# QCD matter on a lattice

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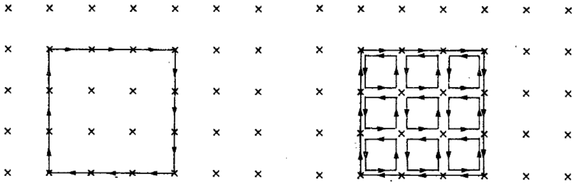
1 February, 2024

# Applications

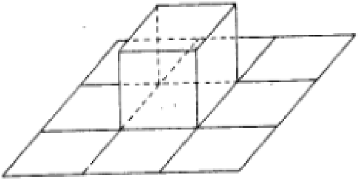
- Spectroscopy
- Matrix elements and correlations functions
- Thermodynamic properties of QCD
- Transport properties of QCD
- Phase transitions
- Nuclear physics
- Properties of QCD under extreme conditions (magnetic field, baryon density, relativistic rotation,...)
- Topological properties
- ...

# Wilson loop at strong coupling



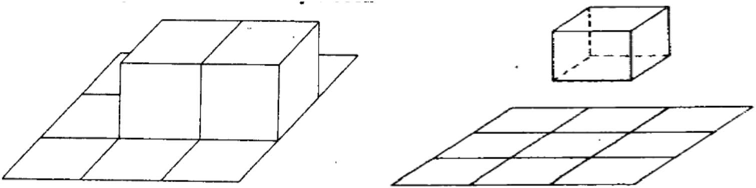
- $S_l = -\sum_p \frac{\beta}{6}(TrU_P + TrU_P^+), \ \beta = \frac{6}{g^2}$
- Strong coupling limit $g \to \infty$, i.e. $\beta \to 0$
- $Z = \int DU e^{-S_l} = \int DU \left(\sum_n \frac{1}{n!}(-S_l)^n\right)$
  $\langle W(C)\rangle = \frac{1}{Z}\int DU \prod_C U_\mu(n)\left(\sum_n \frac{1}{n!}(-S_g)^n\right)$
- $\int dU = 1, \quad \int dUU_{ij} = 0, \quad \int dUU_{ij}U_{kl}^{-1} = \int dUU_{ij}U_{kl}^+ = \frac{1}{3}\delta_{il}\delta_{jk}$
- $W(C) = \left(\frac{\beta}{18}\right)^{S_l} = e^{-\sigma_l S_l}, \quad \sigma_{phys} = \frac{\sigma_l}{a^2} = -\frac{1}{a^2}\log\left(\frac{\beta}{18}\right)$

# Wilson loop at strong coupling



- $W(C) = \left(\frac{\beta}{18}\right)^{S_l} \left(1 + 4S_l(\frac{\beta}{18})^4\right) = \left(\frac{\beta}{18}\right)^{S_l} e^{4S_l(\beta/18)^4}$
- $W(C) = \left(\frac{\beta}{18}\right)^{S_l} \left(1 + 4S_l(\frac{\beta}{18})^4 + 12S_l(\frac{\beta}{18})^5 + \right) = \left(\frac{\beta}{18}\right)^{S_l} e^{4S_l(\beta/18)^4 + 12S_l(\beta/18)^5}$

# Wilson loop at strong coupling

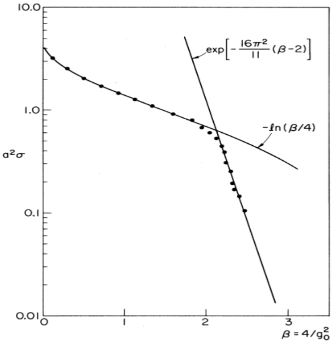


- $\sigma_{phys} a^2 = -log u - 4u^4 - 12u^5 + 10u^6 + ..., \quad u = \frac{\beta}{18}$
- $V(r) = \sigma_{phys} r$
- There is confinement at strong coupling

# Renormalization group at strong coupling

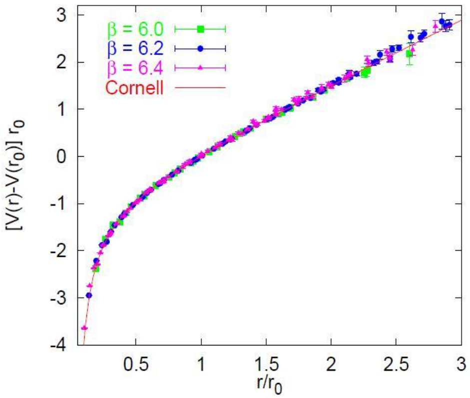
- RG: different lattice spacings give the same physics
  $\{\beta_1, a_1\}$, $\{\beta_2, a_2\}$, $\{\beta_3, a_3\}$, ... but $\sigma_{phys} = \frac{\sigma_l(g(a))}{a^2}$
- $\sigma_{phys} a^2 = -log u - 4u^4 - 12u^5 + 10u^6 + ..., \quad u = \frac{\beta}{18} = \frac{1}{3g^2}$
- $\beta$-функция: $\quad \frac{d\sigma_{phys}}{da}\Big|_{a \to 0} = 0$
  $\frac{dg^2}{dlna^2} = g^2 \ln 3g^2$
- $g$ decreases in the continuum limit
  $g\Big|_{a \to 0} \to \frac{1}{\sqrt{3}}$

# Correct continuum limit



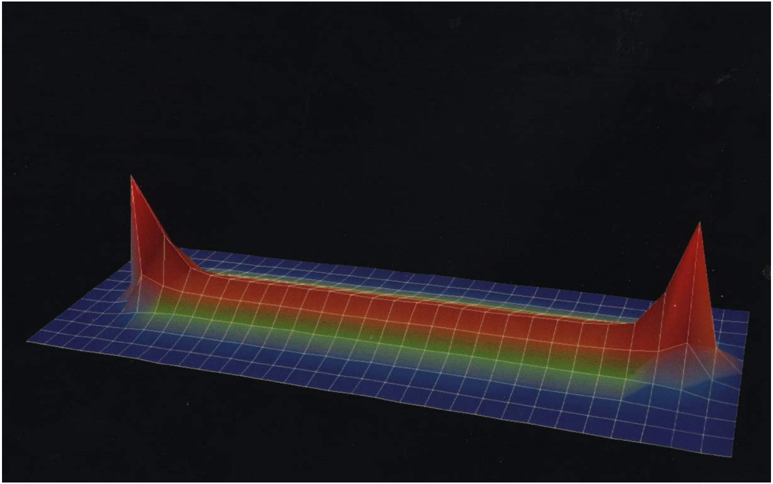

- Weak coupling limit $g \to 0$
- $\sigma_{phys} = \frac{1}{a^2} f(g), \quad \left.\frac{d\sigma_{phys}}{dloga^2}\right|_{a\to 0} = 0$
- $-f(g) + \frac{df}{dg}\frac{dg}{d\log a^2} = 0, \quad \frac{dg}{d\log a^2} = \frac{11}{6}\frac{N}{16\pi^2} g^3$
- $\sigma \sim \frac{1}{a^2} \exp\left(-\frac{6\pi^2}{11}\beta\right), \quad \beta = \frac{4}{g^2}$

# Confinement in lattice simulation

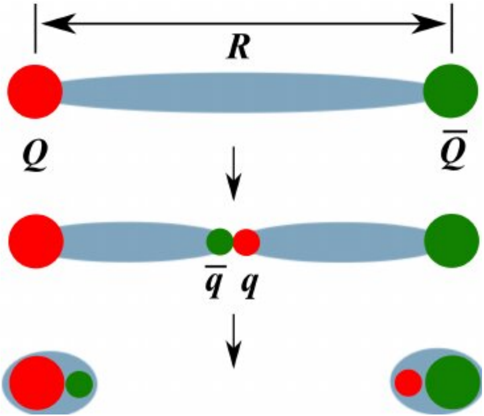



- Small distances: $V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r}$
  Asymptotic freedom $\alpha_s(r) \sim -\frac{1}{\log \Lambda r}|_{r\to 0} \to 0$
- Large distances $V(r) = \sigma_{phys} r$ - Confinement
  $F = \sigma \simeq 160000$ N
- To separate quarks one needs infinite energy

# Confinement in lattice simulation

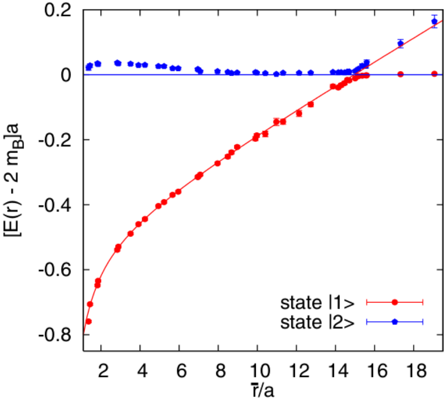


# String breaking



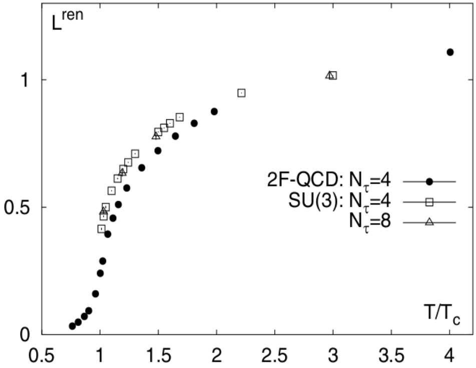

from arXiv:1001.0570

# String breaking




- The string is not broken
- The string is broken

# Polyakov line




- Fermions violate $Z_3$ symmetry

# QCD vacuum

- Is vacuum an empty space ($\epsilon = 0$)?

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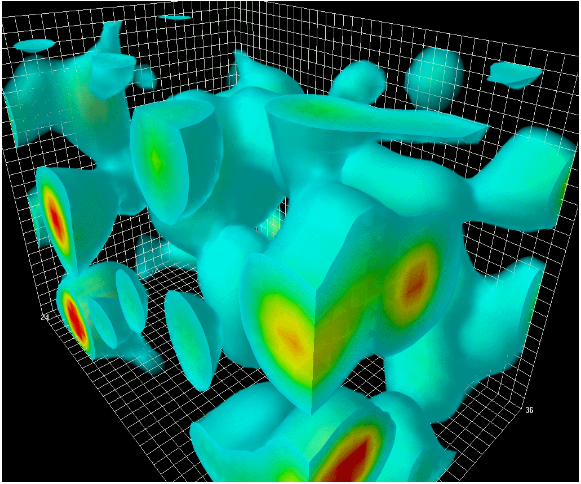
- Is vacuum an empty space ($\epsilon = 0$)?
- Vacuum is the state with the smallest energy
- QCD vacuum: $\epsilon \simeq -(265\ MeV)^4$, $\langle H^2 + E^2 \rangle \neq 0$

# QCD vacuum



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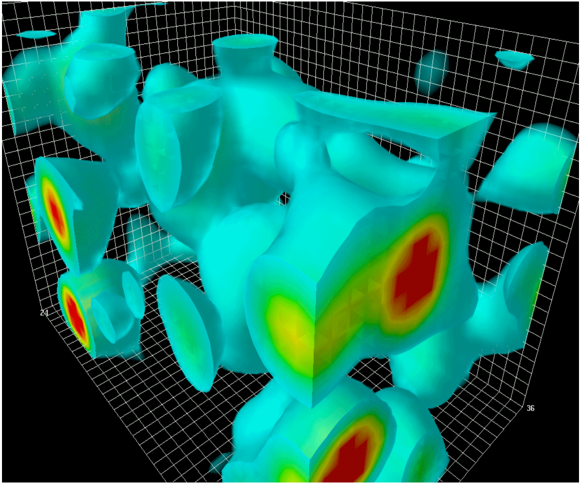

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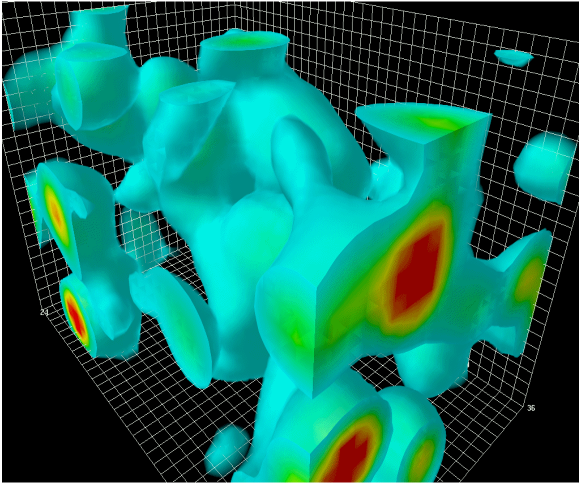

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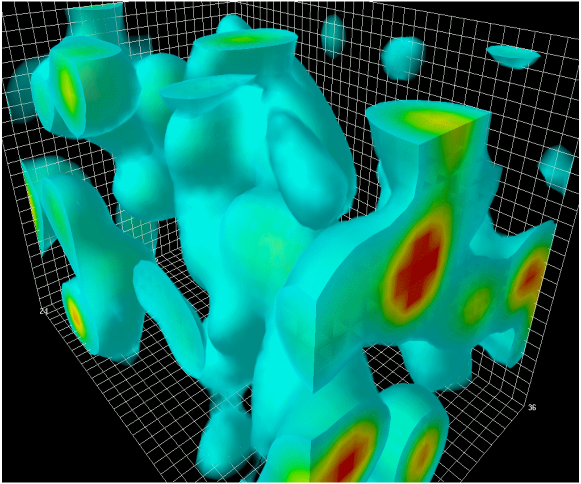

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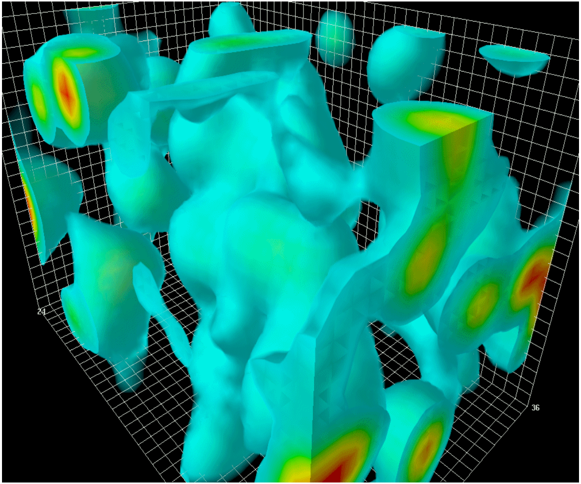

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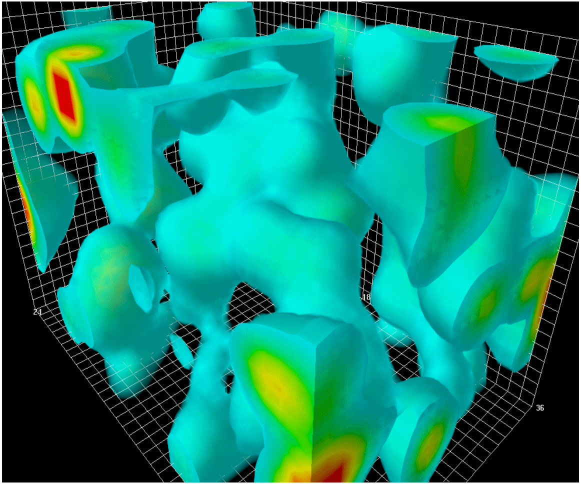
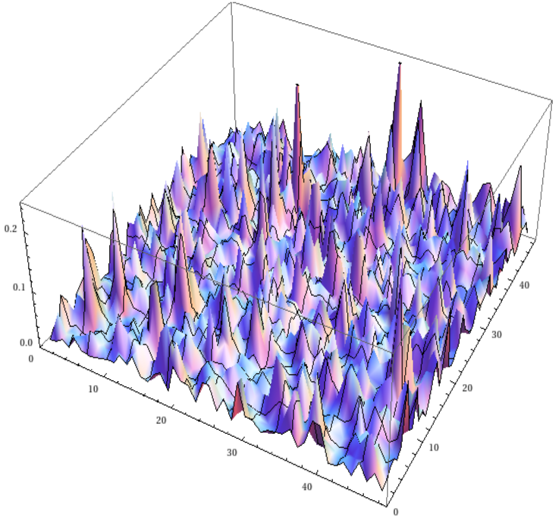
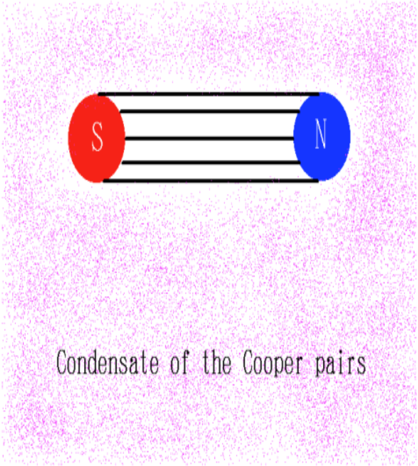


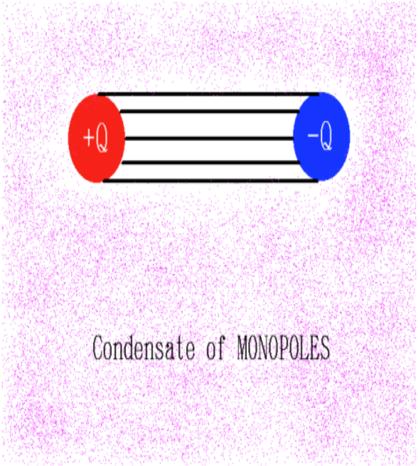
# Quantum (ultraviolet) fluctuations in QCD vacuum




- Classical vacuum is distorted by UV fluctuations
- The fluctuations take place at distances $\sim a$

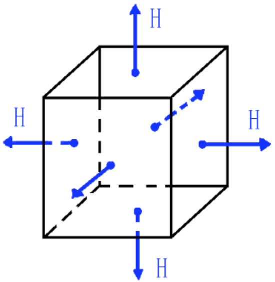
# Model of dual superconductor



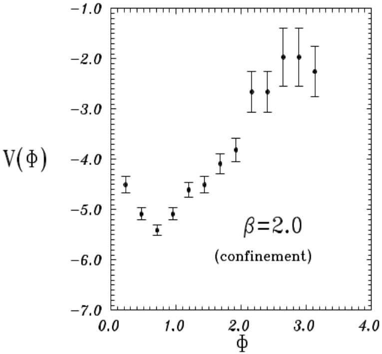



# What is a monopole in SU(N)?

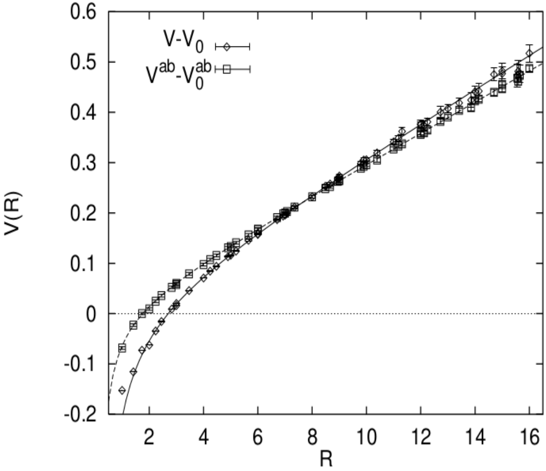
- Consider SU(2) as an example
- $2\hat{A}_\mu = \sigma^1 A^1_\mu + \sigma^2 A^2_\mu + \sigma^1 A^3_\mu = \begin{pmatrix} A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu \end{pmatrix}$
- SU(2)$\rightarrow$U(1)
- $2\hat{A}_\mu = \sigma^3 A^3_\mu = \begin{pmatrix} A^3 & 0 \\ 0 & -A^3 \end{pmatrix}$
- One can introduce: $\vec{E}^3, \vec{H}^3$
- Diagonalization in the whole space is impossible
- Maximal Abelian gauge: $min_S \int d^4x[(A^1_\mu)^2 + (A^2_\mu)^2]$



# Condensation of monopoles



# Abelian dominance




- $W(C) = Tr P \exp\left(i \int_C dx^\mu \hat{A}_\mu\right) \to \exp\left(i \int_C dx^\mu A^3_\mu\right)$

# The other variants of QCD vacuum

- Spaghetti vacuum
- Fluctuation of topology
- Self-dual fields (Sergey Nedelko)
- ...

# Chiral symmetry breaking

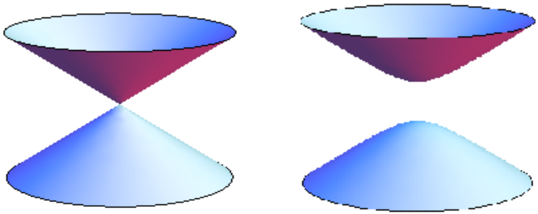
- Left and right sectiors of the theory do not interact

$$\mathcal{L} = \bar{\Psi} i\hat{D}\Psi = \bar{\Psi} i\hat{D}\left(\frac{1+\gamma_5}{2}+\frac{1-\gamma_5}{2}\right)\Psi = \bar{\Psi} i\hat{D}\frac{1+\gamma_5}{2}\Psi + \bar{\Psi} i\hat{D}\frac{1-\gamma_5}{2}\Psi = \bar{\Psi}_R i\hat{D}\Psi_R + \bar{\Psi}_L i\hat{D}\Psi_L$$

- For $N_f$ quarks chiral symmetry is $SU_L(N_f)\times SU_R(N_f)\times U_V(1)\times U_A(1)$
- Chiral condensate $\langle\bar{\Psi}\Psi\rangle = \langle\bar{\Psi}_L\Psi_R\rangle + \langle\bar{\Psi}_R\Psi_L\rangle$ breaks chiral symmetry
- Dynamical chiral symmetry breaking $SU_L(N_f)\times SU_R(N_f)\to SU_V(N_f)$
- The mechanism of chiral symmetry breaking is unknown
- Some ideas can be gained from NJL model

$$\mathscr{L}_S = \bar{\psi}\left[i\partial\!\!\!/ + g\left(\sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5\right)\right]\psi + \frac{1}{2}\left[(\partial\boldsymbol{\pi})^2 + (\partial\sigma)^2\right] - \frac{\mu^2}{2}\left(\sigma^2+\boldsymbol{\pi}^2\right) - \frac{\lambda}{4}\left(\sigma^2+\boldsymbol{\pi}^2\right)^2.$$

# Chiral symmetry breaking



- NJL models are based on BCS theory
- The interaction term $(\bar{\psi}\psi)^4$
- $\alpha_{NJL} < 1$ no solutions, $M = 0$, $E^2 = \vec{p}^2$
- $\alpha_{NJL} > 1$ there is solution $M \neq 0$, $E^2 = \vec{p}^2 + M^2$
- Gap equation for the dynamical mass
- Dynamical symmetry breaking
- The condensate of Cooper pairs: $\langle \bar{\psi}\psi \rangle \neq 0$
- Too simple model: no confinement

# What is matter composed of?

- The following law is well satisfied in nature
  $M \simeq \sum_i M_i$
- In QCD
  $p(uud) \quad M_p c^2 = 938 \text{ MeV} \gg (m_u + m_u + m_d)c^2 = 12 \text{ MeV}$
  $n(udd) \quad M_n c^2 = 940 \text{ MeV} \gg (m_u + m_d + m_d)c^2 = 15 \text{ MeV}$

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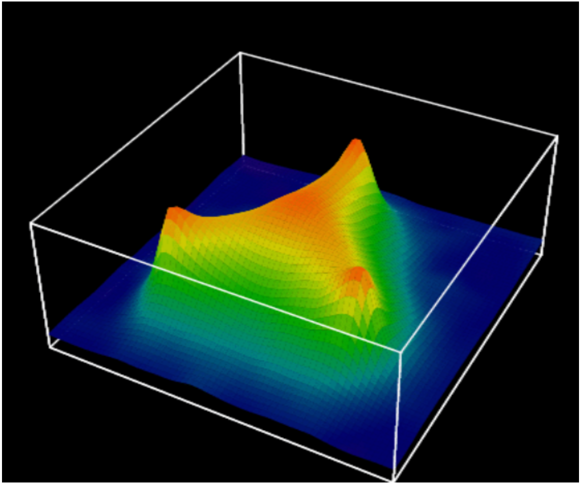
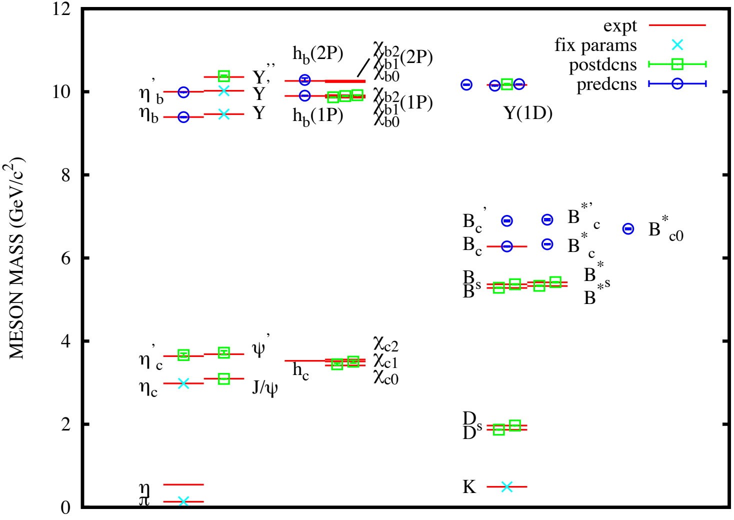
  $n(udd) \quad M_n c^2 = 940 \text{ MeV} \gg (m_u + m_d + m_d)c^2 = 15 \text{ MeV}$
- Where is the rest of mass?

# Chromoelectric fields in proton

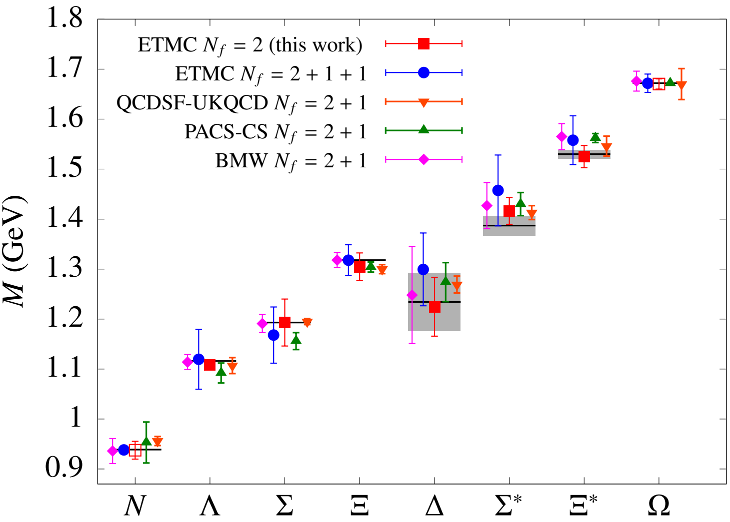


- We are composed of gluons to 98%!

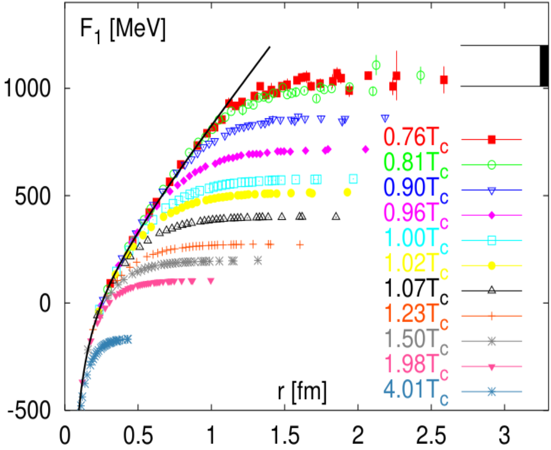
# Spectroscopy: Mesons



# Spectroscopy: Baryons

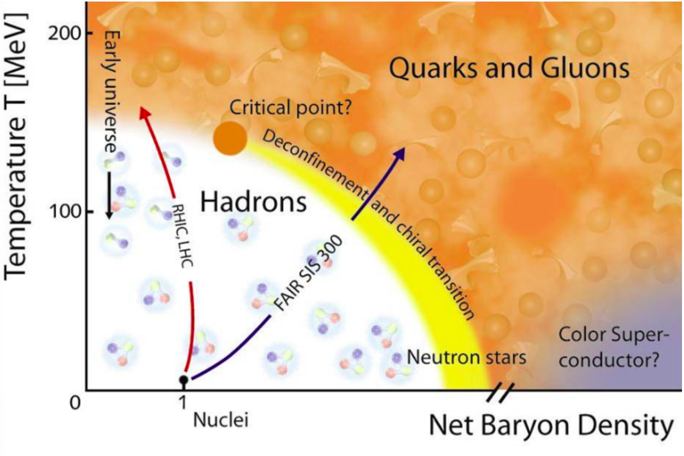


# Static potential at finite temperature



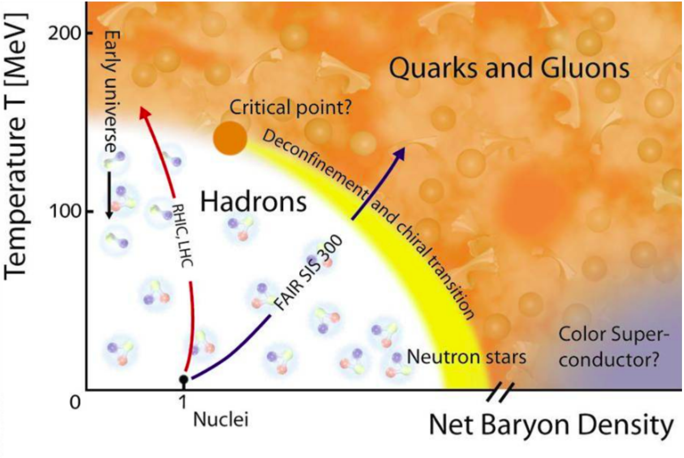

- One needs the temperature
  $T \sim 150$ MeV $\sim 1.5 \times 10^{12}$ degrees

# QCD under extreme conditions



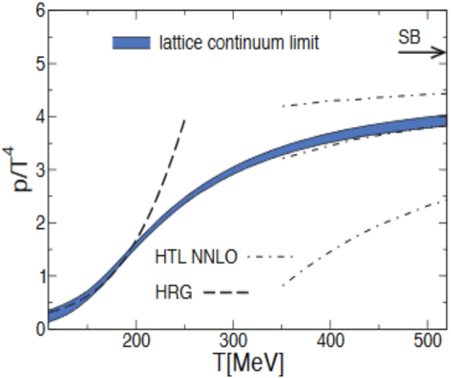

- Modern experiments: **LHC**(Switzerland), **RHIC**(USA), **FAIR**(Germany), **NICA**(Russia, Dubna, JINR)

# QCD under extreme conditions




- Temperature $T \sim 150$ MeV $\sim 1.5 \times 10^{12}$ degrees
- Baryon density $n > n_0$
- Magnetic fields $eB \sim 10^{13}$ T
- Rotation with angular velocity $\omega \sim 10^{22}$ c$^{-1}$
- ...

# QCD equation of state




- Low temperature: HRG
- High temperature: SB - Stefan Boltzmann: $p = \sigma T^4$
- At very high temperature QGP is gas of quarks and qluons?

# High temperatures

- Consider a limit $T \to \infty$
- $g(T) \to 0 \Rightarrow$ QGP is a free gas of quarks and gluons?
- Matsubara frequency:
  - For bosons: $p_0 = 2\pi T n$
  - For fermions: $p_0 = \pi T(2n+1)$
- The propagator at nonzero temperature
  $G_0(x,y) = T\sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \frac{e^{ip(x-y)}}{(2\pi T n)^2 + \vec{p}^2 + m^2}$
- Fermions and $n \neq 0$ bosons can be integrated out

# High temperatures

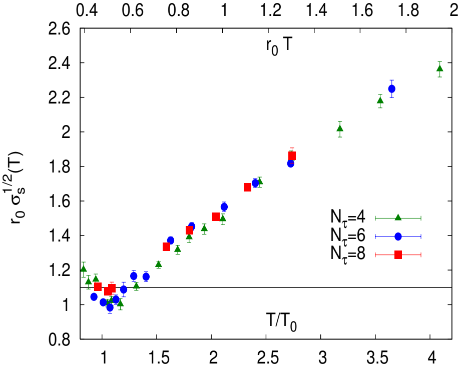
- We are left with $A_4, A_i$ at $n = 0$
- Propagator
  $G_0(x,y) = T\sum_{n=-\infty}^{+\infty}\int \frac{d^3p}{(2\pi)^3}\frac{e^{ip(x-y)}}{(2\pi Tn)^2+\vec{p}^2+m^2}\Big|_{T\to\infty} \to$
  $\int \frac{d^3p}{(2\pi)^3}\frac{e^{i\vec{p}(\vec{x}-\vec{y})}}{\vec{p}^2+m^2}$
- Modes with $n = 0$ becomes tree dimentional
- Dimensional reduction $D = 4 \to 3$
- Effective action for $A_4, A_i$
  $S = \int d^3x \left( \frac{1}{2}Tr(F_{ij}^2) + Tr((D_iA_4)^2) + m_D^2 Tr(A_4^2) + ... \right)$
- $\langle P(\vec{x})\rangle = TrP\exp\left(i\int_0^T dx^4 \hat{A}_4(\vec{x},x^4)\right) \neq 0$
- $A_4$ is like a Higgs: $\langle A_4\rangle \neq 0$

# High temperatures

- Debye mass $m_D \sim g(T)T$ is a scale for $A_4$
- In the limit $T \to \infty$ magnetic scale $g^2(T)T$
- $A_4$ can be integrated out
- 4D QCD is equvivalent to 3D Yang-Mills with chromo-magnetic degrees of freedom

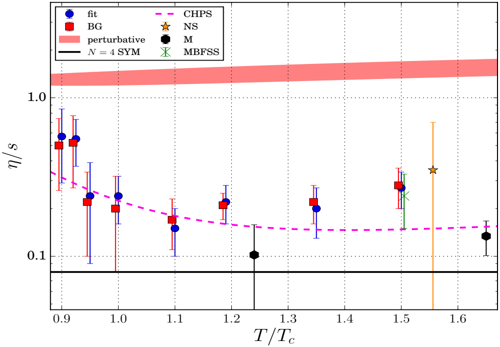
$$S = \int d^3x \left( \frac{1}{2} Tr(F_{ij}^2) + ... \right)$$

- Nonperturbative theory with spatial confinement
- The scale is determined by spatial string tension $\sigma_s$
- QCD in nonperturbative at any temperature! It never becomes a gas of quarks and gluons!



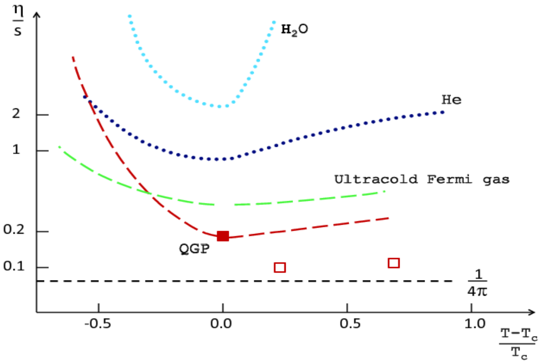

- Spatial string tension $W(C) \sim e^{-\sigma_s S}$
- Confinement of magnetic field
- In the region $T > T_c$, $\sigma_s$ rises as $\sim T$

# Shear viscosity of QGP




- QGP is close to the ideal liquid ($\frac{\eta}{s} = (1-3)\frac{1}{4\pi}$)
- Considerable deviation from gas of quarks and gluons
- The result is close to the N=4 SYM $\frac{\eta}{s} = \frac{1}{4\pi}$

# Shear viscosity of QGP




- QGP is the most superfluid liquid

THANK YOU!