Bjorken Sum Rule With New Analytic Coupling

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Abstract: We analyze recently obtained experimental data for the polarized Bjorken sum rule in the region of small values of Q^2. Our investigation is based on a new form of coupling constant which doesn't contain the Landau pole. We found an excellent agreement between the experimental data and the predictions of analytic QCD, as well as a strong difference between these data and the results obtained in the framework of perturbative QCD.

Introduction

We introduce the derivatives (in the k-order of perturbation theory (PT)) [1]

$$\tilde{a}^{(k)}_{n+1}(Q^2) = \frac{(-1)^n}{n!}\frac{d^n a^{(k)}_s(Q^2)}{(dL)^n}, \quad a^{(k)}_s(Q^2) = \frac{\beta_0 \alpha^{(k)}_s(Q^2)}{4\pi},$$

where $L = \ln\frac{Q^2}{\Lambda^2}$, $Q^2 = -q^2$, q^2 – transferred momentum in the Euclidean domain for spacelike processes. The series of derivatives $\tilde{a}_n(Q^2)$ can successfully replace the corresponding series of $a_s(Q^2)$-powers.

Analytic Coupling

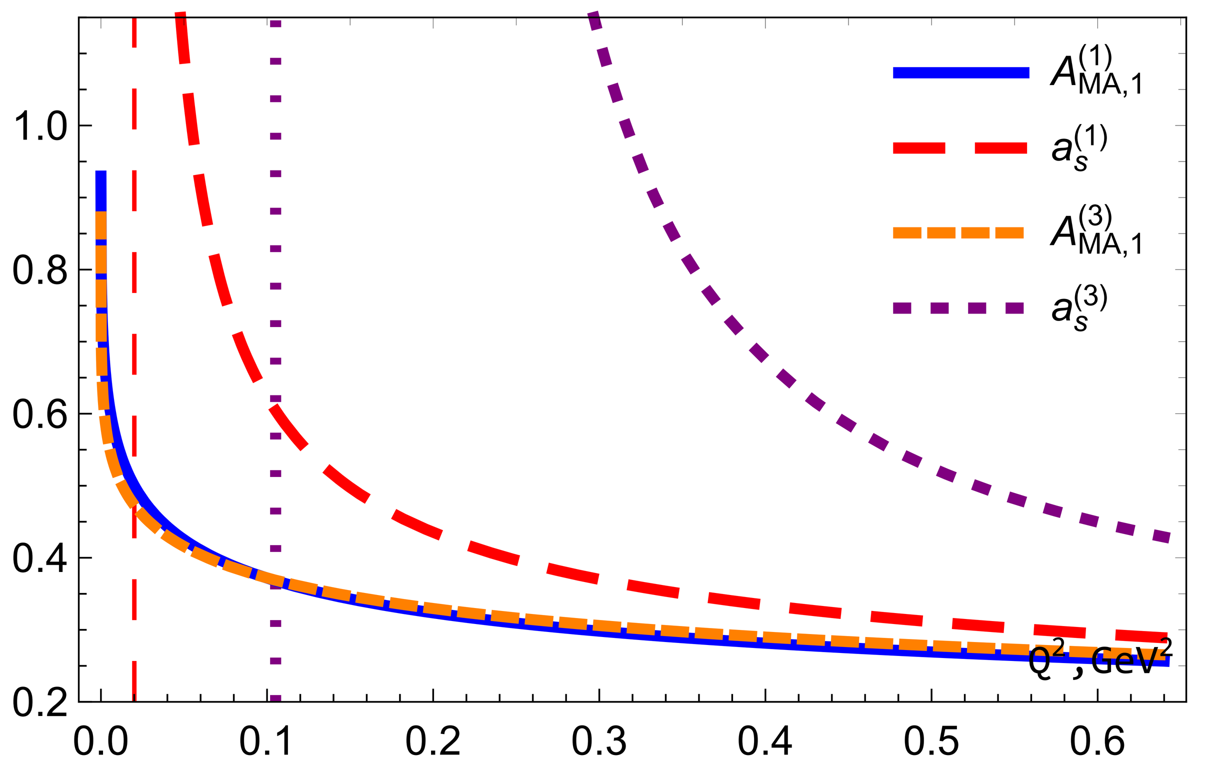

Fig.1 Comparison $a^{(i)}_s(Q^2)$ and $A^{(i)}_{\rm MA,\it i}(Q^2)$. Vertical lines indicate the appropriate values of Λ_i.

In the frame of analytical perturbation theory (APT) [2] one can construct new holomorphic couplant $A^{(i)}_{\rm MA}(Q^2)$

$$A^{(i)}_{\rm MA}(Q^2) = \frac{i}{\pi}\int_0^{+\infty}\frac{d\sigma}{(\sigma+Q^2)}\, r^{(i)}_{\rm pt}(\sigma), \quad r^{(i)}_{\rm pt}(\sigma) = {\rm Im}\; a^{(i)}_s(-\sigma - i\epsilon)\,.$$

The final expressions for $\tilde{a}^{(i+1)}_\nu(Q^2)$ are represented as the sum of LO expression and the high order corrections [3]

$$\tilde{a}^{(i+1)}_\nu(Q^2) = \tilde{a}^{(1)}_\nu(Q^2) + \sum_{m=1}^{i}\frac{\Gamma(\nu+m)}{m!\Gamma(\nu)}\left(\hat{R}_m\, \tilde{a}^{(i+1)}_{\nu+m}(Q^2)\right),$$

where $\hat{R}_m$ – differential operators ($\sim d^m/d\nu^m$). The structure of spectral integral allows to permorf the same operation for $\tilde{A}^{(i)}_{\rm MA}(Q^2)$:

$$\tilde{A}^{(i+1)}_{\rm MA,\nu}(Q^2) = \tilde{A}^{(1)}_{\rm MA,\nu}(Q^2) + \sum_{m=1}^{i}\frac{\Gamma(\nu+m)}{m!\Gamma(\nu)}\left(\hat{R}_m\, \tilde{A}^{(1)}_{\rm MA,\nu+m}(Q^2)\right).$$

Bjorken Sum Rule

The definition of polarized Bjorken sum rule (BSR)

$$\Gamma^{p-n}_1(Q^2) = \int_0^1 dx\,\left[g^p_1(x,Q^2) - g^n_1(x,Q^2)\right].$$

BSR in the OPE form (twist-2+massive twist-4) reads [4]

$$\Gamma^{p-n}_1(Q^2) = \frac{g_A}{6}\left(1 - D_{\rm BS}(Q^2)\right) + \frac{\hat{\mu}_4 M^2}{Q^2+M^2}$$

and another form of twist-4 term for small Q^2 values [5]

$$\Gamma^{p-n}_1(Q^2) = \frac{g_A}{6}\left(1 - D_{\rm BS}(Q^2)\right) + \frac{\hat{\mu}_4 M^2(Q^2+M^2)}{(Q^2+M^2)^2 + M^2\sigma^2}.$$

The twist-2 term $D_{\rm BS}(Q^2)$ in PT and APT takes the form

$$D^{(k)}_{\rm BS}(Q^2) = \frac{4}{\beta_0}\left(\tilde{a}^{(k)}_1 + \sum_{m=2}^{k}\tilde{d}_{m-1}\tilde{a}^{(k)}_m\right),$$

$$D^{(k)}_{\rm MA,BS}(Q^2) = \frac{4}{\beta_0}\left(A^{(k)}_{\rm MA} + \sum_{m=2}^{k}\tilde{d}_{m-1}\,\tilde{A}^{(k)}_{\rm MA,\nu=m}\right).$$

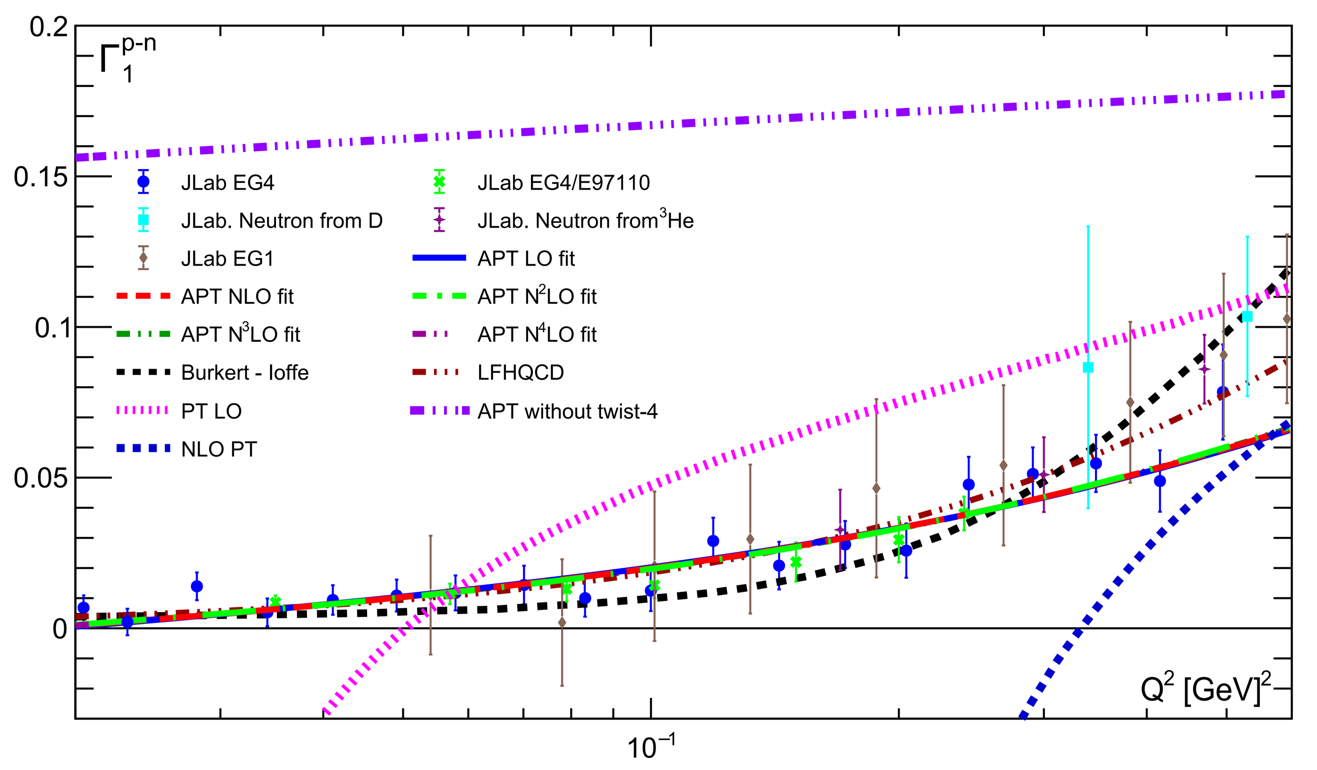

Fig.2 The results for $\Gamma^{p-n}_1(Q^2)$ in the first four orders of APT with $\sigma = \sigma_\rho$.

The values of the fit parameters with $\sigma = \sigma_\rho = 145$ MeV (the ρ-meson decay width) and $\sigma = 0$:

	M^2 for $\sigma=\sigma_\rho$ (for $\sigma=0$)	$\hat{\mu}_{\rm MA,4}$ for $\sigma=\sigma_\rho$ (for $\sigma=0$)	χ^2/(d.o.f.) for $\sigma=\sigma_\rho$ (for $\sigma=0$)
LO	1.592 ± 0.300 (1.631 ± 0.301)	-0.168 ± 0.002 (-0.166 ± 0.001)	0.788 (0.789)
NLO	1.505 ± 0.286 (1.545 ± 0.287)	-0.157 ± 0.002 (-0.155 ± 0.001)	0.755 (0.757)
N^{2}LO	1.378 ± 0.242 (1.417 ± 0.241)	-0.159 ± 0.002 (-0.156 ± 0.002)	0.728 (0.728)
N^{3}LO	1.389 ± 0.247 (1.429 ± 0.248)	-0.159 ± 0.002 (-0.157 ± 0.002)	0.747 (0.747)
N^{4}LO	1.422 ± 0.259 (1.462 ± 0.259)	-0.159 ± 0.002 (-0.157 ± 0.001)	0.754 (0.754)

Summary

- The calculation results taking into account only statistical uncertainties.
- The cases $\sigma = 0$ and $\sigma = \sigma_\rho$ lead to very similar values for the fitting parameters and χ^2-factor.
- The quality of the APT fits is very good (as evidenced quantitatively by the values of χ^2/(d.o.f.)) and much better than PT fits.

Acknowledgments

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