

Breaking higher-spin symmetry in the bulk

V.E. Didenko
(with A.V. Korybut)

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- String theory comes from higher-spin symmetry breaking?
Old ideas [D.Gross'88; S.Konshtein, M.Vasiliev'89]
Not that old [R.Metsaev'99; B.Sundborg'01; E.Witten'01;
M.Bianchi, J.F.Morales, H.Samtleben'03]
Recent [A.Sagnotti'11; M.Vasiliev'18]

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- Mathematical tools to approach nonlinear HS dynamics
Generating formulation [M.Vasiliev'89'92'03; V.D'22; V.D.,
A.Korybut'23]
Locality problem [S.Prokushkin, M.Vasiliev'98; S.Giombi,
X.Yin'10; S.Sleight, M.Taronna'18; V.D., O.Gelfond,
A.Korybut, M.Vasiliev'18-24; D.Ponomarev'17; Y.Neiman'23]

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- **Goal:** Recover field spectrum in d dimensions by spontaneous breaking of HS symmetry in $d + 1$. A proper HS vacuum is required.

- Metric like approach

Dates back to (Fang-)Fronsdal'78

$$\square \phi^{\alpha(s)} - D^\alpha D_\mu \phi^{\mu\alpha(s-1)} + \frac{1}{2} D^\alpha D^\alpha \phi^{\alpha(s-2)\mu}{}_\mu - \\ - m_s^2 \phi^{\alpha(s)} + 2\lambda g^{\alpha\alpha} \phi^{\alpha(s-2)\mu}{}_\mu = O(\phi^2) + O(\phi^3) + \dots, \\ m_s^2 = -\lambda((s-2)(d+s-3) - s)$$

[Huge list of people. Different tools including the holographic reconstruction. Stuck at quartic order]

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- Unfolded approach [M.Vasiliev] $d^2 = 0$

$$dw + [W_0, w]_* = \mathbf{e} \wedge \mathbf{e}C'$$

$$dC + W_0 * C - C * \pi(W_0) = 0$$

$w = w(Y|x)$ and $C = C(Y|x)$, $W_0(Y|x)$ is the AdS vacuum

- nonlinear corrections

$$dw + w * w = \Upsilon(w, w, C) + \Upsilon(w, w, C, C) + \dots$$

$$dC + w * C - C * \pi(w) = \Upsilon(w, C, C) + \Upsilon(w, C, C, C) + \dots$$

Υ 's are determined from $d^2 = 0$ modulo field redefinition
(locality issue!)

$$w \rightarrow w + f_w(w, C..C), \quad C \rightarrow C + f_C(C..C)$$

or systematically using **Vasiliev's equations**.

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- Simplest vacuum

$$w = W_0 \quad - \text{AdS}, \quad C = 0$$

- Goal: Find w and C that solve HS e.o.m manifesting Poincaré symmetry in one dimension less.

A hint from classical double copies

- Kerr-Shield ansatz for a black hole

$$g_{mn} = g_{0\,mn} + M\phi k_m k_n$$

Solves the linearized and full Einstein eqs. at a time.

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solves Klein-Gordon for $s = 0$, Maxwell for $s = 1$

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- Kerr-Shield multicopy for any s (VD, A.Matveev, M.Vasiliev, 2008); for $s = 0, 1, 2$ (Monteiro, O'Connell, White, 2014); in any d (VD, N.Dosmanbetov'23)

$$\square\phi_{m_1\dots m_s} - sD_n D_{(m_1}\phi^n{}_{m_2\dots m_s)} = -\frac{2}{\ell^2}(s-1)(s+1)\phi_{m_1\dots m_s}$$

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- Weyl multicopy (VD, A.Matveev, M.Vasiliev, 2008)

$$\text{Weyl}_s = \frac{(\text{Maxwell})^s}{(\text{scalar})^{s-1}}$$

A hint from classical double copies

- Multicopies as a free data for HS interactions

$$w^{(1)} = \sum_s a_s w_s, \quad C^{(1)} = \sum_s a_s C_s$$

w_s, C_s stand for s -multicopy.

- Leading order HS nonlinearities (Vasiliev)

$$D_0 C^{(2)} + [w^{(1)}, C^{(1)}]_* = -\frac{\eta}{2} \mathbf{e}^{\alpha\dot{\alpha}} y_\alpha \int d\bar{u} d\bar{v} \int_0^1 d\tau$$
$$e^{i\bar{u}\dot{\alpha}\bar{v}\dot{\alpha}} ((1-\tau)\partial_1 - \tau\partial_2)_{\dot{\alpha}} C^{(1)}(\tau y, \bar{y} + \bar{u}) C^{(1)}((1-\tau)y, \bar{y} + \bar{v}) + h.c.$$

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- Zeroth copy $s = 0$ forms a closed sector at this order [VD, A.Korybut'22]

$$a_s = 0, \quad s > 0$$

HS dynamics of symmetric fields

Fields w , C should contain traceless diagrams

$$\omega^{a(s-1), b(n)} = dx^\mu \omega_\mu^{a(s-1), b(n)} = \begin{array}{|c|c|c|c|} \hline & \bullet & \bullet & \bullet \\ \hline & \bullet & \bullet & \\ \hline \end{array}, \quad 0 \leq n \leq s-1$$

$$C^{a(m), b(s)} = \begin{array}{|c|c|c|c|} \hline & \bullet & \bullet & \bullet \\ \hline & \bullet & \bullet & \\ \hline \end{array}, \quad m \geq s$$

Generating realization

$$w = w(y_\alpha, \mathbf{y}_\alpha^a | x), \quad C = C(y_\alpha, \mathbf{y}_\alpha^a | x), \quad \alpha = 1, 2, \quad a = 0 \dots d$$

w , C are then two-row diagrams provided (Howe duality)

$$[w, t_{\alpha\beta}]_* = [C, t_{\alpha\beta}]_* = 0, \quad t_{\alpha\beta} = \frac{1}{4i} (\mathbf{y}_\alpha^a \mathbf{y}_{a\beta} + y_\alpha y_\beta)$$

$$(f * g)(y, \vec{\mathbf{y}}) = \int f(y + u, \vec{\mathbf{y}} + \vec{\mathbf{u}}) g(y + v, \vec{\mathbf{y}} + \vec{\mathbf{v}}) e^{iu_\alpha v^\alpha + i\vec{\mathbf{u}}_\alpha \vec{\mathbf{v}}^\alpha},$$

HS dynamics of symmetric fields

Off-shell HS algebra: $Sp(2)$ singlets of $y_\alpha, \mathbf{y}_\alpha^a$ endowed with *-product

On-shell HS algebra: Factor of the off-shell algebra over a two-sided ideal

$$\text{Id} = t_{\alpha\beta} * A^{\alpha\beta} = A^{\alpha\beta} * t_{\alpha\beta}$$

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On-shell HS algebra: Factor of the off-shell algebra over a two-sided ideal

$$\text{Id} = t_{\alpha\beta} * A^{\alpha\beta} = A^{\alpha\beta} * t_{\alpha\beta}$$

If $f_{1,2}$ are on-shell, then

$$f_1 * f_2 = f_3 + \text{Id}$$

HS algebra deforms in interaction leading to a deformation of the ideal.

Off-shell form $Y = (y, \mathbf{y})$

$$dw + w * w = \Upsilon(w, w, C) + \Upsilon(w, w, C, C) + \dots$$

$$dC + w * C - C * \pi(w) = \Upsilon(w, C, C) + \Upsilon(w, C, C, C) + \dots$$

$$\pi f(y, \mathbf{y}) = f(-y, \mathbf{y})$$

Generating interactions

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Vasiliev's trick:

$$w \rightarrow W(z; Y|x) := \omega(Y|x) + W_1(z; Y|x) + W_2(z; Y|x) + \dots,$$

$$d_x W + W * W = 0$$

star product is extended to (z, Y) – space

Generating interactions

Generating equations [VD'22; VD, A.Korybut'23]

star product:

$$(f * g)(z; Y) = \int f(z + u', y + u; \mathbf{y}) * g(z - v, y + v + v'; \mathbf{y}) e^{iu_\alpha v^\alpha + iu'_\alpha v'^\alpha},$$

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Equations:

$$d_x W + W * W = 0,$$

$$d_z W + \{W, \Lambda\}_* + d_x \Lambda = 0,$$

$$d_x C + (W(z'; y, \vec{\mathbf{y}}) * C - C * W(z'; -y, \vec{\mathbf{y}})) \Big|_{z'=-y} = 0$$

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$$\Lambda = dz^\alpha z_\alpha \int_0^1 d\tau \tau C(-\tau z, \vec{\mathbf{y}}) e^{i\tau z_\alpha y^\alpha}$$

$$\frac{\partial}{\partial z^\alpha} \Lambda^\alpha = C * \delta, \quad \delta = e^{izy}, \quad \delta * f(z) = f(z) * \delta = f(0)\delta$$

- Deformation of the $sp(2)$

$$[t_{\alpha\beta}, t_{\gamma\delta}]_* = \epsilon_{\alpha\gamma} t_{\beta\delta} + \epsilon_{\beta\gamma} t_{\alpha\delta} + \epsilon_{\alpha\delta} t_{\beta\gamma} + \epsilon_{\beta\delta} t_{\alpha\gamma},$$

$$d_x t_{\alpha\beta} + [W, t_{\alpha\beta}]_* = 0,$$

$$d_z t_{\alpha\beta} + [\Lambda, t_{\alpha\beta}]_* = 0$$

- Deformation of the $sp(2)$

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- Manifest form

$$t_{\alpha\beta} = \frac{1}{4i} (\vec{\mathbf{y}}_\alpha \cdot \vec{\mathbf{y}}_\beta + y_\alpha y_\beta) - z_\alpha z_\beta \int_0^1 d\tau \tau (1-\tau) C(-\tau \vec{\mathbf{y}}_z; \vec{\mathbf{y}} \otimes \vec{\mathbf{y}}) e^{i\tau z y}$$

Quest for a new vacuum

- Restriction imposed – no spins other than $s = 0$
- Poincaré global symmetry

$$ds^2 = \frac{1}{\rho^2} (d\rho^2 + \eta^{ij} dx_i dx_j)$$

$$\mathbf{y}^a = \begin{cases} \mathbf{y}^j = \vec{\mathbf{y}}, & a = j < d \\ i\bar{y}, & a = d \end{cases},$$

- On-shell representative (traceless condition)

$$\Delta_{\alpha\beta} C := \left(\eta^{ij} \frac{\partial}{\partial \mathbf{y}^{i\alpha}} \frac{\partial}{\partial \mathbf{y}^{j\beta}} - \frac{\partial}{\partial \bar{y}^\alpha} \frac{\partial}{\partial \bar{y}^\beta} - y_\alpha y_\beta \right) C = 0.$$

- Protected AdS

$$W_0 = \frac{i}{2\rho} (dx^j \mathbf{y}_j^\alpha (y - \bar{y})_\alpha + d\rho y_\alpha \bar{y}^\alpha)$$

Quest for a new vacuum

- Ansatz

$$C = \rho^2 e^{iy_\alpha \bar{y}^\alpha} T(\mathbf{p}, \mathbf{q}; \rho), \quad \mathbf{p} = -\rho^2 \bar{\mathbf{y}}^\alpha \cdot \bar{\mathbf{y}}^\beta y_\alpha y_\beta, \quad \mathbf{q} = 2i\rho^2 y_\alpha \bar{y}^\alpha$$

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- Fock projector

$$e^{iy\bar{y}} * e^{iy\bar{y}} = \frac{1}{4} e^{iy\bar{y}}$$

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- Fock projector

$$e^{iy\bar{y}} * e^{iy\bar{y}} = \frac{1}{4} e^{iy\bar{y}}$$

- Equations to solve

$$\cancel{d_z W_0} + \{W_0, \Lambda[C]\}_* + d_x \Lambda[C] = 0,$$
$$\left(\eta^{ij} \frac{\partial}{\partial \mathbf{y}^{i\alpha}} \frac{\partial}{\partial \mathbf{y}^{j\beta}} - \frac{\partial}{\partial \bar{y}^\alpha} \frac{\partial}{\partial \bar{y}^\beta} - y_\alpha y_\beta \right) C = 0$$

$$\Lambda_\alpha = \rho^2 \int_0^1 d\tau \tau z_\alpha e^{i\tau z(y-\bar{y})} T(\tau^2 \mathbf{p}, -\tau \mathbf{q}; \rho)$$

- Algebraic conditions

$$[D, \Lambda_\alpha]_* = -i\rho \frac{\partial}{\partial \rho} \Lambda_\alpha,$$

$$[\vec{P}, \Lambda_\alpha]_* = 0$$

$$[\vec{P}, \bullet]_* = i\vec{y}^\alpha \left(\frac{\partial}{\partial \bar{y}^\alpha} + \frac{\partial}{\partial y^\alpha} \right) - i \left(y - \bar{y} - i \frac{\partial}{\partial z} \right)^\alpha \frac{\partial}{\partial \bar{y}^\alpha},$$

$$[D, \bullet]_* = -iy^\alpha \frac{\partial}{\partial \bar{y}^\alpha} - i\bar{y}^\alpha \frac{\partial}{\partial y^\alpha} + \epsilon^{\alpha\beta} \frac{\partial^2}{\partial z^\alpha \partial \bar{y}^\beta}$$

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- Explicit result ($x_1 = -\vec{y}^\alpha \cdot \vec{y}^\beta y_\alpha y_\beta$, $x_2 = 2iy_\alpha \bar{y}^\alpha$)

$$C = e^{iy_\alpha \bar{y}^\alpha} \left(\nu_1 \rho^2 + \nu_2 \rho^{d-2} \oint dz \frac{e^z}{z^2} \left(1 + \frac{x_1}{z^2} + \frac{x_2}{z} \right)^{\frac{d-4}{2}} \right)$$

- The scalar vacuum is linearly exact and on-shell

$$\phi(\vec{x}, \rho) := C(0|x) = \nu_1 \rho^2 + \nu_2 \rho^{d-2},$$
$$\Upsilon(w, C..C) = \Upsilon(w, w, C..C) = 0$$

- Global symmetry is the Poincaré algebra at the boundary
- T is polynomial in even dimensions

$$T_{d=8} = \nu_1 \rho^2 + \nu_2 \rho^4 \left(1 + \frac{x_1^2}{120} + \frac{x_2^2}{6} + \frac{x_1}{3} + x_2 + \frac{x_1 x_2}{12} \right),$$

- Gegenbauer polynomials

$$\oint dz \frac{e^z}{z^2} \left(1 - \frac{2xy}{z} + \frac{y^2}{z^2} \right)^{-\alpha} = \sum_n \frac{y^n}{(n+1)!} C_n^{(\alpha)}(x).$$

Conclusion

- The natural vacuum of the HS theory is AdS space with zero values of the rest of the fields. It is shown AdS remains protected even in the presence of a nonzero scalar ϕ stretched along radial direction

$$\phi(\vec{x}, \rho) = \nu_1 \rho^2 + \nu_2 \rho^{d-2}$$

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Thank you!