Crust of compact stars
Lecture 4
Aren’t crustal models simpler, than it is possible?

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Ioffe Institute

Many-particle systems: from condensed matter to quarks and stars

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Crust of compact stars

Selection of topics and plan

"Everything should be made as simple as possible, but not simpler"

Attributed to Albert Einstein

According to Robinson [*Nature* **557**, 30 (2018)], it can be a compressed version of lines from a 1933 lecture by Einstein:
"*It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.*"

My preference in these lectures: *models, which can be solved analytically*

Lecture 1: Introduction and basic models of outer crust at T=0

Lecture 2: Outer crust: thermodynamics and elasticity

Lecture 3: Inner crust

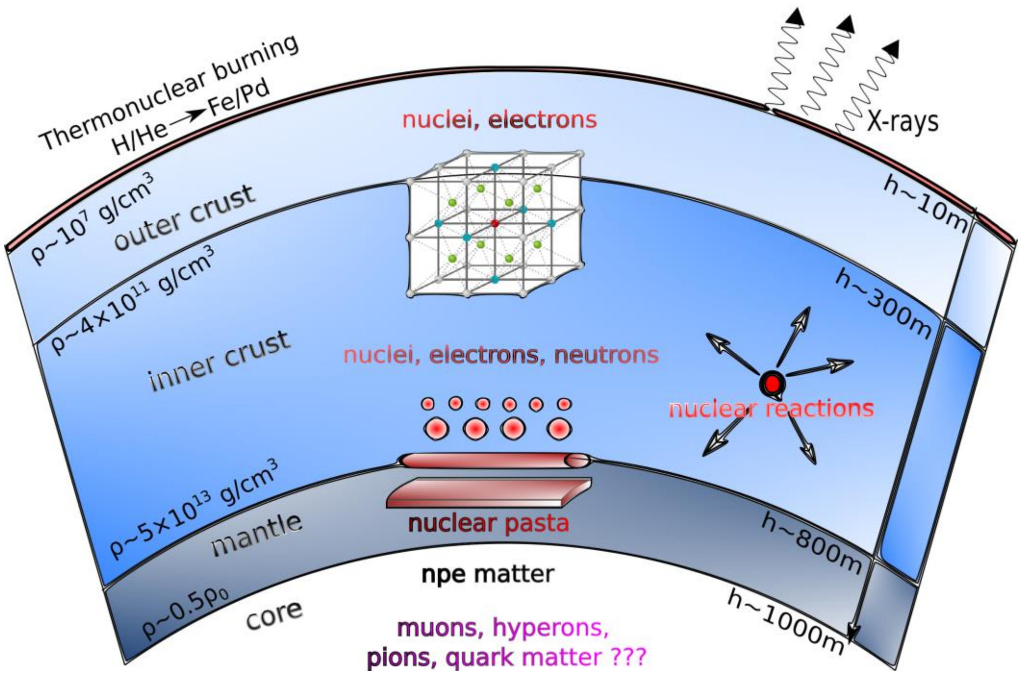
Lecture 4: Aren't crustal models simpler, than it is possible?
+ M(R) not dealing with crust

Crust of compact stars

<u>Lecture 4:</u> Are crustal models as simple as possible or they are (still) too simple?
+ M(R) not dealing with crust

Let's inspect our spherical cows

What we want to know about the crust?



N. N. Shchechilin ©

- **Composition**
  - Equilibrium
  - Nonequilibrium
- **Equation of state**
  - T=0
  - Thermal properties
  - State of matter (solid/liquid)
- **Dynamical properties One/two liquid hydro (magneto) dynamics**
- **Transport properties (kinetic coefficients)**
- **Elasticity, strength**
- …

D.G. Yakovlev, HEA2017(?)
Crust as Cinderella of NS

Why???

These properties affect observations, and thus they are required for adequate interpretation of observations

Main mystery of NSs is the core. The crustal properties should be known accurately to avoid biases in conclusions on the core properties

Pressure within ion sphere model

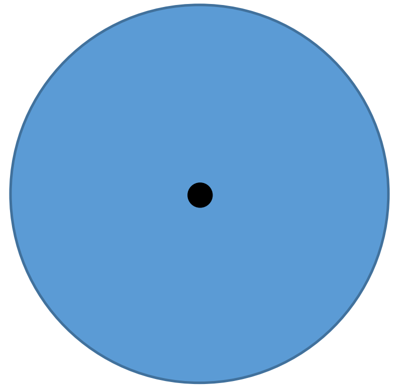
Thermodynamics

$$F = F_e + F_i$$

$$P = P_e + P_i$$

$$P_\mathrm{i} = -\frac{3}{10}\frac{Z^2 e^2}{a} n$$

Ion sphere model



Pressure is given by pressure at the boundary.
There are only electrons

$$P = P_e$$

Step back: small polarization

Screening cloud

$$\mu_e - e\phi = \mathrm{const}$$

$$\delta n_e = \frac{\partial n_e}{\partial \mu_e}\delta\mu_e = \frac{\partial n_e}{\partial \mu_e} e\phi$$

Potential

$$\phi = \frac{Ze^2}{a}\left[\frac{a}{r} - \left(\frac{3}{2} - \frac{1}{2}\frac{r^2}{a^2}\right)\right]$$

$$\int n_e \mathrm{d}^3 r = Z \Rightarrow n_e(r=a) = n_e^{\mathrm{un}} - \frac{3}{10}\frac{\partial n_e}{\partial \mu_e}\frac{Ze^2}{a}$$

$$\frac{\partial P_e}{\partial \mu_e} = n_e = Zn$$

$$P = P_e(n_e^{\mathrm{un}}) - \frac{3}{10}\frac{Ze^2}{a}\frac{\partial P_e}{\partial n_e}\frac{\partial n_e}{\partial \mu_e}$$

Note for students

- If you write down expression for thermodynamic function, all other thermodynamic functions should be calculated from this expression by thermodynamic relations

- If you have a brilliant idea how to calculate another thermodynamic quantity easily, check that it follows from thermodynamic relations and expression for thermodynamic function, mention in the first line.

A model, there thermodynamic functions are not in thermodynamically consistent is dangerous: it (potentially) can allow everything (perpetuum mobile?)

Crust/Mantle – core transition

Two approaches:

n^t_{cc}
- From the crust side. Thermodynamic (as before): calculate energy density for whole phases and find the optimal one!

n^i_{cc}
- From the core side: analyze stability of uniform matter for small (plane wave?) perturbations while decreasing of the density. If it is unstable – the nonuniform phase should start (crust or mantle)

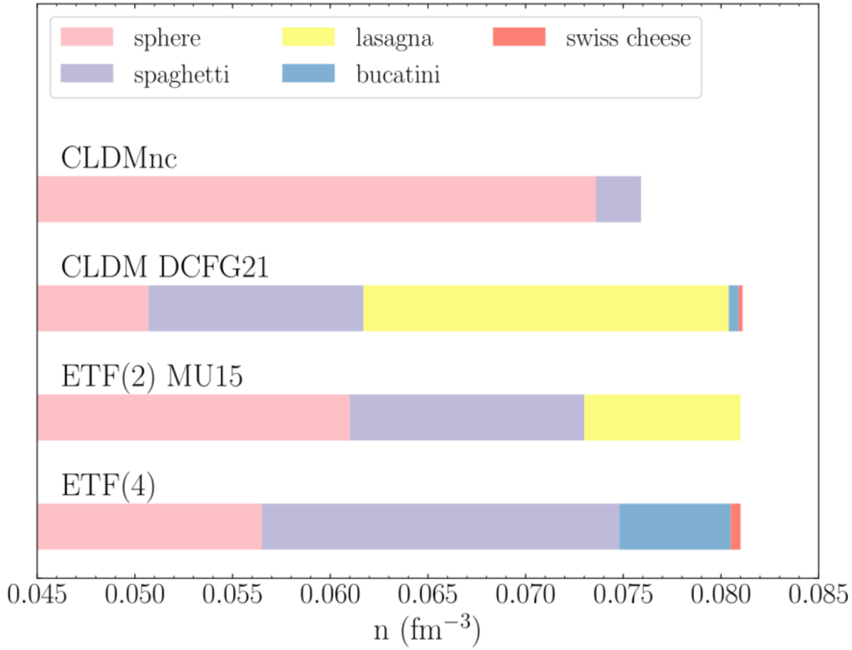
Obviously, if $n^t_{cc} < n^i_{cc}$ there is a problem: between these values uniform core is preferred, but it is unstable…

Douchin&Haensel (2000): SLy4, CLDM $n^t_{cc} = 0.076\,\mathrm{fm}^{-3} < n^i_{cc} = 0.079\,\mathrm{fm}^{-3}$

Reason: CLDM was simpler than it is required for these densities.

Within ETF(4) calculations we don't have this problem

Note: the published version had minor numerical problem and the crust ends earlier



Shchechilin et al. (2022)

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Likely reason: CLDM was simpler than it is required for these densities

Pearson et al. (2019): BSK22-26, ETFSI $\mathrm{BSK24}: n^t_{cc} = 0.079\,\mathrm{fm}^{-3} < n^i_{cc} = 0.081\,\mathrm{fm}^{-3}$

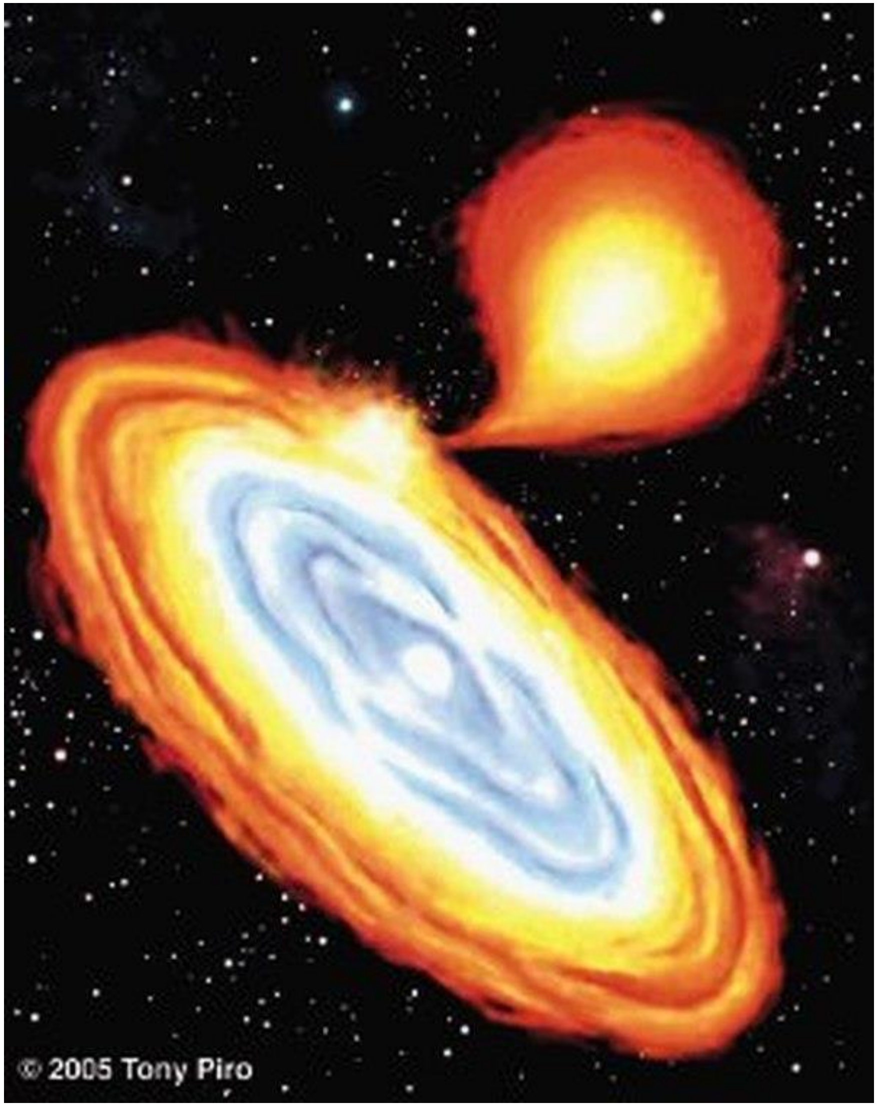
“The instabilities that we do see in our calculations involve something more significant than a change in Z, and an obvious possibility is that changes to non-spherical pasta phases…”

Schechilin et al. (2023): ETFSI for pasta

	$\bar{n}_{\rm sp}$ [fm^{-3}]	$\bar{n}_{\rm sp}^{\rm SI}$ [fm^{-3}]	$\bar{n}_{\rm cc}^{\rm s}$ [fm^{-3}]	$\bar{n}_{\rm cc}$ [fm^{-3}]	$\bar{n}_{\rm cc}^{*}$ [fm^{-3}]
BSk22	0.056	0.066	0.071	0.071	0.072
BSk24	0.050	0.076	0.081	0.082	0.081
BSk25	0.042	0.065	0.086	0.089	0.086
BSk26	0.056	0.079	0.085	0.086	0.085

Problem resolved for most of EOS (not for BSK22…)

Models of accreting neutron star crust



Many neutron stars are observed accreting, i.e. they are located in the close binary system with transfer of matter from (Roche-lobe overflow) companion star to the neutron star

- What happens with matter after accretion?
- How it affect observations?
- Which information on superdense matter can be inferred from these observations?

What happens with matter after accretion?

(traditional approach)

1. Thermonuclear burning in surface layers leads to formation of heavy elements («ashes»)

$^{56}\mathrm{Fe}, \quad \mathrm{e}^-$

<u>Traditional apporach [Sato 1979, Haensel&Zdunik 1990,….]:</u>

Consider nuclear reaction on course of compression (increase of the pressure)

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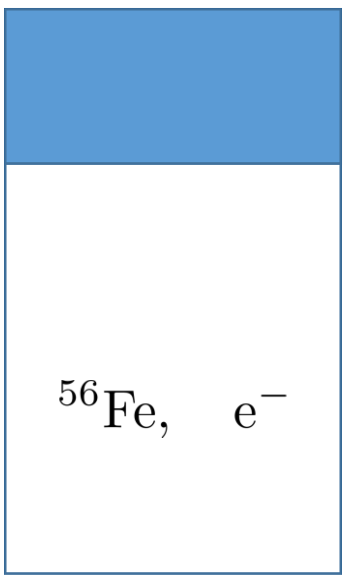
2. The subsequent portions of the accreted matter compress underlying layers after arrival to the surface



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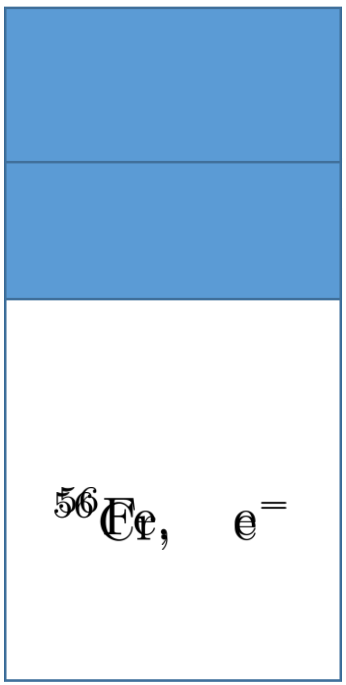

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2. The subsequent portions of the accreted matter compress underlying layers after arrival to the surface
3. Compression leads to beta-captures

$$^{A}_{Z}X + e^{-} \rightarrow ^{A}_{Z-1}X + \nu_e$$

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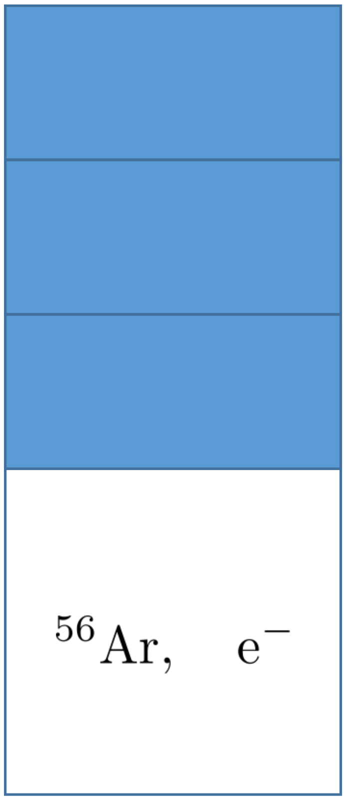

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4. Beta-captures move nuclei to the neutron drip line and, finally, beta-capture is accompanied by neutron emission

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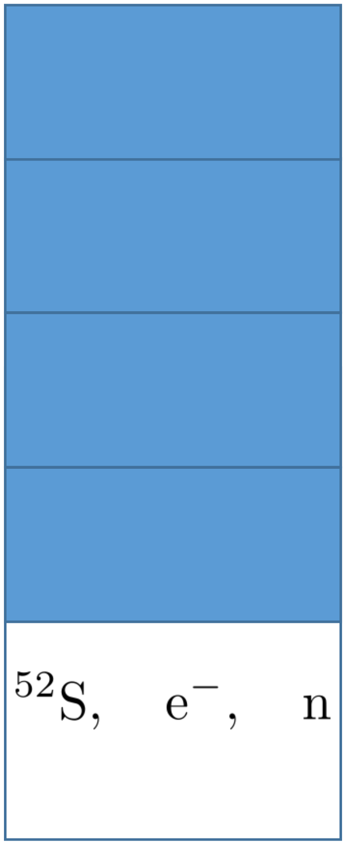

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$^{56}\mathrm{Ti}, \quad \mathrm{e}^-$
$^{56}\mathrm{Ca}, \quad \mathrm{e}^-$
$^{56}\mathrm{Ar}, \quad \mathrm{e}^-$
$^{52}\mathrm{S}, \quad \mathrm{e}^-, \quad \mathrm{n}$
$^{46}\mathrm{Si}, \quad \mathrm{e}^-, \quad \mathrm{n}$

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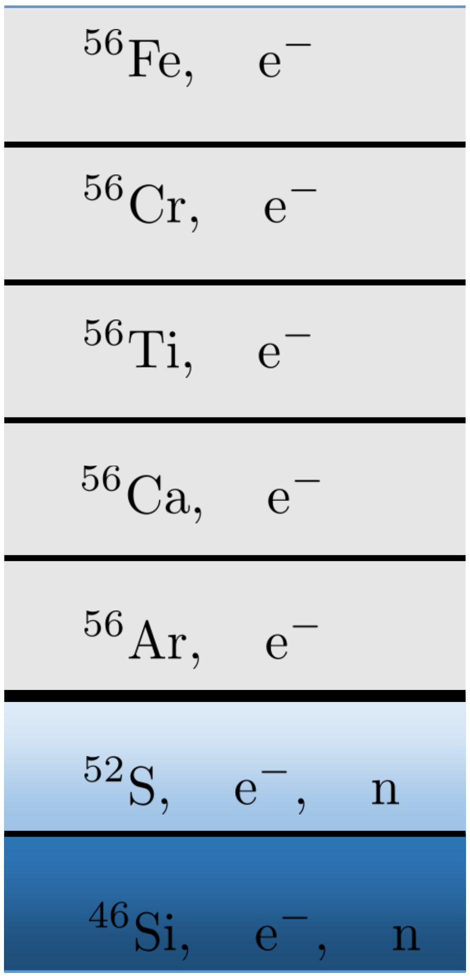
5. ***Fully accreted crust is composed of matter at different compression stages***

Initial value problem: initial composition determines whole evolution

Neutronization is associated with neutron drip $\quad P_{\mathrm{oi}} = P_{\mathrm{drip}}$

What happens with matter after accretion?

(traditional approach)



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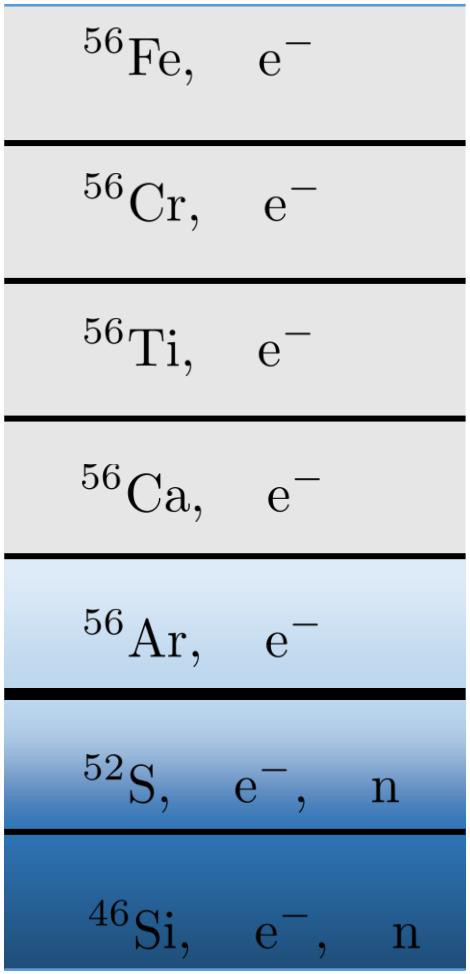
5. ***Fully accreted crust is composed of matter at different compression stages***

Unbound neutrons are not at the equilibrium at the inner crust
Their redistribution among crust layers is energetically favorable

What happens with matter after accretion?

(traditional approach)



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5. ***Fully accreted crust is composed of matter at different compression stages***

The first indication of the importance of neutron diffusion
Bisnovatyi-Kogan G. S., Chechetkin V. M., 1979, UFN, 127, 263

Unbound neutrons are not at the equilibrium at the inner crust
Their redistribution among crust layers is energetically favorable

Thermodynamically consistent model: neutron Hydrostatic/Diffusion equilibrium (nHD)

The statement of problem is crucially modified in the inner crust (=region, where free neutrons exist):

Instead of considering of nuclear reactions in compressing volume element one should consider whole inner crust, accounting for redistribution of neutrons between the layers

Two conditions:

Newtonian gravity

General hydrostatic equation

$$\nabla P = \rho \boldsymbol{g}$$

---

neutron Hydrostatic/Diffusion equilibrium

$$\boldsymbol{\nabla} \mu_n = m_n \boldsymbol{g}$$

Transfer of a neutron from one layer to another does not lead to any energy gain

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Instead of considering of nuclear reactions in compressing volume element one should consider whole inner crust, accounting for redistribution of neutrons between the layers

<u>Two conditions:</u>

General hydrostatic equation

General relativity

$$P' = -\frac{P+\epsilon}{2}\nu'$$

---

neutron Hydrostatic/Diffusion equilibrium

$$\mu_n' = -\frac{\mu_n}{2}\nu' = \frac{\mu_n}{P+\epsilon}P'$$

$$\boxed{\mu_n^\infty = \mu_n \mathrm{e}^{\nu/2} = \mathrm{const}}$$

Transfer of a neutron from one layer to another does not lead to any energy gain

Modeling of fully accreted crust (schematically)

Traditional approach
(Haensel & Zdunik 1990, ….)

Switch to the next, j+1, layer:

Increase of pressure

$$P_{j+1} = P_j + \Delta P$$

Determine composition of this layer, specified by pressure P

Minimization of the Gibbs potential Φ

$$\delta Q^{\infty} = \Phi_j^{\infty} - \Phi_{j+1}^{\infty}$$

Initial composition determines evolution

nHD approach (for $P > P_{\rm oi}$)
(Gusakov&AIC, Gusakov, Kantor&AIC)

Switch to the next, j+1, layer:

Increase of pressure

$$P_{j+1} = P_j + \Delta P$$

and neutron chemical potential $\mu_{\rm n}$

$$\mu_{n,j+1} = \mu_{n,j} + \frac{\mu_n}{\epsilon + P} \Delta P$$

$$\frac{d\mu_n}{\mu_n} = \frac{dP}{\epsilon + P}$$

TOV equation + nHD condition

Determine composition of this layer, specified by pressure P ***and neutron chemical potential*** $\mu_{\rm n}$

Minimization of the appropriate thermodynamic potential Ψ

$$\delta Q^{\infty} = \Psi_j^{\infty} - \Psi_{j+1}^{\infty}$$

Family of models, parametrized by $P_{\rm oi}$

Thermodynamics of nHD inner crust

Gusakov, Kantor, AIC (2021)

In the fully accreted inner crust nuclear reactions take place at the conditions, specific for a given layer:

- Pressure (hydrostatic equilibrium with overlying layers) P
- Neutron chemical potential (nHD condition) $\mu_{\rm n}$

↓

Appropriate thermodynamic potential is **not the Gibbs energy** Φ, but:

$$\Psi = \Phi - \mu_{\rm n} N_b = E + PV - \mu_{\rm n} N_b - TS$$

Total number of baryons (beta equilibrium is assumed)

The heat release equals to the decrease of thermodynamic potential

$$Q = \Psi_{\rm before} - \Psi_{\rm after}$$

(see GKA21 for 'textbook' proof)

Ground state:

$\Psi = 0$

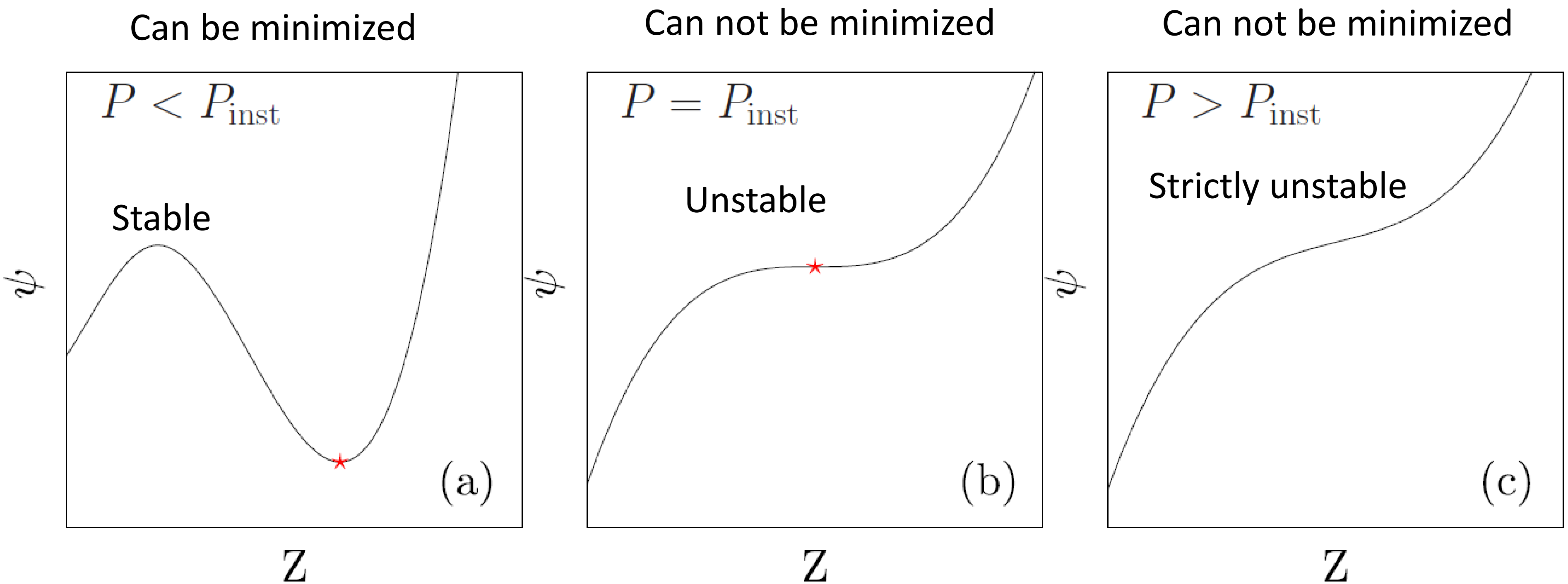
Where does accreted crust end? At the instability!

Construction of the crust within nHD approach:

Increase P and μ_n + minimization of the appropriate thermodynamic potential Ψ

One component example:

$\psi(Z)$ with grow of P



Instability:

Nuclei dissociation as a result of beta-capture/neutron emission sequence

Similar instability without neutrons:
G. S. Bisnovatyi-Kogan&V. M. Chechetkin, Astrophys. Space Sci. 26, 25 (1974).

How to chose P_{oi}? Fully accreted crust

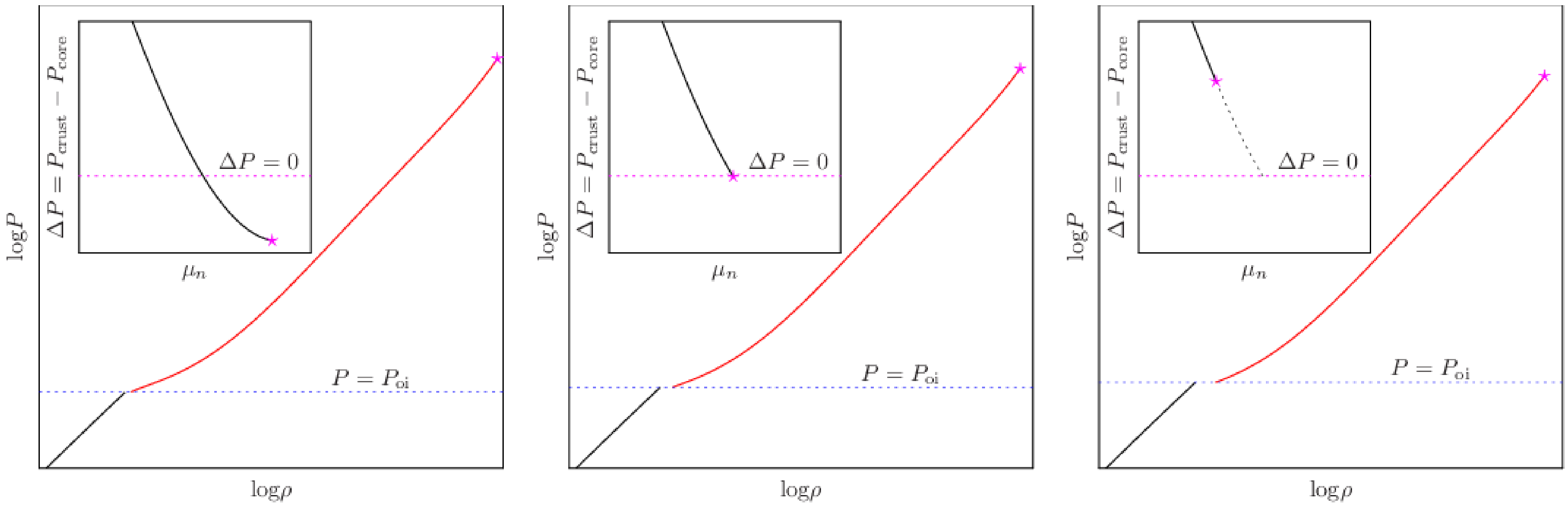
- Accretion supplies nuclei into the crust
- Instability can dissociate nuclei => stationary state can exist
- Our aim is to describe neutron star: the crust and the core should be connected in thermodynamically consistent way (continuity of the pressure and neutron chemical potential)

Accretion leads to the formation of fully accreted crust:

- Instability is active (and compensate nuclei supply by accretion)
- The crust in connected with the core

Construction of FAC model via shooting method

Depend on nuclear physics of innermost crustal layers. Example: (smooth) SLY4 case



<u>Can not be FAC state</u>

The instability does not take place in the crust

$P_{\rm oi}$ is too low

<u>The FAC state</u>

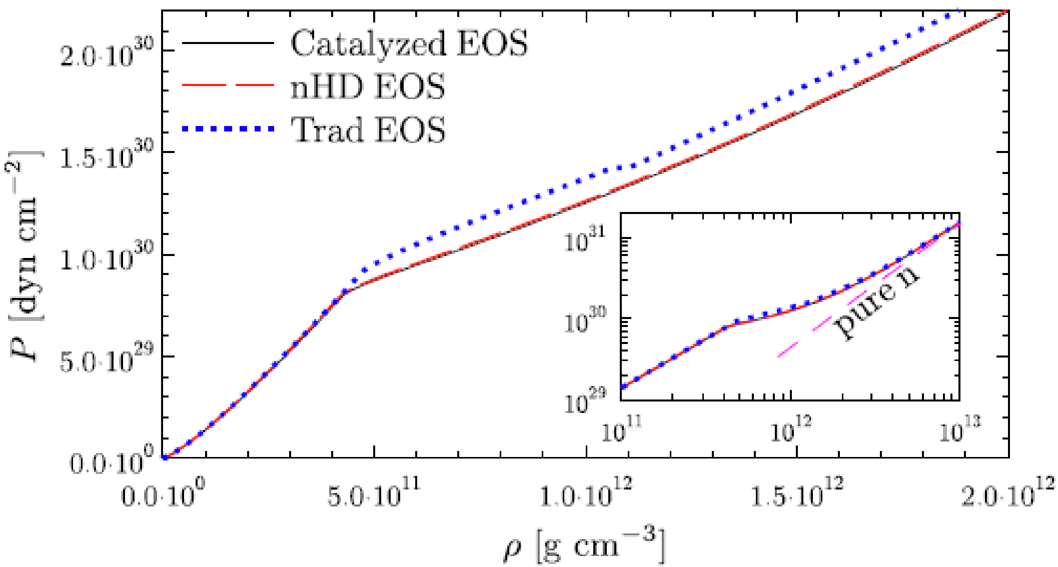
The instability take place exactly at the crust-core boundary

<u>Can not be FAC state</u>

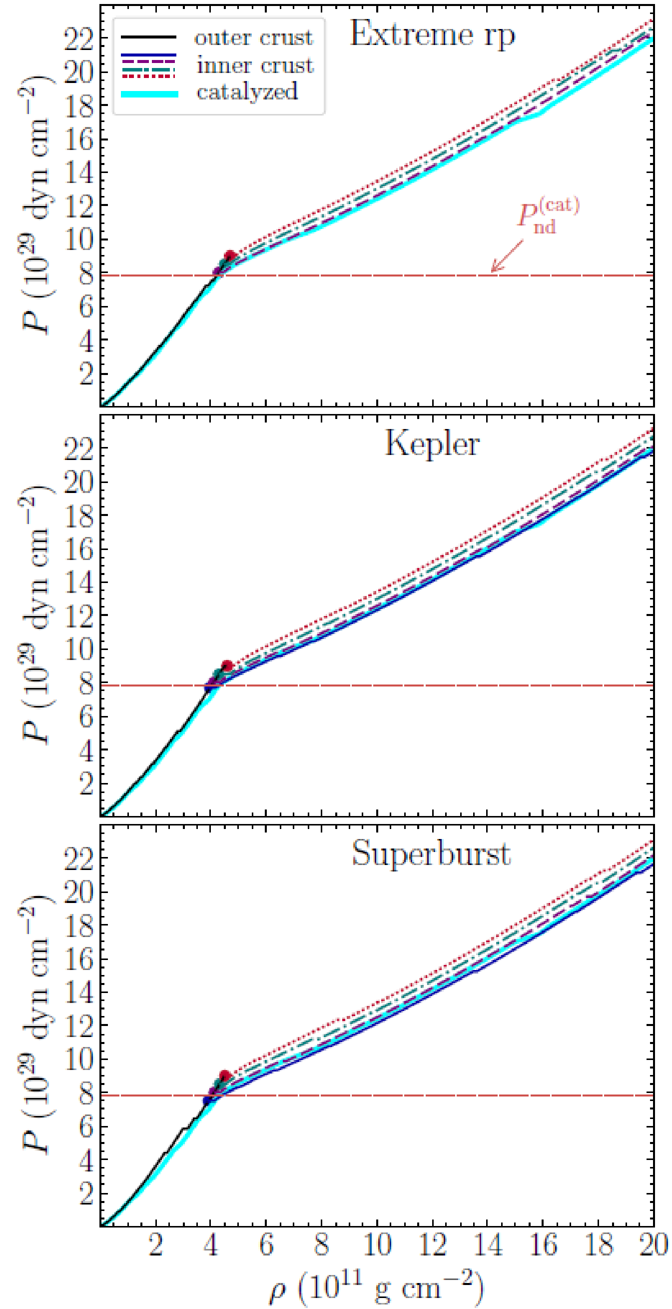
The instability takes place, but crust EOS can not be connected with core

$P_{\rm oi}$ is too high

Few examples of FAC EOS



M.E. Gusakov & AIC, Phys. Rev. Lett. 124, 191101 (2020)



N.N. Shchechilin, M.E. Gusakov & AIC, submitted

Accreted neutron star crust within nHD models

(almost) everything is not as it was generally belived before

- Inner and outer crust boundary is not associated with neutron drip-out from nuclei
- The main reactions at the inner crust are: beta-decay (electron emission) and neutron captures (the reverse reactions were crucial within the traditional approach: electron captures and neutron emissions!)
- There is upward diffusion/superfluid flow of neutrons, which supplies neutrons to the shallow layers of inner crust (to compensate neutron captures)
- The instability threshold is reached at the bottom of the inner crust, which leads to dissociation of nuclei (via sequence of beta captures and neutron emissions). It allows stationary crust structure
- It is this threshold, that determines position of the outer-inner crust interface

Efficiency of the deep crustal heating

$$\langle H \rangle = \frac{q}{m_u} \langle \dot{M} \rangle$$

<u>Straightforward approach:</u>

(applied previously)

Sum of the heat release over reactions:

$$q = \sum_i q_i$$

Detailed information on the reaction kinetics is required

<u>Thermodynamic approach</u>

(here)

Two assumptions:

- *Equilibrium composition of NS core*
- *Fully accreted crust (stationary structure)*

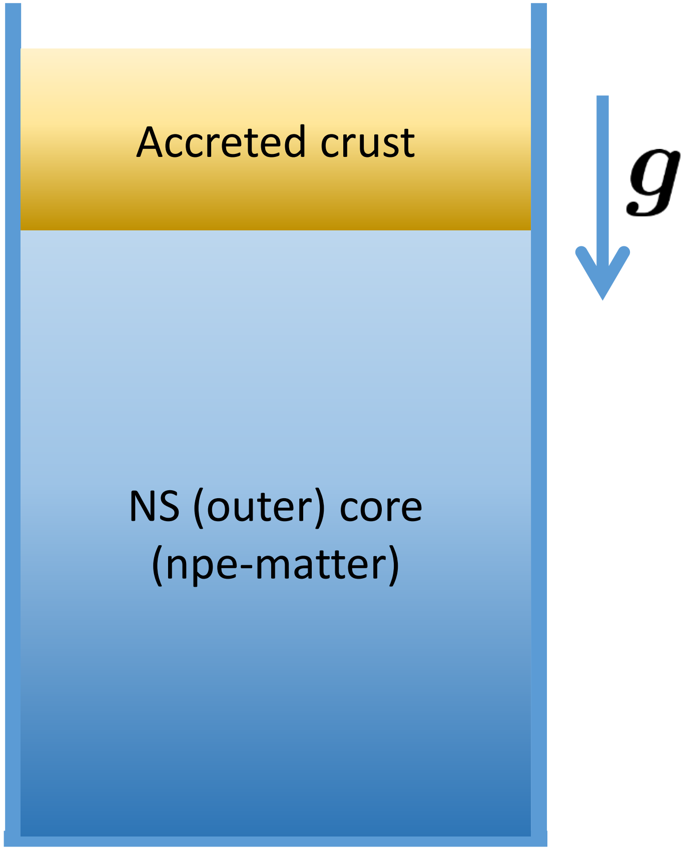
$$q^\infty = \overline{m}_{b,\mathrm{ash}}\, e^{\nu_\mathrm{s}/2} - \mu^\infty_{b,\mathrm{core}}$$

Almost no information on the reaction kinetics

Neutrino energy is included into the heating

M.E. Gusakov & AIC
[Phys. Rev. D:Letters, 103 (2021), L101301]

Heating efficiency: general consideration



Plane-parallel consideration (to simplify presentation)

<u>Thought experiment I:</u>

«The accretion process»

a) Let us add δN_b baryons (in form of hydrogen) to the surface and keep them there `by hands'.
The energy of the system is

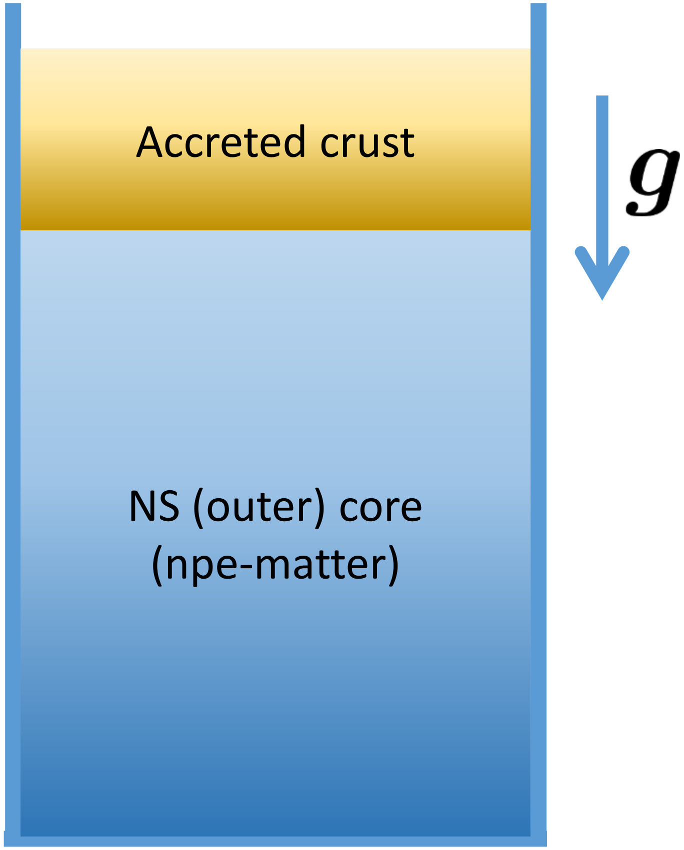
$$E(N_b) + \mu^{\infty}_{b,\,\mathrm{H}} \delta N_b$$

Hydrogen energy per baryon, including gravitational energy

b) Release the baryons
The baryons compress the crust and core, initiate reactions, but total energy should be conserved

$$E_I(N_b + \delta N_b) = E(N_b) + \mu^{\infty}_{b,\,\mathrm{H}} \delta N_b$$

Heating efficiency: general consideration



Plane-parallel consideration (to simplify presentation)

Thought experiment II:

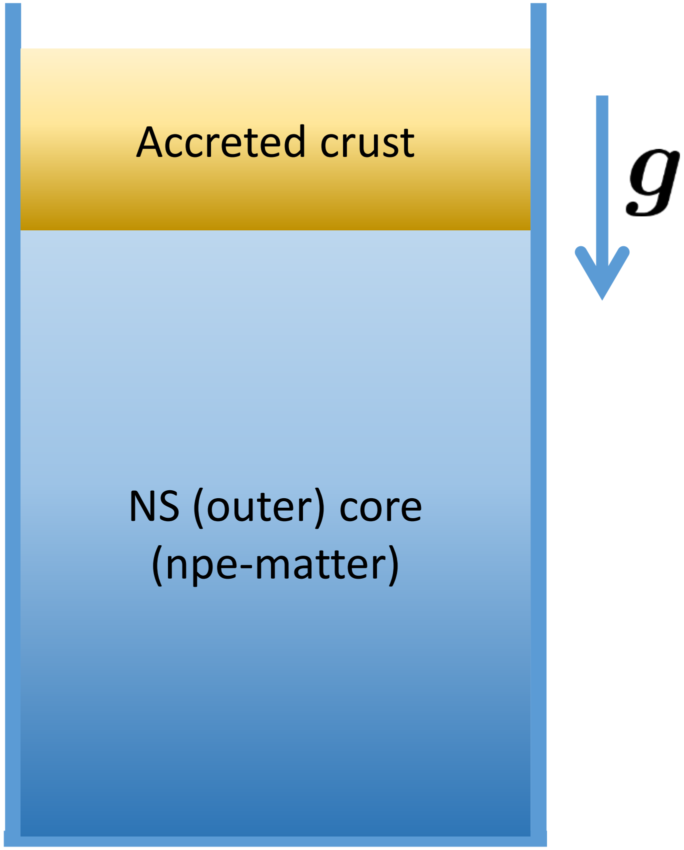
«The result of accretion»

- For fully accreted crust the EOS is fixed
  $\Rightarrow$ the number of baryons in the crust is fixed
  $\Rightarrow$ Additional δN_b baryons appear in the core of NS

- The core is in equilibrium, thus the change of the energy is

$$E_{II}(N_b+\delta N_b) = E(N_b) + \mu^{\infty}_{b,\,\mathrm{core}}\delta N_b$$

Heating efficiency: general consideration



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<u>Thought experiment II:</u>

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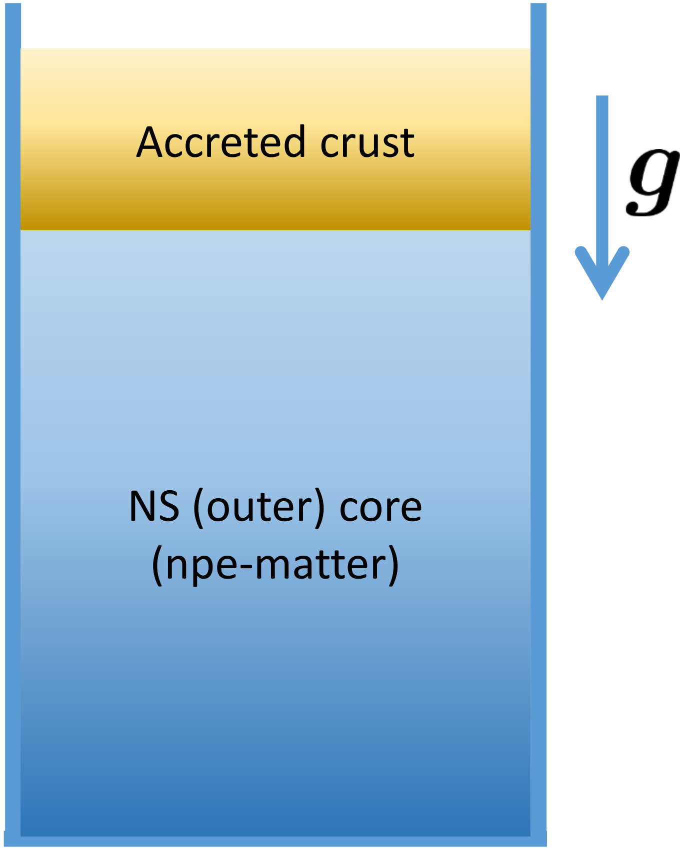
$$E_{II}(N_b+\delta N_b)=E(N_b)+\mu^{\infty}_{b,\,\mathrm{core}}\delta N_b$$

Which answer is the correct one?

Heating efficiency: general consideration



Plane-parallel consideration (for simplicity of the talk)

Thought experiment I:

«The accretion process»

$$E_I(N_b+\delta N_b)=E(N_b)+\mu^{\infty}_{b,\,\mathrm{H}}\delta N_b$$

Thought experiment II:

«The result of accretion»

$$E_{II}(N_b+\delta N_b)=E(N_b)+\mu^{\infty}_{b,\,\mathrm{core}}\delta N_b$$

Which answer is the correct one?

Both are accurate!!!

The heat was released in the first experiment!

$$Q=E_I(N_b+\delta N_b)-E_{II}(N_b+\delta N_b)=\left(\mu^{\infty}_{b,\,\mathrm{H}}-\mu^{\infty}_{b,\,\mathrm{core}}\right)\delta N_b$$

Heating efficiency: general consideration

The result is the same for a spherical general relativistic star

M.E. Gusakov & AIC [Phys. Rev. D:Letters, 103 (2021), L101301]

$$\mu_{b,\mathrm{H}}^{\infty} = m_{b,\mathrm{H}}\, e^{\nu_\mathrm{s}/2}$$

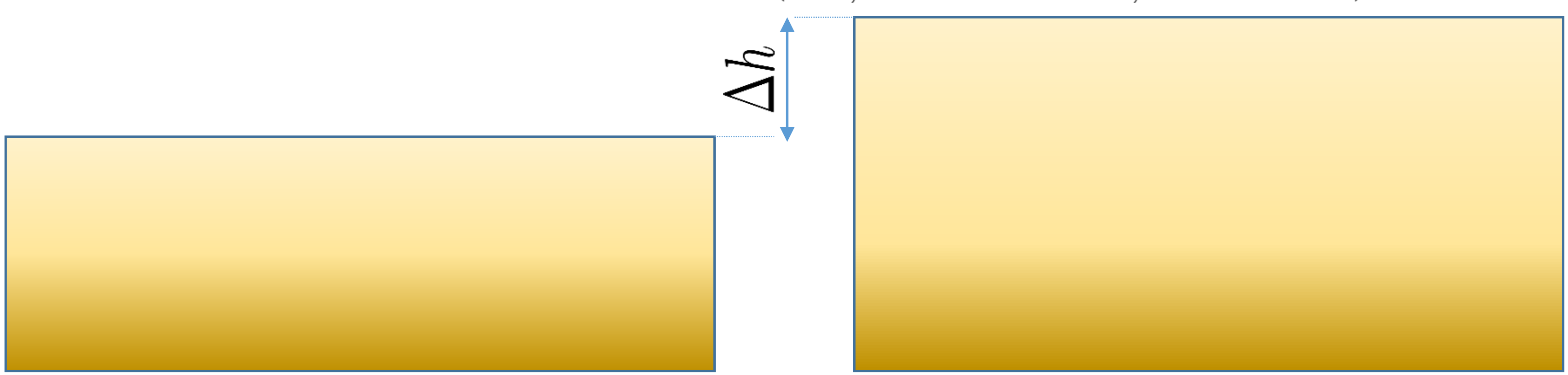
$$\mu_{b,\,\mathrm{core}}^{\infty} = m_{b,\mathrm{Fe}}\, e^{\nu_\mathrm{s}^{(\mathrm{cat})}/2}$$

$$\begin{aligned} q_{\mathrm{tot}}^{\infty} &\equiv Q/N_\mathrm{b} = m_{b,\mathrm{H}} e^{\nu_\mathrm{s}/2} - \mu_{b,\mathrm{core}}^{\infty} \\ &= \left(m_{b,\mathrm{H}} - \overline{m}_{b,\mathrm{ash}}\right) e^{\nu_\mathrm{s}/2} + \left(\overline{m}_{b,\mathrm{ash}}\, e^{\nu_\mathrm{s}/2} - \mu_{b,\mathrm{core}}^{\infty}\right) \end{aligned}$$

The «nuclear» part, associated with thermonuclear burning to the ashes (≈ ^{56}Fe).
Released from the surface, does not heat up the NS core

The gravitational energy, associated with larger thickness of the accreted crust. Released in the depths of the crust, heats up NS core

$$\left(m_{b,\mathrm{Fe}}\, e^{\nu_\mathrm{s}/2} - m_{b,\mathrm{Fe}}\, e^{\nu_\mathrm{s}^{(\mathrm{cat})}/2}\right) \approx m_U\, g\, \Delta h$$



Catalyzed crust

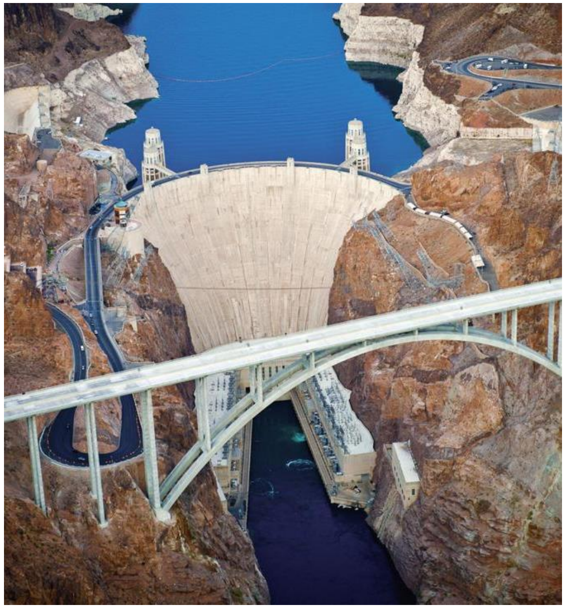
Fully accreted crust

Heating efficiency: general consideration

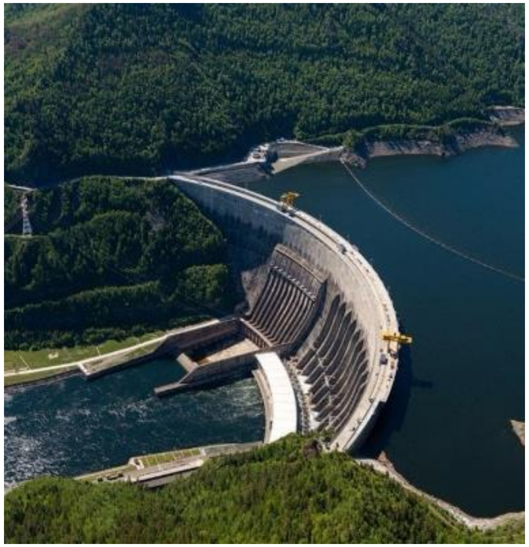
$$q \approx m_u\, g\, \Delta h \Rightarrow \Delta h \approx 25\ \mathrm{m}\, \frac{q}{0.5\ \frac{\mathrm{MeV}}{\mathrm{nucleon}}} \left(\frac{g}{2\times 10^{14}\ \frac{\mathrm{cm}}{\mathrm{s}^2}}\right)^{-1}$$

Within traditional one component approach derived by Zdunik et al. [A&A, 599 (2017), 119]

Deep crustal heating: conversion of the gravitational energy into the heat
(in some sense similar to the gravity dams)



Hoover dam
221 m



Sayano-Shushenskaya Dam
242 m

Three Gorges Dam
181 m

Heating efficiency: nHD crust

$$q^{\infty} = \overline{m}_{b,\mathrm{ash}}\, e^{\nu_{\mathrm{s}}/2} - \mu^{\infty}_{b,\mathrm{core}}$$

$$\mu^{\infty}_{b,\mathrm{core}} = \mu_b e^{\nu_{\mathrm{cc}}/2} = \mu_n e^{\nu_{\mathrm{cc}}/2} = m_n e^{\nu_{\mathrm{oi}}/2}$$

$$q^{\infty} = \boxed{\overline{m}_{b,\mathrm{ash}}\, e^{\nu_{\mathrm{s}}/2} - \mu_{b,\mathrm{o}}(P_{\mathrm{oi}})\, e^{\nu_{\mathrm{oi}}/2}} + \mu_{b,\mathrm{o}}(P_{\mathrm{oi}})\, e^{\nu_{\mathrm{oi}}/2} - \mu_n\, e^{\nu_{\mathrm{oi}}/2}$$

q^{∞}_{o}

Baryon chemical potential at the bottom of outer crust

$$q^{\infty} = e^{\nu_{\mathrm{oi}}/2}\left\{q_{\mathrm{o}} + \left[\mu_{b,\mathrm{o}}(P_{\mathrm{oi}}) - m_n\right]\right\}$$

For nHD crust the energy release is given by

- EOS in the **outer** crust
- Pressure at the outer-inner crust interface P_{oi}

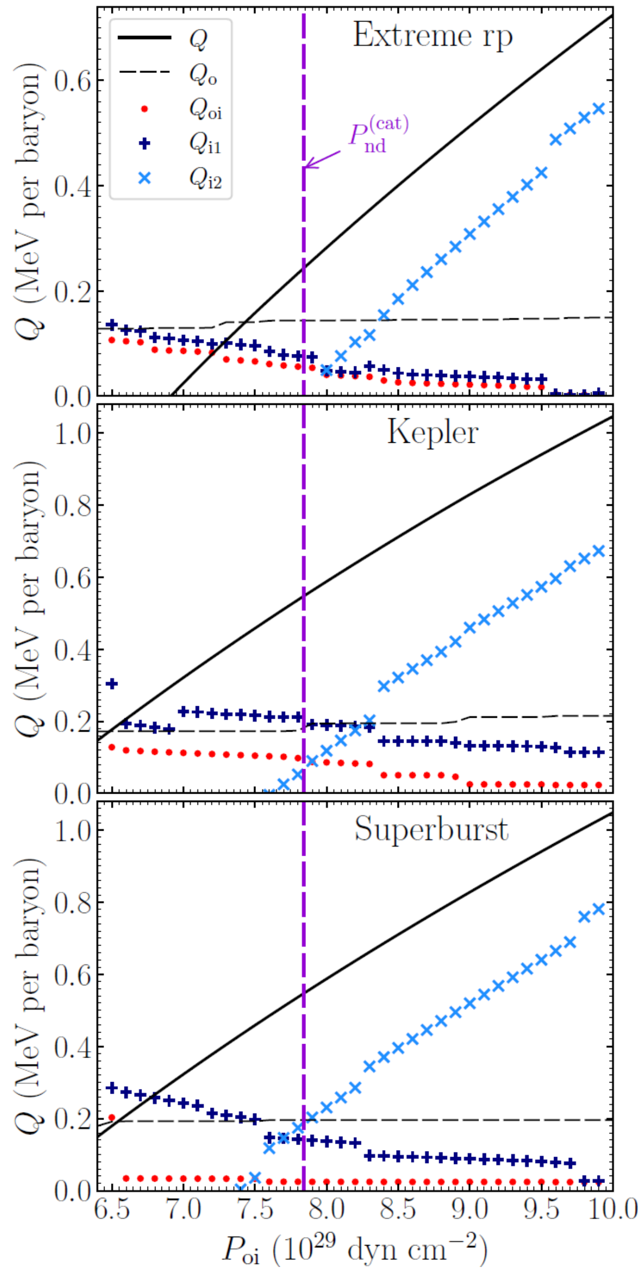
(details on the inner crust EOS are not required!)

Heating efficiency: nHD crust

$$q \lesssim 0.5 \frac{\text{MeV}}{\text{nucleon}}$$

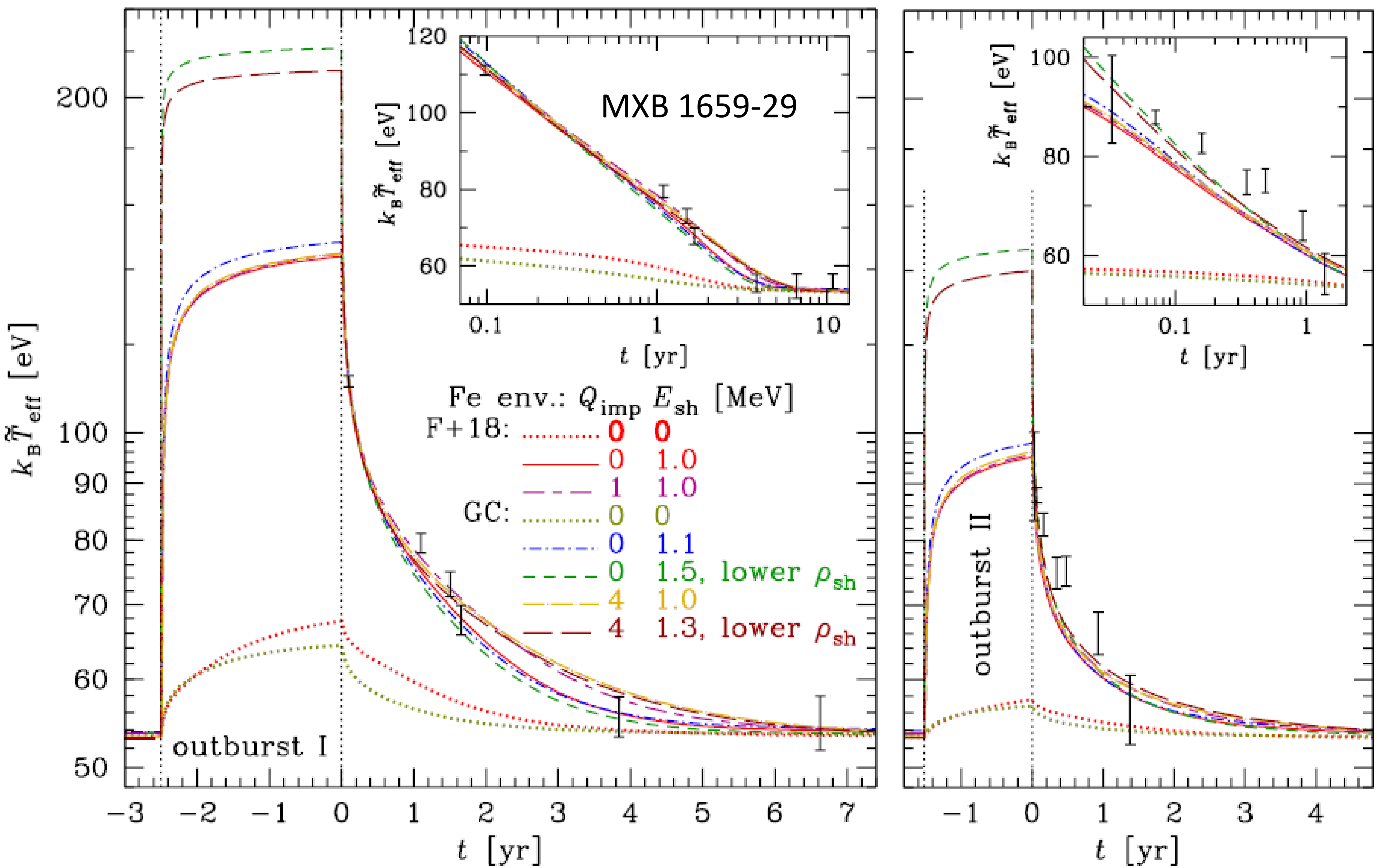
Deep crustal heating is factor of few less efficient,

than it was supposed in traditional models!



Shchechilin, Gusakov, AIC,
MNRAS: Letters, 515 (2022), L6

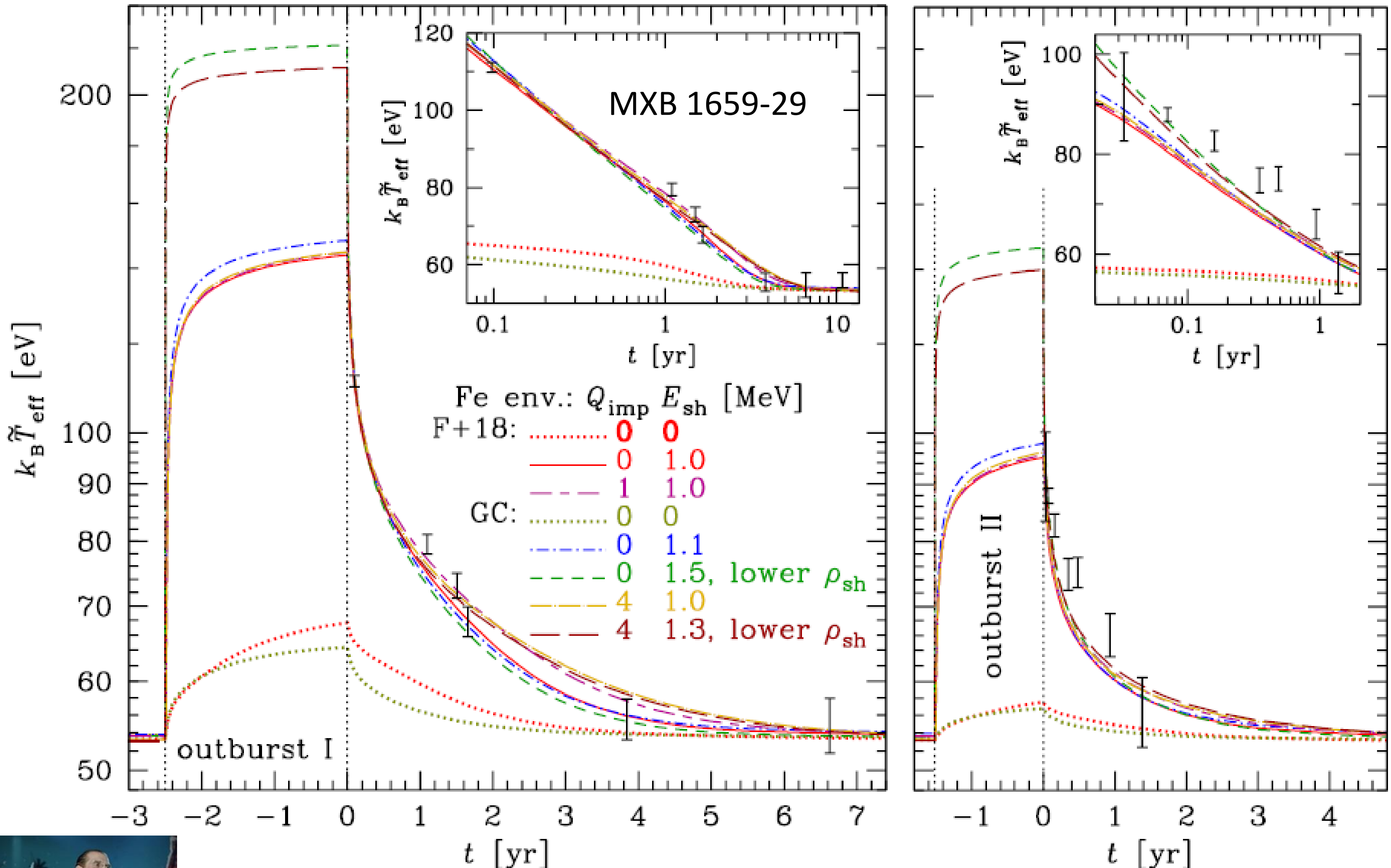
Can we explain observations?

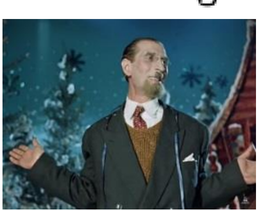


Potekhin et al., MNRAS522 (2023), 4830

Additional heating is shallow layers is required (as before)

Can we explain observations?





Our model is simpler, than it is possible….
We don't know nature of this source….

Potekhin et al., MNRAS522 (2023), 4830

Can we build NS model without specifying crust?

Zdunik et al., A&A 599 (2017), A119

Tolman-Oppenheimer-Volkoff equations

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G\rho m}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{mc^2}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1},$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho,$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}r} = \frac{2Gm}{c^2\, r^2}\left(1+\frac{4\pi P r^3}{mc^2}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1}.$$

Thermodynamics: $\mu_b^\infty = \mu_b \exp(\nu/2) = \mathrm{const}$

$$\mathrm{d}P = n_b \mathrm{d}\mu_b;\ \ \rho c^2 + P = \Phi = n_b \mu_b$$

$$n_b \frac{\mathrm{d}\mu_b}{\mathrm{d}r} = \frac{\mathrm{d}P}{\mathrm{d}r} = -2\frac{\rho c^2 + P}{\rho c^2}\frac{\mathrm{d}\nu}{\mathrm{d}r} = 2 n_b \mu_b \frac{\mathrm{d}\nu}{\mathrm{d}r}$$

Can we build NS model without specifying crust?

Zdunik et al., A&A 599 (2017), A119

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Thermodynamics: $\mu_b^\infty = \mu_b \exp(\nu/2) = \mathrm{const}$

Crust-core interface: well predicted by instability of uniform matter ➡ μ_b^{cc}

Surface: ground state composition is ^{56}Fe $\mu_b^{\mathrm{surf}} = m(^{56}\mathrm{Fe})/56$

<u>Approximation:</u>

Neglect mass of the crust => vacuum solution for redshift within the crust $\nu = \log(1-2GM/c^2 r)$

Can we build NS model without specifying crust?

Zdunik et al., A&A 599 (2017), A119

Tolman-Oppenheimer-Volkoff equations

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G\rho m}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{mc^2}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1},$$

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Surface: ground state composition is ^{56}Fe $\mu_b^{\mathrm{surf}} = m(^{56}\mathrm{Fe})/56$

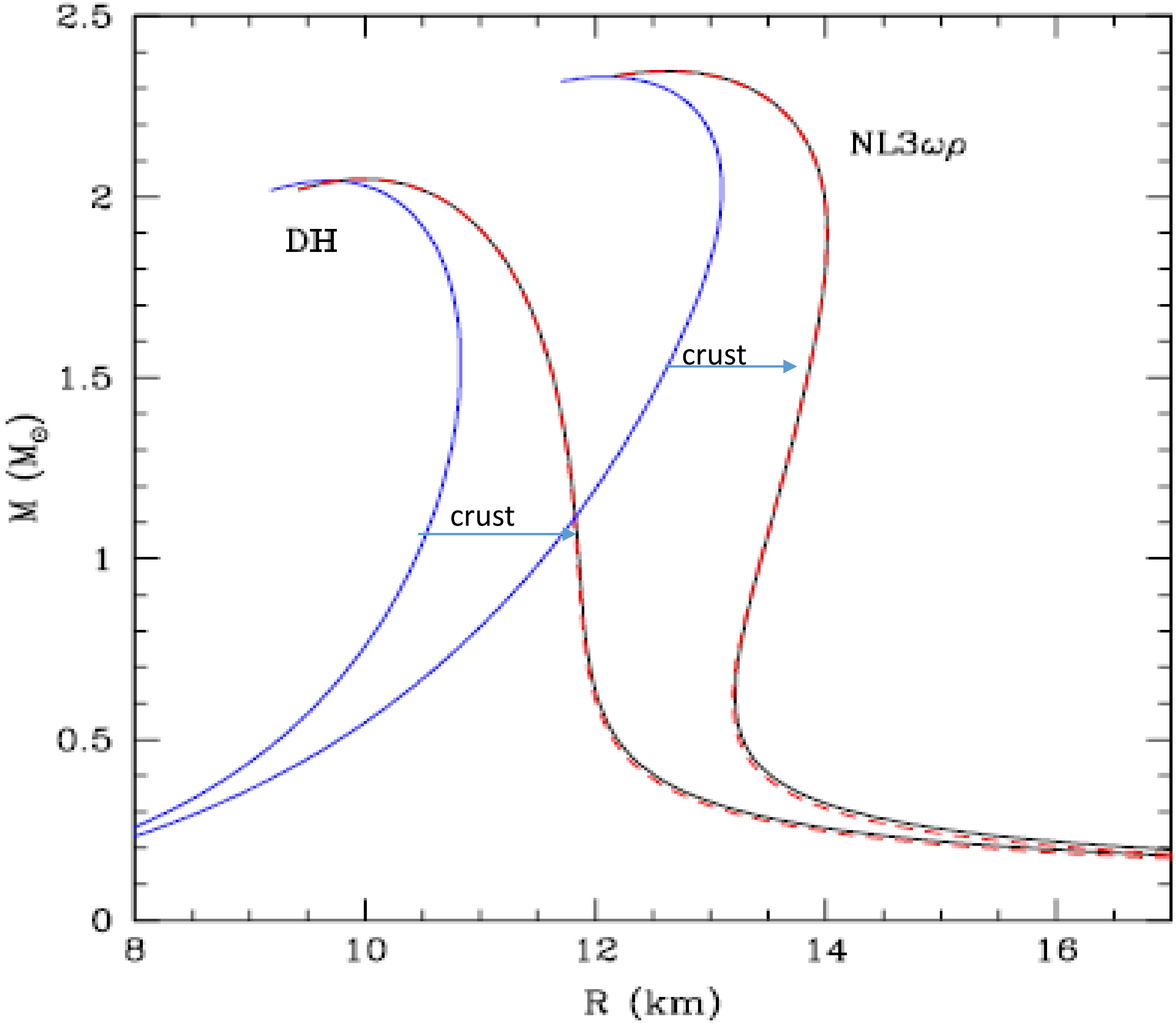
Neglect mass of the crust => vacuum solution for redshift $\nu = \log(1 - 2GM/c^2 r)$

For given central density calculate mass and radius of the core $m^{\mathrm{cc}},\ r^{\mathrm{cc}}$ $\int \rho \mathrm{d}r$

Determine NS radius R by equation: $\dfrac{\mu_b^{\mathrm{surf}}}{\sqrt{1-2GM/c^2R}} = \dfrac{\mu_b^{\mathrm{cc}}}{\sqrt{1-2GM/c^2 r^{\mathrm{cc}}}}$

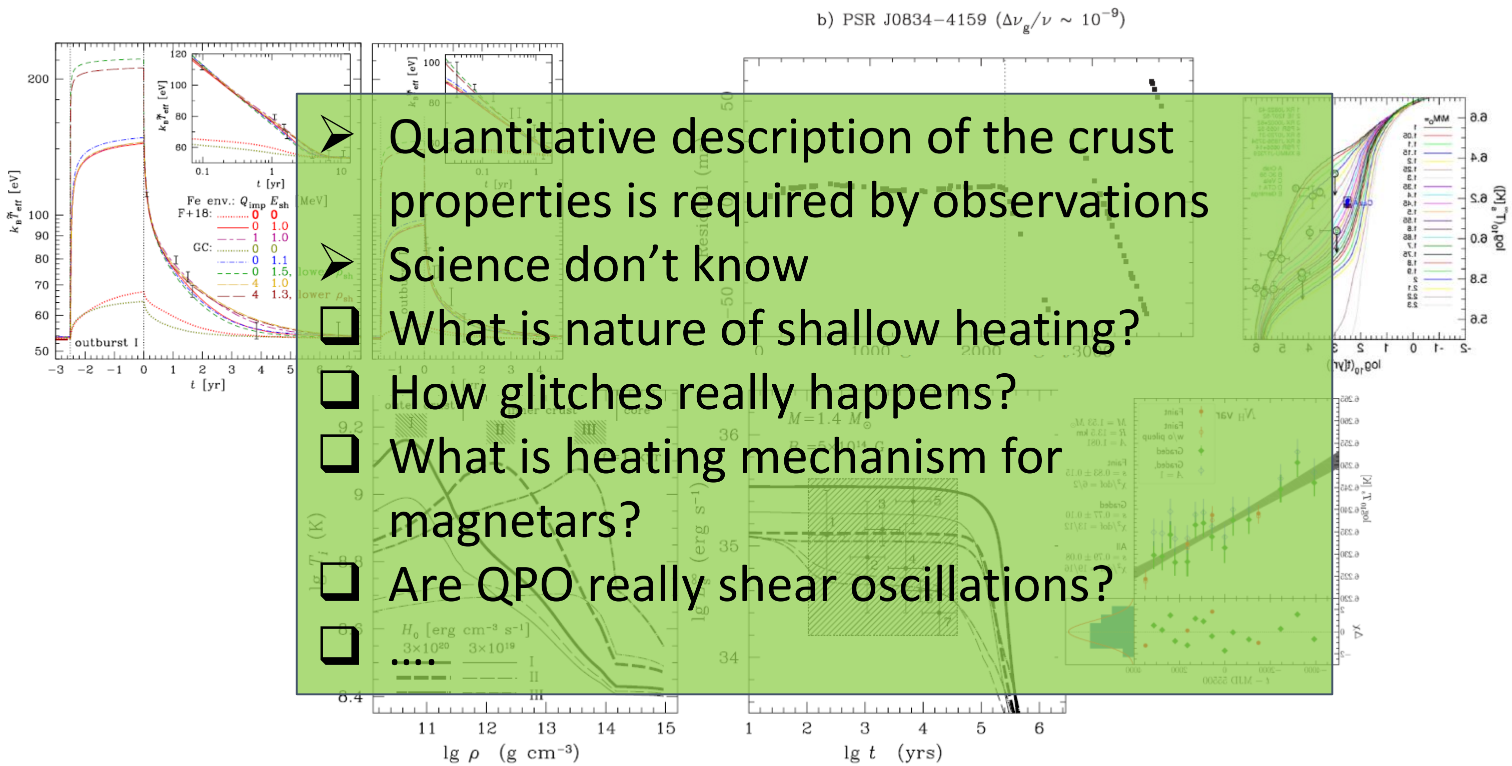
Check for accuracy: $M = m^{\mathrm{cc}}$ vs $M = m^{\mathrm{cc}} + M^{\mathrm{crust}}$, where $M^{\mathrm{crust}} \approx 4\pi R^2 \left(\dfrac{P^{\mathrm{cc}}}{g^{\mathrm{cc}}}\right)$

Can we build NS model without specifying crust?



○ Zdunik et al., A&A 599 (2017), A119

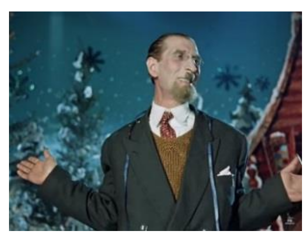
Future of crust physics



- Quantitative description of the crust properties is required by observations
- Science don't know
- ❑ What is nature of shallow heating?
- ❑ How glitches really happens?
- ❑ What is heating mechanism for magnetars?
- ❑ Are QPO really shear oscillations?
- ❑ ….

D.G. Yakovlev, HEA2017(?)

Crust as Cinderella of NS

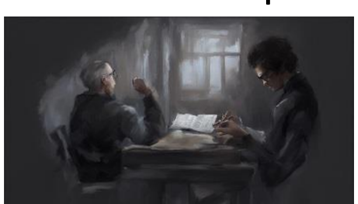


Summary

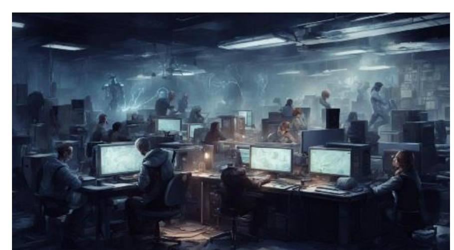
"Everything should be made as simple as possible, but not simpler"

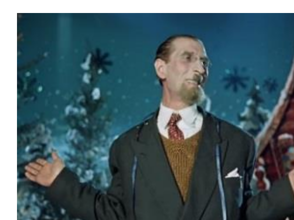
Attributed to Albert Einstein

- Physics of NS crust deals with almost the same problems as terrestrial solid state physics. Similarly, the (unattainable) goal is to describe all properties on the base of ab initio simulations, and we done it for OCP (but don't forget the limitations). Generally we need to appeal to simplified models.
- Some models (degenerate electrons, ion sphere models, effective shear modulus) can be so simple, that allow for analytical solution (nowadays). It's amazing.
- We need carefully inspect models: Even a small details can reveal inconsistencies/errors. If it is a case, one should make a back step
- The progress in observations put a tighter requirements for the models: models which was 'possible' yesterday can become too simple tomorrow. We need to inspect our models.
- Observations of accreting neutron reveal that our models are simpler, than it is possible. We need to work!



One can calculate M(R) without dealing with crust





Questions?

- Who is comrade Nikodimov?

- Do I understand correctly, that outer crust is composed of fully ionized nuclei and degenerate electrons?
- Do I understand correctly, that unbound neutrons are present in inner crust?
- Do I understand correctly, one should include crust while calculating M(R)?
- Do I understand correctly, that pressure is monotonically increasing with depth?
- Do I understand correctly, that density is monotonically increasing with depth?
- Do I understand correctly, that pressure is smooth function of radius?
- Do I understand correctly, that density is smooth function of radius?

- Are there a density jump between outer and inner crust?

- You demonstrate plot there different authors predict different result within CLDM. Why different results can be obtained within one model?

- Are there any heat realize in the crust of accreting neutron star?
- You mentioned instability. Are neutron stars stable?
- Are you sure that mantle is located in all neutron stars?

- Are there any test for a huge set of complicated models, presented in your lections?