Crust of compact stars
Lecture 3.
Inner crust

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Crust of compact stars
Selection of topics and plan

"Everything should be made as simple as possible, but not simpler"

Attributed to Albert Einstein

According to Robinson [*Nature* **557**, 30 (2018)], it can be a compressed version of lines from a 1933 lecture by Einstein:
"*It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.*"

My preference in these lectures: *models, which can be solved analytically*

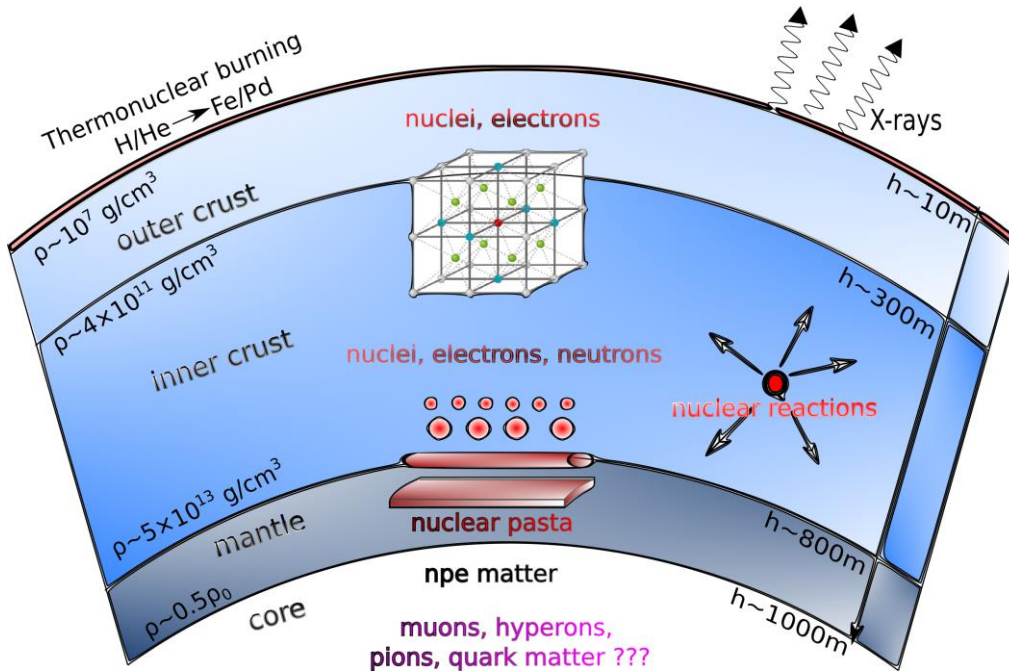Lecture 1: Introduction and basic models of outer crust at T=0

Lecture 2: Outer crust: thermodynamics and elasticity

Lecture 3: Inner crust

Lecture 4: Aren't crustal models simpler, than it is possible?
+ M(R) not dealing with crust

What we want to know about the crust?

N. N. Shchechilin ©

- Thermonuclear burning H/He→Fe/Pd
- ρ~10^7 g/cm³ outer crust
- ρ~$4×10^{11}$ g/cm³
- inner crust
- ρ~$5×10^{13}$ g/cm³ mantle
- ρ~$0.5ρ_0$ core
- nuclei, electrons
- nuclei, electrons, neutrons
- nuclear reactions
- nuclear pasta
- npe matter
- muons, hyperons, pions, quark matter ???
- X-rays
- h~10m
- h~300m
- h~800m
- h~1000m

➢ **Composition**
  ▪ Equilibrium
  ▪ Nonequilibrium
➢ **Equation of state**
  ▪ T=0
  ▪ Thermal properties
  ▪ State of matter (solid/liquid)
➢ **Dynamical properties One/two liquid hydro (magneto) dynamics**
➢ **Transport properties (kinetic coefficients)**
➢ **Elasticity, strength**
➢ **…**

Why???

These properties affect observations, and thus they are required for adequate interpretation of observations

Main mystery of NSs is the core. The crustal properties should be known accurately to avoid biases in conclusions on the core properties
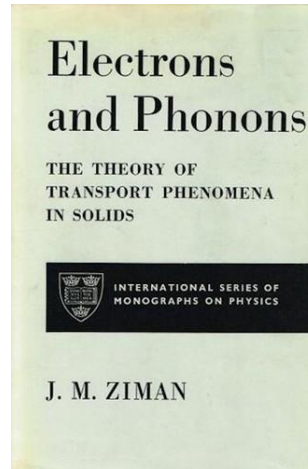
D.G. Yakovlev, HEA2017(?)
Crust as Cinderella of NS

What we need to know to study the crust?

- Nuclear physics

- Statistical physics (many-particle system)

- Solid state physics

  - Transport theory

  - Theory of elasticity

- Superfluidity

- Magnetohydrodynamics

- …

Seems to be extremely complicated:
we need simplified models

Electrons and Phonons

THE THEORY OF TRANSPORT PHENOMENA IN SOLIDS

INTERNATIONAL SERIES OF MONOGRAPHS ON PHYSICS
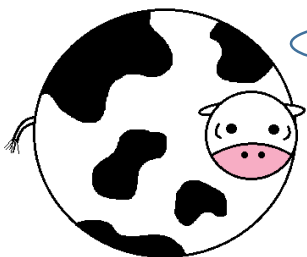
J. M. ZIMAN

- **Composition**
  - Equilibrium
  - Nonequilibrium
- **Equation of state**
  - $T=0$
  - Thermal properties
  - State of matter (solid/liquid)
- **Dynamical properties**
  One/two liquid hydro (magneto) dynamics
- **Transport properties (kinetic coefficients)**
- **Elasticity, strength**
- …

Some problems can be solved analytically!

Thermodynamics of the outer crust
(school topic: theoretical description of interacting fermion systems)

$$F = F_{\mathrm{e}} + F_{\mathrm{i}} + F_{\mathrm{ie}} + \sum_i M_i N_i$$

Nuclei mass
(nuclear physics)

Model system
Ideal degenerate gas of electrons

Electrons: relativistic noninterracting fermions

Solved analytically!

Model system
Nuclei on the **uniform** neutralizing background of electrons

OCP: Ab initio calculations +fits
Mixtures: calculations+fints+intuition

Corrections

➤ No uniform electron background (screening)

Liquid:
Potekhin & Chabrier (2000)
Crystall: Baiko (2002)

The corrections, as predicted today, affects melting temperature. BUT it can be byproduct of different approaches, applied to calculate these corrections….

➤ ….
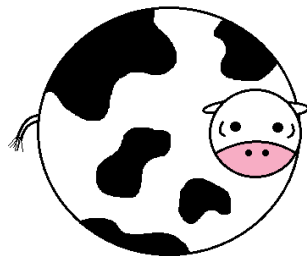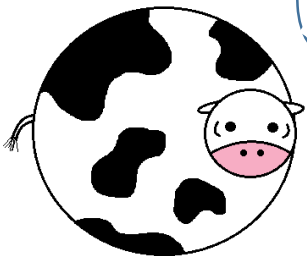
Inner crust

➤ Nuclear physics

Atomic mass tables are unavailable for nuclei beyond the drip line: we need to calculate nuclei properties specially for inner crust conditions

- Compressible liquid drop model CLDM
- Exteded Thomas-Fermi (ETF)
- ETF+Strutinsky integral (ETFSI)
- HFB
- …

➤ Thermodynamics

To calculate required thermodynamic functions

- Finite temperature effects:
  - Relatively low temperature: application of outer crust thermodynamics of solid phase (lattice of clusters with Coulomb interaction among them)
  - High temperature: Finite temperature CLDM, ETF,…

➤ **Composition**

> Common introduction:
> Inner crust is composed of degenerate electrons, unbound neutrons + nuclei (clusters)

Not enough for quantitative description!!!

That are the nuclei types???

- Equilibrium

Optimal energy density

- Nonequilibrium

Formation history + nuclei reaction

Inner crust EOS: approaches
Wigner-Seitz approximation (Ion sphere model), T=0

General aim: _Determine structure of the cluster_
Consider a spherical cell at given n_b and
determine states of the protons and neutrons
inside it (+ determine optimal size of the cell)

Parameter space for nuclear approaches:

1. Liquid drop model:

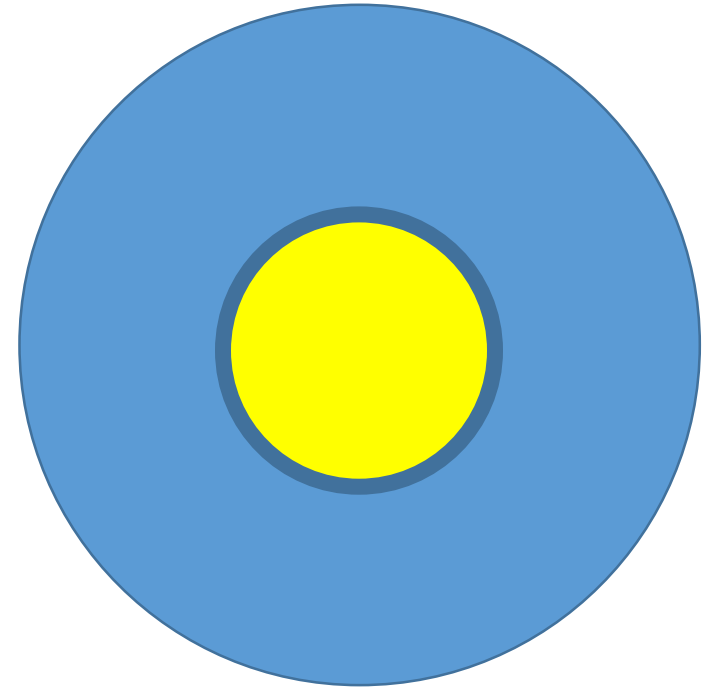   6 (+/- a few) parameters

2. Extended Thomas-Fermi:

   Two functions

3. ETF+Strutinsky integral:

   Two functions

4. HFB:

   Wave functions

+ Nuclear physics:
Nucleon interaction model
 (not just atomic mass table)

Very limited calculations
beyond WZ model

Compressible liquid drop model for inner crust

"Everything should be made as simple as possible, but no simpler"

Attributed to Albert Einstein

(Overconfident) physicist of nowadays : Liquid drop model is obsolete! You are not in 1960s!!! It is simpler, than possible!!!

What is CLDM in 2020-s and why it is still applicable? Just do not request too much from it

Compressible liquid drop model for inner crust

➢ CLDM is rather a class of modes, than one model

➢ CLDM does not assume that step-like profile for proton and neutron density is real. Rather, general feature of CLDMs is that they start from explicit analytical expression (for energy) and adjust available parameters to make this expression as simple as possible, but (reasonable) accurate

$$E = E_{\mathrm{in}} + E_{\mathrm{out}} + E_{\mathrm{surf}} + E_C + E_e$$

➢ Explicit expressions for all thermodynamic quantities can be obtained analytically. It guaranties absolute (up to numerical accuracy) thermodynamic consistency of the model (if properly applied)
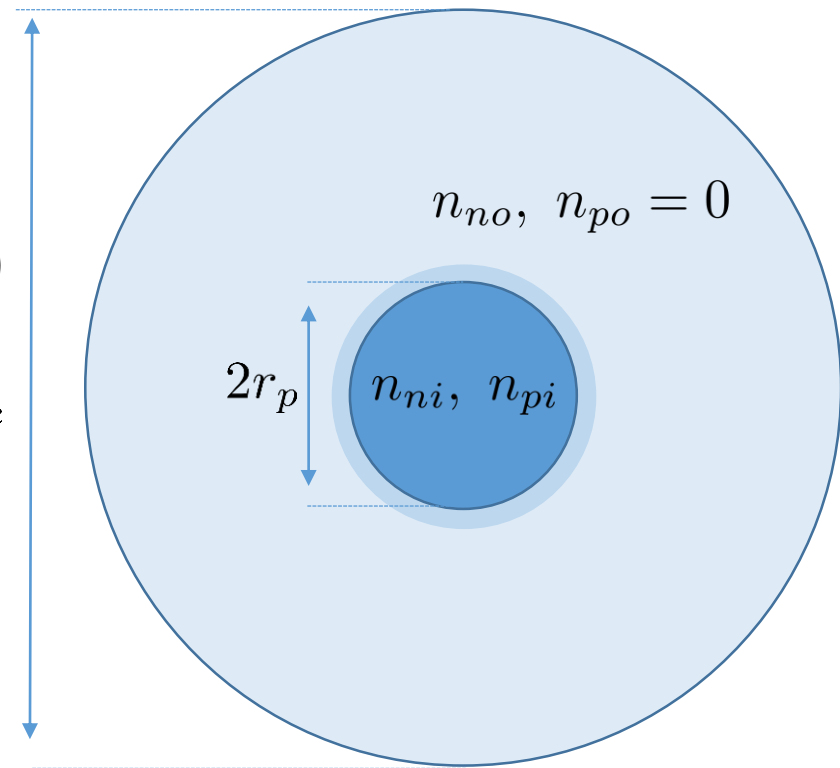
$2r_c$

$2r_p$

$n_{no}, \; n_{po} = 0$

$n_{ni}, \; n_{pi}$

"Everything should be made as simple as possible, but no simpler"

Attributed to Albert Einstein

Compressible liquid drop model for inner crust

➢ CLDM is rather a class of modes, than one model

➢ General feature of these models is that they start from explicit analytical expression (for energy)

$$E = E_{\text{in}} + E_{\text{out}} + E_{\text{surf}} + E_C + E_e$$

Nuclear energy inside cluster:
$$E_{\text{in}} = \epsilon_{\text{nm}}(n_{ni}, n_{pi})V_i$$

Nuclear energy outside cluster:
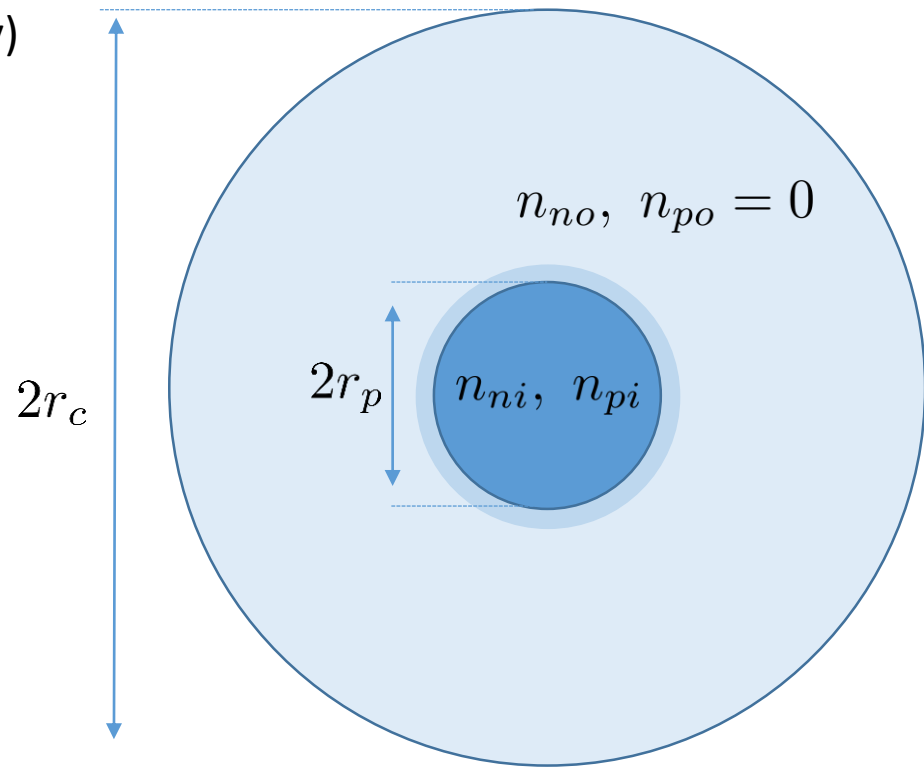$$E_{\text{out}} = \epsilon_{\text{nm}}(n_{no}, n_{po})V_o$$

Surface energy
$$E_{\text{surf}} = \sigma(\ldots)S + \ldots$$

Coulomb energy: two uniformly charged balls
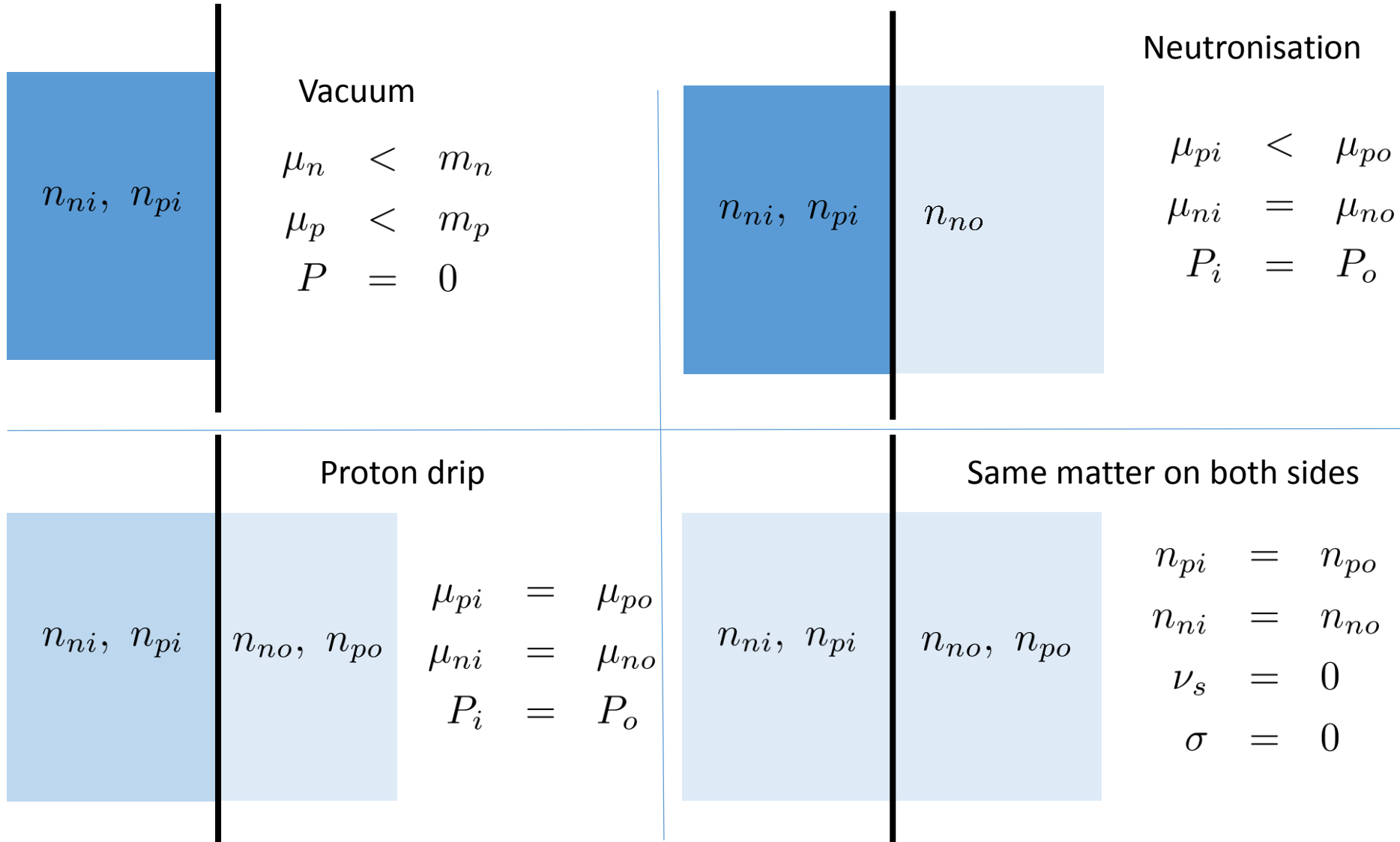$$E_C = \frac{3}{5}\frac{Z^2 e^2}{R_N}\left(1 - \frac{3}{2}w^{1/3} + \frac{1}{2}w\right) + \ldots$$

Energy of electrons (uniform degenerate ideal gas)
$$E_e = \epsilon_e(n_e)(V_o + V_i)$$

$n_{no},\ n_{po} = 0$

$2r_c$

$2r_p$

$n_{ni},\ n_{pi}$

Surface energy? What is it? Something artificial???

Equilibrium of the plain interface

Vacuum

$n_{ni}, \ n_{pi}$

$$\mu_n < m_n$$
$$\mu_p < m_p$$
$$P = 0$$

Neutronisation

$n_{ni}, \ n_{pi}$ | n_{no}

$$\mu_{pi} < \mu_{po}$$
$$\mu_{ni} = \mu_{no}$$
$$P_i = P_o$$

Proton drip

$n_{ni}, \ n_{pi}$ | $n_{no}, \ n_{po}$

$$\mu_{pi} = \mu_{po}$$
$$\mu_{ni} = \mu_{no}$$
$$P_i = P_o$$

Same matter on both sides

$n_{ni}, \ n_{pi}$ | $n_{no}, \ n_{po}$

$$n_{pi} = n_{po}$$
$$n_{ni} = n_{no}$$
$$\nu_s = 0$$
$$\sigma = 0$$

Keller et al., arXiv:2401.13461: unavoidable

Two phase equilibrium is governed by one parameter (for plain interface)

Surface energy is natural. It can be calculated

Real two-phase system

Reference two-phase system

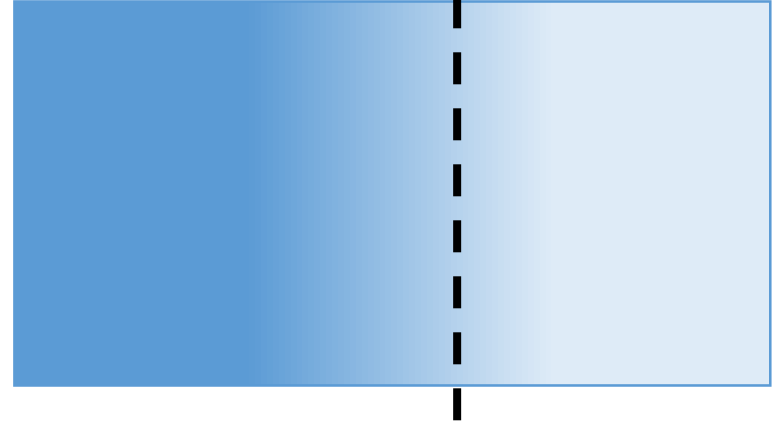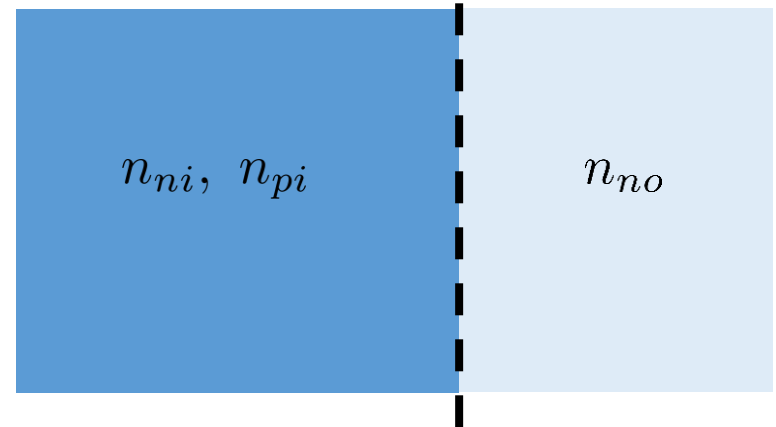$E_{\text{two phase}}, N_{\text{two phase}}, Z_{\text{two phase}}$

Surface energy is natural. It can be calculated

Centelles et al. Nucl. Phys. A, 635 (1998), 193

Real two-phase system

Reference two-phase system

$$n_{ni}, \; n_{pi}$$

$$n_{no}$$

$$E_{\text{two phase}}, N_{\text{two phase}}, Z_{\text{two phase}}$$

$$
\begin{aligned}
E_{\text{ref}} &= \epsilon_i V_i + \epsilon_o V_o, \\
N_{\text{ref}} &= n_{ni} V_i + n_{no} V_o, \\
Z_{\text{ref}} &= n_{pi} V_i
\end{aligned}
$$

$$
\begin{aligned}
E_{\text{surf}} &= E_{\text{two phase}} - E_{\text{ref}} \\
N_{\text{surf}} &= N_{\text{two phase}} - N_{\text{ref}} \\
Z_{\text{surf}} &= Z_{\text{two phase}} - Z_{\text{ref}}
\end{aligned}
$$

Can be nulled by choose of the reference system

- ➤ Surface energy is a correction, required to reproduce two phase system energy
- ➤ (Neutron) adsorption is required for thermodynamically consistent description of two phase boundary
- ➤ Surface energy describe two-phase thermodynamics precisely

Compressible liquid drop model for inner crust

➢ CLDM is rather a class of modes, than one model

➢ General feature of these models is that they start from explicit analytical expression (for energy)

$$E = E_{\text{in}} + E_{\text{out}} + E_{\text{surf}} + E_C + E_e$$

➢ The models differ my parametrization of the surface energy: for plane interface it is one parameter function. One can chose arbitrary property from (denser) phase: proton fraction (often), neutron chemical potential, …

➢ Many models accounts for curvature corrections for surface energy (reduce surface tension)

➢ Some models accounts for proton diffusivity (not step-like profile of proton density)

➢ Numerically, surface tension can be extrapolated from terrestrial nuclei, fitted to plane interface properties, ETF calculations for inner crust, …

$n_{no}, \ n_{po} = 0$

$2r_c$

$2r_p$ $\quad n_{ni}, \ n_{pi}$
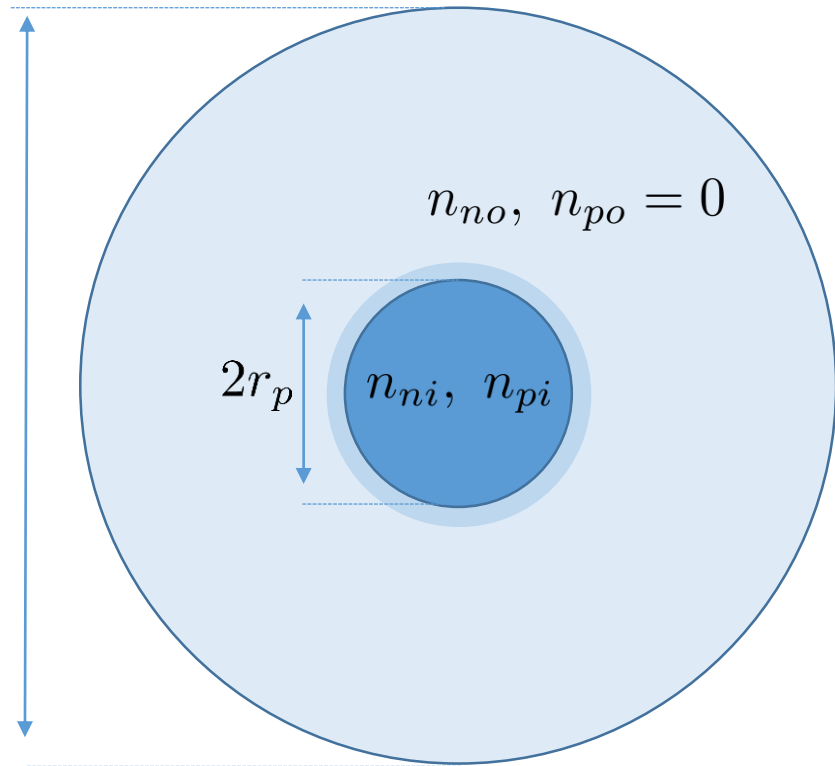
$$E_{\text{surf}} = 4\pi\sigma(\ldots)S + \ldots$$

Compressible liquid drop model: example

$$\epsilon = w\,\epsilon_{\mathrm{nm}}(n_{ni},\,n_{pi}) + (1-w)\,\epsilon_{\mathrm{nm}}(n_{no},\,0) + \frac{E_s(\nu_s, r_p)}{V_c} + \frac{E_C(n_{pi}, r_p, w)}{V_c} + \epsilon_e(n_e).$$

Explicit (algebraic) expression for the energy density

6 parameters:

$$n_{ni},\ n_{pi},\ n_{no},\ \nu_s,\ V_c,\ r_p$$
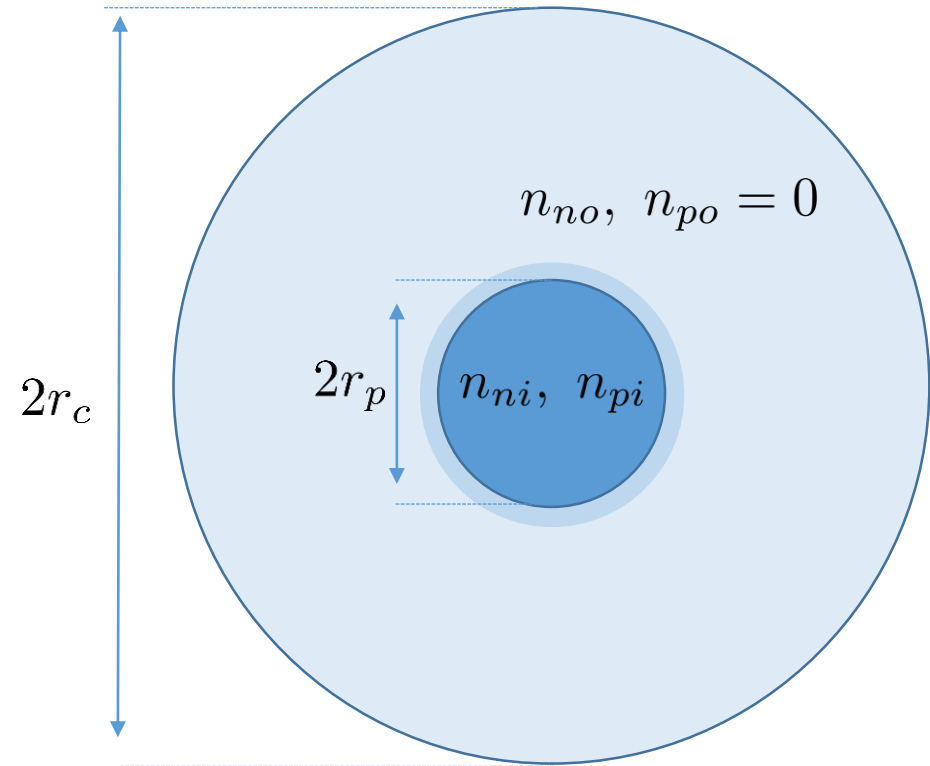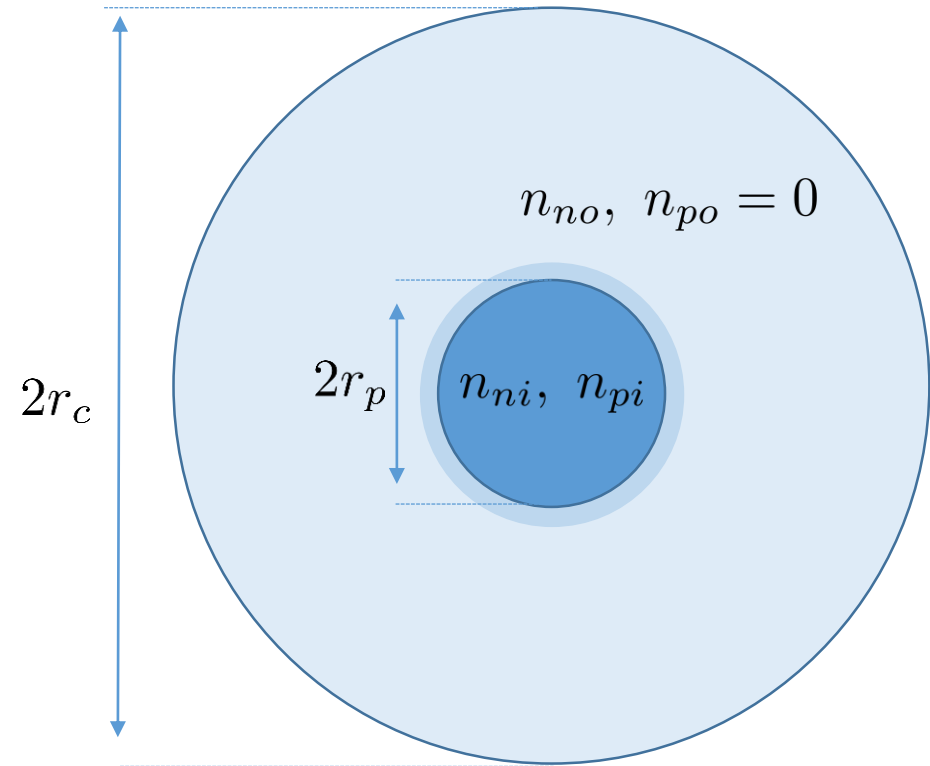
Minimization at fixed n_b

System of 5 algebraic (nonlinear) equations
With clear physical meaning:
- Chemical equilibrium inside the cell (2 eqs.)
- Beta-equilibrium (1 eq)
- Mechanical equilibrium (1 eq)
- Optimal size of the cell (1 eq)

$2r_c$

$n_{no},\ n_{po} = 0$

$2r_p$

$n_{ni},\ n_{pi}$

Explicit formulae for thermodynamic quantities – absolute thermodynamic consistency!

$$\mu_n,\ P,\ \ldots$$

(Gusakov, AIC 2020)

Compressible liquid drop model: example

$$\epsilon = w\,\epsilon^{\mathrm{bulk}}(n_{ni},\,n_{pi}) + (1-w)\,\epsilon^{\mathrm{bulk}}(n_{no},\,0) + \frac{E_s(\nu_s, r_p)}{V_c} + \frac{E_C(n_{pi}, r_p, w)}{V_c} + \epsilon_e(n_e).$$

Explicit (algebraic) expression for the energy density

6 parameters:

$$n_{ni},\ n_{pi},\ n_{no},\ \nu_s,\ V_c,\ r_p$$

Minimization at fixed n_b

System of 5 algebraic (nonlinear) equations
With clear physical meaning:
- Chemical equilibrium inside the cell (2 eqs.)
- Beta-equilibrium (1 eq)
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- Optimal size of the cell (1 eq)

In this formulation neutron chemical potential is equal to the bulk value inside and outside cluster and at the surface

$n_{no},\ n_{po} = 0$

$2r_c$

$2r_p$

$n_{ni},\ n_{pi}$

(Gusakov, AIC 2020)

Extended Thomas-Fermi approach

Density functional (4th order Wigner-Kirkwood expansion)

$$E_{\mathrm{ETF}} = E_{\mathrm{nuc}} + E_{\mathrm{C}} + E_{\mathrm{e}}$$

nuclear   Coulomb   electron

$$E_{\mathrm{nuc}} = \frac{1}{A} \int \mathcal{E}(\rho_q(\mathbf{r}), \nabla\rho_q(\mathbf{r}), \triangle\rho_q(\mathbf{r}))\mathrm{d}\mathbf{r}$$
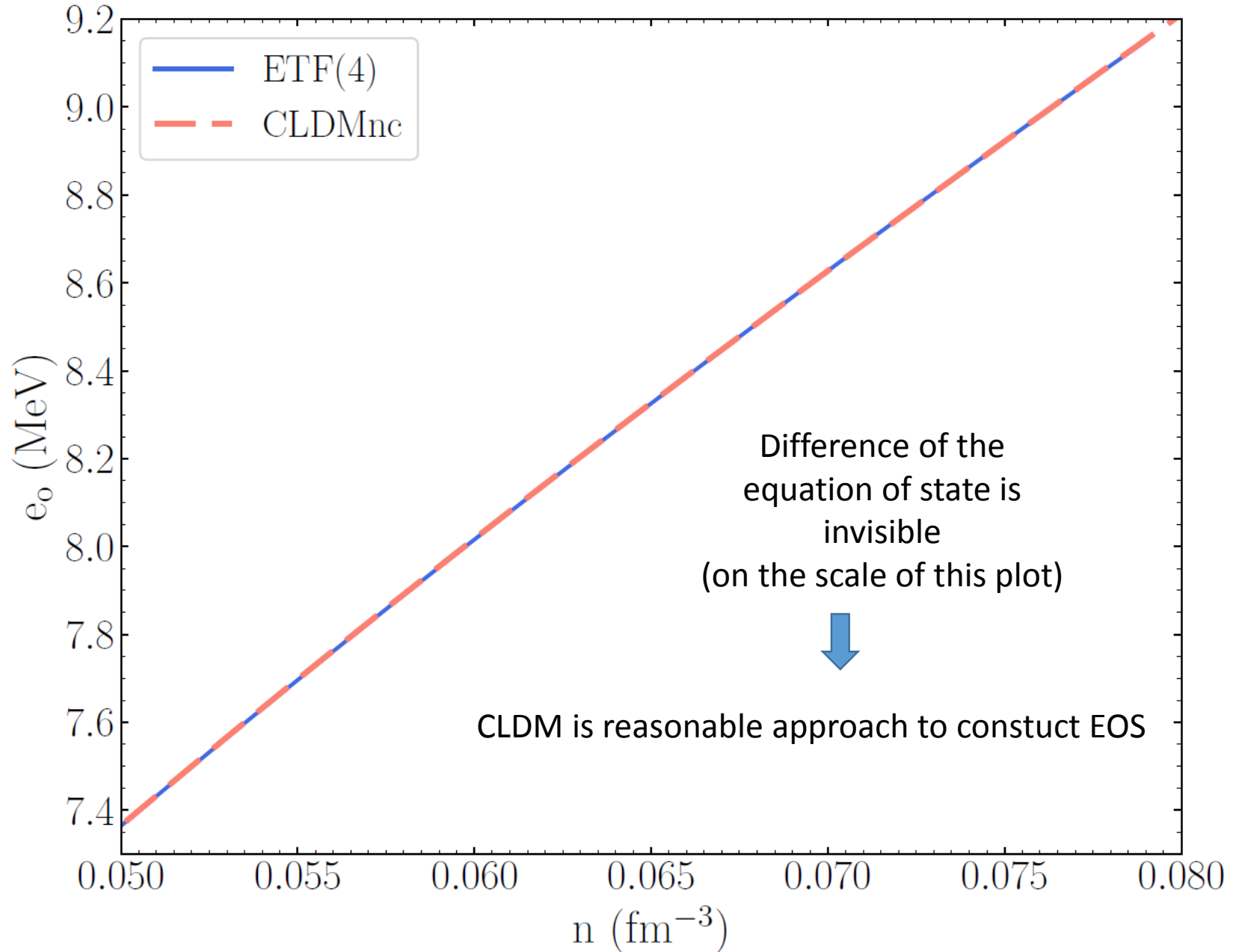
Nucleon profile optimization: (accurate) Euler–Lagrange equations

$$\frac{\partial \delta e_{\mathrm{ETF}}}{\partial \delta \rho_p} = \mu_p; \qquad \frac{\partial \delta e_{\mathrm{ETF}}}{\partial \delta \rho_n} = \mu_n; \qquad \mu_n - \mu_p = \mu_e$$

Real life nucleon profile optimization: minimization over parametrized profiles

$$\rho_{\mathrm{q}}(r) = \rho_{\mathrm{B}q} + \frac{\rho_{\Lambda q}}{1 + \exp\left[\left(\dfrac{C_q - R}{r - R}\right)^2 - 1\right] \exp\left(\dfrac{r - C_q}{a_q}\right)}$$

Extended Thomas-Fermi approach

Density functional (4th order Wigner-Kirkwood expansion)

$$E_{\mathrm{ETF}} = E_{\mathrm{nuc}} + E_{\mathrm{C}} + E_{\mathrm{e}}$$

nuclear   Coulomb   electron

$$E_{\mathrm{nuc}} = \frac{1}{A} \int \mathcal{E}(\rho_q(\mathbf{r}), \nabla\rho_q(\mathbf{r}), \triangle\rho_q(\mathbf{r}))\mathrm{d}\mathbf{r}$$

Nucleon profile optimization: (accurate) Euler–Lagrange equations

$$\frac{\partial\delta e_{\mathrm{ETF}}}{\partial\delta\rho_p} = \mu_p; \qquad \frac{\partial\delta e_{\mathrm{ETF}}}{\partial\delta\rho_n} = \mu_n; \qquad \mu_n - \mu_p = \mu_e$$

Real life nucleon profile optimization: minimization over parametrized profiles

- ➢ In some sense similar to CLDM: optimization over a few parameters
- ➢ Much more computation extensive even in this case (integrals should be taken numerically)
- ➢ Choose of the functional form of the profiles:
    Conclusion 'results are not sensitive to the functional form of profiles' depends on the required accuracy... Even predictions for structure of the mantle can be affected!

Inner crust: Equation of state

Difference of the
equation of state is
invisible
(on the scale of this plot)

CLDM is reasonable approach to constuct EOS

Thermodynamics of inner crust

Thermal corrections for electrons

$$F = F(T = 0) + F_{\text{clust}} + F_{\text{e}} + F_{\text{n}}$$

Previous slides
(CLDM, ETF, ETFSI)

Thermal excitations for neutrons

Thermodynamics for clusters:
System of charged spheres on neutralizing background
As for outer crust!!

$$F_{\text{i}}(T, n_{\text{i}}, Z, A) \neq F_{\text{id}}(T, n_{\text{i}})$$

Calculated in three assumptions

All nucleons
inside cluster

All nucleons above
background density

Band structure
theory (Chamel
2008+)

$$C \propto (T/T_{\text{p}})^3 \propto A^{3/2}$$
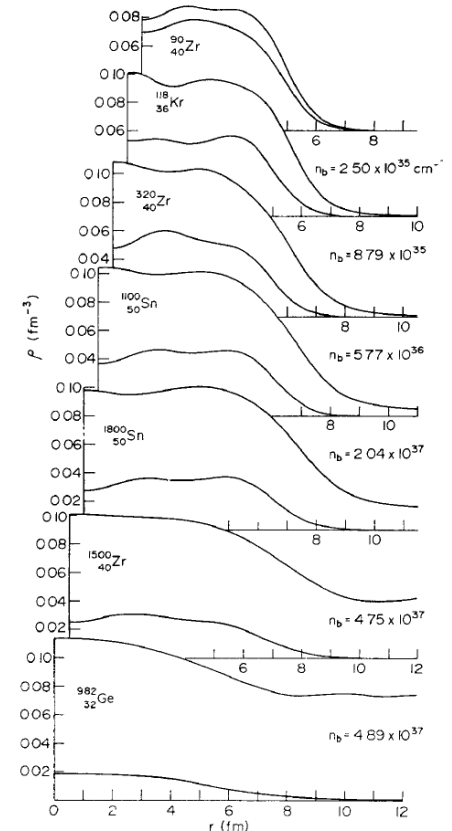
The mantle: funny phases between crust and core

Ravenhall et al, PRL 50 (1983), 2066

"It is found that just below nuclear saturation density more stable forms of dense matter exist than the near-spherical nuclei or bubbles customarily assumed. Because of the large effect of the Coulomb lattice energy, cylindrical and planar geometries can occur, both as nuclei and as bubbles."
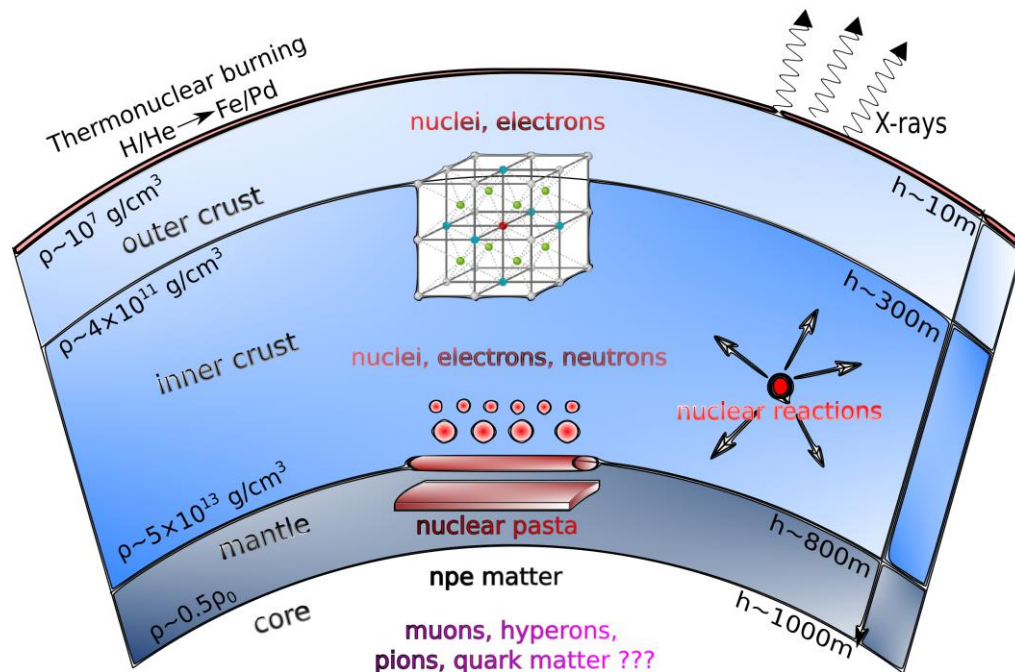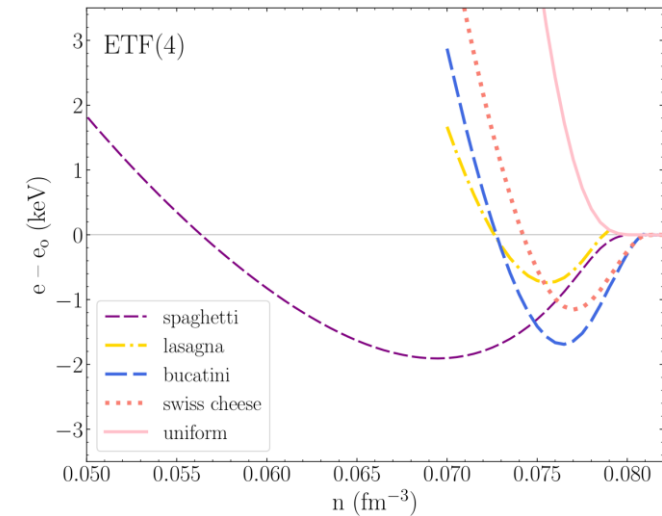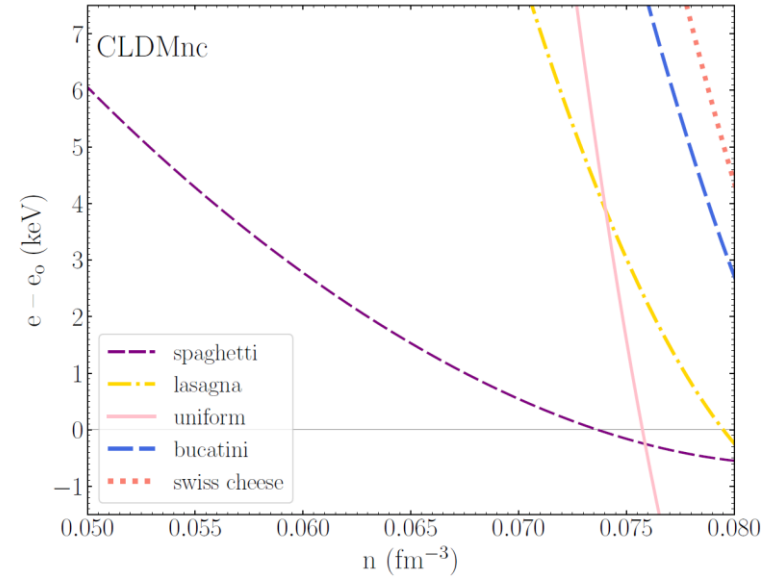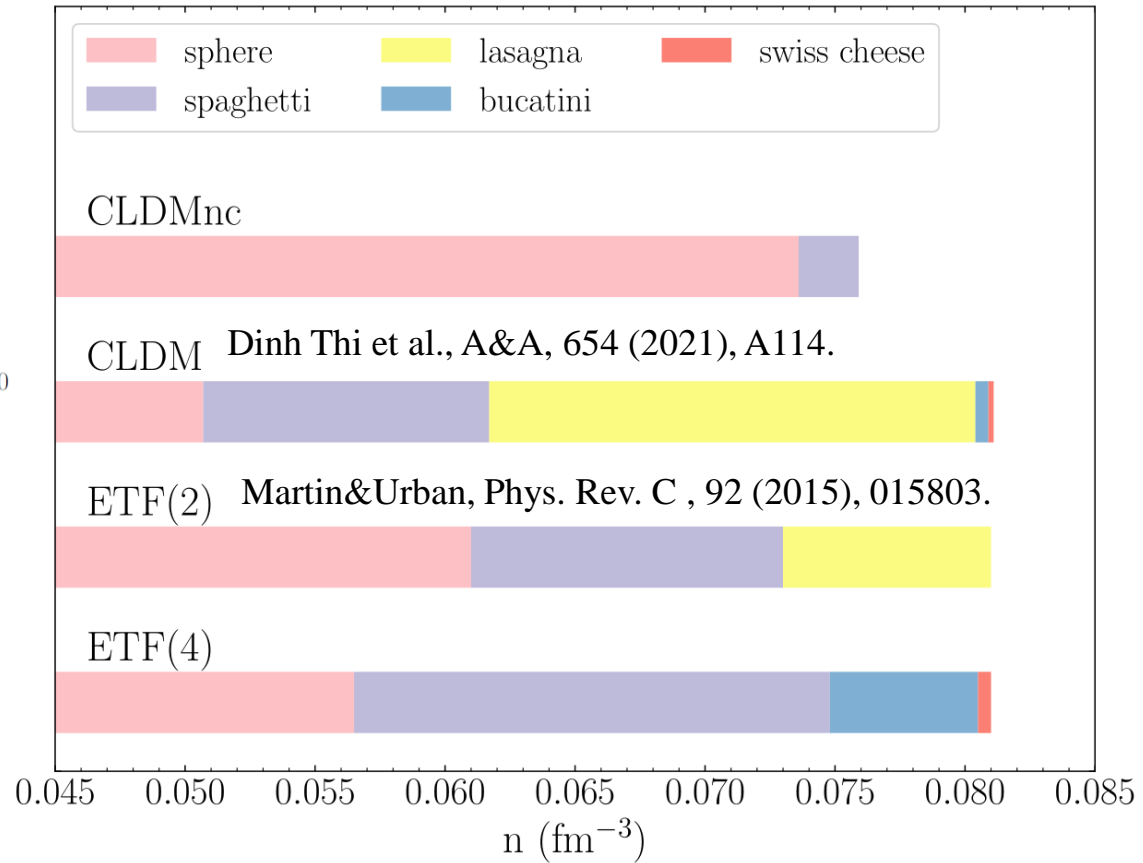

N. N. Shchechilin ©

How to study?

Calculate energy density assuming different cluster shape and *compare*!

Result depends on nucleon interaction model and, even worse, depends on the approach for a given interaction model!

Example: mantle for SLy4

Douchin&Haensel: no mantle for Sly4

Dinh Thi et al., A&A, 654 (2021), A114.

Martin&Urban, Phys. Rev. C , 92 (2015), 015803.

Shchechilin et al. (2022)

Pasta shapes are determined by few keV/nucleon!

Extended Thomas-Fermi approach

Conclusion 'results are not sensitive to the functional form of profiles' depends on the required accuracy…

Shchechilin et al. (under review):

Three parametrizations:

$$\rho_{q}^{i}(r) = \rho_{Bq} + \rho_{\Lambda q} f^{i}(r)$$

Strong damping

$$f_{q}^{\text{StrD}}(r) = \frac{1}{1 + \exp\left[\left(\dfrac{C_q - R}{r - R}\right)^2 - 1\right]\exp\left(\dfrac{r - C_q}{a_q}\right)}$$

Soft damping

$$f_{q}^{\text{SoftD}}(r) = \frac{1}{1 + \left(\dfrac{C_q - R}{r - R}\right)^2 \left(\dfrac{r}{C_q}\right)^2 \exp\left(\dfrac{r - C_q}{a_q}\right)}$$
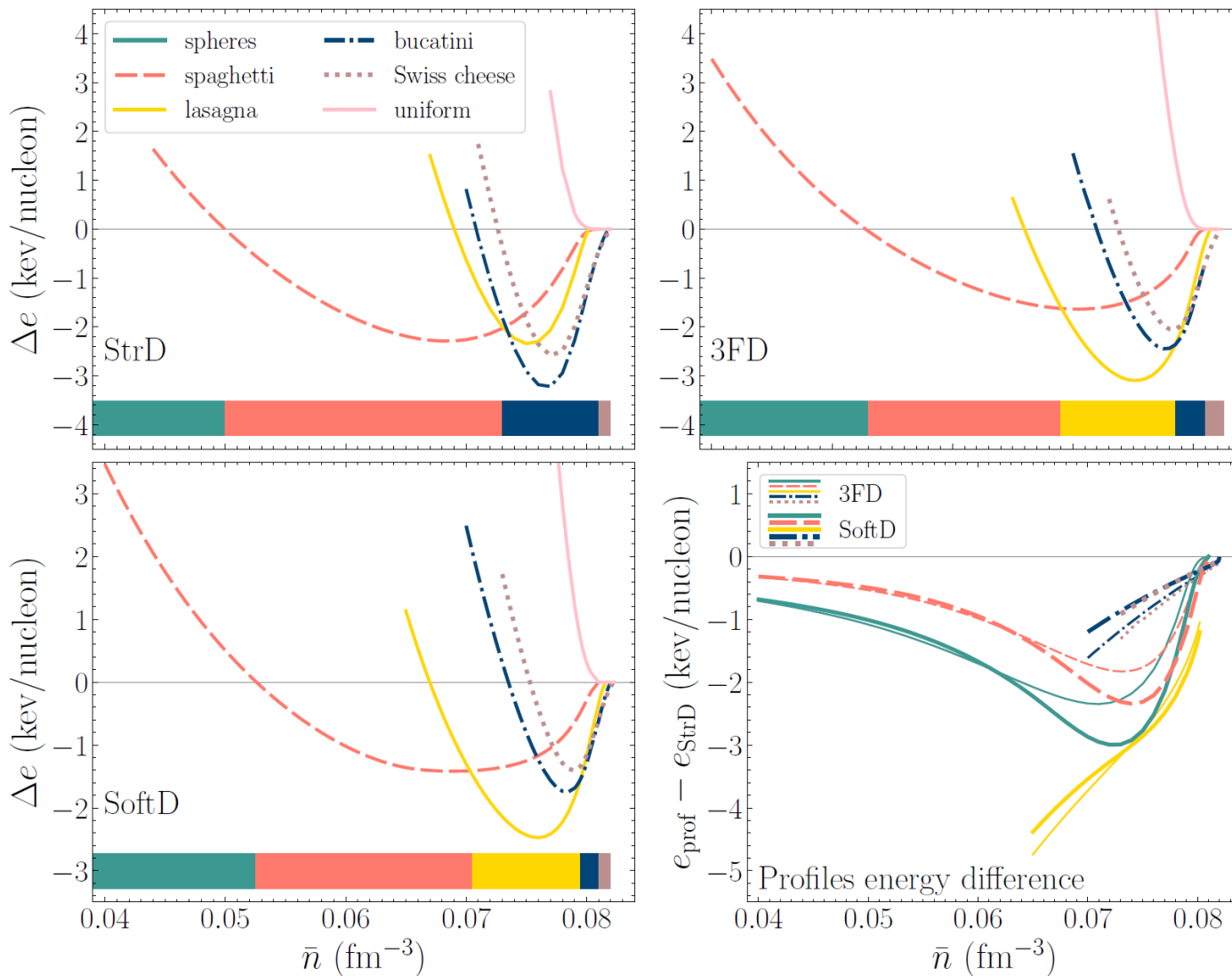
3FD damping

$$f_{q}^{\text{3FD}}(r) = f_{q}^{FD}(-r) + f_{q}^{FD}(r) + f_{q}^{FD}(2R - r) + f_{q}^{FD}(-R) + 2\, f_{q}^{FD}(R)$$

$$f_{q}^{\text{FD}}(r) = \frac{1}{1 + \exp\left(\dfrac{r - C_q}{a_q}\right)}$$

The differences are large enough to alter the pasta sequence

Extended Thomas-Fermi approach

Conclusion 'results are not sensitive to the functional form of profiles' depends on the required accuracy.

Shchechilin et al. (under review):

BSK24

The differences are large enough to alter the pasta sequence

Extended Thomas-Fermi approach

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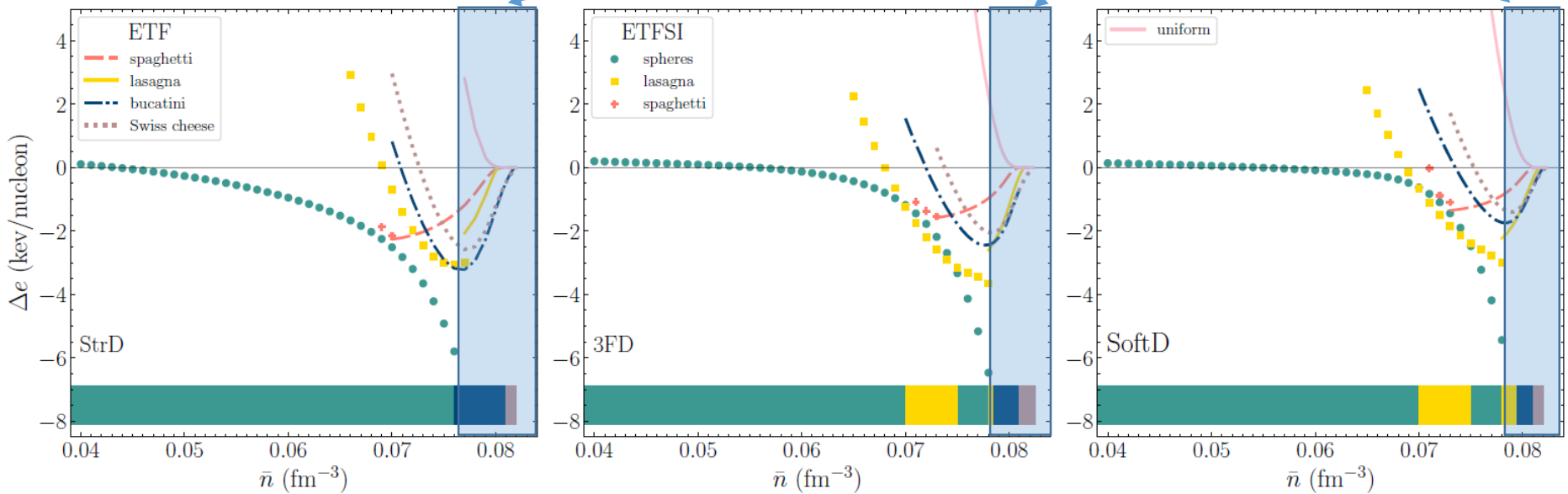
Shchechilin et al. (under review):
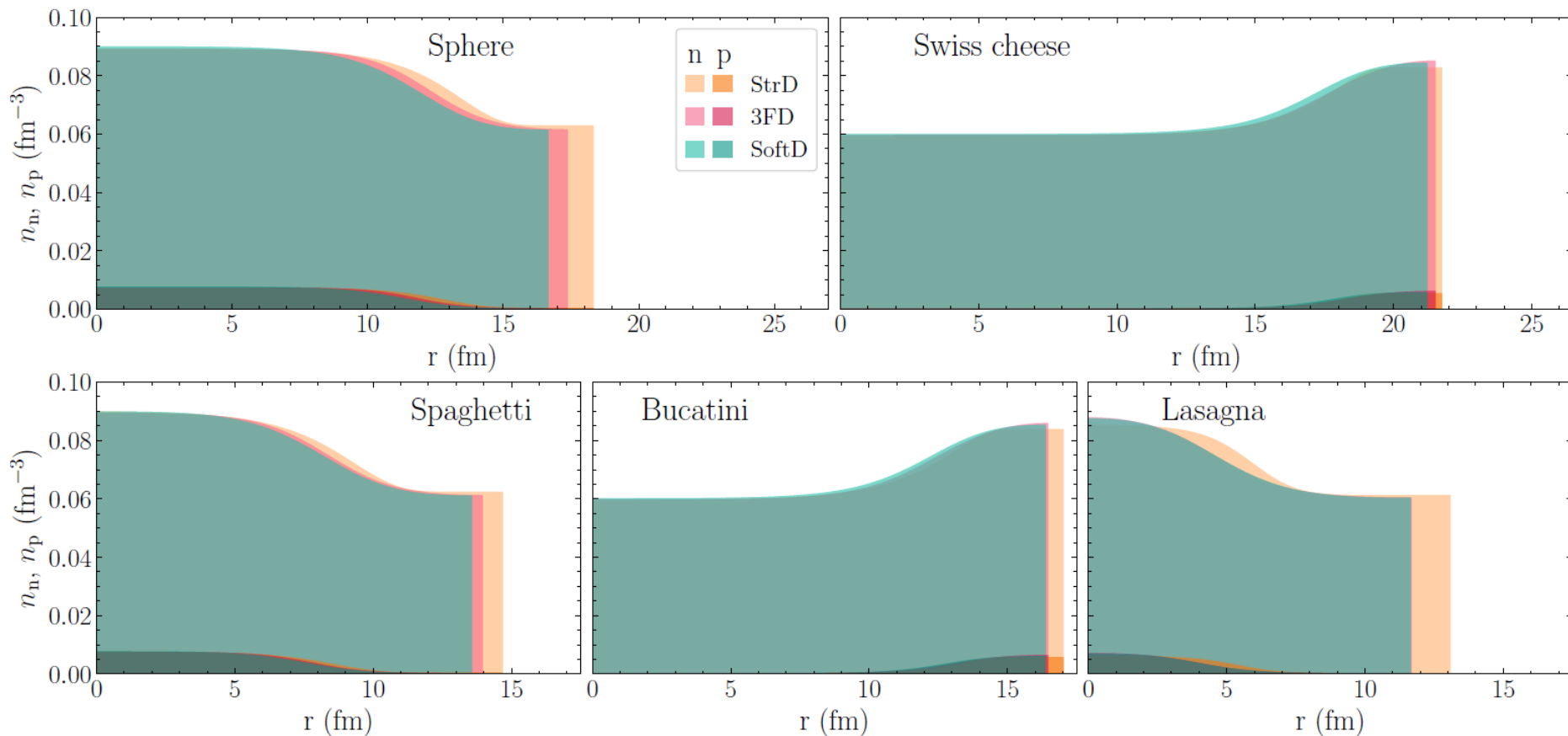
New profiles are in rather good agreement

Note: profile variations (with respect to real optimal profile) affect the energy only in the second order

Extended Thomas-Fermi + Strutinsky integral

Shchechilin et al. (under review):

Proton drip => no shells

- ➢ Account for Strutinsky Integral reduces difference between profiles
- ➢ The spaghetti are disfavored, if Strutinsky Integral is included (Peason&Chamel 2022)
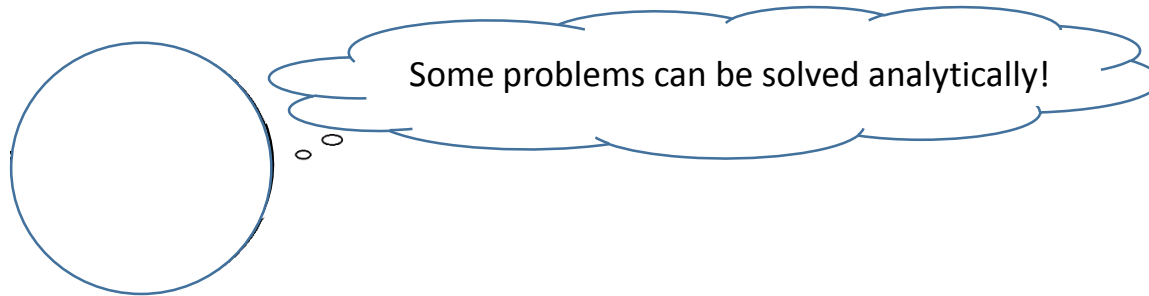- ➢ Difference can be less, than 1 keV/nucleon!!! (theory for terrestrial nuclei RMS ~500 keV)

It's not just for fun: shape of the clusters affects transport properties, elasticity, pining (for glitches), ….
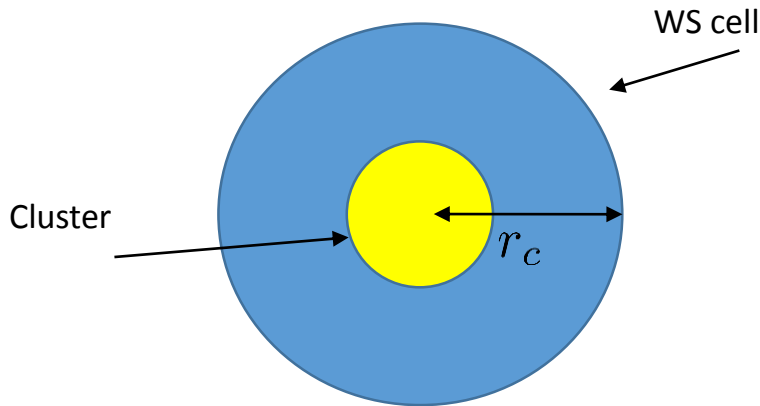Hopefully, EOS is almost unaffected (few keV correction for 10 MeV)

Inner crust: elasticity (CLDM + WZ model)

Why in these lectures?

Some problems can be solved analytically!

Inner crust: elasticity (CLDM + WZ model)

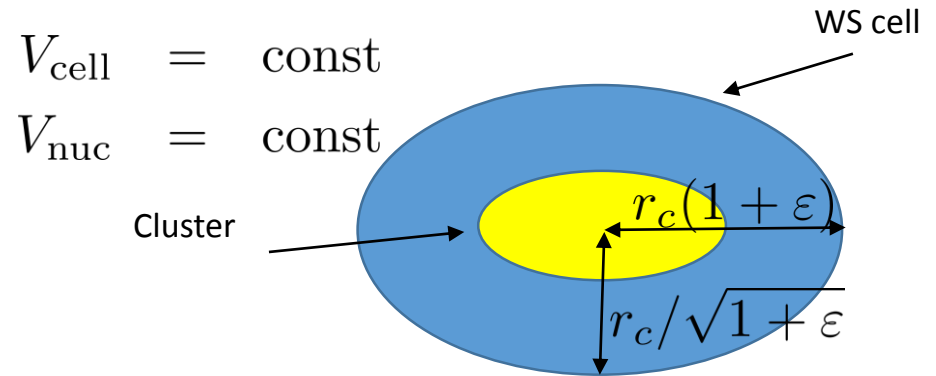## Undeformed crust:

WS cell

Cluster

r_c

Spherical cell with
spherical nucleus inside

Minimization of internal parameters
of CLDM model at fixed n_b (and n_N)

$$n_{pi}, \ n_{ni}, \ n_{no}, \ r_p, \ \nu_s, \ r_c$$

Deformed crust:

$$V_{\text{cell}} = \text{const}$$
$$V_{\text{nuc}} = \text{const}$$

WS cell

Cluster

$r_c(1+\varepsilon)$

$r_c/\sqrt{1+\varepsilon}$

Eccentricity:

The cell (ε):
Driving parameter.
Given by deformation of the crust
(mimics deformation of primitive cell)

The nucleus (ε_p):
Driven parameter.
Optimal (minimize the cell energy)

$$\epsilon = u\,\epsilon^{\text{bulk}}(n_{ni}, n_{pi}) + (1-u)\,\epsilon^{\text{bulk}}(n_{no}, 0) + \frac{E_s}{V_c} + \frac{E_C}{V_c} + \epsilon_e(n_e).$$

WZ: analytical corrections for finite nuclei

$$E = E_0 + \frac{1}{2}E_{\varepsilon\varepsilon}\varepsilon^2 + E_{\varepsilon p}\varepsilon\varepsilon_p + \frac{1}{2}E_{pp}\varepsilon_p^2$$

$$\frac{\partial E}{\partial \varepsilon_p} = 0 \qquad\Longrightarrow\qquad \varepsilon_p = \frac{u}{2 + 3\,u - 4\,u^{1/3}}\varepsilon$$

$$\Delta E = \frac{3}{2}\mu V_c \varepsilon^2$$
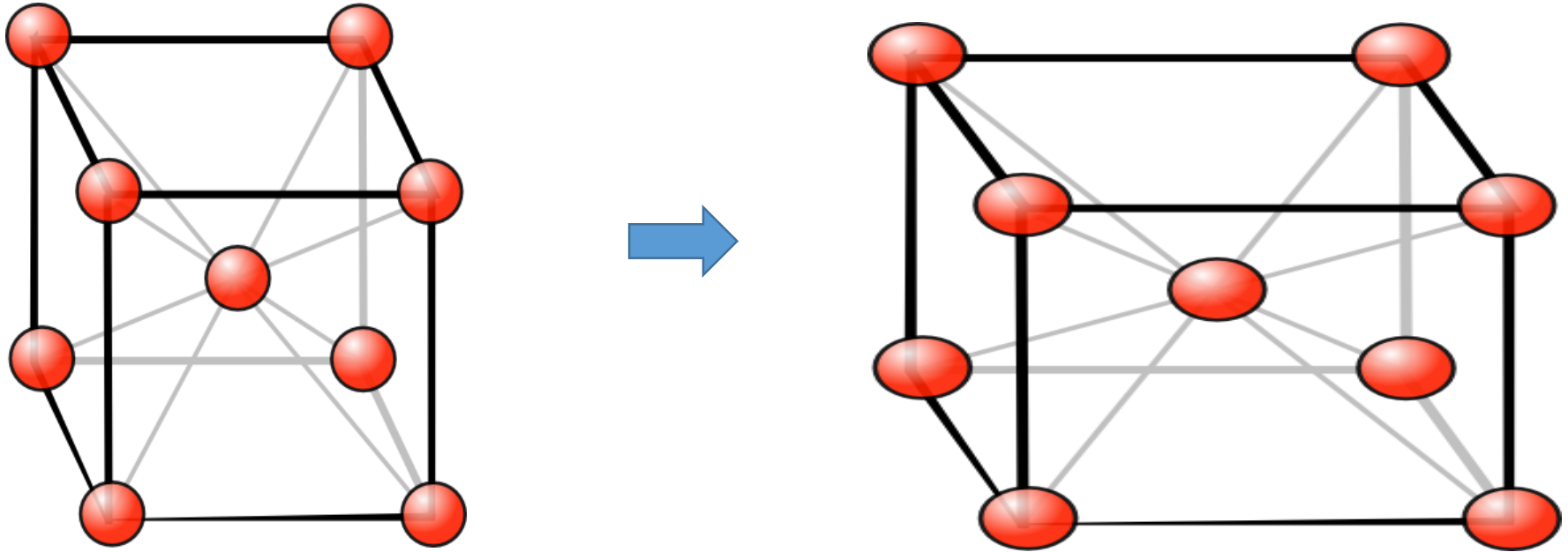
Accurate calculations for bcc lattice [Baiko 2011]

Nuclei deformation reduces the shear modulus!

$$\mu = 0.1194572 \frac{Z^2 e^2}{r_c V_c}$$

$$\mu = \frac{3}{25}\frac{(Ze)^2}{r_c V_c}\left(1 - \frac{u^{5/3}}{2 + 3u - 4u^{1/3}}\right)$$

Is WZ approximation applicable for elasticity? How accurate is it?
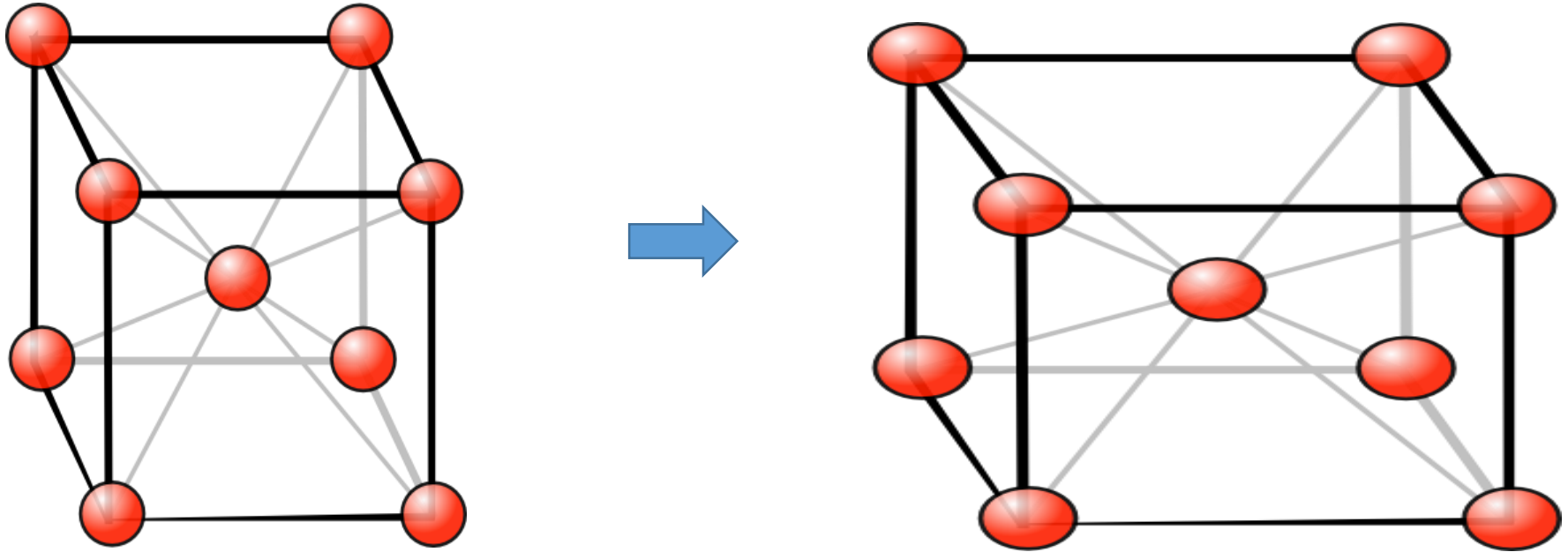
Inner crust elasticity: account for lattice

Ordered system of proton – neutron clusters.

General expansion for small perturbations

$$\frac{E}{V_{0c}} = \frac{E_0}{V_{0c}} + C^b \delta N_b + C^Z \delta Z - P u_{ii} + \frac{1}{2} S_{ijkl} u_{ij} u_{kl} + C^{ZZ} \delta Z^2 +$$
$$+ C^{bb} \delta N_b^2 + C^{bZ} \delta N_b \delta Z + C^{bu} \delta N_b u_{ii} + C^{Zu} \delta Z u_{ii},$$

Inner crust elasticity: account for lattice

$$E \equiv E_{\mathrm{CLDM}} = \boxed{V_{\mathrm{p}}\varepsilon^{\mathrm{nuc}}\left(n_{\mathrm{pi}}, n_{\mathrm{ni}}\right) + (V_{\mathrm{c}} - V_{\mathrm{p}})\varepsilon^{\mathrm{nuc}}\left(0, n_{\mathrm{no}}\right) + E_{\mathrm{s}}} + E_{\mathrm{e}}\left(n_{\mathrm{e}}\right) + E_{\mathrm{C}}$$

<u>Compressible liquid drop model:</u>
Explicit analytical dependence on the parameters

$$n_{\mathrm{pi}} = \frac{Z}{V_{\mathrm{p}}} \qquad n_{\mathrm{ni}} = \frac{N_{\mathrm{i}}}{V_{\mathrm{p}}} \qquad n_{\mathrm{no}} = \frac{N_{\mathrm{o}}}{V_{\mathrm{c}} - V_{\mathrm{p}}} \qquad n_{\mathrm{e}} = \frac{Z}{V_{\mathrm{c}}}$$

Inner crust elasticity: account for lattice
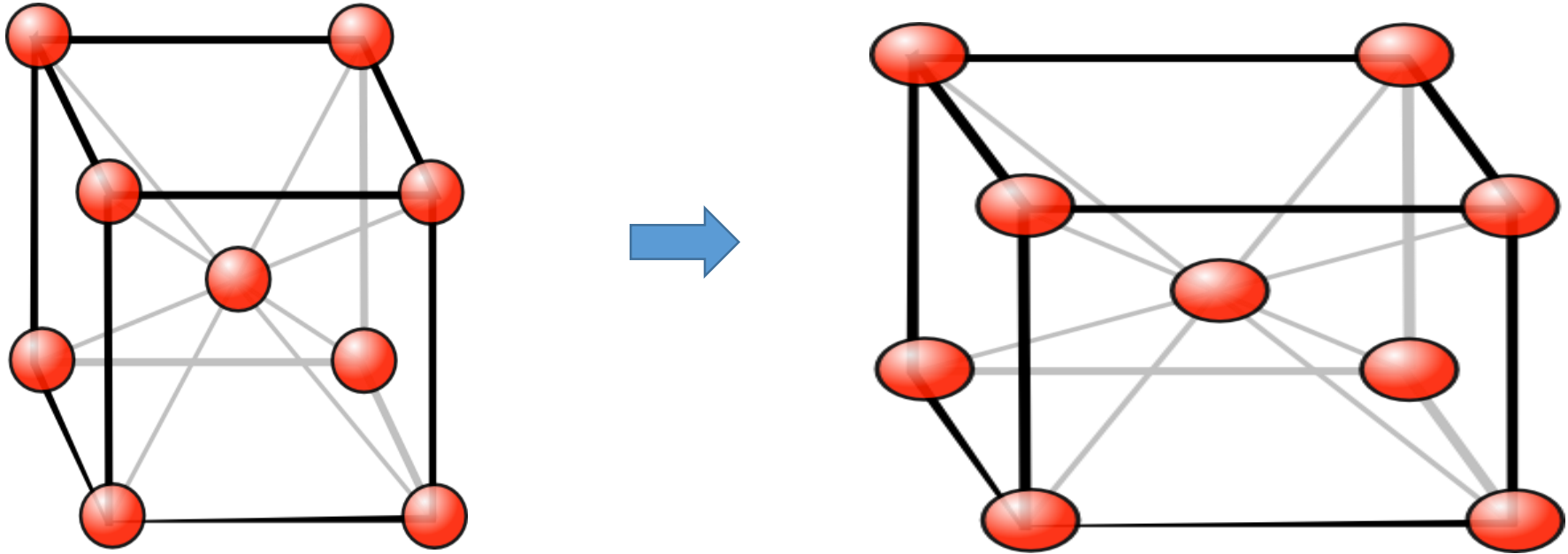
$$E \equiv E_{\mathrm{CLDM}} = \boxed{V_{\mathrm{p}}\varepsilon^{\mathrm{nuc}}\left(n_{\mathrm{pi}}, n_{\mathrm{ni}}\right) + (V_{\mathrm{c}} - V_{\mathrm{p}})\varepsilon^{\mathrm{nuc}}\left(0, n_{\mathrm{no}}\right) + E_{\mathrm{s}}} + E_{\mathrm{e}}\left(n_{\mathrm{e}}\right) + \boxed{E_{\mathrm{C}}}$$

<u>Coulomb energy:</u> The most complicated. Energy of lattice of ellipsoids on uniform background

<u>Calculation in reciprocal space?</u> For OCP diverge due to point-like charges (associated with self interaction energy of point-like charge) => Ewald summation is required (lection 1)
Here: finite size clusters=> good convergence and no problems (just long formulae)!
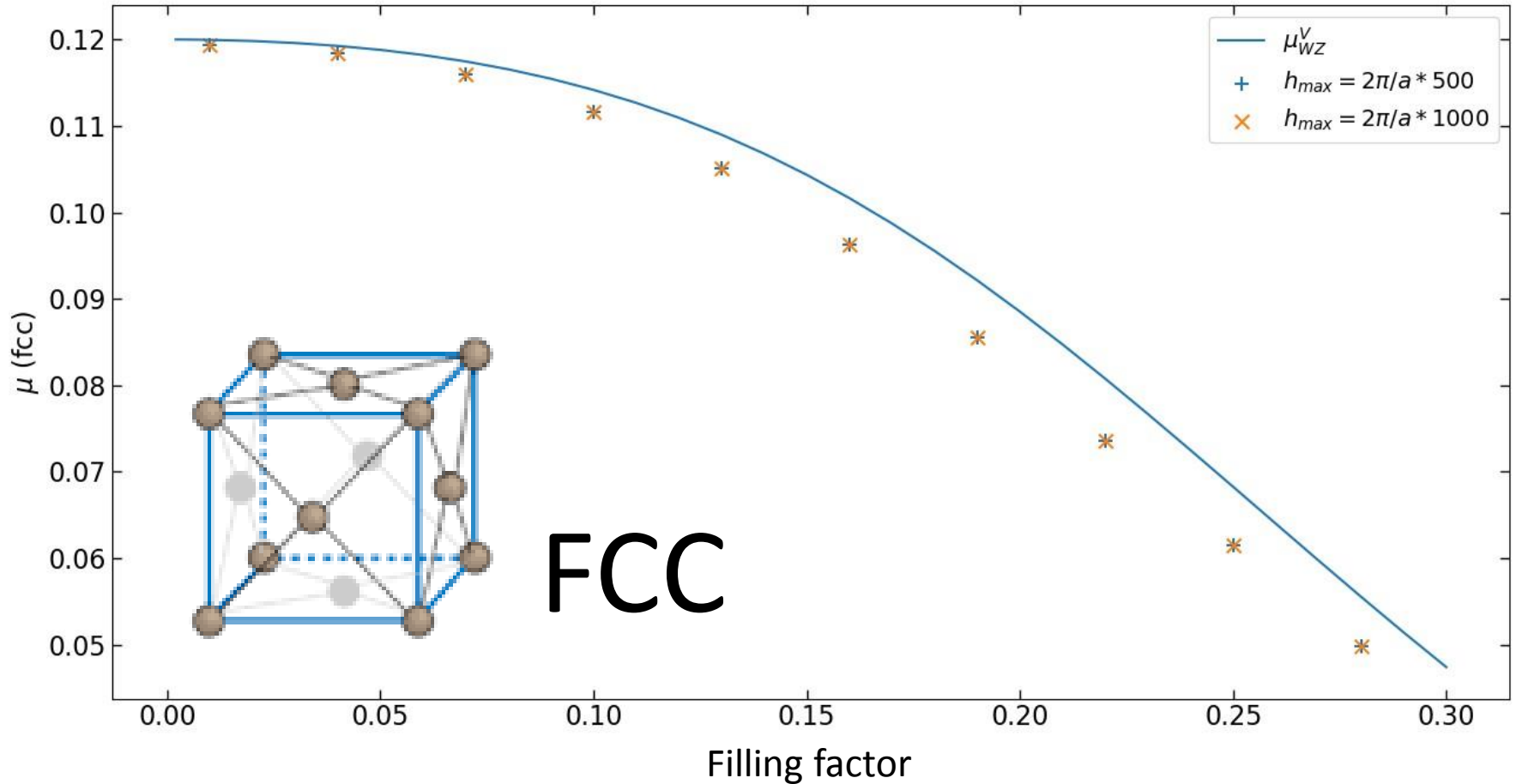
Inner crust: shear modulus

$$\mu_{\mathrm{WZ}}^{\mathrm{V}} = \frac{3}{25} \frac{Z^2 e^2 n}{a} \left(1 - \frac{u^{5/3}}{2 + 3u - 4u^{1/3}} \right)$$
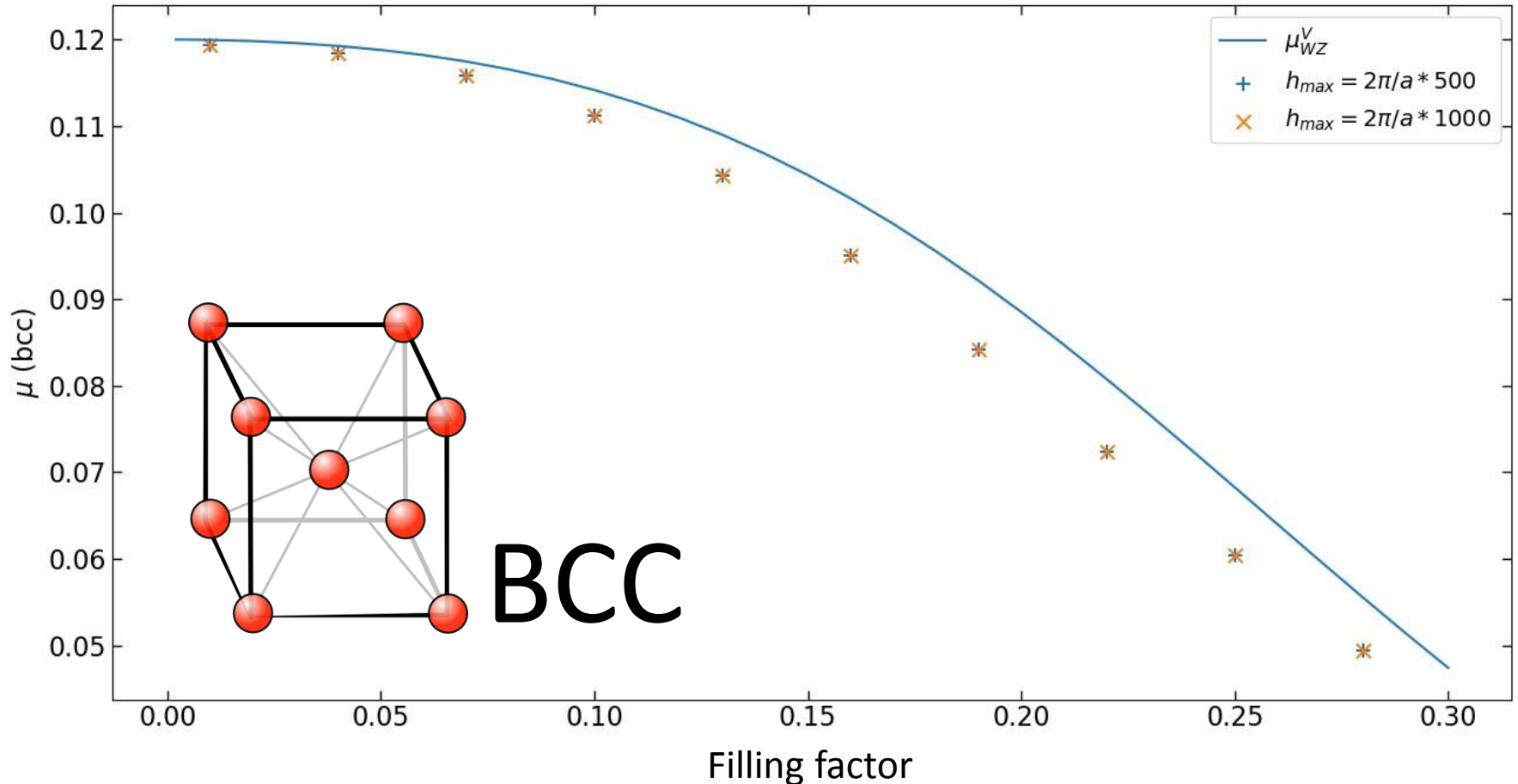
Wigner-Seitz model works very good!!!

Inner crust: shear modulus

$$\mu_{\mathrm{WZ}}^{\mathrm{V}} = \frac{3}{25} \frac{Z^2 e^2 n}{a} \left(1 - \frac{u^{5/3}}{2 + 3u - 4u^{1/3}} \right)$$
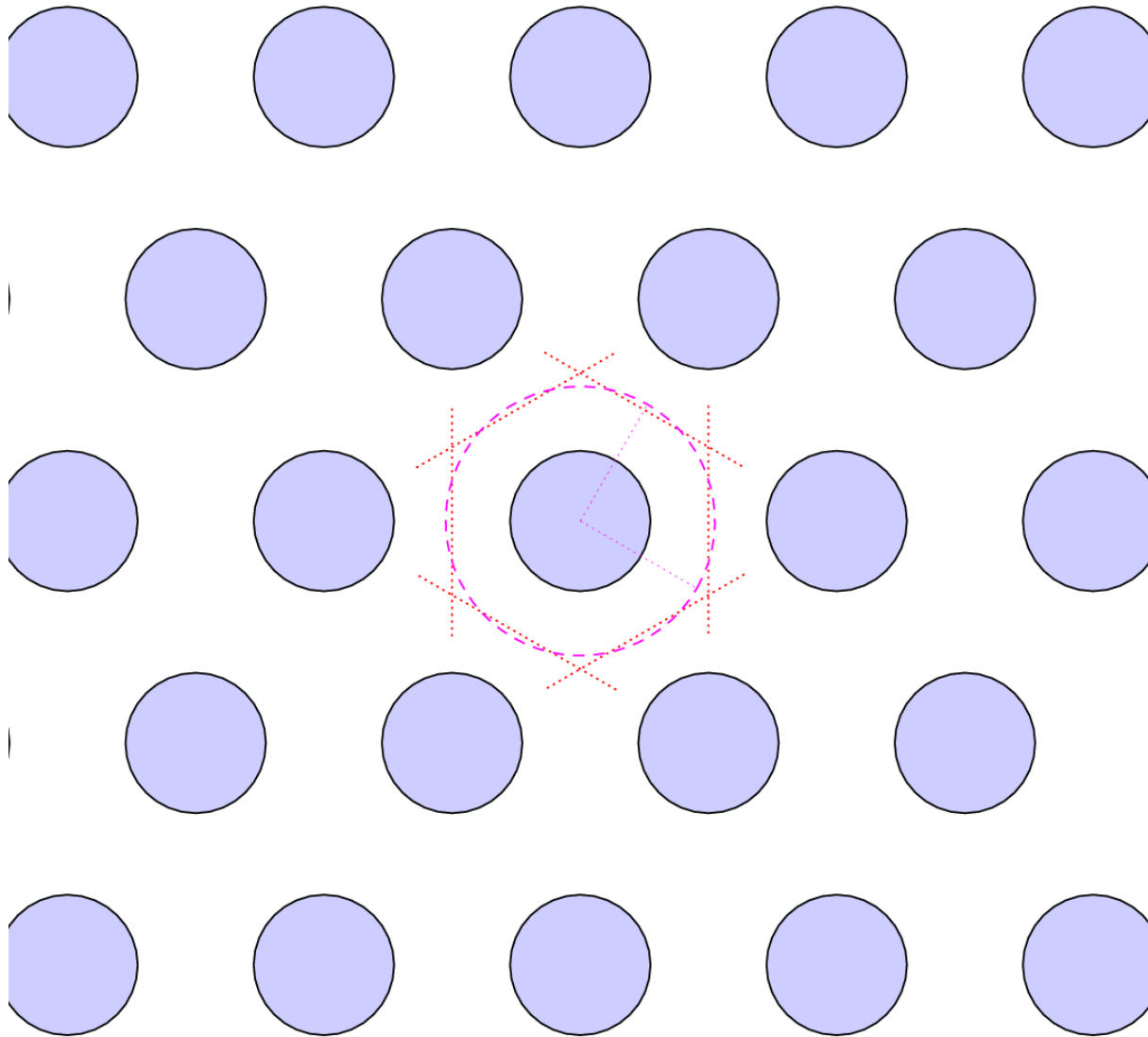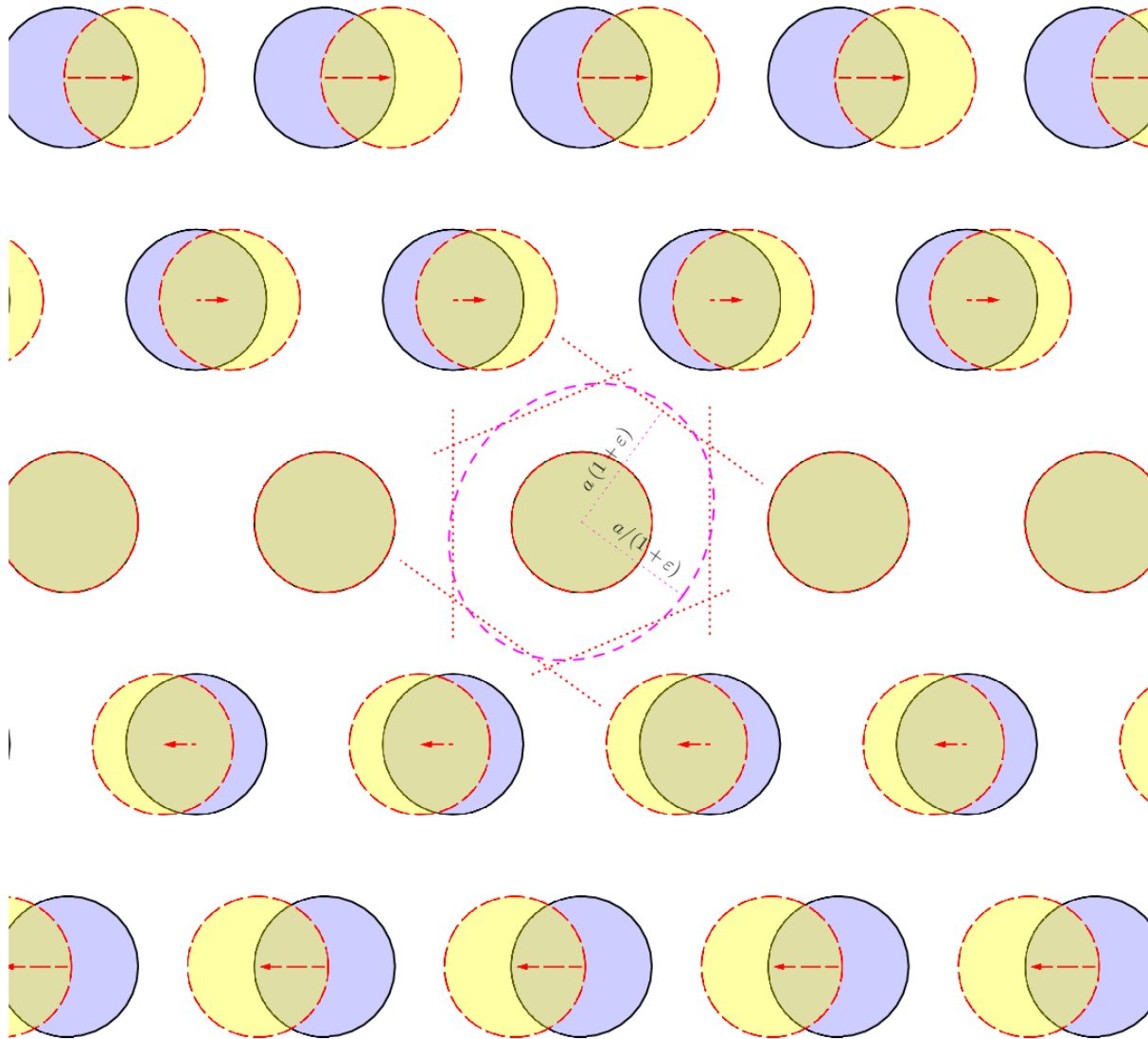

FCC

Analytical Wigner-Seitz model works very good!!!
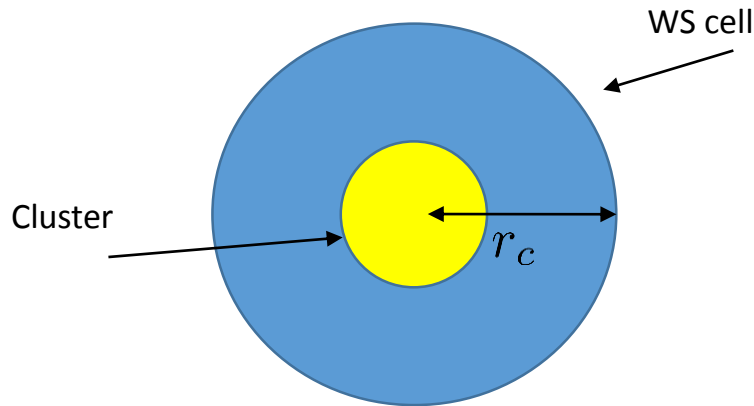
Elasticity of the spaghetti phase

Elasticity of the spaghetti phase

Elasticity of the spaghetti phase
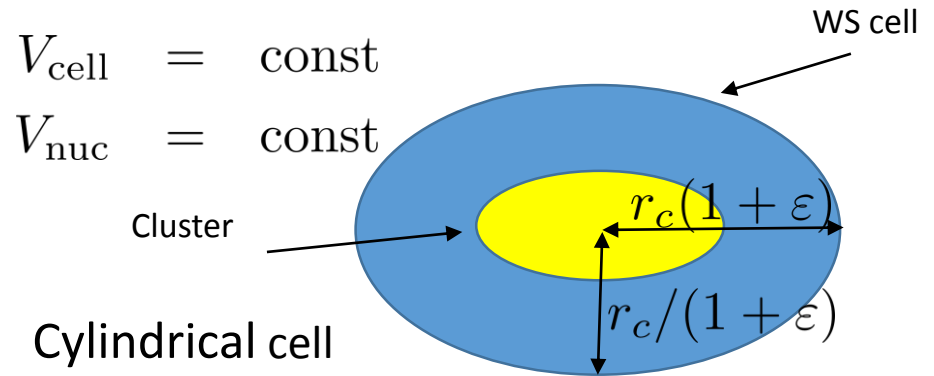
Undeformed crust:

WS cell

Cluster $\longrightarrow$ r_c

Cylindrical cell with circular base

Minimization of internal parameters of CLDM model at fixed n_b (and n_N)

$$n_{pi}, \ n_{ni}, \ n_{no}, \ r_p, \ \nu_s, \ r_c$$

Deformed crust:

$$V_{\text{cell}} = \text{const}$$
$$V_{\text{nuc}} = \text{const}$$

WS cell

Cluster $\longrightarrow$ $r_c(1+\varepsilon)$ $r_c/(1+\varepsilon)$

Cylindrical cell with elliptical base

Eccentricity:

The cell (ε):
Driving parameter.
Given by deformation of the crust (mimics deformation of primitive cell)

The nucleus (ε_p):
Driven parameter.
Optimal (minimize the cell energy)

$$\epsilon = u\,\epsilon^{\text{bulk}}(n_{ni},\,n_{pi}) + (1-u)\,\epsilon^{\text{bulk}}(n_{no},\,0) + \frac{E_s}{V_c} + \frac{E_C}{V_c} + \epsilon_e(n_e).$$

Elasticity of the spaghetti phase

As for spherical clusters:

$$E = E_0 + \frac{1}{2}E_{\varepsilon\varepsilon}\varepsilon^2 + E_{\varepsilon p}\varepsilon\varepsilon_p + \frac{1}{2}E_{pp}\varepsilon_p^2$$
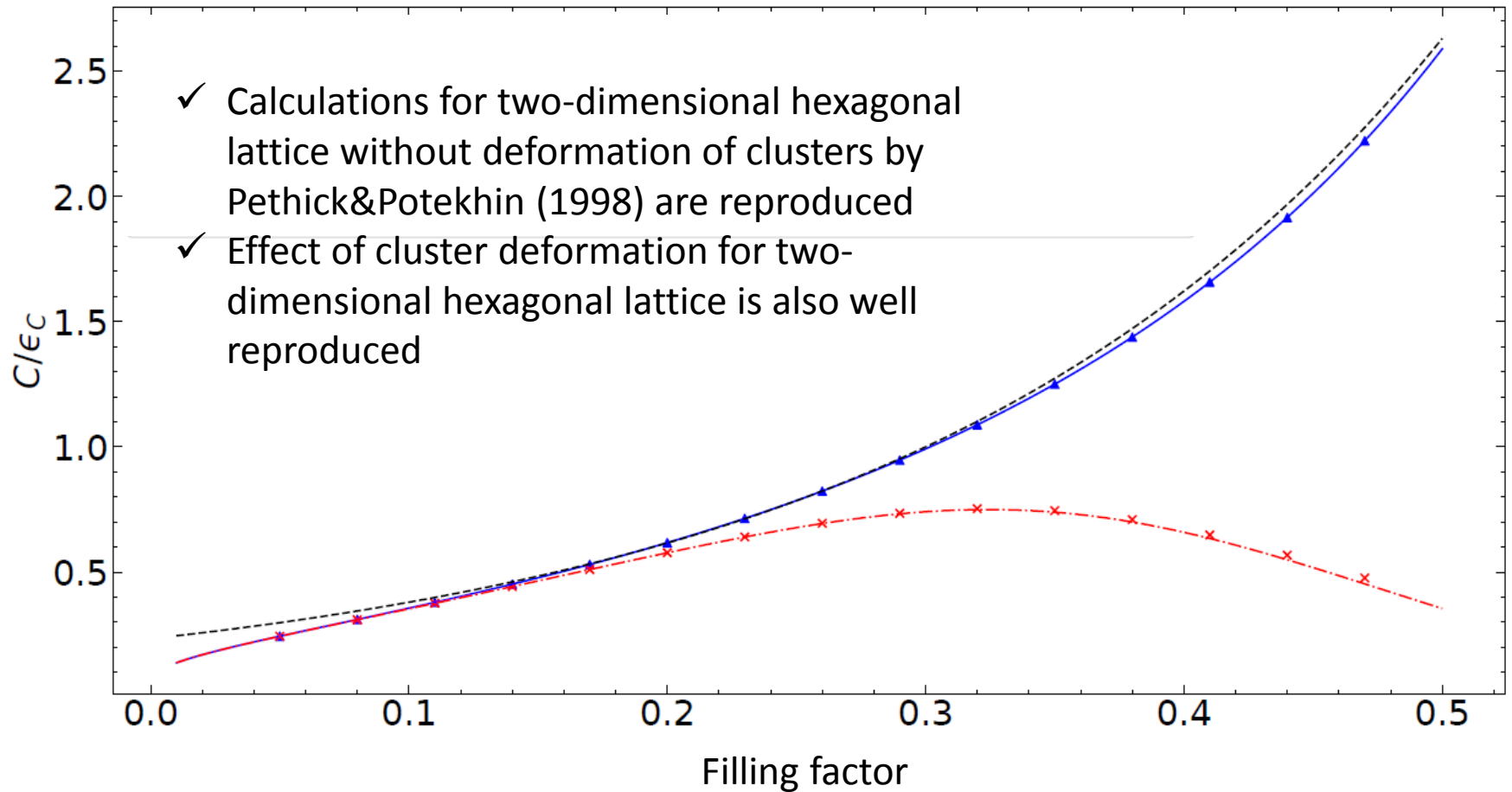
$$\frac{\partial E}{\partial \varepsilon_p} = 0 \qquad \Longrightarrow \qquad \varepsilon_p = \frac{2u}{7u - 5 - 3\ln u}\varepsilon$$

$$C = \frac{\pi}{4}\rho_p^2 r_p^2 u \left\{ 1 - \frac{2u^2}{7u - 5 - 3\ln u} \right\}$$

Deformation of cluster reduce shear modulus

Elasticity of the spaghetti phase

$$C = \frac{\pi}{4} \rho_p^2 r_p^2 u \left\{ 1 - \frac{2u^2}{7u - 5 - 3\ln u} \right\}$$

✓ Calculations for two-dimensional hexagonal lattice without deformation of clusters by Pethick&Potekhin (1998) are reproduced
✓ Effect of cluster deformation for two-dimensional hexagonal lattice is also well reproduced

Filling factor

Wigner-Seitz model works very good!!!

Elasticity of the mantle
Effective shear modulus

Pethick et al. (2020): "We consider the elastic constants of phases with nonspherical nuclei, so-called pasta phases ... We then turn to pasta phases without long-range order and calculate upper (Voigt) and lower (Reuss) bounds on the effective shear modulus and find that the lower bound is zero, but the upper bound is nonzero. To obtain better estimates, we then apply the self-consistent formalism and find that this predicts that the shear modulus of the phases without long-range order is zero if the pasta elements are spatially uniform"
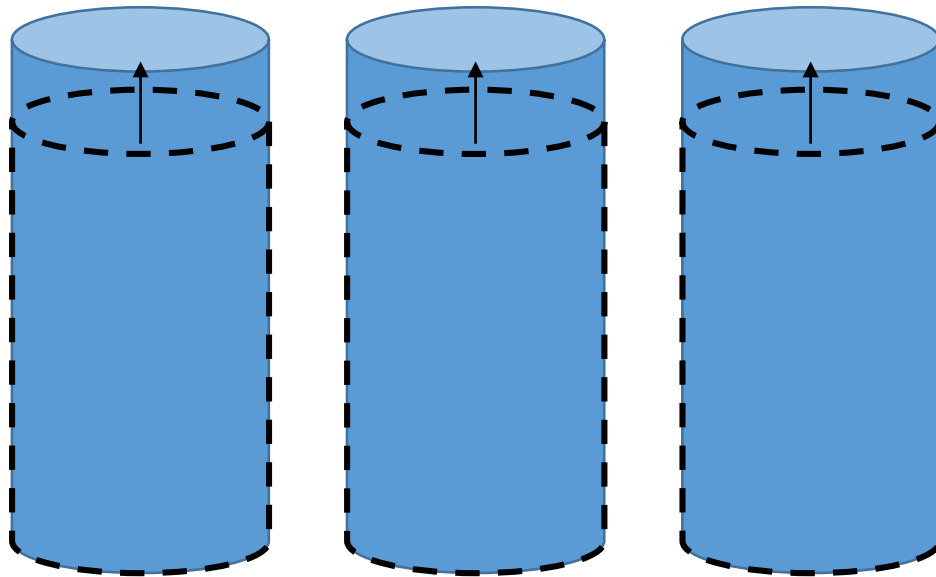
Reason: Deformation of the pasta phases, *considered at constant n_p and n_n* along symmetry axis don't change anything (in particular energy)

Can mantle support shear stress???

Crust of compact stars
Summary of lecture 3

➢ <u>Inner crust composition</u>
✓ CLDM is not obsolete and it is still a good tool to study inner crust EOS
✓ ETFSI is more accurate and it is required accuracy to analyze transition to the mantle: preference of mantle phases are governed by ~ keV/nucleon

➢ <u>Inner crust elasticity</u>
✓ Analytical effective shear modulus for inner crust within static (+Voigt) approximation
❑ Can mantle support elastic stress?

➢ <u>Not mentioned</u>
✓ Neutrons in inner crust are superfluid (critical temperatures $\sim 10^9$ K)

Lecture 4

➢ Are crustal models as simple as possible or they are (still) too simple?
• Internal consistency (few examples)
• Confronting observations
➢ M(R) without specifying crust