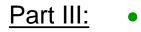
Dense fermion systems in the center of compact stars

E.E. Kolomeitsev

BLTP, JINR, Dubna



- Neutron stars
- Nuclear Equation of state

Infinite nuclear matter

1) make A, V, big by keeping
$$n_p = \frac{Z}{V}, \ n_n = \frac{A-Z}{V} = n - n_p$$
 fixed

2) switch off electromagnetic interaction

3)
$$m_n = m_p = m_N$$

Energy density of the infinite nuclear matter as function of the proton and neutron densities:

$$\lim_{A \to \infty} \frac{M(A,Z)c^2}{A} = E(n_p, n_n) = m_N - \lim_{A \to \infty} \frac{B(A,Z)c^2}{A}$$

Binding energy per nucleon:

$$\varepsilon(n,x) = E(n,x)/n - m_N$$

where, $n = n_p + n_n$ total density, $x = n_p/n$ proton fraction chemical potentials: $\mu_n = \frac{\partial E(n_p, n_n)}{\partial n_n} = \frac{\partial E(n, x)}{\partial n} - \frac{x}{n} \frac{\partial E(n, x)}{\partial x}$ $\mu_p = \frac{\partial E(n_p, n_n)}{\partial n_p} = \frac{\partial E(n, x)}{\partial n} + \frac{1 - x}{n} \frac{\partial E(n, x)}{\partial x}$

Pressure:
$$P = \mu_n n_n + \mu_p n_p - E = n \frac{\partial E}{\partial n} - E$$
 T=0

Infinite nuclear matter. Symmetry energy

$$arepsilon(n,x)=arepsilon_0(n)+arepsilon_{
m S}(n)\,(1-2\,x)^2+\dots$$

<u>ISM energy:</u> $\varepsilon_0(n)$ <u>Symmetry energy:</u> $\varepsilon_S(n)$

Two definitions of the symmetry energy: (1) $\varepsilon_{\rm S}(n) = \frac{1}{8} \frac{\partial^2 \varepsilon(n,x)}{\partial x^2} \Big|_{x=1/2}$ local (x~1/2)

(2)
$$\varepsilon_{\rm S}(n) = \varepsilon(n, x = 0) - \varepsilon(n, x = 1/2)$$

NS applications

If the derivative $\frac{\partial^4 \varepsilon(n, x)}{\partial x^4}$ is very small, then both definitions are equivalent

Equation of state of nuclear matter

 $E(n_p, n_n) = \varepsilon_0(n) + \varepsilon_S(n) \frac{(n_p - n_n)^2}{n^2}$ The energy per nucleon of the nuclear matter

 n_p – proton number density $n = n_p + n_n$ n_n – neutron number density

 $\varepsilon_0(n) = E_0 + 0 + \frac{K}{18} \frac{(n - n_0)^2}{n_0^2} + \frac{Q}{162} \frac{(n - n_0)^3}{n_0^3} + O\left(\frac{(n - n_0)^4}{n_0^4}\right)$ nuclear matter parameters

symmetry energy
$$\varepsilon_{S}(n) = J + \frac{L}{3} \frac{n - n_{0}}{n_{0}} + \frac{K_{\text{sym}}}{18} \frac{(n - n_{0})^{2}}{n_{0}^{2}} + \frac{Q_{\text{sym}}}{162} \frac{(n - n_{0})^{3}}{n_{0}^{3}} + O\left(\frac{(n - n_{0})^{4}}{n_{0}^{4}}\right)$$

saturation density n_0 and binding energy E_0 $n_0 \simeq 0.16 \pm 0.015 \text{ fm}^{-3}$ $E_0 \simeq -15.6 \pm 0.6 \text{ MeV}$

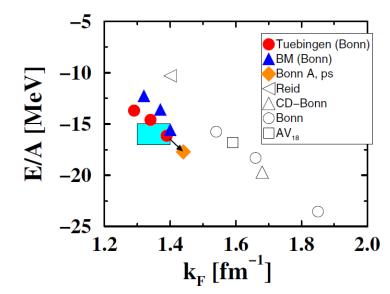
Correlations among parameters •

 n_0 vs E_0 –Coester line problem: role of TNF, relativistic effects, chiral forces

Stiffness of EoS

frequently characterized by the compressibility modulus K $K = 240 \pm 20 \,\mathrm{MeV}$





Correlations among parameters L-J
$$\varepsilon_S[n] = J + \frac{L}{3} \frac{n - n_0}{n_0} + \frac{K_{\text{sym}}}{18} \frac{(n - n_0)^2}{n_0^2} + \dots$$



Masses: UNEDF0 Skyrme DF+BHF [Kortelainen *et al.*, PRC **82**, 024313 (2010)]

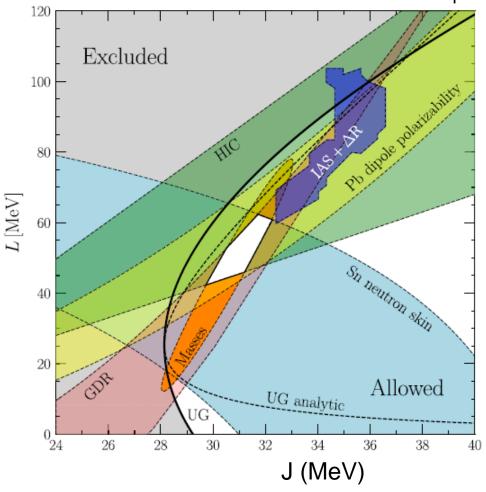
Isobaric analog states+isovector skin: [Danielewicz et al. NPA 958, 147 (2017)]

Pb dipole polarizability: [Roca-Maza *et al.*, PRC **88**, 024316 (2013)]

Sn neutron skin: [Chen et al., PRC 82, 024321 (2010)]

GDR: [Trippa et al., PRC **77**, 061304 (2008)]

Isospin defusion in HIC [Tsang et al., PRL 102, 122701 (2009)]



Behind all calculation are particular models for NN interactions and many-body techniques

Relativistic mean-field models

nucleon-nucleon interaction

vacuum: one boson-exchange for NN-potential+ Lippmann-Schwinger equations

Euler-Lagrange equations for $q \equiv q(\vec{x}, t) = \{\Psi, \sigma, \omega, \rho\}$ $\partial_{\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}q)}\right] - \frac{\partial \mathcal{L}}{\partial q} = 0$

 $[i\gamma_{\mu}(\partial^{\mu} + ig_{\omega N}\omega^{\mu} + ig_{\rho N}\boldsymbol{\tau}\boldsymbol{\rho}^{\mu}) - (m_N - g_{\sigma N}\sigma)]\Psi_N = 0$

$$\begin{aligned} (\partial^{2} + m_{\sigma}^{2})\sigma + \frac{dU}{d\sigma} &= g_{\sigma N} \sum_{N=p,n} \bar{\Psi}_{N} \Psi_{N} & \text{nucleon sources} \\ (\partial^{2} + m_{\omega}^{2})\omega_{\mu} &= g_{\omega N} \sum_{N=p,n} \bar{\Psi}_{N} \gamma_{\mu} \Psi_{N} & \text{for meson fields} \\ (\partial^{2} + m_{\rho}^{2})\boldsymbol{\rho}_{\mu} &= g_{\rho N} \sum_{N=p,n} \bar{\Psi}_{N} \boldsymbol{\tau} \gamma_{\mu} \Psi_{N} & \text{for meson fields} \end{aligned}$$

$$\Psi_N = \left(\begin{array}{c} \Psi_p \\ \Psi_n \end{array}\right)$$

medium: mean-field approximation



$$egin{aligned} &\sigma(r,t) = \sigma \ &\omega_\mu(r,t) = \delta_{\mu,0}\,\omega_0 \ &
ho_\mu^a(r,t) = \delta^{a,3}\,\delta_{\mu,0}\,
ho_0^{(3)} \ & ext{constant fields} \end{aligned}$$

$$\omega_0 = \frac{g_{\omega N}}{m_{\omega}^2} < \Psi^{\dagger} \Psi > \equiv \frac{g_{\omega N}}{m_{\omega}^2} \ n_B = \frac{g_{\omega N}}{m_{\omega}^2} \ (n_p + n_n)$$

(vector) density

$$\rho_0^{(3)} = \frac{g_{\rho N}}{m_{\rho}^2} < \Psi^{\dagger} \tau^{(3)} \Psi > \equiv \frac{g_{\rho N}}{m_{\rho}^2} \ n_{\rm iso} = \frac{g_{\rho N}}{m_{\rho}^2} \ (n_p - n_n)$$

$$m_{\sigma}^{2}\sigma_{0} + \frac{dU}{d\sigma}\Big|_{\sigma_{0}} = g_{\sigma N} < \bar{\Psi}\Psi > \equiv g_{\sigma N}n_{s} = g_{\sigma N}(n_{s,p} + n_{s,n}) \quad scalar \ density$$

$$\begin{bmatrix} i\gamma_{\mu}\partial^{\mu} - (g_{\omega N}\omega_{0} + (g_{\rho N}\rho_{0}^{(3)})\gamma^{0} - (m_{N} - g_{\sigma N}\sigma_{0})]\Psi = 0 \\ V \\ m_{N}^{*} = m_{N} - g_{\sigma N}\sigma \\ m_{N}^{*} \end{bmatrix} \Psi_{N} = 0 \longrightarrow -(p_{0} - V)^{2} + m_{N}^{*2} + (p\sigma)^{2} = 0 \\ (p_{0} - V)^{2} = m_{N}^{*2} + p^{2}$$

nucleon spectrum in MF approximation

$$\epsilon_N(p) = \sqrt{m_N^{*2} + p^2} + g_{\omega N} \,\omega_0 + g_{\rho N} \,I_N \,\rho_{03}$$

[Serot, Walecka]

pion dynamics falls out completely in this approx.

The energy-momentum tensor
$$T_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + \sum_{i} \frac{\partial\mathcal{L}}{\partial(\partial q_{i}/\partial x_{\mu})} \cdot \frac{\partial q_{i}}{\partial x^{\nu}} \qquad E = \langle T_{00} \rangle \quad P = \frac{1}{3} \langle T_{ii} \rangle$$

$$T_{\mu\nu} = \left[-\frac{1}{2}m_{\omega}^{2}\omega_{0}^{2} - \frac{1}{2}m_{\rho}^{2}\rho_{0}^{(3)2} + \frac{1}{2}m_{\sigma}^{2}\sigma^{2} + U(\sigma) \right]g_{\mu\nu} + i\bar{\Psi}_{N}\gamma_{\mu}\partial_{\nu}\Psi_{N}$$

$$E = \frac{m_{\sigma}^{2}\sigma^{2}}{2} + U(\sigma) - \frac{m_{\omega}^{2}\omega_{0}^{2}}{2} - \frac{m_{\rho}^{2}\rho_{0}^{(3)2}}{2} + \sum_{N} \int_{0}^{p_{\text{F,N}}} \frac{2\mathrm{d}^{3}p}{(2\pi)^{3}}\epsilon_{N}(p) \qquad \epsilon_{N}(p) = \sqrt{m_{N}^{*2} + p^{2}} + g_{\omega N}\omega_{0} + g_{\rho N}I_{N}\rho_{03}$$

$$= \frac{m_{\sigma}^{2}\sigma^{2}}{2} + U(\sigma) + \frac{m_{\omega}^{2}\omega_{0}^{2}}{2} + \frac{m_{\rho}^{2}\rho_{0}^{(3)2}}{2} + \sum_{N} \int_{0}^{p_{\text{F,N}}} \frac{2\mathrm{d}^{3}p}{(2\pi)^{3}}\sqrt{m_{N}^{*2} + p^{2}}$$

m ...

Energy-density functional

$$E[n_p, n_n; \sigma] = \frac{m_{\sigma}^2 \sigma^2}{2} + U(\sigma) + C_{\omega}^2 \frac{(n_n + n_p)^2}{2m_N^2} + C_{\rho}^2 \frac{(n_n - n_p)^2}{8m_N^2} + \sum_N \int_0^{M_r, N} \frac{dp \, p^2}{\pi^2} \sqrt{(m_N - g_{\sigma N} \, \sigma)^2 + p^2}$$

evaluated for σ -field following from the equation $\frac{\delta E[n_p, n_n, \sigma]}{\delta \sigma} = 0$ \Rightarrow $\sigma = \sigma(n_p, n_n)$
Paremeters $C_i^2 = \frac{g_{iN}^2 m_N^2}{m_i^2}$ are adjusted to properties of nuclear matter at saturation
Dimensionless scalar field $f = \frac{g_{\sigma N} \sigma}{m_N}$ $\frac{m_{\sigma}^2 \sigma^2}{2} \rightarrow \frac{m_N^4 f^2}{2C_{\sigma}^2}$
 $U(f) = m_N^4 (bf^3/3 + cf^4/4)$ $\begin{pmatrix} n_0 & \simeq & 0.16 \pm 0.015 \text{ fm}^{-3} \\ E_0 & \simeq & -15.6 \pm 0.6 \text{ MeV} \\ m_N^*(n_0) & \simeq & (0.75 \pm 0.1) m_N \\ K & \simeq & 240 \pm 40 \text{ MeV} \\ J & \simeq & 32 \pm 4 \text{ MeV} \end{pmatrix}$

If we add gradient terms this energy density functional can be used for a description of properties of atomic nuclei.

 $U(f) = m_N^4 (bf^3/3 + cf^4/4)$

ISM:
$$n_p = n_n = n/2$$

condition for the minimum

condition for the minimum

$$\frac{\partial E[n/2, n/2; f]}{\partial n} \bigg|_{n_0, f_0} = \frac{1}{n_0} E[n_0; f_0]$$

$$E(n,x) = (m_N + \varepsilon(n,x))n$$

binding energy

$$\frac{1}{n_0} E[n_0; f_0] = m_N + E_0$$

 f_0 is the scalar field at the density $n_p = n_n = n_0/2$

effective nucleon mass

$$K = 9 n_0 \left[\frac{\partial^2 E}{\partial n^2} \bigg|_{n_0, f_0} - \left(\frac{\partial^2 E}{\partial n \partial f} \bigg|_{n_0, f_0} \right)^2 \left[\frac{\partial^2 E}{\partial f^2} \bigg|_{n_0, f_0} \right]^{-1} \right] \qquad 1 - f_0 = \frac{m_N^*}{m_N}$$

~

symmetry energy
$$\varepsilon_{\text{sym}}(n) = \frac{n}{8} \frac{\partial^2}{\partial n_p^2} E[n - n_p, n_p] \Big|_{n_p = n/2} = \frac{C_\rho^2 n}{8m_N^2} + \frac{\pi^2 n}{4 p_{\text{FN}} \sqrt{m_N^{*2} + p_{\text{FN}}^2}}$$

ρ-meson repulsion Pauli exclusion principle

$$p_{\rm FN} = (3\pi^2 n/2)^{1/3}$$

$$n_0 \simeq 0.16 \pm 0.015 \text{ fm}^{-3}$$

 $E_0 \simeq -15.6 \pm 0.6 \text{ MeV}$
 $m_N^*(n_0) \simeq (0.75 \pm 0.1) m_N$
 $K \simeq 240 \pm 40 \text{ MeV}$
 $J \simeq 32 \pm 4 \text{ MeV}$

equation for the scalar field

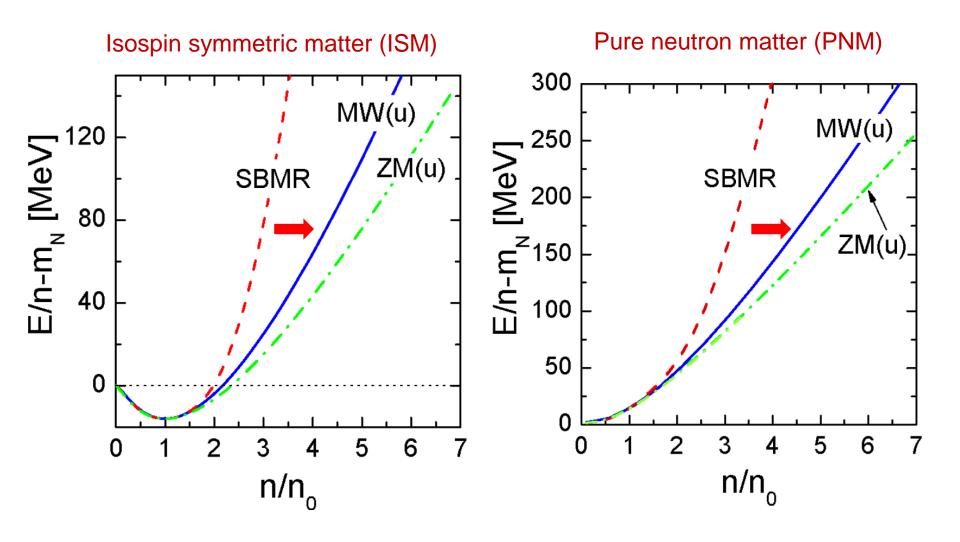
$$\frac{m_N^4 f}{C_{\sigma}^2} + \frac{\mathrm{d}U}{\mathrm{d}f} = 2 \int_{0}^{p_{\mathrm{FN}}} \frac{2\mathrm{d}^3 p}{(2\pi)^3} \frac{m_N(1-f)}{\sqrt{m_N^2(1-f)^2 + p^2}}$$

Input: $n_0 = 0.16 \text{ fm}^{-3}$, $E_0 = -16 \text{ MeV}$, J = 32 MeV, $m_N = 938 \text{ MeV}$ Walecka model U(f) = 0 W: $C_{\sigma}^2 = 329.70$, $C_{\omega}^2 = 249.40$, $C_{\rho}^2 = 68.09$ \longrightarrow $K \simeq 553 \text{ MeV}$, $m_N^*(n_0)/m_N \simeq 0.54$

Modified Walecka model

$$U(f) = m_N^4 (bf^3/3 + cf^4/4)$$

Additional input: K = 270 MeV, $m_N^*(n_0)/m_N = 0.8$ 750 1,0 QHD-I (Walecka) QHD-I (Walecka) mod. Walecka mod. Walecka 500 vector 0,8 250 m[^]/m 0,6 U, MeV sum 0,4 -250 0,2 scalar -500 -750 └─ 0 0,0 2 0 8 4 n_B/n_o $n_{\rm B}/n_0$



Neutron stars

Courageous theorists



Lev Landau (Phys. Z. Sowjetunion, 1, 285, 1932) speculated that one could compose a stellar object out of neutral particles held by gravity

Discovery of neutron 1932 by James Chadwick



Neutron would be a good candidate to build up "unheimliche Sterne" (weird stars) Landau @ seminar in NBI, 1932





Baade and Zwicky @ Stanford Meeting, 15-16 Dec. 1933

"... With all reserve we advance the view that <u>supernovae</u> represent the transitions from ordinary stars into <u>neutron stars</u>, which in their final stages consist of extremely closely packed neutrons..." **Courageous theorists**



Friedrich Hund 1936 role of inverse β -decays in the stability of neutron stars $n \rightarrow p + e + \tilde{\nu}_e$

Landau, Nature 1938, "Origin of stellar energy"





Tolman, Oppenheimer & Volkoff 1939 calculated properties of neutron stars: (only neutrons) R~10 km and M~1 M_{sol}



J.J.L. Duyvendak

Courageous sinologist in Leiden. 1942

"Amice, I have succeeded in finding another place where your Nova is mentioned. There exists an extensive work, of which a facsimile edition was published only a few years ago (and which could not have been known to earlier researchers), treating the institutions of the Sung dynasty, which includes the year 1054. The name is Sung Hui Yao. In vol. 54 of this work,..."

letter to Jan Oort, July 30, 1940

On this, Oort added in pencil,

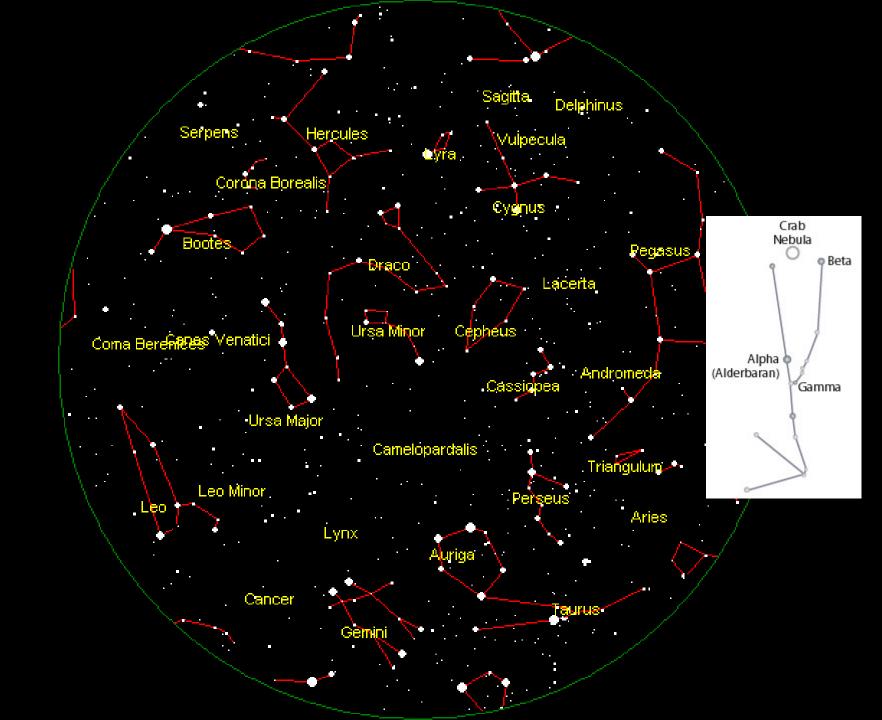
"Must write an airmail letter about this to Mayall and Baade, as soon as I am back in Leiden."

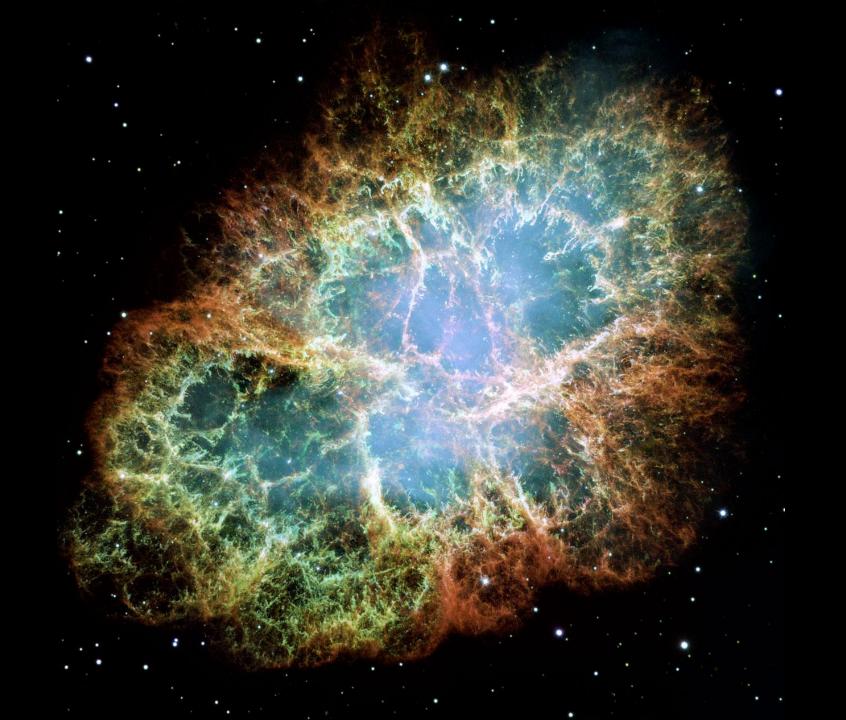
The original text (quoted by Biot from Ma Tuan-lin's Wenhsien T'ung-k'ao) is found in the Sung-shih ("History of the Sung Dynasty") by T'o-t'o (1313 to 1355).² It runs:

In the 1st year of the period *Chih-ho* [1054], the 5th moon, the day *chi-ch'ou* [July 4] [a guest-star] appeared approximately several inches south-east of *T'ien-kuan* [ζ Tauri]. After more than a year³ it gradually became invisible.



Jan Oort





🗳 Baade, Astrophysical J. 98, 188 (1942) |

Fig. CC.

Mufea

ries

probed deeper into the nebula, observing that a prominent star near the nebula's center might be related to its origin.

Auriga

Gemi

enadude

Crab Nebula can be a result of an explosion happened about 1000 years ago

Six years later, scientists discovered that the Crab was emitting among the strongest radio waves of any celestial object.

Baade noticed in 1954 that the Crab possessed powerful magnetic fields.

1963 a high-altitude rocket detected X-rays from the nebula.

On the verge of discovery...



... somewhere in UK



While studying distant galaxies, Jocelyn Bell noticed small pulses of radiation when the telescope was looking at a particular position in the sky.

For a short time, she and A. Hewish thought the pulses might be coming from an extra-terrestrial civilizations.

£6)



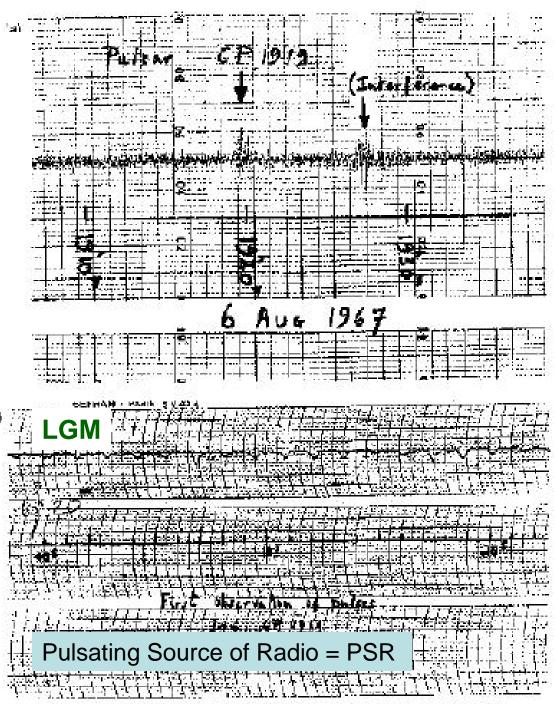


FIGURE 1: Chart on which Jocelyn Bell discovered her first pulsar.

Lighthouse model

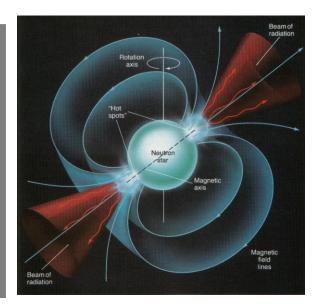
Energy Emission from a Neutron Star

It seems more rewarding therefore to look for some mechanisms by which the neutron star can release either its magnetic or its rotational energy or both. In this communication I would like to outline the principal features of a possible model of this kind.

F. PACINI

Center for Radiophysics and Space Research, Cornell University, New York.

[Nature, 216 (1967) 567]



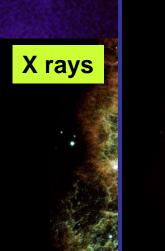
Rotating Neutron Stars as the Origin of the [Nature, 218 (1968) 731] **Pulsating Radio Sources**

bу

T. GOLD

Center for Radiophysics and Space Research, Cornell University, Ithaca, New York

The constancy of frequency in the recently discovered pulsed radio sources can be accounted for by the rotation of a neutron star. Because of the strong magnetic fields and high rotation speeds, relativistic velocities will be set up in any plasma in the surrounding magnetosphere, leading to radiation in the pattern of a rotating beacon.



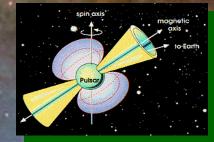


radio

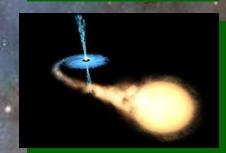
Crab's central pulsar was discovered in 1968 by radio astronomers. The pulsar was then identified as a source of periodic optical and X-ray radiation.

Neutron Star Zoo

>2000 neutron stars in isolated rotation-powered pulsars ~ 30 millisecond pulsars



>100 neutron stars in accretion-powered X-ray binaries ~ 50 x-ray pulsar intense X-ray bursters (thermonuclear flashes)



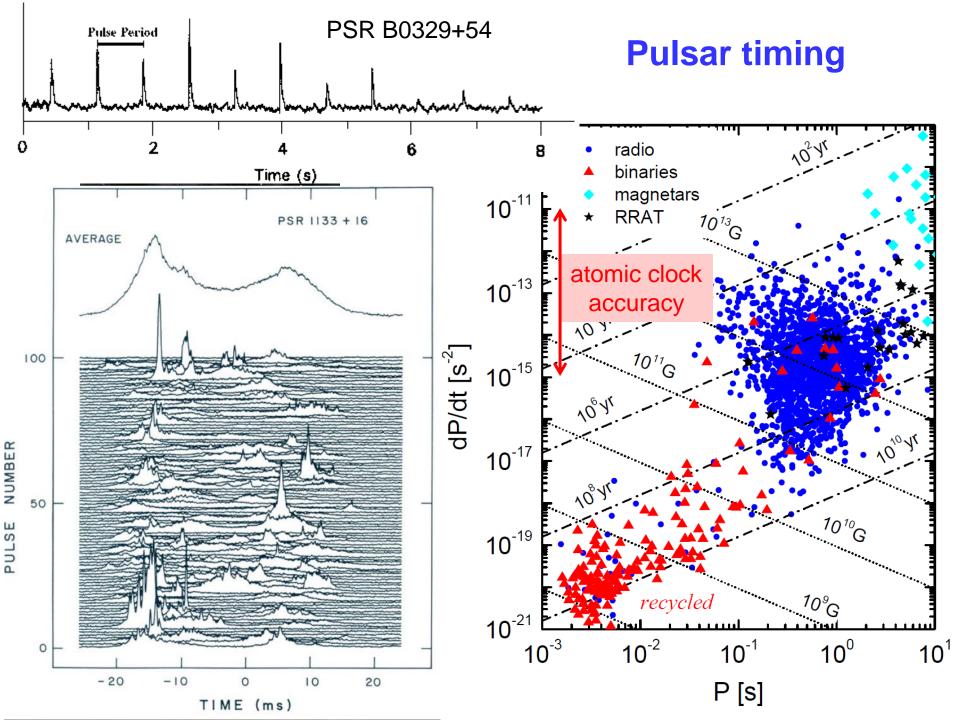
short gamma-ray bursts neutron star -- neutron star,

neutron star -- black-hole mergers

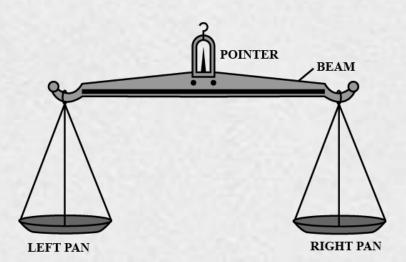


soft gamma-ray repeaters – magnetars (super-strong magnetic fields)

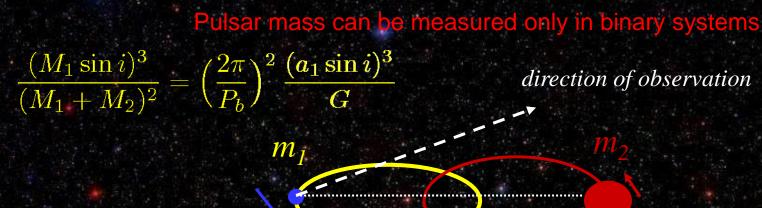




Neutron star masses



Measuring pulsar mass

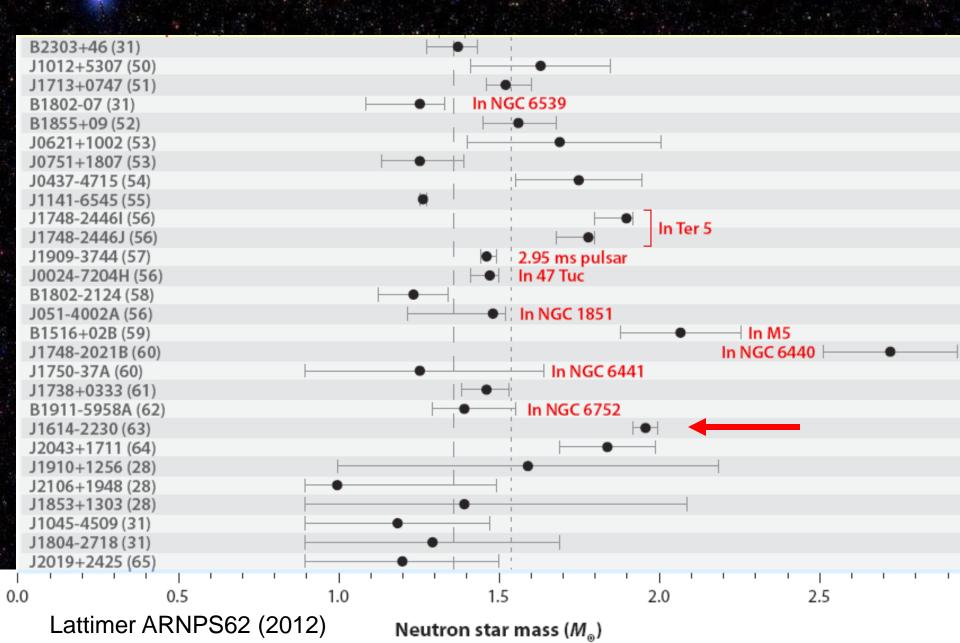


Newton gravity \longrightarrow 5 Keplerian orbital parameters: orbital period, semi-major axis length, excentricity,

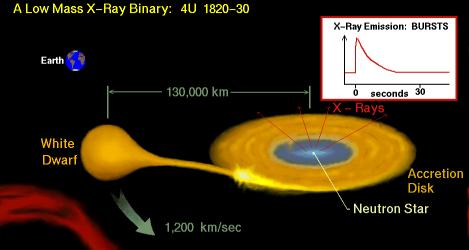
Do not determine individual masses of stars and the orbital inclination.

Measurement of any 2 post-Keplerian parameters allows to determine the mass of each star.

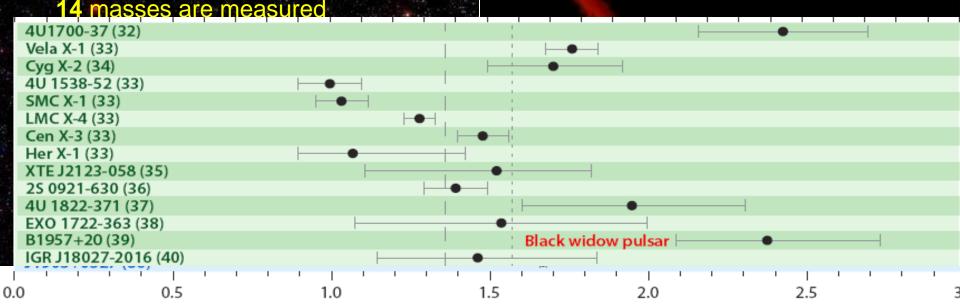
White dwarf -- neutron star binaries



Measuring pulsar mass



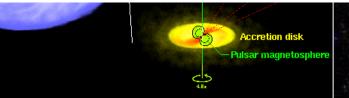
X-ray binaries



Neutron star mass (M_{\odot})

Dany Page





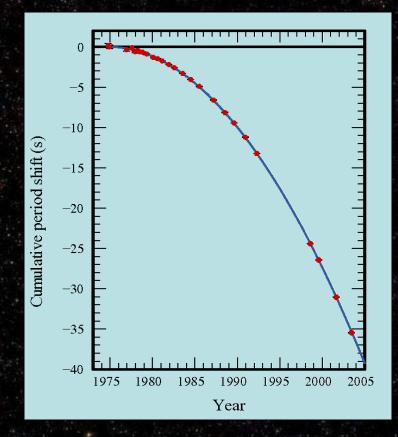
Measuring pulsar mass

Double neutron star binaries

1974 PSR B1913+16 Hulse-Taylor pulsar

First precise test of Einstein gravitation theory

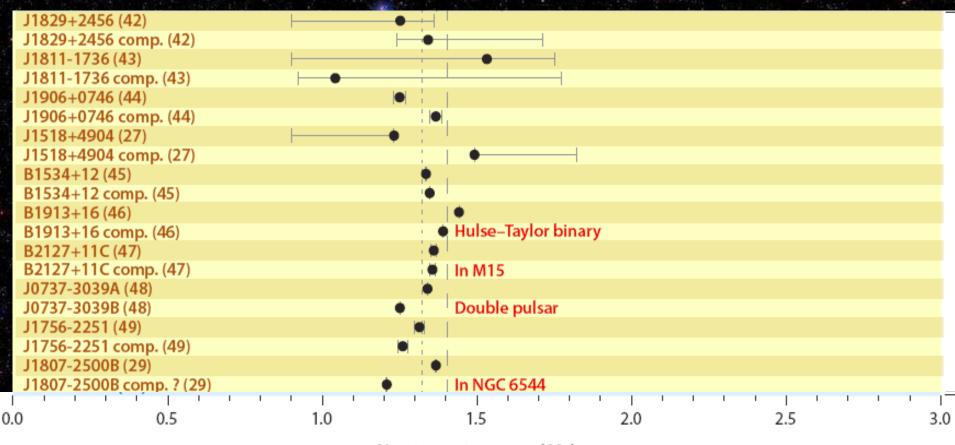
2003 J0737-3039 first double pulsar



Pulsar A: $P^{(A)}=22.7$ ms, $M^{(A)}=1.338$ M_{sol} Pulsar B: $P^{(B)}=2.77$ ms, $M^{(A)}=1.249\pm0.001$ M_{sol} Orbiting period 2.5 hours

[Nature 426, 531 (2003), Science 303, 1153 (2004)]

Double neutron star binaries



Neutron star mass (M_o)



Most massive neutron stars



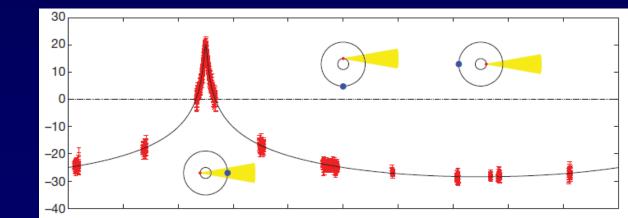
Pulsar J1614-2230

 $\mathbf{M} = (1.97 \pm 0.04) \ \mathbf{M_{sol}}$

P.Demorest et al., Nature 467, 1081 (2010)

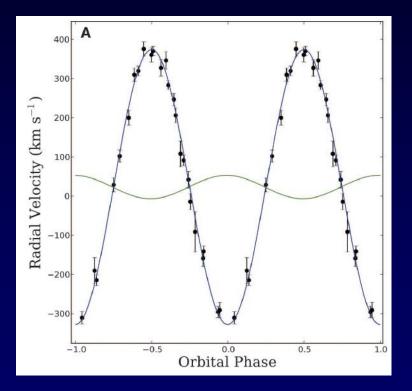
Measured Shapiro delay with high precision

Time signal is getting delayed when passing near massive object.



Pulsar J0348+0432

$\mathbf{M} = (2.01 \pm 0.04) \ \mathbf{M_{sol}}$



Measured phase-resolved spectra of the optical counterpart. hydrogen Balmer lines

Antonadis et al, Science 340, 1233232

pulsar-white dwarf binary

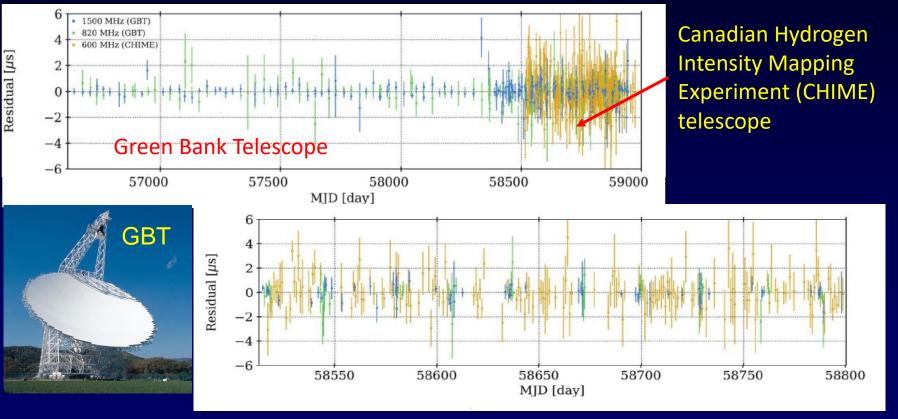
pulsar-white dwarf binary

Pulsar J0740+6620

$\mathbf{M} = (2.08 \pm 0.07)~\mathbf{M}_{\rm sol}$

Highest well-known mass of NS





Fonseca et al, Astrophys. J. Lett., 915 (2021) L12

Shapiro delay

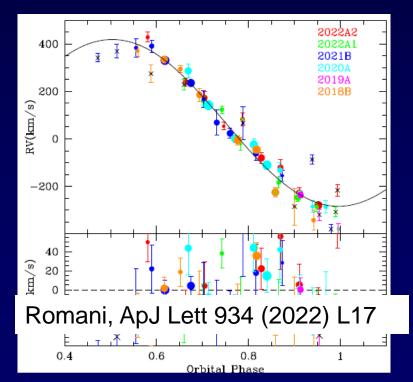
Pulsar J0952-0607

$\mathbf{M} = (\mathbf{2.35} \pm \mathbf{0.17}) \ \mathbf{M}_{\rm sol}$

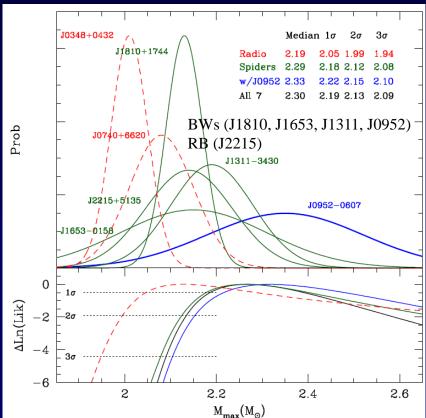
"Black widow" pulsar

the companion is being destroyed by the strong powerful outflows, or winds, of high-energy particles caused by the neutron star.

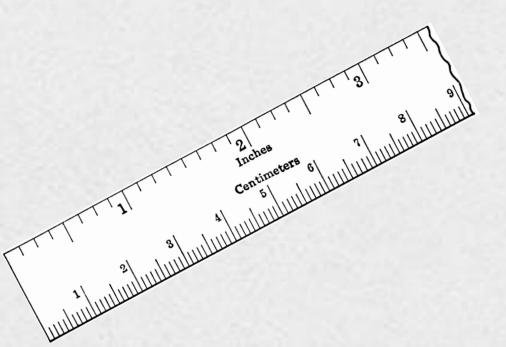
Radial velocity from the companion spectrum



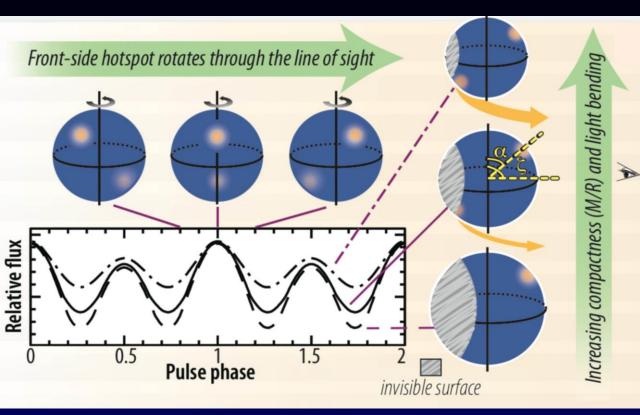




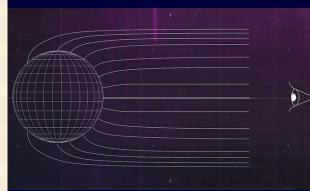
Neutron star radius



The Neutron Star Interior Composition Explorer Mission (NICER)



Lightcurve modeling constrains the compactness (GM/Rc²) and viewing geometry of a non-accreting millisecond pulsar



PSR J0740+6620

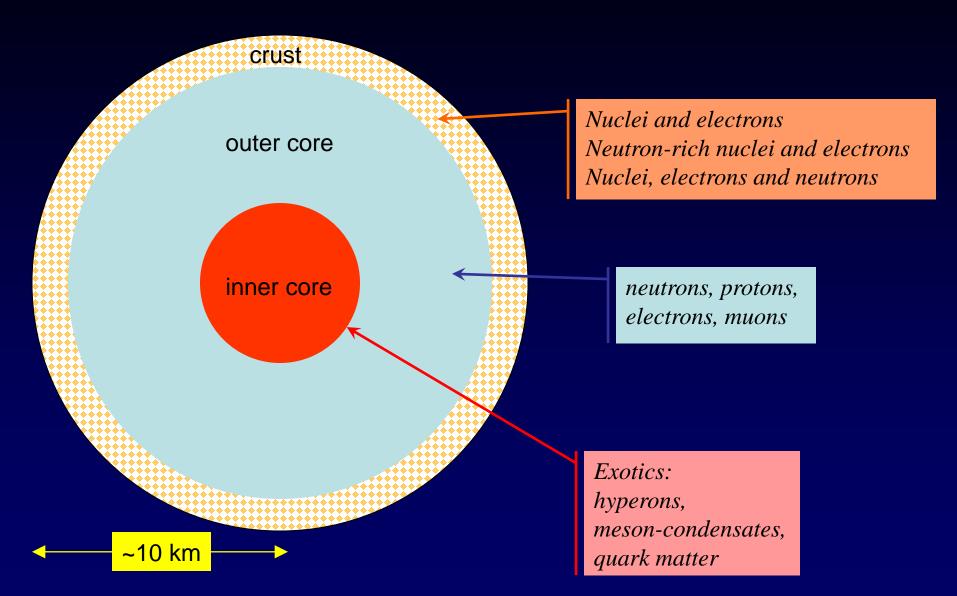
Thomas E. Riley et al 2021 ApJL 918 L27

$$R = 12.39^{+1.30}_{-0.98}$$
 km and $M = 2.072^{+0.067}_{-0.066} M_{so}$

M. C. Miller et al 2021 ApJL 918 L28

$$R = 13.7^{+2.6}_{-1.5} \,\mathrm{km}$$
 and $M = 2.08 \pm 0.07 \,M_{\mathrm{sol}}$

Cross section of a neutron star

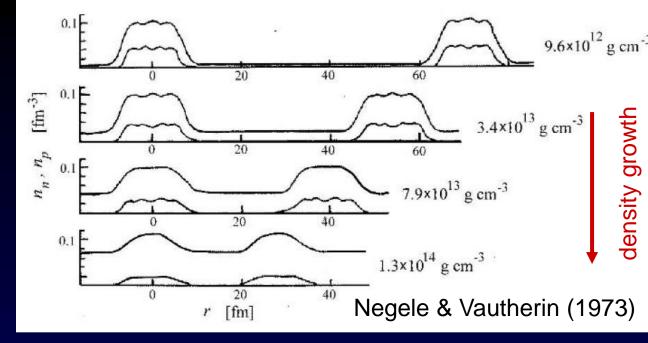


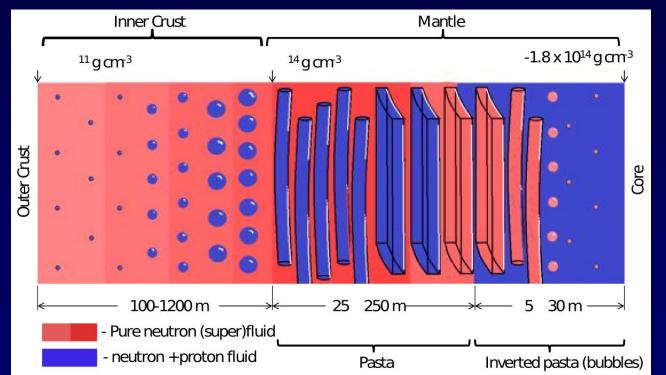


Nucleus melting

Pasta structure

interplay of Coulomb energy and surface tension



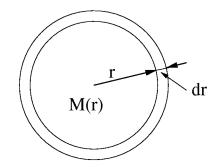


saturation density $\rho_0 = 2.8 \times 10^{14} \frac{\mathrm{g}}{\mathrm{cm}^3}$

 $M_{crust} \sim 0.1 M_{sol}$ $R_{crust} \sim 10^2 - 10^3 m$

Tolman-Oppenheimer-Volkov equation

Equilibrium condition for a shell in a non-rotating neutron star



OUTPUT:

 $S_{\Omega}(r) dp = dF_G$ Newton's Law $4 \pi r^2 dp = G \frac{M(r) dM}{r^2} \qquad dM = 4 \pi r^2 \varepsilon(p) dr$

INPUT: equation of state (EoS)

$$\varepsilon = \varepsilon(p)$$
 or $\begin{cases} p = p(n) \\ \varepsilon = \varepsilon(n) \end{cases}$

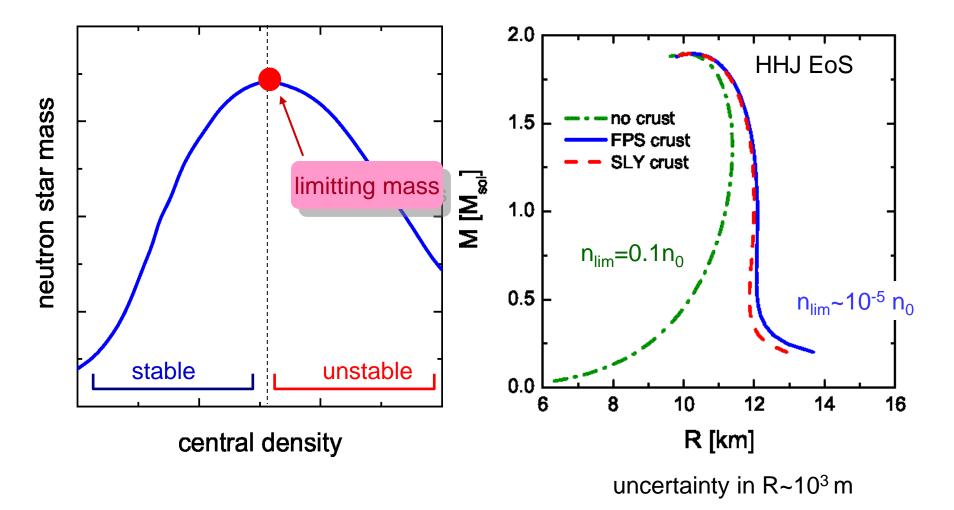
boundary conditions: $\varepsilon(r=0) = \varepsilon_c$, M(r=0) = 0, P(r=R) = 0

neutron star density profile, radius R and mass M

relativistic corrections

$$\frac{dp}{dr} = -\frac{G\rho M}{r^2} \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi P r^3}{Mc^2}\right) \left(1 - \frac{2GM}{c^2 r}\right)^{-1}$$

Neutron star configuration



Two basic physical principles determine the structure of compact stellar objects:

Electroneutrality and Pauli exclusion principle.

Macroscopic object held by gravity must be electrically neutral:

Consider a sphere of a radius R

with the uniform charge density n_0 and the baryon density n

Coulomb energy:

$$E_{\rm C} = e^2 2 \pi R^2 n_Q = 1.5 \times 10^{36} \text{ MeV} \left(\frac{n_Q}{n_0}\right) \left(\frac{R}{1 \text{km}}\right)^2$$

Gravitational energy: (n₀=0.16 fm⁻³)

$$E_{\rm G} = -G \, 2 \, \pi \, R^2 \, n \, m_N = -0.7 \, \operatorname{MeV} \times \left(\frac{n}{n_0}\right) \, \left(\frac{R}{1 \, \rm km}\right)^2$$

$$E_{\rm C} + E_{\rm G} = 0 \qquad \Longrightarrow \qquad \frac{n_Q}{n} \sim 10^{-36}$$

Pauli blocking at work: neutron star composition

Composition is determined by the minimum of energy

Symmetry energy: = $\bar{a}_{Sym} (n_n - n_p)^2 / n \longrightarrow$ It is favorable to have some protons.

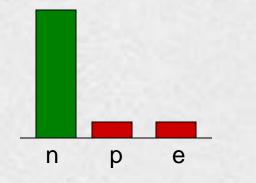
Electroneutrality: $n_e = n_p$ \longrightarrow There will be also some electrons

Electron energy:

$$E_{e} = 2 \int_{0}^{p_{F}e} \frac{d^{3}p}{(2\pi)^{3}} E_{e}(p) = \frac{p_{F}^{4}e}{4\pi^{2}} = \frac{3}{4} (3\pi^{2})^{1/3} n_{e}^{4/3}$$

\$\low Electron energy increases fast

Energy minimization: Mainly neutrons and small admixture of protons and electrons



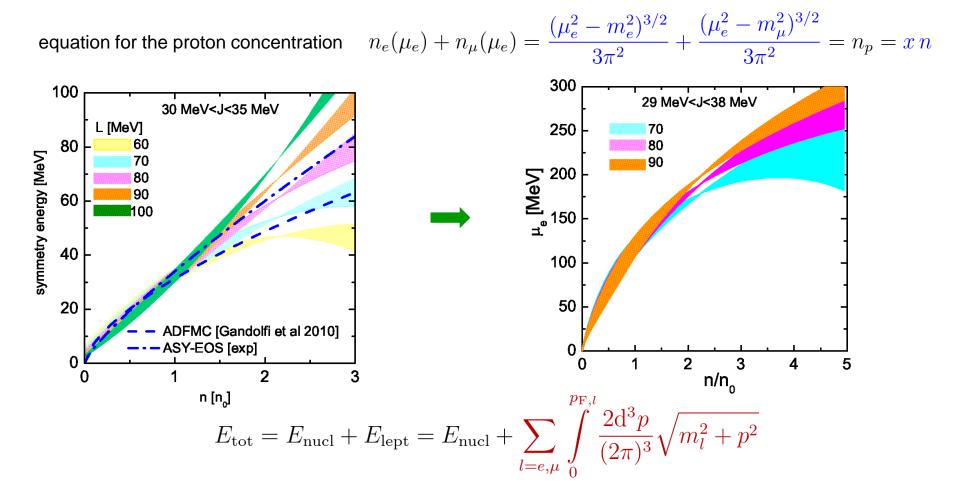
 $\begin{array}{c} \text{medium in } \textbf{\textit{\beta} equilibrium} \\ n \leftrightarrow p + e^- \end{array}$

chemical potentials:

$$\mu_n = \frac{\partial E(n_p, n_n)}{\partial n_n} = \frac{\partial E(n, x)}{\partial n} - \frac{x}{n} \frac{\partial E(n, x)}{\partial x} \qquad \mu_p = \frac{\partial E(n_p, n_n)}{\partial n_p} = \frac{\partial E(n, x)}{\partial n} + \frac{1 - x}{n} \frac{\partial E(n, x)}{\partial x}$$

Condition of the beta equilibrium $n\leftrightarrow p+e^ n\leftrightarrow p+\mu^ \mu^-\leftrightarrow e^-$

$$\mu_n = \mu_p + \mu_e \qquad \mu_\mu = \mu_e$$
$$\mu_e = \mu_n - \mu_p = -\frac{1}{n} \frac{\partial E(n, x)}{\partial x} = 4 \varepsilon_{\rm S}(n) (1 - 2x)$$

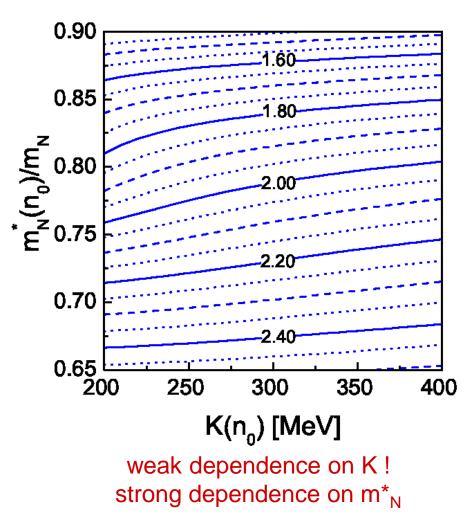


(pure) Walecka model $U(\sigma)=0$ $n_0 = 0.16 \text{fm}^{-3}, E_{\text{bind}} = -16 \text{ MeV}$ $k = 553 \text{ MeV}, m_N^*(n_0) = 0.54 m_N$

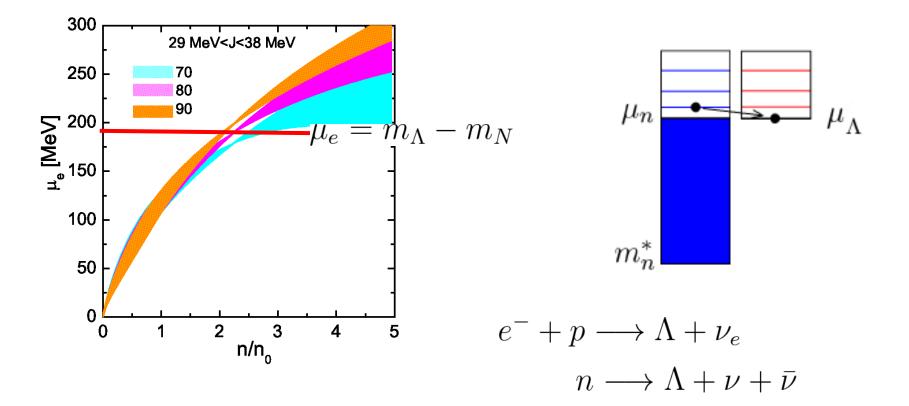
3.0 2.5 PSR J1614-2230 M/M sol 2.0 1.5 1.0 0.5 0.0 2 8 4 6 10 0 n/n_o Hardest EoS among RMF models

<u>modified Walecka</u> $U(\sigma)=a\sigma^3+b\sigma^4$





Hyperonization of nuclear matter



Energy-density functional with hyperons

$$\begin{split} B \in \mathrm{SU}(3) \text{ ground state multiplet} & \text{scalar field } f = g_{\sigma} \chi_{\sigma} \sigma/m_{N} \\ E[f, \{n_{\mathrm{B}}\}] = \sum_{B} E_{\mathrm{kin}} \left(p_{\mathrm{F},B}, m_{B} \Phi_{B}(f)\right) + \frac{m_{N}^{4} f^{2}}{2C_{\sigma}^{2}} + U(f) + \frac{\left[C_{\omega}^{2} \widetilde{n}_{B}^{2} + C_{\rho}^{2} \widetilde{n}_{I}^{2} + C_{\phi}^{2} \widetilde{n}_{S}^{2}\right]}{2m_{N}^{2}} \\ \text{effective densities:} & \widetilde{n}_{B} = \sum_{B} x_{\omega B} n_{B} \quad \widetilde{n}_{I} = \sum_{B} x_{\rho B} t_{3B} n_{B} \quad \widetilde{n}_{S} = \sum_{H} x_{\phi H} n_{H} \\ C_{i} = \frac{g_{iN} m_{N}}{m_{i}}, \ i = \sigma, \omega, \rho \quad C_{\phi} = m_{\omega} C_{\omega}/m_{\phi} \\ \text{with coupling constant ratios} & x_{\omega(\rho)B} = \frac{g_{\omega(\rho)B}}{g_{\omega(\rho)N}} \quad x_{\phi H} = \frac{g_{\phi H}}{g_{\omega N}} \\ \text{mass scaling:} & \frac{m_{N}^{*}}{m_{N}} = \Phi_{N}(f) = 1 - f \quad \frac{m_{H}^{*}}{m_{N}} = \Phi_{H}(f) = 1 - x_{\sigma H} \frac{m_{N}}{m_{H}} f \\ \text{quark counting SU(6)} & g_{\omega N} : g_{\omega \Lambda} : g_{\omega \Sigma} : g_{\omega \Xi} = 3 : 2 : 2 : 1 \\ \text{scalar couplings:} & g_{\rho N} : g_{\rho \Lambda} : g_{\rho \Sigma} : g_{\rho \Xi} = 1 : 0 : 2 : 1 \\ \text{scalar couplings:} \\ \mathbf{s} + \mathbf{V} = \mathbf{U}_{\mathrm{bind}} & x_{\sigma H} = \frac{x_{\omega H} n_{0} C_{\omega}^{2}/m_{N}^{2} - U_{H}(n_{0})}{m_{N} - m_{N}^{*}(n_{0})} \qquad \begin{cases} U_{\Lambda}(n_{0}) = -28 \,\mathrm{MeV} \\ U_{\Sigma}(n_{0}) = -15 \,\mathrm{MeV} \\ U_{\Xi}(n_{0}) = -15 \,\mathrm{MeV} \end{cases} \end{cases} \end{cases}$$

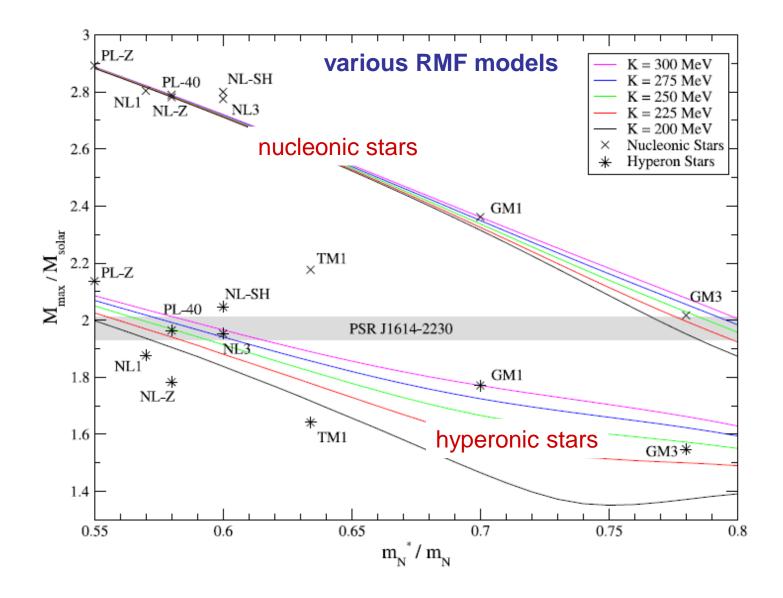
Neutron star composition with hyperons

 $n_0 = 0.17 \,\mathrm{fm}^{-3}, \quad E_0 = -16 \,\mathrm{MeV}, \quad K = 210 \,\mathrm{MeV}, \quad J = 36.8 \,\mathrm{MeV}, \quad m_N^*(n_0) = 0.85 \,m_N$ $x_{\omega\Lambda} = x_{\omega\Sigma} = 2 x_{\omega\Xi} = \frac{z}{2}$ $E_{\text{bind}}^{\Lambda} = -30 \,\text{MeV}$ $E_{\text{bind}}^{\Xi} = -18 \,\text{MeV}$ 1.0 (III)(IV)(II) (I) concentreation 0.8 0.6 0.4 0.2 Λ-0.0 2 3 4 5 0 1 2 3 4 5 0 1 2 3 4 5 0 1 2 3 4 5 6 0 1 n/n_0

CASE ICASE IICASE IIICASE IV $E_{\text{bind}}^{\Sigma} = +30 \text{ MeV}$ $E_{\text{bind}}^{\Sigma} = -10 \text{ MeV}$ $E_{\text{bind}}^{\Sigma} = +30 \text{ MeV}$ $E_{\text{bind}}^{\Sigma} = -10 \text{ MeV}$ $x_{\rho\Sigma} = 2 x_{\rho\Xi} = \frac{2}{3}$ $x_{\rho\Sigma} = 2 x_{\rho\Xi} = \frac{2}{3}$ $x_{\rho\Sigma} = x_{\rho\Xi} = 1$ $x_{\rho\Sigma} = x_{\rho\Xi} = 1$

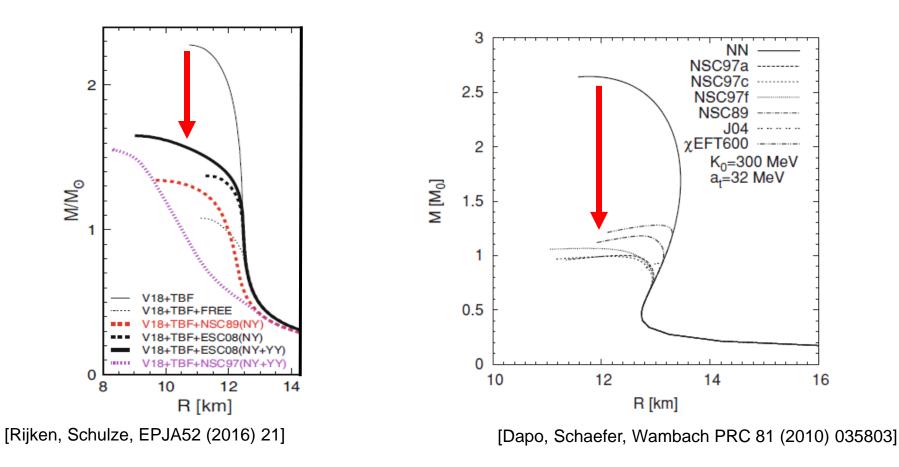
quark counting

SU(3) symmetry



"Hyperon puzzle"

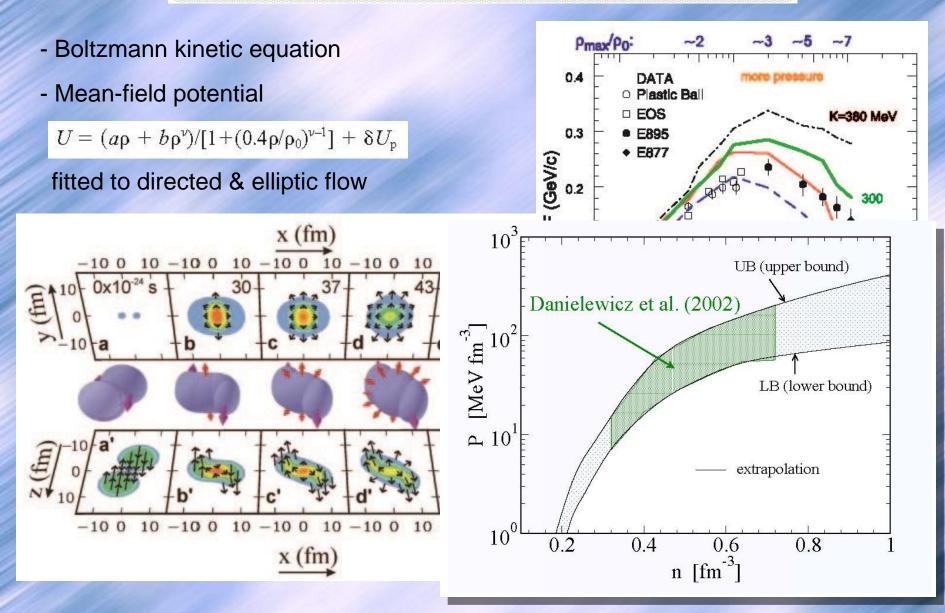
If we allow for a population of new Fermi seas (hyperon, Δ baryons, ...) EoS will be softer and the NS will be smaller



Simple solutions: -- make nuclear EoS as stiff as possible [flow constraint] -- suppress hyperon population (increase repulsion/reduce attraction)

against phenomenology of YN,NN,YY interaction in vacuum +hypernuclear physics

Constraints from heavy-ion collisions



[Danielewicz, Lacey, Lynch, Science 298, 1592 (2002)]

In NLW the scalar field is monotonously increasing function of the density

$$m_{\sigma}^2 \sigma + U'(\sigma) = g_{\sigma} \left(n_{\mathrm{S},p} + n_{\mathrm{S},n} \right)$$

dimensionless scalar field $~~f=g_{\sigma}\sigma/m_N$

$$\frac{f}{C_{\sigma}^2} + \frac{U'(f)}{m_N} = n_{\mathbf{S},p} + n_{\mathbf{S},p}$$

$$\frac{\mathrm{d}f}{\mathrm{d}n} = \frac{2\partial(n_{\mathrm{S},p} + n_{\mathrm{S},n})/\partial n}{m_N^3 C_{\sigma}^{-2} + \tilde{U}''(f)/m_N - 2\partial(n_{\mathrm{S},p} + n_{\mathrm{S},n})/\partial f} - \frac{\partial n_{\mathrm{S},n}}{\partial f}$$

Observation:

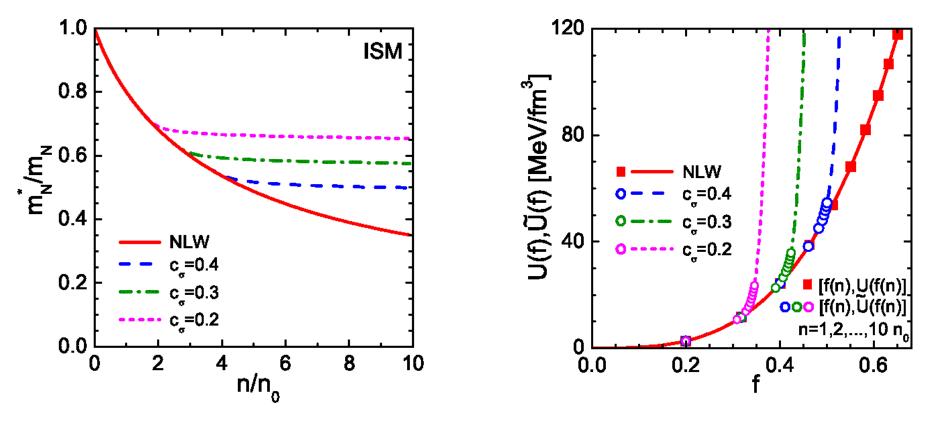
If we modify the scalar potential $\widetilde{U}(\sigma) = U(\sigma) + \Delta U(\sigma)$ so that the m^{*}_N(n) levels off

$$n_{\mathrm{S},i} = \int_{0}^{p_{\mathrm{F},i}} \frac{m_N^* p^2 \mathrm{d}p / \pi^2}{(p^2 + m_N^{*2})^{1/2}}$$

source is the scalar density

$$egin{aligned} m_N^* &= m_N - g_\sigma \sigma \ m_N^* &= m_N (1-f) \ C_\sigma^2 &= g_\sigma^2 m_N^2 / m_\sigma^2 \end{aligned}$$

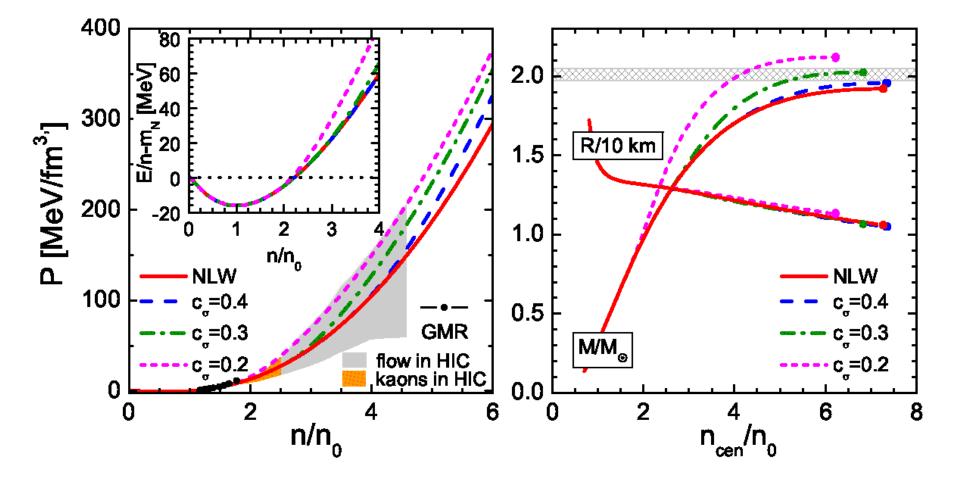
$$egin{aligned} &rac{\partial n_{\mathrm{S},i}}{\partial n} = rac{m_N^*}{2\sqrt{p_{\mathrm{F},i}^2+m_N^{*2}}} \ &rac{\partial n_{\mathrm{S},i}}{\partial f} = \int\limits_{0}^{p_{\mathrm{F},i}} rac{m_N p^4 \mathrm{d} p/\pi^2}{(p^2+m_N^{*2})^{3/2}} \end{aligned}$$



Simulation of excluded volume effect

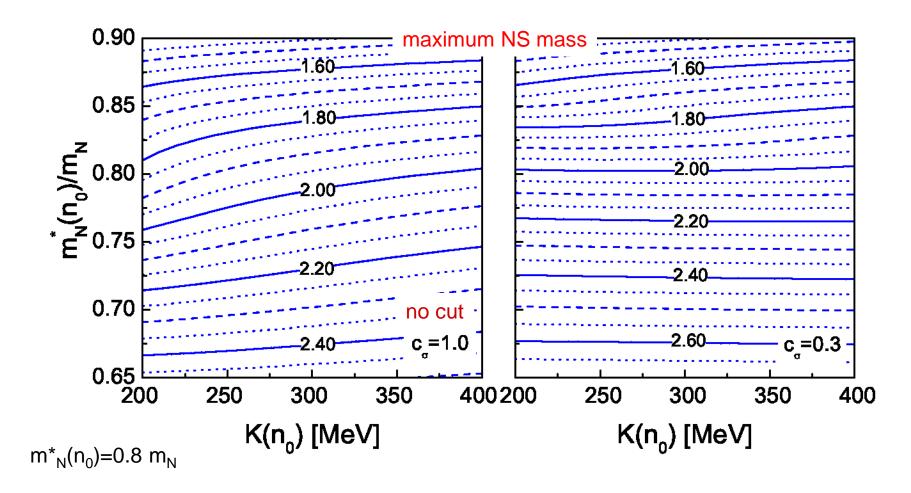
If m*N(n) saturates then the EoS stiffens

$$f_{
m s.core} = f_0 + c_\sigma (1-f_0)$$



P.-G. Reinhard, [Z. Phys. A 329 (1988) 257] introduced a "switch function" to get rid off the scalar field fluctuations

$$\mathscr{U}''(\Phi) = m_{\infty}^2 + \Delta m^2 \cosh^{-2}\left(\frac{\Phi - \Phi_0}{\delta \Phi}\right)$$



The effect is more pronounced if the input parameter of the model $m_N^*(n_0)$ is chosen smaller

New precise measurements of neutron star mass : $M_{crit} > 2.0$

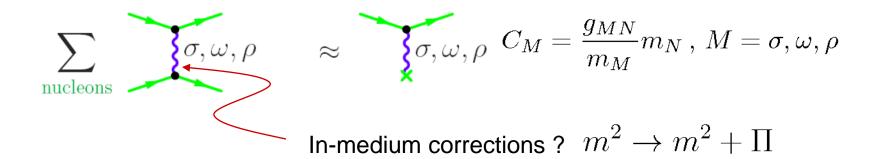
EoS must be sufficiently stiff

Constraints from particle flow in HICS:

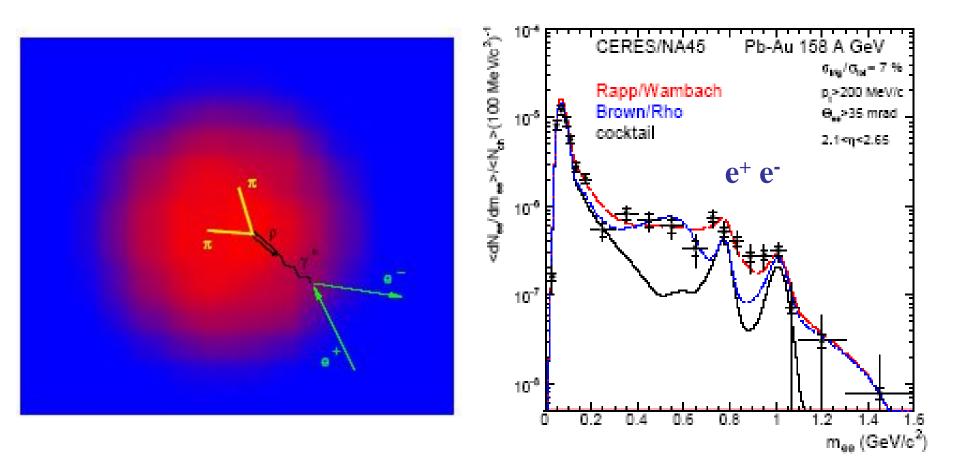
EoS should be not too stiff at $n \sim 4n_0$

R(elativistic)MF EoS based on the mean-field approximation are not flexible enough_

(cut mechanism)



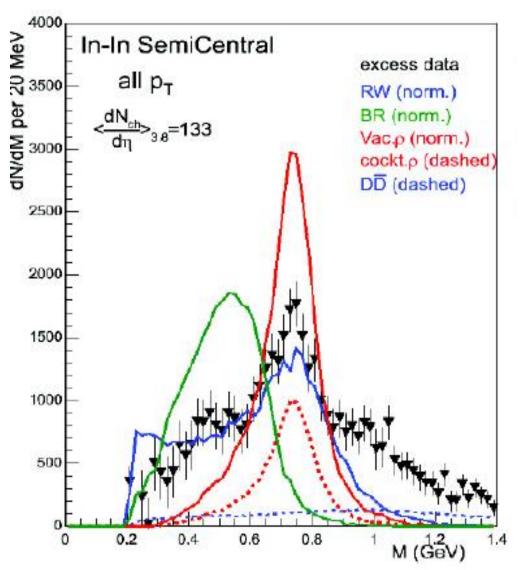
Mesons in medium



$$\frac{d^4 N_{ee}}{dq^4} = -\int dx^4 \ \mathcal{L}(M) \ \frac{\alpha^2}{\pi^3 q^2} \ \frac{Im \Pi_{em}(q, T(x), \mu_B(x))}{e^{q_0/T(x)} - 1}$$

ρ-meson spectral function

NA60 $\mu^+\mu^-$ data at 158 AGeV



G.Brown and M.Rho, Phys. Rev. Lett. **66** (1991) 2720; Phys. Rept. **269** (1996) 333

R.Rapp and J.Wambach, Adv. Nucl. Phys. **25** (2000) 1

Reduction of rho-meson mass

NA60 Collaboration, PRL **96** (2006) 162302 Volker Metag, Medium modifications of mesons in elementary reactions and heavy-ion collisions, Progress in Particle and Nuclear Physics 61 (2008) 245–252

| | KEK | Jlab | CBELSA/TAPS | CERES | NA60 |
|------------------------------------|---|---|--|--|--|
| Reaction Momentum acceptance | pA 12 GeV p >0.5 GeV/c | γA 0.6–3.8 GeV p >0.8 GeV/c | $\gamma A 0.7-2.5 \text{ GeV}$ p > 0.0 GeV/c | Au + Au 158 AGeV $p_t > 0.0 GeV/c$ | $In + In 158 AGeV$ $p_t > 0.0 GeV/c$ |
| ρ | $\frac{\Delta m}{m} = -9\%$ no broadening | $\Delta m \approx 0$ some broadening | | broadening favoured over density dependent mass shift | $\Delta m \approx 0$ strong broadening |
| ω | | | $rac{\Delta m}{m}pprox -14\%$ $rac{\Gamma_{\omega}(ho_0)}{\Gamma_{\omega}}pprox 16$ | 11455 51111 | |
| Φ | $\frac{\Delta m}{m} = -3.4\%$ | | | | |
| | $\frac{\Gamma_{\Phi}(\rho_0)}{\Gamma_{\Phi}} = 3.6$ | | | | |
| | 500 400 300 500 500 400 500 500 500 500 500 500 5 | | | 140 Cu 120 120 750 100 80 40 20 $\chi^2/ndf=83/50$ 0.9 | βγ<1.25 |

KVOR model [EEK and D.Voskresensky NPA 759 (2005) 373]

- in standard RMF model m_{σ} , m_{ω} , and m_{ρ} do not change Can the in-medium modification (decrease) of meson masses be included in an RMF model??
- σ field dependent masses and couplings constant
- decreasing functions of σ : $m^*_{\omega}(\sigma)$, $m^*_{\rho}(\sigma) \leftarrow$ self-consistent σ field results in *increase* of ρ an ω masses
- universal scaling $m_{\sigma}^*/m_{\sigma} \approx m_{\omega}^*/m_{\omega} \approx m_{\rho}^*/m_{\rho} = \Phi(n)$

Lattice QCD (SC-QCD): common drop of meson masses [Ohnishi Miura Kawamoto Mod.Phys.Lett A23, 2459]

Sliding vaccua and double decimation concept [Brown, Rho PR396(2004)1]

"vector manifestation" [Harada, Yamawaki]

Half-skyrmion model of dense nuclear matter [Vento; Rho, Hyun Kyu Lee 1704.02775]

$$\mathcal{L} = \bar{\Psi}_N \left(\partial \cdot \gamma - g_\omega \chi_\omega \omega \cdot \gamma - \frac{1}{2} g_\rho \chi_\rho \rho \cdot \gamma \tau \right) \Psi_N - m_N \Phi_N \bar{\Psi}_N \bar{\Psi}_N$$
$$+ \frac{\partial^\mu \sigma \partial_\mu \sigma}{2} - \Phi_\sigma^2 \frac{m_\sigma^2 \sigma^2}{2} - U(\sigma) - \frac{\omega_{\mu\nu} \omega^{\mu\nu}}{4} + \Phi_\omega^2 \frac{m_\omega^2 \omega_\mu \omega^\mu}{2} - \frac{\rho_{\mu\nu} \rho^{\mu\nu}}{4} + \Phi_\rho^2 \frac{m_\rho^2 \rho_\mu \rho^\mu}{2}$$

Field redefinition

$$\Psi_N \to \Psi_N / \sqrt{a_N} \ , \ \sigma \to \sigma / \sqrt{a_\sigma} \ , \ \omega_\mu \to \omega_\mu / \sqrt{a_\omega}, \rho_\mu \to \rho_\mu / \sqrt{a_
ho}$$

$$\mathcal{L}_{N} = \bar{\Psi}_{N} (i \ D \cdot \gamma) \Psi_{N} - m_{N} \Phi_{N} \bar{\Psi}_{N} \bar{\Psi}_{N},$$

$$D_{\mu} = \partial_{\mu} + ig_{\omega} \chi_{\omega} \omega_{\mu} + \frac{i}{2} g_{\rho} \chi_{\rho} \rho_{\mu} \tau$$

$$\mathcal{L}_{M} = \frac{\partial^{\mu} \sigma \partial_{\mu} \sigma}{2} - \Phi_{\sigma}^{2} \frac{m_{\sigma}^{2} \sigma^{2}}{2} - U(\sigma)$$

$$- \frac{\omega_{\mu\nu} \omega^{\mu\nu}}{4} + \Phi_{\omega}^{2} \frac{m_{\omega}^{2} \omega_{\mu} \omega^{\mu}}{2} - \frac{\rho_{\mu\nu} \rho^{\mu\nu}}{4} + \Phi_{\rho}^{2} \frac{m_{\rho}^{2} \rho_{\mu} \rho^{\mu}}{2}$$

 $m_i^*/m_i = \phi_i(\chi_\sigma\sigma)/\sqrt{a_i(\chi_\sigma\sigma)} = \Phi_i(\chi_\sigma\sigma)$ mass scaling function $\chi_i = \tilde{\chi}_i(\chi_\sigma\sigma)/\sqrt{a_i(\chi_\sigma\sigma)}$ coupling-constant scaling function

where

Energy-density functional

 $B \in SU(3)$ ground state multiplet scalar field $f = g_{\sigma} \chi_{\sigma} \sigma / m_N$

$$\begin{split} E[f, \{n_{\rm B}\}] &= \sum_{B} E_{\rm kin}(p_{{\rm F},B}, m_{B}\Phi_{B}(f)) + \sum_{l=e,\mu} E_{\rm kin}(p_{{\rm F},l}, m_{l}) \\ &+ \frac{m_{N}^{4}f^{2}}{2C_{\sigma}^{2}}\eta_{\sigma}(f) + \frac{1}{2m_{N}^{2}} \Big[\frac{C_{\omega}^{2}\widetilde{n}_{B}^{2}}{\eta_{\omega}(f)} + \frac{C_{\rho}^{2}\widetilde{n}_{I}^{2}}{\eta_{\rho}(f)} + \frac{C_{\phi}^{2}\widetilde{n}_{S}^{2}}{\eta_{\phi}(f)} \Big] , \\ C_{i} &= \frac{g_{iN}m_{N}}{m_{i}} , i = \sigma, \omega, \rho \quad C_{\phi} = m_{\omega}C_{\omega}/m_{\phi} \end{split}$$

effective densities: $\tilde{n}_B = \sum_B x_{\omega B} n_B$ $\tilde{n}_I = \sum_B x_{\rho B} t_{3B} n_B$ $\tilde{n}_S = \sum_H x_{\phi H} n_H$ with coupling constant ratios $x_{\omega(\rho)B} = \frac{g_{\omega(\rho)B}}{g_{\omega(\rho)N}}$ $x_{\phi H} = \frac{g_{\phi H}}{g_{\omega N}}$

mass scaling:

 $\Phi_m(f) \approx \Phi_N(f) = 1 - f$

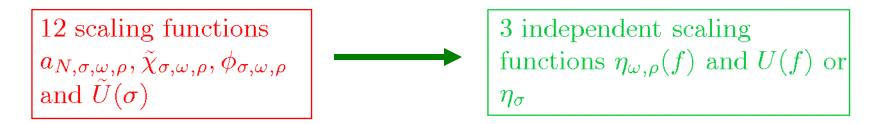
 $\Phi_H(f) = 1 - x_{\sigma H} \frac{m_N}{m_H} \xi_{\sigma H} f$

scaling functions

$$\eta_i(f) = \frac{\Phi_i^2(f)}{\chi_i^2(f)}, \quad i = \sigma, \, \omega, \, \rho$$

The standard sigma potential can be introduced as $\eta_{\sigma}(f) = 1 + rac{2\,C_{\sigma}^2}{m_{\Lambda^*}^4 f^2} U(f)$

Equivalence of RMF models

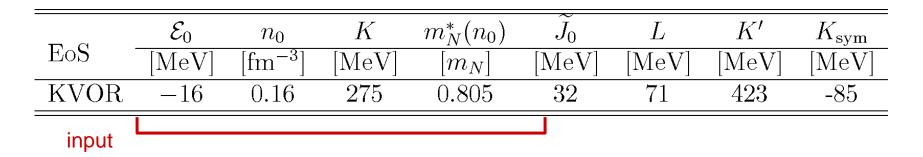


control of EoS stiffness in ISM and BEM

Choice of scaling functions:

- •monotonous increase of the scalar field as a function of density f(n)
- absence of several solutions for f(n) and jumps among them

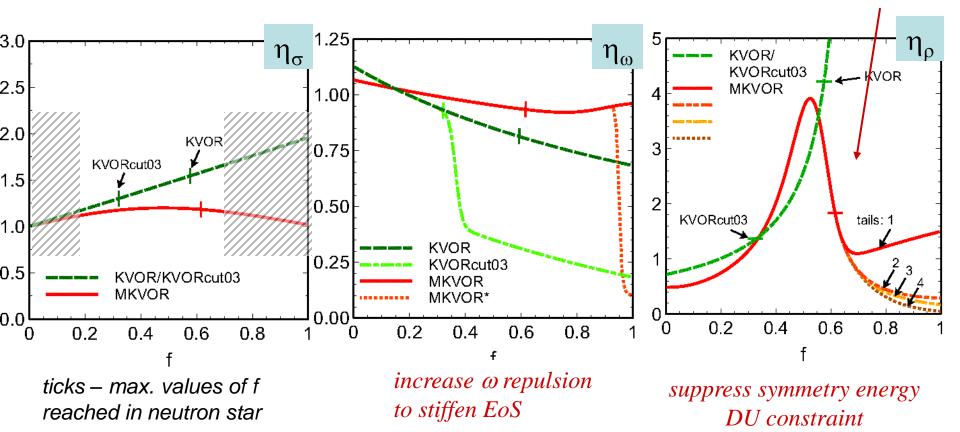
 $\begin{array}{l} \hline \mathsf{KVOR \ model} & [\mathsf{EEK}, \mathsf{Voskresensky} \ \mathsf{NPA759}, \ 373 \ (2005)] \\ \eta_{\sigma}^{\mathrm{KVOR}} = 1 + 2 \frac{C_{\sigma}^2}{f^2} \left(\frac{b}{3} f^3 + \frac{c}{4} f^4 \right) \qquad \eta_{\omega}^{\mathrm{KVOR}} = \left[\frac{1 + z \bar{f}_0}{1 + z f} \right]^{\alpha} \qquad \bar{f}_0 = f(n_0) \\ \eta_{\rho}^{\mathrm{KVOR}} = \left[1 + 4 \frac{C_{\omega}^2}{C_{\rho}^2} \left(1 - [\eta_{\omega}^{\mathrm{KVOR}}(f)]^{-1} \right) \right]^{-1} \qquad \alpha = 1 \quad z = 0.65 \end{array}$

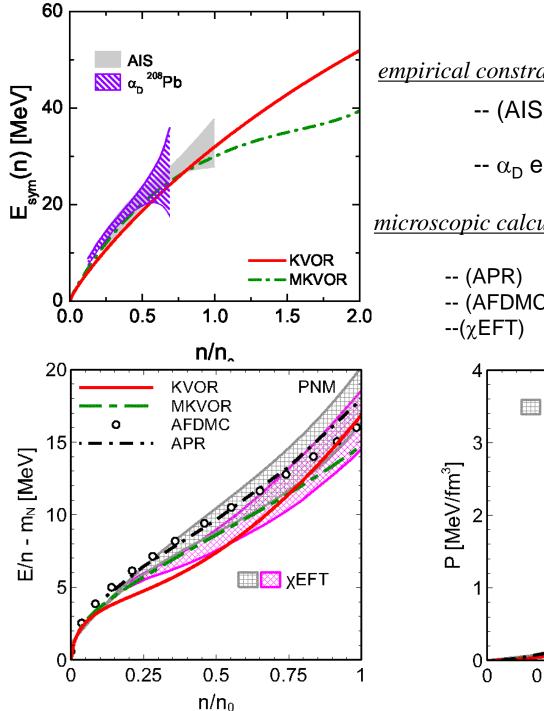


| D O | \mathcal{E}_0 | n_0 | K | $m_N^*(n_0)$ | \widetilde{J}_0 | L | K' | $K_{\rm sym}$ |
|------------|-----------------|-------------|-------|--------------|-------------------|-------|-------|---------------|
| EoS | [MeV] | $[fm^{-3}]$ | [MeV] | $[m_N]$ | [MeV] | [MeV] | [MeV] | [MeV] |
| MKVOR | -16 | 0.16 | 240 | 0.73 | 30 | 41 | 557 | -159 |

scaling functions for coupling constants vs scalar field:

saturate f growth





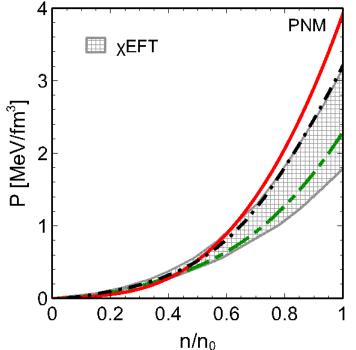
Neutron matter EoS

empirical constraints on symmetry energy

-- (AIS) analog isobar states [Danielewicz, Lee NPA 922 (2014) 1] -- α_D electric dipole polarizability ²⁰⁸Pb [Zhang, Chen 1504.01077]

microscopic calculations

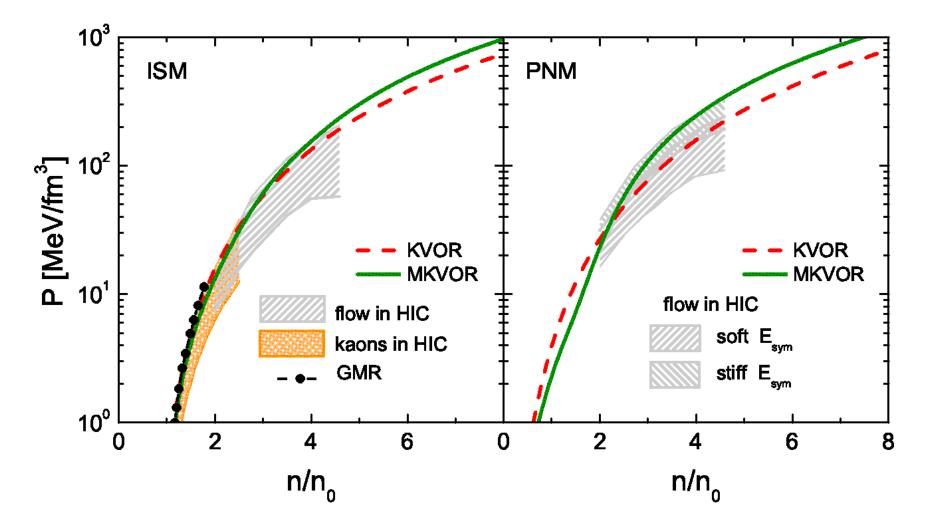
Akmal, Pandharipande, Ravenhall -- (AFDMC) Gandolfi et al.MNRAS 404 (2010) L35 Hebeler, Schwenk EPJA 50 (2014) 11



Constraints on EoS from HICs

Particle flow: Danielewicz, Lacey and Lynch, Science 298 (2002) 1592

Kaon production: Fuchs, Prog. Part. Nucl. Phys. 56 (2006) 1



Attempts to solve the hyperon puzzle

play with hyperon coupling constants

$$g_{\omega N}: g_{\omega \Lambda}: g_{\omega \Sigma}: g_{\omega \Xi} = 3:2:2:1$$

 $g_{\rho N}: g_{\rho \Lambda}: g_{\rho \Sigma}: g_{\rho \Xi} = 1:0:2:1$

$$x_{\sigma H} = \frac{x_{\omega H} n_0 C_{\omega}^2 / m_N^2 - U_H(n_0)}{m_N - m_N^*(n_0)} \qquad \begin{cases} U_{\Sigma}(n_0) = -25 \text{ MeV} \\ U_{\Sigma}(n_0) = +30 \text{ MeV} \\ U_{\Xi}(n_0) = -15 \text{ MeV} \end{cases}$$

extensions

phi meson: HH' repulsion

$$g_{\phi N}:g_{\phi \Lambda}:g_{\phi \Sigma}:g_{\phi \Xi}=0:2:2:1$$
 $g_{\phi \Lambda}=-rac{\sqrt{2}}{3}g_{\omega N}$

[J. Schaffner et al., PRC71 (1993), Ann.Phys. 235 (94), PRC53(1996)]

SU(3) coupling constants: extra parameters to tune. two effects: $|g_{\omega H}|$ increases; $g_{\phi N}$ non zero

[Weissenborn et al., PRC85 (2012);NPA881 (2012); NPA914(2013)]

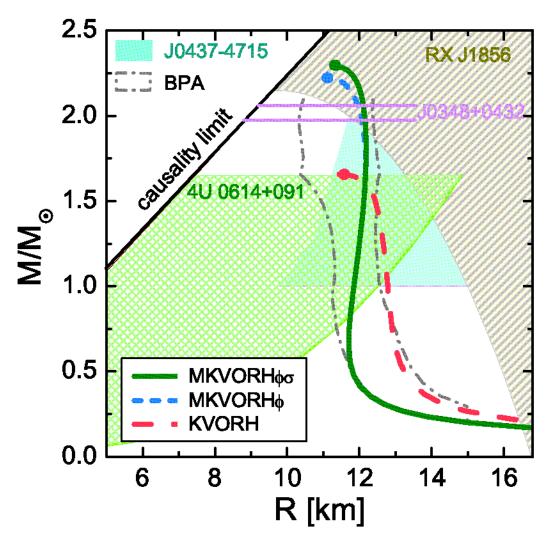
alternative

mass of ϕ meson If we take into account a reduction of the ϕ mass in medium we can increase a HH repulsion

 $x_{mH} = rac{g_{mH}}{g_{mN}}$

 $(U_{1}(n_{0})) = -28 \,\mathrm{MeV}$

Mass-radius constraints



BPA: Bayesian probability analysis [Lattimer, Steiner ...]

msp PSRJ0437-4715: 3σ confidence Bogdanov ApJ 762, 96 (2013)

Conclusion

NSs and HICs are the only sources of the information about properties of the strongly interacting matter under extreme condition.

They provide test for our theories and models in dynamical systems.

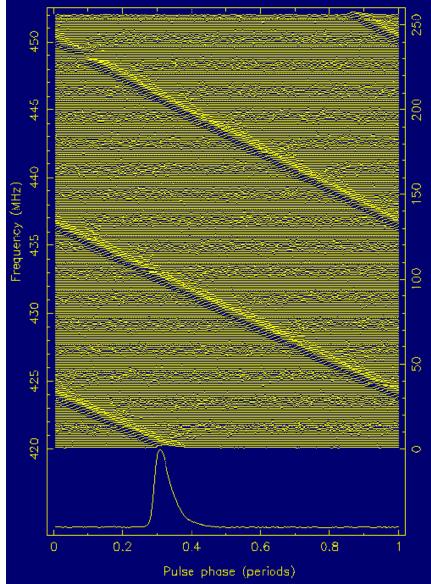
Problems in discussion:

Relativistic equation of state? Inclusion of new particles (hyperons, Deltas)? Meson in medium?

Dispersion Measure

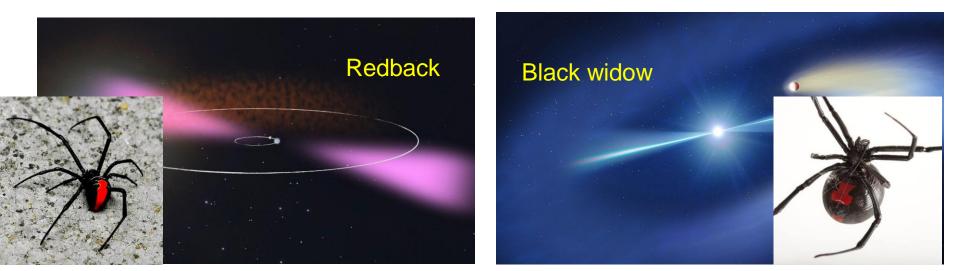
In pulsar astronomy a handy quantity is the dispersion measure (DM) of a pulsar, which manifests itself observationally as a broadening of an otherwise sharp pulse when a pulsar is observed over a finite bandwidth. Technically the **DM** is the "integrated column density of free electrons between an observer and a pulsar". It is perhaps easier to think about dispersion measure representing the number of free electrons between us and the pulsar per unit area. So if we could construct a long tube of cross-sectional area 1 square cm and extending from us to the pulsar, the DM would be proportional to the number of free electrons inside this volume.

Pulses emitted at higher frequencies arrive earlier than those emitted at lower frequencies.



Spider Systems

The most compelling evidence for this 'recycling' scenario comes from the discovery of three transitional millisecond pulsars, which have been seen to switch between rotationally powered millisecond pulsar and accretion-powered low-mass X-ray binary states.



Redback pulsars represent a recently identified and fast growing family of binary, eclipsing radio pulsars whose companion is a low-mass star $0.1 \leq M_2/M_\odot \leq 0.7$ (being M_2 the mass of the companion) on almost circular orbits with periods $0.1 \leq P_{\rm orb}/d \leq 1.0$. Black widows form another family of pulsars with orbital periods in the same range but with companion stars that are appreciably lighter, $M_2 \leq 0.05 M_\odot$. Among detected redbacks, some belong to the Galactic disk population whereas others reside in globular clusters (GCs). A recent listing of these system can be found in A. Patruno's catalogue¹.

So far, astronomers have found at least 18 black widows and nine redbacks within the Milky Way, and additional members of each class have been discovered within the dense globular star clusters that orbit our galaxy.