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World Constants and Limiting Transition*

G. Gamow, D. Ivanenko, and L. Landau

§ 1. In constructing a system of units in physics, there exist two basic methods for choosing units of any new quantity:

(i) One merely specifies an arbitrary standard of measure (this is the way in which one introduces the usual definitions of, say, gram or ohm).

(ii) By employing some law—we denote it by A—that relates the quantity in question to those that are known and which involves a numerical coefficient, one chooses a satndard in such a way as to reduce this coefficient to unity (this is exemplified by the definition of a charge unit in terms of the Coulomb law).

Technical difficulties apart, one can always make use of either method of the above two.[1)] In the first case, we have a new arbitrary satndard; that is, we increase the number of units forming the basis of the theory of dimensions. Moreover, the coefficient in the law A then takes a specific numerical value that appears to be a new world constant.

In the second case, both the number of basic arbitrary standard and the number of world constants remain unchanged; for measuring the quantity in question, we only obtain a unit that is natural with respect to preceding ones. This unit will change in response to changes in basic standard. The character of this variation is studied within a dimensional analysis that introduces the concept of dimensions of a given physical quantity.

Constants of zero dimensions are independent of the choice of basis units and can therefore be treated as mathematical constants (numbers). One can hope that all these numerical constants can be obtained theoretically. Within a given system of dimensions, world constants from which one can compose a combination of zero dimension must therefore obey a mathematical relation, so that they are not independent.

From the aforesaid, it follows that we can always reduce the number of basic standard (number of dimensions) using one of the world constants for this and setting it to unity. Below, this process, which is equivalent to going over from the first definition to the second one, will be referred to as a reduction.

For a complete reduction (that is, a reduction to the number of standard that is equal to zero) to be possible, it is necessary that the number of independent constants not be less than the number of dimensions forming the basis of the system of units being considered. Obviously, the number of independent constants cannot be greater than the number of basic independent basic units in our system of dimensions.

For example, only the reduction to two units was possible in Newtonian mechanics, since, in the presence of three basic dimensions of T, L, and M, there was only one law featuring a world constant; that is,

$$f = \chi \frac{mm'}{r^2}.$$

A second constant, which enables a reduction to one dimension is introduced by the special theory of relativity via the relation

$$x_i = ict.$$

Finally, the last missing constant h appears in the framework of quantum mechanics:

$$\varphi = \frac{2\pi W}{h}$$

(this is the expression for the phase φ in terms of the action W).

Usually, we are dealing with the case where the number of constants known from experiments and not yet reduced to a smaller number by establishing mathematical relations is much greater than the adopted number of basic units. In this case, it is advisable to choose the most general constants for performing complete reduction.

The quartic system $CGS1°$ is employed in modern experimental physics. In technologies, however, practical considerations dictate the use of a much greater number of standards (cm, g, s, 1°, Ω, A, ...); there, one adopts some $CGS1°\Omega\ldots$ system.

Yet another example of choosing a basic system is provided by Planck's natural system of units (c, χ, h, k).

§ 2. We have seen above that each constant is a representative of a physical law (theory), a world

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[1)]Of course, this is so if there is a law that relates the new quantity being considered to some known previously.

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constant symbolizing the generality of a law. More universal constants correspond to more general laws (theories)—to illustrate, one can compare the Rydberg constant with the Planck constant $\hbar$. The introduction of new constants and their reduction to a smaller number were reflected in the history of physics as a changeover of theories and their gradual unification.[2)] By way of example, we can indicate the introduction of the constant $\hbar$ and the reduction of the Rydberg constant. Fixing the number of dimensions as above, we thereby constrain the number of genuine constants: among the available constants, we take n ones (n is equal to the number of dimensions) for basic ones, reducing the remaining to genuine (that is, independent) ones. From the point of view of reduction, it obviously does not matter which constants are taken for basic ones. Here, however, we are guided by two heuristic principles. The first of these is that which is based on the degree of generality of the theory that these constants represent: it is natural to reduce the Rydberg constant to the Planck constant, but not vice versa, because the theory of atomic spectra is obviously of lower order with respect to the general theory of atoms. The other principle tests a constant for a limiting transition (see below). By way of example, we will trace the history of the constant h (that is, the development of the quantum theory from the point of view of the introduction of this constant). Classical mechanics and electrodynamics can be considered as an initial stage. Bohr's theory (old quantum mechanics) introduced h as an empirical constant in its equations, pursuing only ad hoc purposes: h symbolized discontinuity, jumps, etc. Only in Schrödinger–Heisenberg wave mechanics did h appear quite naturally as a constant associated with dimension. No requirements of discontinuity are introduced, and the empirical significance of h is clarified only a posteriori. We are inclined to deem the theory of the constant h completed. Imagine a completed (!) physics. We will construct it on the basis of n dimensions; there will obviously remain n world constants in it that appear in a natural way—that is, as mere dimensional rather than empirical coefficients. All extra constants will be reduced. As to the world constants in question, we can set them to unity according to the proposal of Planck, whereby we go over to physics without dimensions. Let us construct a physics system that is in a limiting relation to the above completed physics. To do this, we apply the limiting-transition method, making the world constant in question tend to zero (of course, such a constant must first be introduced if it was initially equal to unity). The theory obtained via this limiting transition will be referred to as a classical theory with respect to the world constant being considered. For example, conventional mechanics is classical with respect to h, while wave mechanics is completed in the above sense; as to Bohr's theory, with its h introduced in an ad hoc manner, it can be called a vulgar theory. In the same way, the theory of relativity is a completed theory with respect to $1/c$ ($1/c$ appears in the metric as a dimensional coefficient); for a limiting theory, we have here conventional mechanics, as in the preceding case, and nonrelativistic quantum mechanics. It should be emphasized that, for a genuinely basic constant in the sense of the limiting transition, we have here $1/c$ rather than c, since it is the former that is made to tend to zero. As to theories that are vulgar with respect to $1/c$, these include a number of prerelativistic formulations of electrodynamics. Further, geometric optics is a classical theory with respect to the constant of wavelength ($\lambda_0 \to 0$), while wave optics is a completed theory. From this point of view, the Fresnel theory of diffraction is a vulgar theory. On the basis of this method, we can construct new classical theories by introducing new constants and making them tend to zero. Such classical theories can be doubly, triply, etc., limiting ones (rank of a classical theory). For example, conventional mechanics is triply limiting—with respect to quantum theory, the special theory of relativity, and the theory of gravity (the corresponding constants are h, $1/c$, and χ). Since a combination of constants is also a constant, there arises the question of elementary constants.

We have seen that a normal course of the development of a theory was from a limiting through a vulgar to a completed one. Having constructed parallel schemes, we notice gaps—some theories skipped a "vulgar" period, while, in the history of others, there were no limiting case. Historically, we have the L, M, T system of dimensions (temperature apart) and, hence, three genuine world constants. According to the aforesaid, the choice of the three dimensions was accidental from the lofty point of view; as to the choice of "genuine" constants, we may heuristically follow the generality of theories and the limiting-transition principle. From both points of view, one is led to adopt h, $1/c$, and χ for "genuine" constants (all three of them represent the most advanced theories, and all three meet the limiting-transition test).

§ 3. If, following the aforesaid, we lay the basic constants h, $1/c$, and χ in the foundation of the theory of dimensions, we can obtain "natural units" for all other physical quantities, including mass and electric charge. The charge and mass units deduced in this way do not coincide with "elementary" values obtained for these quantities experimentally (the charges and masses of the electron and of the proton).

[2)] In a sense, one can associate each new law with a new irreducible constant, introducing the corresponding new dimension.

However, this coincidence could hardly be expected because the mass of the electron differs from the mass of the proton—it would have been strange if one of them had proved to be a basic one.

The only thing that is natural to expect is that either of these masses will be expressed, in one way or another, in terms of the "natural unit" of mass. The origin of two mass values (m_+, m_-) may be that the equation from which they will be determined has two different roots corresponding to two charge values ($+e$, $-e$).

Not yet having the theory of the electron at our disposal, we may deduce, however, some conclusions about the character of this theory from a dimensional analysis. Let us find the dimensions of charge and mass in terms of our basic dimensions $[h]$, $[1/c]$, and $[\chi]$. After some simple algebra, we obtain

$$[e] = \sqrt{[h]\cdot[c]}; \quad [m] = \sqrt{\frac{[h]\cdot[c]}{[\chi]}}; \quad \left[\frac{e}{m}\right] = \sqrt{[\chi]}$$

or

$$e = \lambda\sqrt{h\cdot c}; \quad m = \nu\sqrt{\frac{hc}{\chi}},$$

where λ and ν are numerical constants that are different for the proton and for the electron. (It is obvious that $\lambda_- = -\lambda_+$.)

The above formulas for the dimensions may also furnish valuable guidelines in constructing the theory of the electron on the basis of an incomplete system of theoretical physics where some world constants are set to zero.

It can easily be seen that the only incomplete system leading to finite values of charge and mass is

$$\left\{h = 0; \quad \frac{1}{c} = 0; \quad \chi \neq 0\right\};$$

that is, this is a nonquantum, nonrelativistic, gravitating electron. In this case, the electron charge becomes a new world constant.

As to other incomplete systems, they lead to indefinitely small (or indefinitely large) charges or masses. In particular, frequent attempts at constructing a theory of a nonquantum electron in the general theory of relativity cannot be successful ($h = 0$, $c \neq \infty$, $\chi \neq 0$, whence it follows that $e = m = 0$).

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