} \omega(\mathcal{T}z + \ldots) C(\mathcal{T}z + \ldots) \ldots C(\mathcal{T}z + \ldots)$$

$$= \int d\mathcal{T} \delta(\mathcal{T}) e^{i\mathcal{T} z_\alpha y^\alpha + \cdots} \ldots \omega(\mathcal{T}z + \ldots) C(\mathcal{T}z + \ldots) \ldots C(\mathcal{T}z + \ldots).$$

Z-dominated nonlocality

$$\Upsilon(\omega, C, \ldots, C) =$$

$$= \sum \int_0^1 d\mathcal{T} \ldots e^{i\mathcal{T} z_\alpha y^\alpha + \underbrace{i\mathcal{T} \partial_{m\alpha} \partial_n{}^\alpha}_{} + \cdots} \omega(\mathcal{T}z + \ldots) C(\mathcal{T}z + \ldots) \ldots C(\mathcal{T}z + \ldots) =$$

$$= \int d\mathcal{T} \delta(\mathcal{T}) \, \omega C^n + \int d\mathcal{T} \delta(\mathcal{T}) \, (\partial_m \partial_n) \omega C^n + \int d\mathcal{T} \delta(\mathcal{T}) \, (\partial_m \partial_n)^2 \omega C^n + \ldots$$

Spin-locality and Z-dominance

From holomorphic vertices with Z-dominated nonlocalities obtained in 2009.02811 (V.Didenko, O.Gelfond, AK, M.Vasiliev) manifestly spin-local vertices were indeed extracted in 2101.01683 (O.Gelfond, AK). Perhaps some sort of symmetry is responsible for truncation!

Combined all the contribution to the holomorphic third order vertex can written in the remarkable factorized form

$$\Upsilon(y, t, p_j) = \int \mathscr{D}\rho \, \Pi^\alpha(y, t, p_j \,|\, \rho) \circledast \int_0^1 \mathrm{d}\mathcal{T} \, (1 - \mathcal{T}) z_\alpha \, e^{i\mathcal{T} z_\alpha (y - P_z)^\alpha} \times$$

$$\times \, \omega(\mathsf{y}_\omega) C(\mathsf{y}_1) \dots C(\mathsf{y}_3) \Big|_{\mathsf{y} = 0}.$$

Here

$$t = -i\frac{\partial}{\partial \mathsf{y}_\omega}, \quad p_i = -i\frac{\partial}{\partial \mathsf{y}_i}, \quad P_z(t, p_j \,|\, \rho) = \rho_t^z t + \rho_1^z p_1 + \dots \rho_3^z p_3 \,, \, P_y = \dots, P_t = \dots.$$

$$\Pi^\alpha(y, t, p_j \,|\, \rho) := \sum_k \mathcal{R}_k(\rho) \pi_k^\alpha(y, t, p_j) \exp\left\{ -iy_\alpha P_y{}^\alpha - it_\alpha P_t^\alpha \right\},$$

$$\int \mathscr{D}\rho := \int \mathrm{d}^n \rho \,.$$

«Half-product»

$$\phi(y) \circledast \Gamma(z, y) := \frac{1}{(2\pi)^2} \int \mathrm{d}^2 u \, \mathrm{d}^2 v \, e^{iuv} \, \phi(y + u) \Gamma(z - v, y) \,.$$

Function Π^α does not contain any nonlocalities! π^α is a polynomial in each argument.

Z-dominated nonlocality comes after half-product computation

$$\int \mathscr{D}\rho\, \Pi^\alpha(y,t,p_j|\,\rho) \circledast \int_0^1 d\mathcal{T}\,(1-\mathcal{T})z_\alpha\, e^{i\mathcal{T}z_\alpha(y-P_z)^\alpha} =$$

$$= \int_0^1 d\mathcal{T} \int \mathscr{D}\rho \ldots \exp\left\{i\mathcal{T}z_\alpha(y-P_z)^\alpha - \underbrace{i\mathcal{T}P_{z\alpha}P_y{}^\alpha}_{} + i(1-\mathcal{T})y^\alpha P_{y\alpha} + \ldots\right\}$$

Generating system guarantees that $\Upsilon(y,t,p_j)$ is z-independent

$$\frac{\partial}{\partial z^\alpha}\left(\int \mathscr{D}\rho\, \Pi^\beta(y,t,p_j|\,\rho) \circledast \int_0^1 d\mathcal{T}\,\frac{1-\mathcal{T}}{\mathcal{T}}\mathcal{T}z_\beta\, e^{i\mathcal{T}z_\alpha(y-P_z)^\alpha}\right) = 0$$

In terms of half-product this condition can be rewritten as

$$\int \mathscr{D}\rho\, \Pi^\beta(y,t,p_j|\,\rho)(y-P_z)_\beta \circledast \int_0^1 d\mathcal{T}\,\mathcal{T}z_\alpha e^{i\mathcal{T}z_\alpha(y-P_z)^\alpha} = 0\,.$$

Since it is true for all z it should be true in particular for $z=0$ which turns into y-independence condition

$$\frac{\partial}{\partial y^\alpha}\int_0^1 d\mathcal{T} \int \mathscr{D}\rho\, \mathcal{T}(y-P_z)^\beta \Pi_\beta\big((1-\mathcal{T})(y-P_z) + P_z, t, p_j|\,\rho\big) = 0\,.$$

Scenario for obtaining manifestly Z-independent expression

If one manages to rewrite left part of half-product in the form

$$\Pi^\beta(y,t,p_j|\,\rho) = (y - P_z)^\beta \Pi(y,t,p_j|\,\rho)\,,$$

then after simple algebra it is possible to integrate by parts and bring the expression to the form

$$\int \mathscr{D}\rho\,(y-P_z)^\beta \Pi(y,t,p_j|\,\rho) \circledast \int_0^1 d\mathcal{T}\, \frac{1-\mathcal{T}}{\mathcal{T}} \mathcal{T} z_\beta\, e^{i\mathcal{T} z_\alpha (y-P_z)^\alpha} =$$

$$= i \int \mathscr{D}\rho\, \Pi(y,t,p_j|\,\rho)\,.$$

I.e. one needs to «divide» $\Pi^\beta(y,t,p_j|\,\rho)$ over $(y-P_z)^\beta$.

Division formula

Consider an identity which rests on Schouten identity and partial integration

$$\Pi_\sigma(y,t,p_j|\rho) \equiv (y-P_z)_\sigma \int_0^1 d\xi \, \frac{\partial}{\partial(y-P_z)^\beta} \Pi^\beta\big(\xi(y-P_z)+P_z,t,p_j|\rho\big) +$$

$$+ \frac{\partial}{\partial y^\sigma} \int_0^1 d\xi \, (y-P_z)^\beta \Pi_\beta\big(\xi(y-P_z)+P_z,t,p_j|\rho\big) .$$

It turns out that

$$\int \mathscr{D}\rho \, (y-P_z)^\alpha \int_0^1 d\xi \, \frac{\partial}{\partial y^\beta} \Pi^\beta\big(\xi(y-P_z)+P_z,t,p_j|\rho\big) \circledast \int_0^1 d\mathcal{T} \, \frac{1-\mathcal{T}}{\mathcal{T}} \mathcal{T} z_\alpha \, e^{i\mathcal{T} z_\alpha(y-P_z)^\alpha} =$$

$$= \int \mathscr{D}\rho \, \Pi^\alpha(y,t,p_j|\rho) \circledast \int_0^1 d\mathcal{T} \, (1-\mathcal{T}) z_\alpha \, e^{i\mathcal{T} z_\alpha(y-P_z)^\alpha} .$$

$$\int \mathscr{D}\rho \, \Pi^\alpha(y,t,p_j|\rho) \circledast \int_0^1 d\mathcal{T} \, (1-\mathcal{T}) z_\alpha \, e^{i\mathcal{T} z_\alpha(y-P_z)^\alpha} \bigg|_{z=0} =$$

$$= i \int \mathscr{D}\rho \int_0^1 d\xi \, \frac{\partial}{\partial y^\beta} \Pi^\beta\big(\xi(y-P_z)+P_z,t,p_j|\rho\big)$$

Shift symmetry

For C^2 holomorphic vertices one can see

$$\Upsilon_{C^2}(y, t, p_1 + a, p_2 - a) = e^{ia_\alpha (t+y)^\alpha} \Upsilon_{C^2}(y, t, p_1, p_2).$$

Analogous property is valid for cubic vertices

$$\Upsilon_{C^3}(y, t, p_1 + a, p_2 - a, p_3 + a) = e^{ia_\alpha (t+y)^\alpha} \Upsilon_{C^3}(y, t, p_1, p_2, p_3).$$

To see that such property indeed take place note that under above transformation derivatives effectively transform as

$$P_z(t, p_1 + a, p_2 - a, p_3 + a) = P_z(t, p_1, p_2, p_3),$$

$$P_y(t, p_1 + a, p_2 - a, p_3 + a) = a + P_y(t, p_1, p_2, p_3),$$

$$P_t(p_1 + a, p_2 - a, p_3 + a) = a + P_t(p_1, p_2, p_3),$$

$$\pi(y, t, p_1 + a, p_2 - a, p_3 + a) = \pi(y, t, p_1, p_2, p_3).$$

From all above properties it follows that left part of half-product transforms as

$$\Pi_\sigma(y, p_1 + a, p_2 - a, p_3 + a) = e^{ia_\alpha (t+y)^\alpha} \Pi_\sigma(y, p_1, p_2, p_3),$$

which eventually leads to the shift symmetry of the cubic vertex.

Reminder equals zero

Recall the division formula (ξ was changed to $(1 - \mathcal{T})$)

$$\Pi_\sigma(y, t, p_j \,|\, \rho) \equiv (y - P_z)_\sigma \int_0^1 d\mathcal{T} \, \frac{\partial}{\partial y^\beta} \Pi^\beta\big((1 - \mathcal{T})(y - P_z) + P_z, t, p_j \,|\, \rho\big) +$$
$$+ \frac{\partial}{\partial y^\sigma} \int_0^1 d\mathcal{T} \, (y - P_z)^\beta \Pi_\beta\big((1 - \mathcal{T})(y - P_z) + P_z, t, p_j \,|\, \rho\big) \,.$$

and z-indepedence condition (y-independence)

$$\frac{\partial}{\partial y^\alpha} \int_0^1 d\mathcal{T} \int \mathscr{D}\rho \, \mathcal{T}(y - P_z)^\beta \Pi_\beta\big((1 - \mathcal{T})(y - P_z) + P_z, t, p_j \,|\, \rho\big) = 0 \,.$$

Introduce two auxilary functions

$$\mathscr{L}(t, p_i) := \int \mathscr{D}\rho \int_0^1 d\mathcal{T} \, \mathcal{T}(y - P_z)^\beta \Pi_\beta\big((1 - \mathcal{T})(y - P_z) + P_z, t, p_i\big) \,,$$

$$\mathscr{R}(y, t, p_i) := \int \mathscr{D}\rho \int_0^1 d\mathcal{T} \, (y - P_z)^\beta \Pi_\beta\big((1 - \mathcal{T})(y - P_z) + P_z, t, p_i\big) \,.$$

which are different only in measure in $\mathcal{T}$.

Due to shift symmetry one can deduce that $\mathscr{R}$ and $\mathscr{Z}$ are related by differential equation

$$\left(1 + (y + \Delta)^\alpha \frac{\partial}{\partial y^\alpha}\right) \mathscr{R}(y, t, p) = \mathscr{Z}(t, p), \quad \Delta_\alpha := \left(t_\alpha + i(-1)^{j+1} \frac{\partial}{\partial p_j^\alpha}\right).$$

Generic solution has the form

$$\mathscr{R}(y, t, p) = \mathscr{R}_0(y, t, p) + \mathscr{Z}(t, p).$$

Solutions to homogeneous equation $\mathscr{R}_0$ are pathological, i.e. they are either not compatible with Lorentz symmetry or non analytic $\Longrightarrow$

$$\mathscr{R}(y, t, p) = \mathscr{Z}(t, p).$$

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/ 14
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Spin locality

After integration over all ρs division formula turns into

$$\int \mathscr{D}\rho \, \Pi_\alpha \left(y, t, p_j \,|\, \rho\right) = \int \mathscr{D}\rho \, (y - \underline{P_z})_\alpha \int_0^1 d\mathcal{T} \frac{\partial}{\partial y^\beta} \Pi^\beta \left((1 - \mathcal{T})(y - P_z) + P_z, t, p_j \,|\, \rho\right),$$

where l.h.s. is local by assumption. R.h.s. is a combination of two distinctive parts

$$y_\alpha \int \mathscr{D}\rho \int_0^1 d\mathcal{T} \frac{\partial}{\partial y^\beta} \Pi^\beta \left((1 - \mathcal{T})(y - P_z) + P_z, t, p_j \,|\, \rho\right) -$$

$$- \int \mathscr{D}\rho \, P_{z\alpha} \int_0^1 d\mathcal{T} \frac{\partial}{\partial y^\beta} \Pi^\beta \left((1 - \mathcal{T})(y - P_z) + P_z, t, p_j \,|\, \rho\right),$$

the first term here is simply $y_\alpha \Upsilon(y, t, p)$.

$$\int \mathscr{D}\rho \, y^\alpha \Pi_\alpha \left(y, t, p_j \,|\, \rho\right) = - \int \mathscr{D}\rho \, y^\alpha P_{z\alpha} \int_0^1 d\mathcal{T} \frac{\partial}{\partial y^\beta} \Pi^\beta \left((1 - \mathcal{T})(y - P_z) + P_z, t, p_j \,|\, \rho\right).$$

Hence the only non local contribution might be proportional to y_α

Term that causes problems can be rewritten in the corresponding way and initial equations turns into

$$\int \mathscr{D}\rho \, \Pi_\alpha(y,t,p_j|\,\rho) = -\int \mathscr{D}\rho \, P_{z\alpha} \int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})\partial_\beta \Pi^\beta(\mathcal{T}P_z,t,p_j|\,\rho)+$$

$$+ \, y_\alpha \int \mathscr{D}\rho \int_0^1 \mathrm{d}\xi \, \partial_\beta \Pi^\beta(\xi y,t,p_j|\,\rho)-$$

$$- \, y_\alpha \int \mathscr{D}\rho \int_0^1 \mathrm{d}\xi \int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})\partial_\beta \Pi^\beta\big((1-\mathcal{T})\xi y + \mathcal{T}P_z,t,p_j|\,\rho\big)-$$

$$- \int \mathscr{D}\rho \int_0^1 \mathrm{d}\xi \int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})^2 (y^\sigma P_{z\sigma})\partial_\alpha \partial_\beta \Pi^\beta\big((1-\mathcal{T})\xi y + \mathcal{T}P_z,t,p_j|\,\rho\big). \quad (1)$$

Hence

$$\widetilde{\Upsilon}(y,t,p_j) = i \int \mathscr{D}\rho \int_0^1 \mathrm{d}\xi \int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})\partial_\beta \Pi^\beta\big((1-\mathcal{T})\xi y + \mathcal{T}P_z,t,p_j|\,\rho\big) \quad (2)$$

is spin local by virtue of spin locality of the others. However it is not exactly the vertex. Vertex is given by

$$\Upsilon(y,t,p_j) = i \int \mathscr{D}\rho \int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})\partial_\beta \Pi^\beta\big((1-\mathcal{T})y + \mathcal{T}P_z,t,p_j|\,\rho\big). \quad (3)$$

Connection between $\widetilde{\Upsilon}$ and Υ

$$\widetilde{\Upsilon}(y,t,p_j) = \int_0^1 \mathrm{d}\xi \, \Upsilon(\xi y,t,p_j), \quad \Upsilon(y,t,p_j) = \left(y^\alpha \frac{\partial}{\partial y^\alpha}+1\right)\widetilde{\Upsilon}(y,t,p_j). \quad (4)$$

Conclusion (Results and open questions)

- Shift symmetry of the holomorphic vertices was discovered

$$\Upsilon^{\eta}_{C^n}(y, t, p_1 + a, p_2 - a, ...) = e^{i a_{\alpha}(y+t)^{\alpha}} \Upsilon^{\eta}_{C^n}(y, t, p_1, p_2, ...).$$

  This property was also found in the vertices that comes from generating system developed by Didenko (2209.01966). Counterpart for mixed vertices is not developed yet

- Shift symmetry turns to be the missing assumption for the proof of the Z-dominance conjecture.

- Effective method to obtain manifestly spin local vertices form Z-dominated is still lacking

- What is the realtion between shift symmetry and compact spin locality?

- How shift symmetry restrics differential homotopy?