On unfolded approach to gauge theories

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main statements

- application of unfolded dynamics approach (formalism of higher-spin gravity) to lower-spin theories is studied
- a new method of unfolding field theories is proposed
- the method is used to unfold scalar electrodynamics
- SSB is formulated in terms of a deformation of unfolded modules

outline

- unfolded dynamics approach of higher-spin gravity
- example: unfolding self-interacting scalar
- unfolding scalar electrodynamics
- Higgs mechanism

unfolded dynamics approach

- **higher-spin (HS) gravity** - a nonlinear gauge theory of interacting massless fields of all spins (including graviton) possessing ∞-dim HS gauge symmetry;
- to formulate the theory in a manifestly diffeomorphism- and gauge-invariant way [Vasiliev'89-94], a special first-order formalism was developed called **unfolded dynamics approach** [Vasiliev'06];
- it is interesting to apply it to different theories
- constructing unfoldeded formulations is not easy
- many examples of unfolding linear theories are known, e.g. [Shaynkman, Vasiliev' 00; Ponomarev, Vasiliev '12; Khabarov, Zinoviev'20; Buchbinder, Snegirev, Zinoviev'20; NM'19-'23].
- but not so many nonlinear theories beyond HS
- method of quantization of unfolded field theories was proposed [NM'23].

unfolded dynamics approach

- **Unfolded equations** are first-order exterior-form equations

$$\mathrm{d}W^A(x) + G^A(W) = 0, \tag{1}$$

- **Unfolded fields** are exterior forms; dynamical field theories require ∞ of unfolded fields encoding all d.o.f. (auxiliary generating variables are handy)
- **Consistency** condition

$$\mathrm{d}^2 \equiv 0 \quad \Rightarrow \quad G^B \frac{\delta G^A}{\delta W^B} \equiv 0. \tag{2}$$

- **Unfolded gauge symmetries**

$$\delta W^A = \mathrm{d}\varepsilon^A(x) - \varepsilon^B \frac{\delta G^A}{\delta W^B}. \tag{3}$$

- Unfolded fields form modules for all symmetries of the theory, realized in algebraic ways.
- Gauge symmetries are associated with $(n > 0)$-forms, 0-forms correspond to physical d.o.f.

unfolded dynamics approach

- Maximally-symmetric background: the zero-curvature equation on a 1-form

$$\mathrm{d}\Omega + \frac{1}{2}[\Omega, \Omega] = 0. \qquad (4)$$

- Global symmetries are residual symmetries of some particular Ω_0,

$$\delta\Omega = \mathrm{d}\varepsilon + [\Omega, \varepsilon] \quad \Rightarrow \quad \mathrm{d}\varepsilon_0 + [\Omega_0, \varepsilon_0] = 0. \qquad (5)$$

- $4d$ Minkowski space: $\Omega \in iso(1,3)$

$$\Omega = e^{\alpha\dot{\beta}} P_{\alpha\dot{\beta}} + \omega^{\alpha\beta} M_{\alpha\beta} + \bar{\omega}^{\dot{\alpha}\dot{\beta}} \bar{M}_{\dot{\alpha}\dot{\beta}}. \qquad (6)$$

- Greek indices correspond to two-component $sl(2,\mathbb{C})$-spinors.
- The simplest non-degenerate solution – Cartesian coordinates

$$e_{\underline{m}}{}^{\alpha\dot{\beta}} = (\bar{\sigma}_{\underline{m}})^{\dot{\beta}\alpha}, \quad \omega_{\underline{m}}{}^{\alpha\beta} = 0, \quad \bar{\omega}_{\underline{m}}{}^{\dot{\alpha}\dot{\beta}} = 0. \qquad (7)$$

- Global Poincaré symmetry in Cartesian coordinates

$$\varepsilon^{\alpha\dot{\alpha}} = c^{\alpha\dot{\alpha}} + c^{\alpha}{}_{\beta}(\bar{\sigma}_{\underline{m}})^{\dot{\alpha}\beta} x^{\underline{m}} + \bar{c}^{\dot{\alpha}}{}_{\dot{\beta}}(\bar{\sigma}_{\underline{m}})^{\dot{\beta}\alpha} x^{\underline{m}}, \quad \varepsilon^{\alpha\alpha} = c^{\alpha\alpha}, \quad \bar{\varepsilon}^{\dot{\alpha}\dot{\alpha}} = \bar{c}^{\dot{\alpha}\dot{\alpha}}. \qquad (8)$$

unfolded self-interacting scalar field

- We want to unfold the theory

$$(\Box + m^2)\phi + \mathrm{U}'(\phi) = 0. \tag{9}$$

- Introduce unfolded master-field

$$\Phi(Y|x) = \sum_{n=0}^{\infty} \Phi_{\alpha(n),\dot{\alpha}(n)}(x) y^{\alpha_1} \dots y^{\alpha_n} \bar{y}^{\dot{\alpha}_1} \dots \bar{y}^{\dot{\alpha}_n}, \tag{10}$$

  and a pair of auxiliary commuting Weyl spinors $Y = (y^\alpha, \bar{y}^{\dot{\alpha}})$.
- Spinorial Euler operators

$$N = y^\alpha \partial_\alpha, \quad \bar{N} = \bar{y}^{\dot{\alpha}} \bar{\partial}_{\dot{\alpha}}. \tag{11}$$

- Primary scalar field

$$\phi(x) = \Phi(Y = 0|x). \tag{12}$$

unfolded self-interacting scalar field

- Consider the following Ansatz [NM'23]

$$\mathrm{d}_L\Phi - a_N e\partial\bar{\partial}\Phi + b_N e y\bar{y} m^2\Phi + c_N e y\bar{y}\mathrm{U}'(f_N\Phi) = 0, \tag{13}$$

$$\mathrm{d}_L := \mathrm{d} + \omega^{\alpha\beta} y_\alpha \partial_\beta + \bar{\omega}^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{\partial}_{\dot{\beta}}, \quad e\partial\bar{\partial} := e^{\alpha\dot{\beta}}\partial_\alpha\bar{\partial}_{\dot{\beta}}, \quad ey\bar{y} := e^{\alpha\dot{\beta}} y_\alpha \bar{y}_{\dot{\beta}}. \tag{14}$$

- A general solution for consistency in terms of arbitrary (non-zero) a_N

$$b_N = \frac{1}{N(N+1)a_{N-1}}, \quad c_N = \frac{1}{(N+1)!\prod_{i=0}^{N-1} a_i}, \quad f_N = N!\prod_{i=0}^{N-1} a_i. \tag{15}$$

- Y-dependence of Φ is manifestly resolved as ($\mathrm{d}_L = e^{\alpha\dot{\alpha}}\nabla_{\alpha\dot{\alpha}}$)

$$\Phi(Y|x) = \sum_{n=0}^{\infty} \frac{(y^\alpha \bar{y}^{\dot{\alpha}} \nabla_{\alpha\dot{\alpha}})^n}{(n!)^2 \prod_{i=0}^{N-1} a_i}\phi(x), \quad (\Box + m^2)\phi + \mathrm{U}'(\phi) = 0. \tag{16}$$

- Y-dependent components are descendants (traceless space-time derivatives) of the primary scalar $\phi(x)$.

unfolded self-interacting scalar field

- Now a new method of unfolding a field theory is proposed: first postulate some concrete form of an unfolded master-field and then look for a corresponding unfolded equation, identically satisfied by this master-field.
- Step 1. Postulate a form of an unfolded field

$$\Phi(Y|x) = e^{y^{\alpha}\bar{y}^{\dot{\alpha}}\nabla_{\alpha\dot{\alpha}}}\phi(x), \quad (\Box + m^2)\phi + \mathrm{U}'(\phi) = 0. \tag{17}$$

- Step 2. Calculate a second spinorial derivative

$$\partial_{\alpha}\bar{\partial}_{\dot{\alpha}}\Phi(Y|x) = (N+1)\nabla_{\alpha\dot{\alpha}}e^{\nabla y\bar{y}}\phi(x) - y_{\alpha}\bar{y}_{\dot{\alpha}}\Box e^{\nabla y\bar{y}}\phi(x). \tag{18}$$

- Step 3. Apply constraints for primaries and Fierz identities to get rid of all but one space-time derivatives

$$\partial_{\alpha}\bar{\partial}_{\dot{\alpha}}\Phi = (N+1)\nabla_{\alpha\dot{\alpha}}\Phi + y_{\alpha}\bar{y}_{\dot{\alpha}}(m^2\Phi + \mathrm{U}'(\Phi)). \tag{19}$$

- Step 4. Contract the expression with the vierbein to get a desired unfolded equation

$$\mathrm{d}_L\Phi - \frac{1}{N+1}e\partial\bar{\partial}\Phi + \frac{1}{N+1}ey\bar{y}\left(m^2\Phi + \mathrm{U}'(\Phi)\right) = 0. \tag{20}$$

- Resulting system is consistent by construction (arose as the identity).

unfolded self-interacting scalar field

- Resulting equation is manifestly integrable in Y's by construction.
- Via unfolded field redefinition, this allows one to generate all other solutions to consistency and/or immediately obtain their solutions. For a redefinition

$$\Phi(Y|x) = \rho_N \widetilde{\Phi}(Y|x). \tag{21}$$

the system turns to

$$\mathrm{d}_L \widetilde{\Phi} - \frac{\rho_{N+1}}{\rho_N (N+1)} e \partial \bar{\partial} \widetilde{\Phi} + \frac{\rho_{N-1}}{\rho_N (N+1)} e y \bar{y} m^2 \widetilde{\Phi} + \frac{1}{\rho_N (N+1)} e y \bar{y} \mathrm{U}'(\rho_N \widetilde{\Phi}) = 0. \tag{22}$$

$$\widetilde{\Phi}(Y|x) = \frac{1}{\rho_N} e^{\nabla_{y\bar{y}}} \phi(x). \tag{23}$$

unfolding scalar electrodynamics

- We apply this recipy to the problem of unfolding scalar electrodynamics, determined by e.o.m.

$$\frac{1}{2}\mathrm{D}_{\alpha\dot{\alpha}}\mathrm{D}^{\alpha\dot{\alpha}}\phi+(m^2+\mathrm{U}'(\phi\phi^*))\phi=0,\quad \frac{1}{2}\mathrm{D}^*_{\alpha\dot{\alpha}}\mathrm{D}^{*\alpha\dot{\alpha}}\phi^*+(m^2+\mathrm{U}'(\phi\phi^*))\phi^*=0, \tag{24}$$

$$\nabla_{\beta\dot{\alpha}}F^{\beta}{}_{\alpha}=iq(\phi\mathrm{D}^*_{\alpha\dot{\alpha}}\phi^*-\phi^*\mathrm{D}_{\alpha\dot{\alpha}}\phi),\quad \nabla_{\alpha\dot{\beta}}\bar{F}^{\dot{\beta}}{}_{\dot{\alpha}}=iq(\phi\mathrm{D}^*_{\alpha\dot{\alpha}}\phi^*-\phi^*\mathrm{D}_{\alpha\dot{\alpha}}\phi), \tag{25}$$

where q is an electric charge and covariant derivatives are defined as

$$\mathrm{D}_{\alpha\dot{\alpha}}:=\nabla_{\alpha\dot{\alpha}}-iqA_{\alpha\dot{\alpha}},\quad \mathrm{D}^*_{\alpha\dot{\alpha}}:=\nabla_{\alpha\dot{\alpha}}+iqA_{\alpha\dot{\alpha}}, \tag{26}$$

$$[\mathrm{D}_{\alpha\dot{\alpha}},\mathrm{D}_{\beta\dot{\beta}}]=-iq\epsilon_{\alpha\beta}\bar{F}_{\dot{\alpha}\dot{\beta}}-iq\epsilon_{\dot{\alpha}\dot{\beta}}F_{\alpha\beta},\quad [\mathrm{D}^*_{\alpha\dot{\alpha}},\mathrm{D}^*_{\beta\dot{\beta}}]=iq\epsilon_{\alpha\beta}\bar{F}_{\dot{\alpha}\dot{\beta}}+iq\epsilon_{\dot{\alpha}\dot{\beta}}F_{\alpha\beta}. \tag{27}$$

unfolding scalar electrodynamics

- Postulate the form of unfolded modules as

$$\Phi(Y|x) = e^{y^{\beta}\bar{y}^{\dot{\beta}}\mathrm{D}_{\beta\dot{\beta}}}\phi(x), \quad \Phi^*(Y|x) = e^{y^{\beta}\bar{y}^{\dot{\beta}}\mathrm{D}^*_{\beta\dot{\beta}}}\phi^*(x). \tag{28}$$

$$F(Y|x) = e^{\nabla_{y\bar{y}}}F_{\alpha\alpha}y^{\alpha}y^{\alpha}, \quad \bar{F}(Y|x) = e^{\nabla_{y\bar{y}}}\bar{F}_{\dot{\alpha}\dot{\alpha}}\bar{y}^{\dot{\alpha}}\bar{y}^{\dot{\alpha}}. \tag{29}$$

- To make $U(1)$ gauge symmetry manifest, introduce 1-form A

$$A(x) = e^{\alpha\dot{\alpha}}A_{\alpha\dot{\alpha}}, \quad \mathrm{d}A = \frac{1}{4}e^{\alpha}{}_{\dot{\beta}}e^{\alpha\dot{\beta}}\partial_{\alpha}\partial_{\alpha}F|_{\bar{y}=0} + \frac{1}{4}e_{\beta}{}^{\dot{\alpha}}e^{\beta\dot{\alpha}}\bar{\partial}_{\dot{\alpha}}\bar{\partial}_{\dot{\alpha}}\bar{F}|_{y=0}. \tag{30}$$

- Here unfolding of Φ is strongly nonlinear (the exponent contains $A_{\alpha\dot{\alpha}}$), since there is gauge interactions now.

unfolding scalar electrodynamics

Skipping all calculations details, one arrives at unfolded formulation of scalar electrodynamics

$$\mathbf{d}\boldsymbol{A(x)} = \frac{1}{4}e^{\alpha}{}_{\dot\beta}e^{\alpha\dot\beta}\partial_\alpha\partial_\alpha F|_{\bar y=0} + \frac{1}{4}e_\beta{}^{\dot\alpha}e^{\beta\dot\alpha}\bar\partial_{\dot\alpha}\bar\partial_{\dot\alpha}\bar F|_{y=0}, \tag{31}$$

$$\mathbf{d}_{\boldsymbol{L}}\boldsymbol{F(Y|x)} - \frac{1}{(N+1)(\bar N+1)}\Big\{ \nu e\partial\bar\partial F - iq(\nu+2)ey\bar y\bar\partial_{\dot\alpha}\Phi\cdot\bar\partial^{\dot\alpha}\Phi^* -$$
$$-q(\nu+2)ey\bar y(\bar N+2)(\Phi\Phi^*\frac{2q}{(\bar N+2)}F) + 2iqey\bar\partial(\Phi N\Phi^* - \Phi^* N\Phi)\Big\} = 0, \tag{32}$$

$$\boldsymbol{(N+1)}\mathbf{D}\boldsymbol{\Phi(Y|x)} - e\partial\bar\partial\Phi + ey\bar y(m^2 + \mathrm{U}'(\Phi\Phi^*))\Phi + q^2ey\bar y\Phi\cdot\frac{1}{(\bar N+2)}F\cdot\frac{1}{(N+2)}\bar F +$$
$$+q^2ey\bar y\left(\Phi\frac{1}{(N+2)}(\Phi N\Phi^* - \Phi^* N\Phi) + (N+1)\Phi\cdot\frac{2}{(N+1)(N+2)}(\Phi N\Phi^* - \Phi^* N\Phi)\right) +$$
$$+iq\left(\frac{1}{(\bar N+2)}F\cdot e\bar y\partial\Phi - (\bar N+1)\Phi\cdot e\bar y\partial\frac{1}{(\bar N+1)(\bar N+2)}F\right) +$$
$$+iq\left(\frac{1}{(N+2)}\bar F\cdot ey\bar\partial\Phi - (N+1)\Phi\cdot ey\bar\partial\frac{1}{(N+1)(N+2)}\bar F\right) = 0, \tag{33}$$

plus conjugate equations for $\bar F$ and Φ^*. Here $\nu = \frac{N+\bar N}{2}$.

unfolding scalar electrodynamics

- Cubic terms, consisting purely of scalar fields, might seem like charged-current interactions. They are not real vertices, but artifacts of nonlinearity of unfolding: only U′-term contributes to the e.o.m. of the primary ϕ.
- Unfolded $U(1)$ gauge symmetry with a gauge parameter $\varepsilon(x)$

$$\delta A = \mathrm{d}\varepsilon, \quad \delta F = 0, \quad \delta \bar{F} = 0, \quad \delta \Phi = iq\varepsilon\Phi, \quad \delta\Phi^* = -iq\varepsilon\Phi^*. \tag{34}$$

- Unfolded Poincaré symmetry: according to the general formula (but too long)
- Manifest solution for Y-dependence:

$$F_{\alpha\alpha} := \nabla_{\alpha\dot{\beta}} A_{\alpha}{}^{\dot{\beta}}, \quad F = e^{\nabla y\bar{y}} F_{\alpha\alpha} y^{\alpha} y^{\alpha}, \quad \Phi = e^{\mathrm{D}y\bar{y}}\phi(x). \tag{35}$$

$$\frac{1}{2}\mathrm{D}_{\alpha\dot{\alpha}}\mathrm{D}^{\alpha\dot{\alpha}}\phi + (m^2 + \mathrm{U}'(\phi\phi^*))\phi = 0, \quad \nabla_{\beta\dot{\alpha}} F^{\beta}{}_{\alpha} = iq(\phi\mathrm{D}^*_{\alpha\dot{\alpha}}\phi^* - \phi^*\mathrm{D}_{\alpha\dot{\alpha}}\phi). \tag{36}$$

Higgs Mechanism

- Consider the massless case with the "Mexican hat" potential

$$\mathrm{U}(\phi\phi^*) = -\mu^2\phi\phi^* + \frac{\lambda}{2}(\phi\phi^*)^2, \quad m^2 = 0. \tag{37}$$

- Consider fluctuations over a particular (real) vacuum

$$\phi_0 = \frac{\mu}{\sqrt{\lambda}}, \quad \Phi(Y|x) = \phi_0 + \mathrm{X}(Y|x) + i\Theta(Y|x). \tag{38}$$

- Linearization of unfolded equations gives

$$\mathrm{d}_L F - \frac{1}{(N+1)(\bar{N}+1)}\left\{ve\partial\bar{\partial}F - (\nu+2)2q^2\phi_0^2 ey\bar{y}F + 4q\phi_0 ey\bar{\partial}N\Theta\right\} = 0, \tag{39}$$

$$(N+1)\mathrm{d}_L\Theta - q\phi_0 A - e\partial\bar{\partial}\Theta - 2q^2\phi_0^2 ey\bar{y}\frac{N(N+3)}{(N+1)(N+2)}\Theta -$$

$$-q\phi_0 e\bar{y}\partial\frac{1}{(\bar{N}+1)(\bar{N}+2)}F - q\phi_0 ey\bar{\partial}\frac{1}{(N+1)(N+2)}\bar{F} = 0, \tag{40}$$

$$(N+1)\mathrm{d}_L\mathrm{X} - e\partial\bar{\partial}\mathrm{X} + 2\mu^2 ey\bar{y}\mathrm{X} = 0. \tag{41}$$

- The last equation indeed describes Higgs boson with $m_X^2 = 2\mu^2$.

Higgs Mechanism

- Inspecting Θ-equation at $Y = 0$, one sees

$$\theta_{\alpha\dot{\alpha}} = \nabla_{\alpha\dot{\alpha}}\theta - q\phi_0 A_{\alpha\dot{\alpha}}, \quad \delta_{U(1)}\theta(x) = q\phi_o\varepsilon(x), \tag{42}$$

i.e. $\theta(x)$ can be gauged away, while $\theta_{\alpha\dot{\alpha}}$ is gauge-invariant.

- Introduce a new unfolded gauge-invariant field via non-invertible field redefinition, that switches the primary

$$B = -\frac{N}{q\phi_0}\Theta, \tag{43}$$

$$\mathrm{d}_L F = \frac{1}{(N+1)(\bar{N}+1)}\left\{\nu e\partial\bar{\partial}F - (\nu+2)m_V^2 ey\bar{y}F - 2m_V^2 ey\bar{\partial}B\right\}, \tag{44}$$

$$\mathrm{d}_L B = \frac{1}{(N+1)(\bar{N}+1)}\left\{\nu e\partial\bar{\partial}B - (\nu+2)m_V^2 ey\bar{y}F - e\bar{y}\partial F - ey\bar{\partial}\bar{F}\right\}. \tag{45}$$

– unfolded massive vector field with $m_V^2 = 2q^2\phi_0^2$.

conclusions

- a new method of unfloding is put forward: first postulating an unfolded master-field, then deriving an unfolded equation as an identity
- using this, unfolded formulation of scalar electrodynamics was constructed
- SSB was analyzed, revealing a structure changeover of unfolded modules
- perspectives:
  - to apply the new method to more complicated theories (Yang-Mills, GR, strings etc.)
  - to investigate the integrability problem: how to inverse $x \to y$ integrability to $y \to x$
  - to extend the analysis to HS SSB