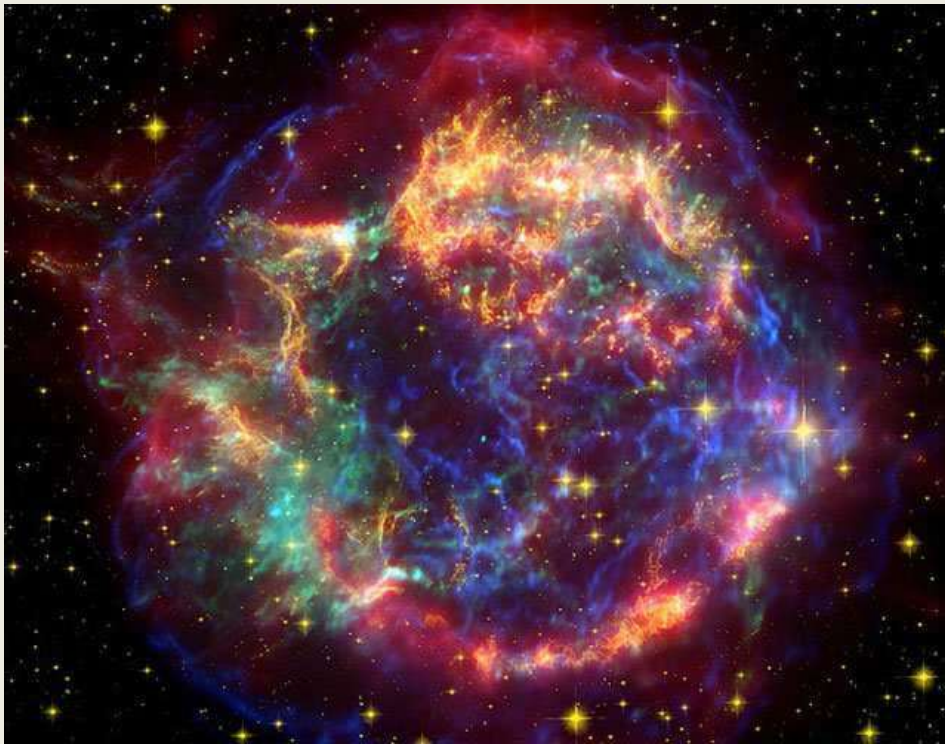


# Neutron Stars Mass estimations from Cooling Evolution



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Yerevan State University,  
AANL CP&IT  
(Yeravan, Armenia)*

**IT School – 2023  
17 October  
Dubna, Russia**

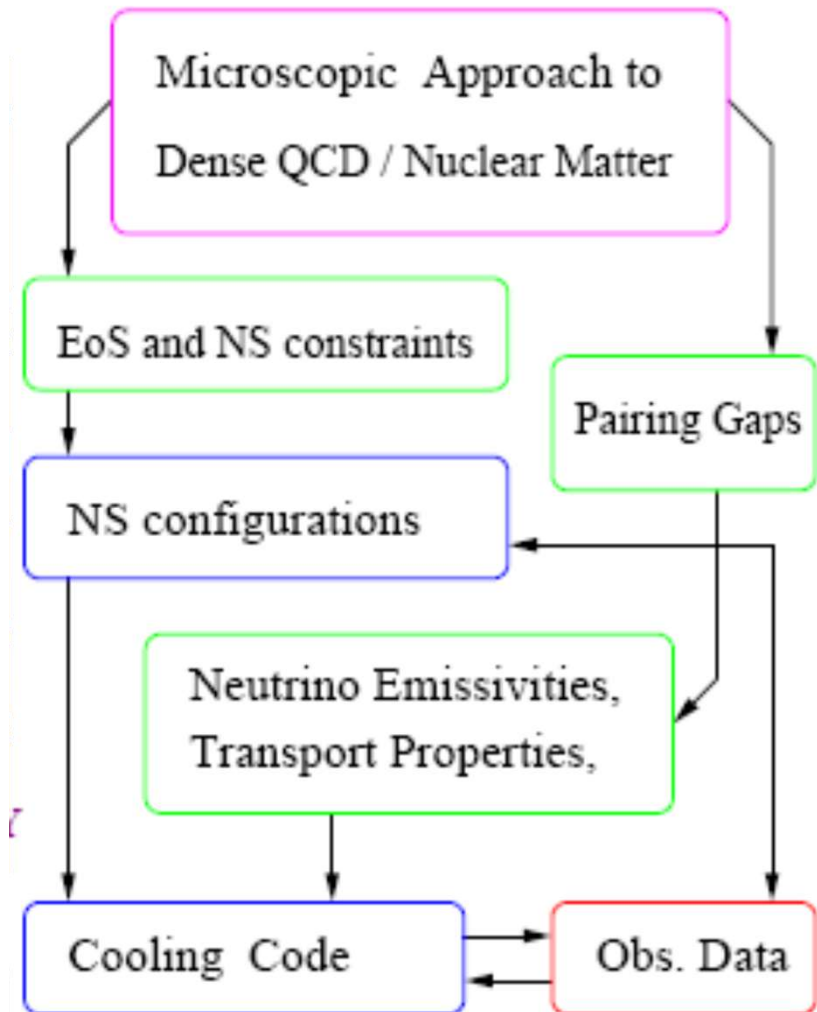
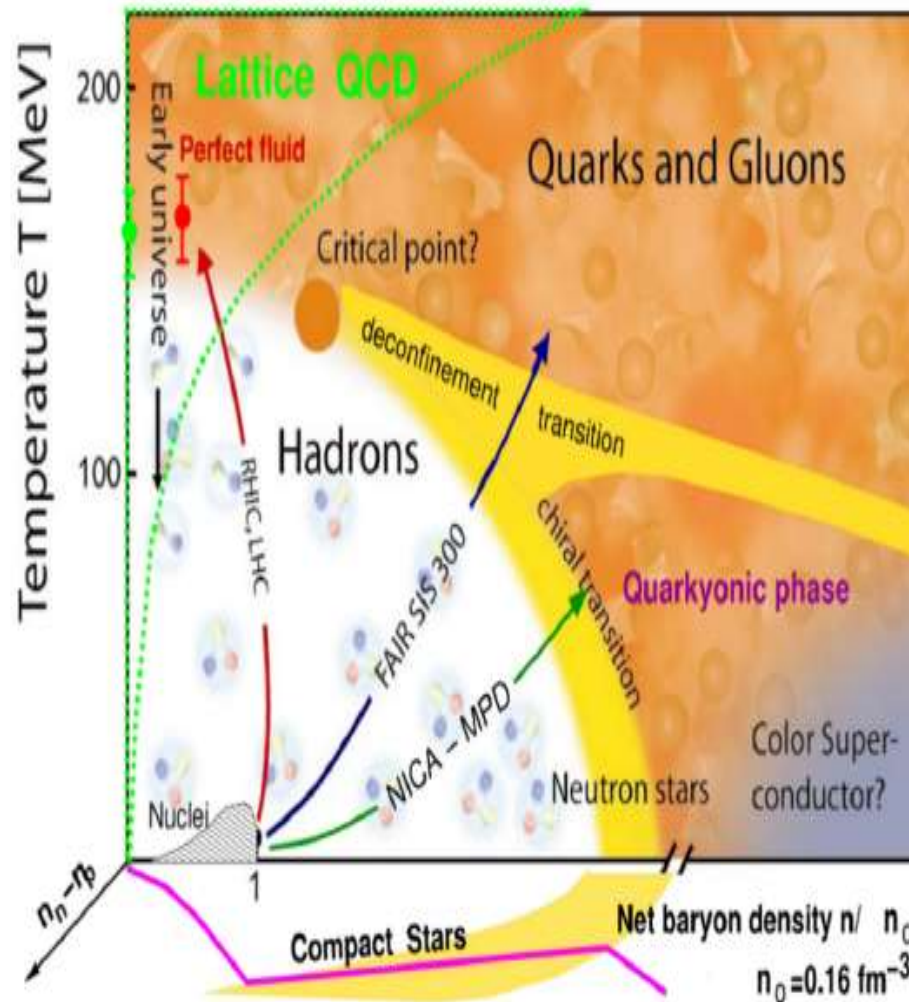
my co-authors: D.Blaschke, D.Voskresensky,  
A. Ayriyan E. Kolomeitsev, K. Maslov,

# Simulation of Cooling Evolution of Neutron Stars

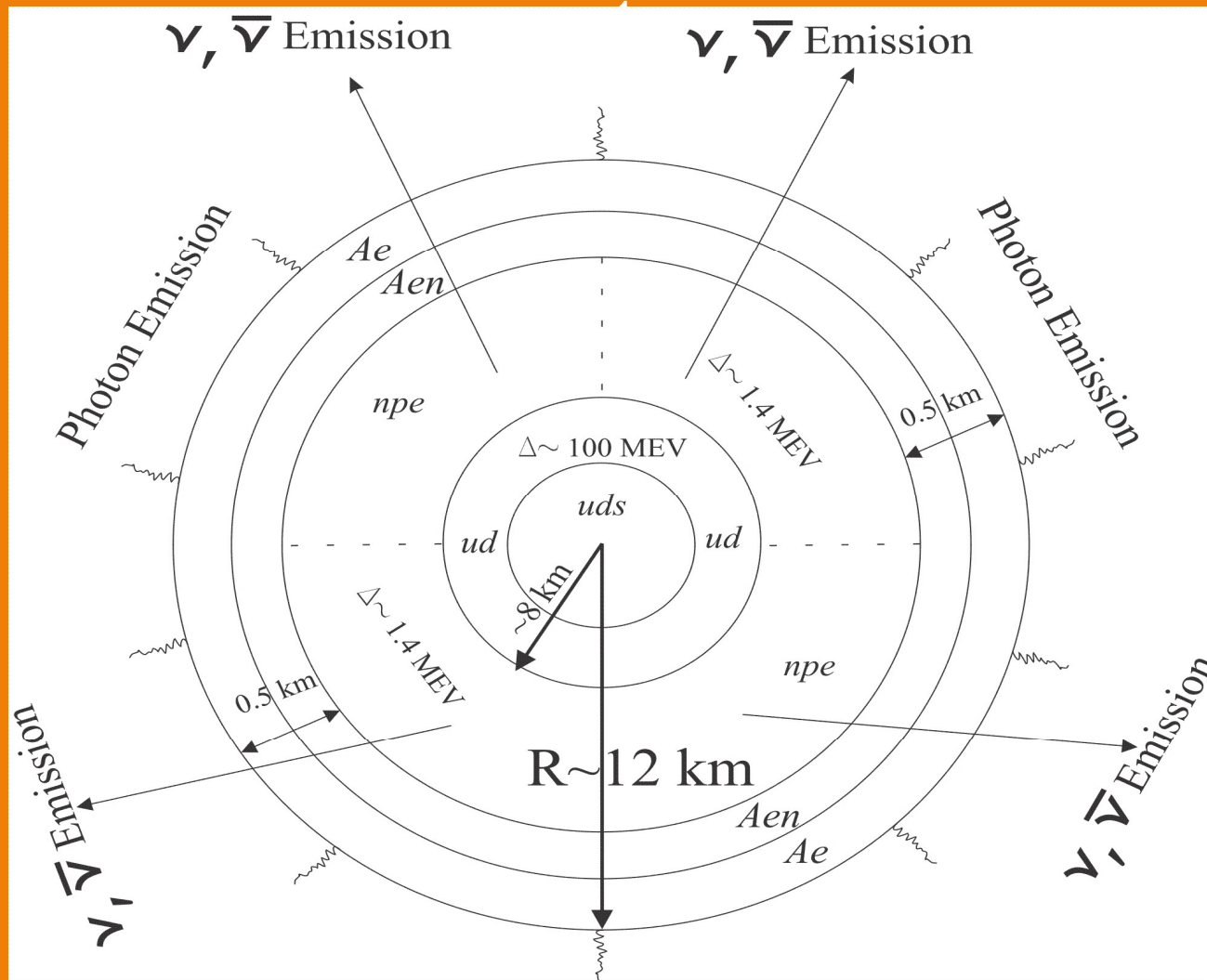
- Motivation
- Neutron Stars structure
- Neutron Stars cooling problem
- ***Results for NS cooling***
- ***Mass extraction***

H. Grigorian, D. N. Voskresensky and D. Blaschke  
Eur. Phys. J. A **52**: 67 (2016).

# Phase Diagramm & Cooling Simulation



# Structure Of Hybrid Star



## Static neutron star mass and radius

The structure and global properties of compact stars are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations<sup>1,2</sup>:

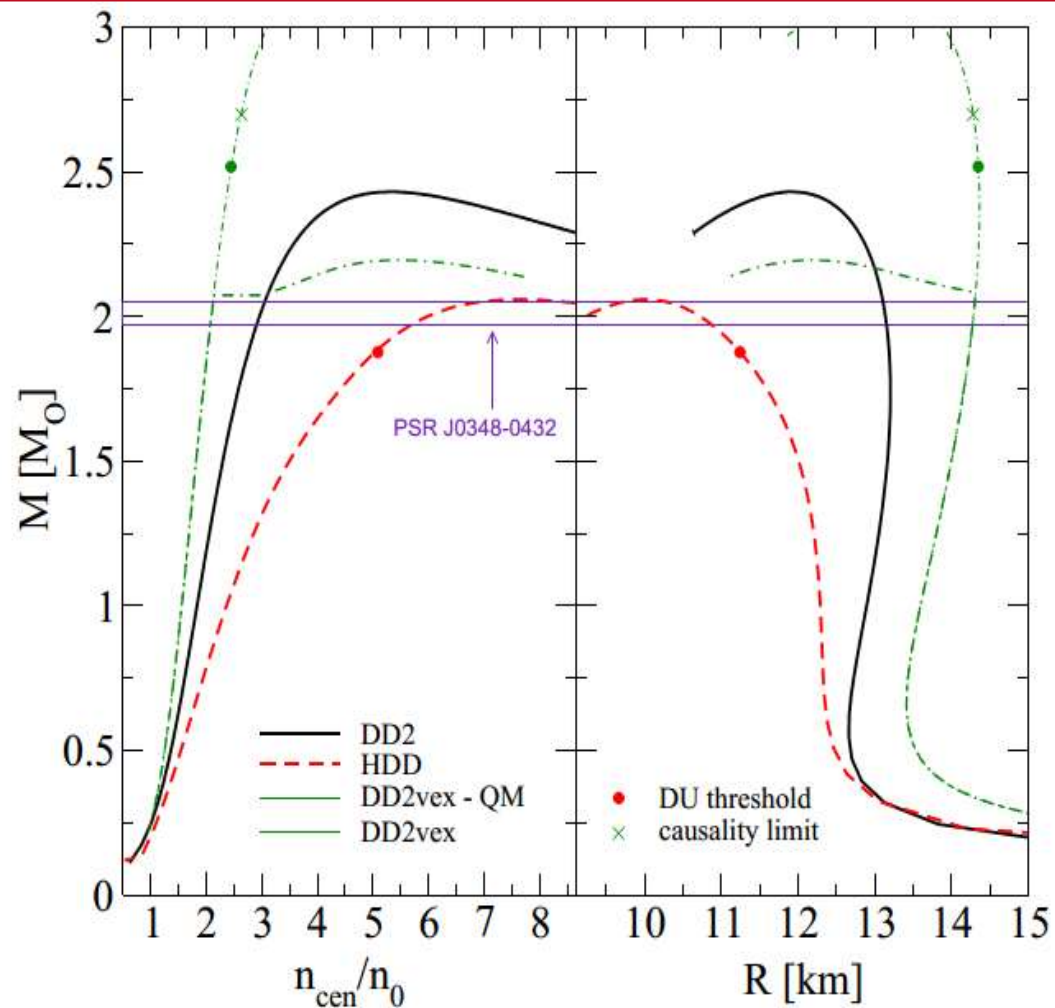
$$\left\{ \begin{array}{l} \frac{dP(r)}{dr} = -\frac{GM(r)\varepsilon(r)}{r^2} \left(1 + \frac{P(r)}{\varepsilon(r)}\right) \left(1 + \frac{4\pi r^3 P(r)}{M(r)}\right) \\ \frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r); \\ \frac{dN_B(r)}{dr} = 4\pi r^2 \left(1 - \frac{2GM(r)}{r}\right)^{-1/2} n(r). \end{array} \right.;$$

<sup>1</sup>R. C. Tolman, Phys. Rev. **55**, 364 (1939).

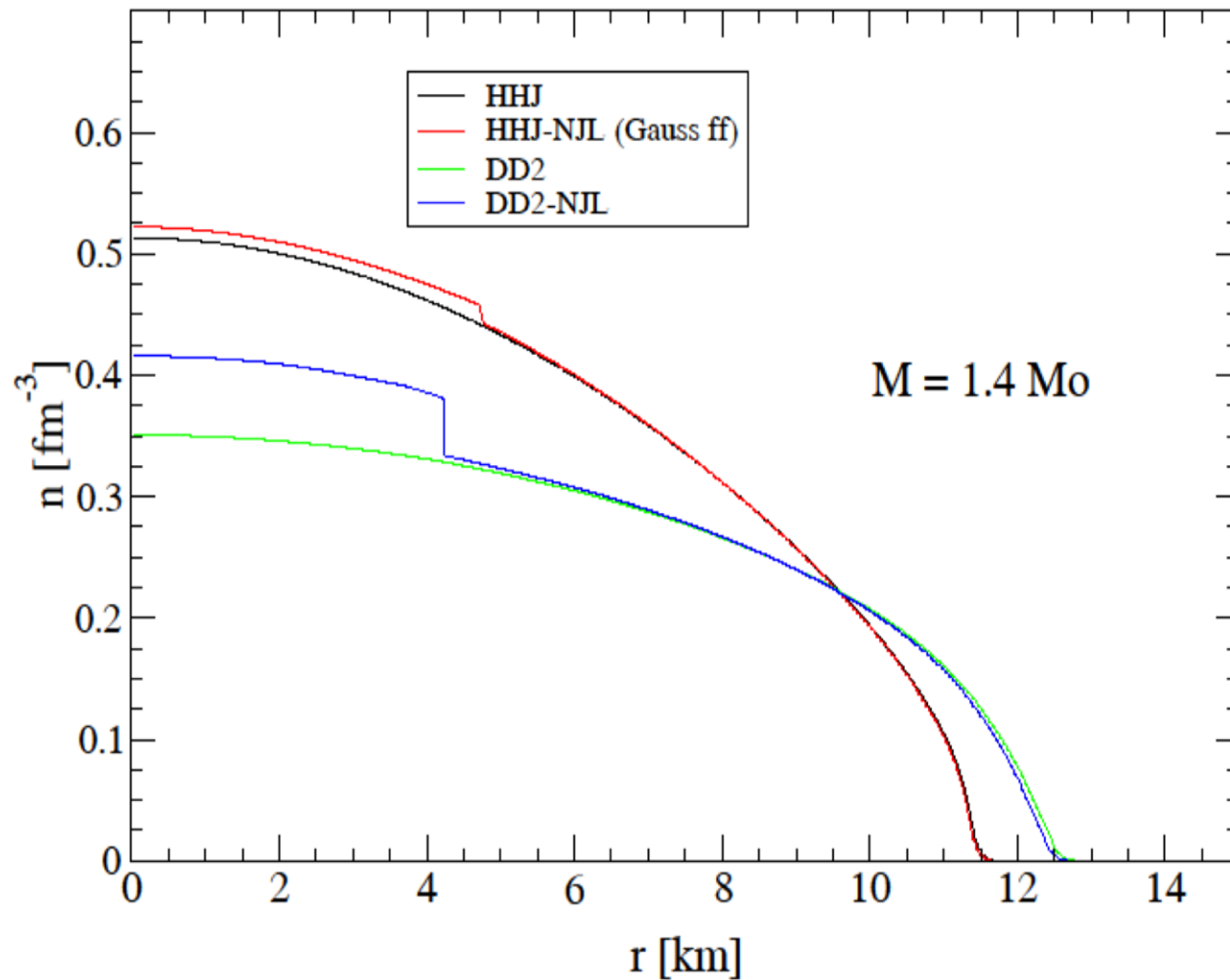
<sup>2</sup>J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).

# Stability of stars

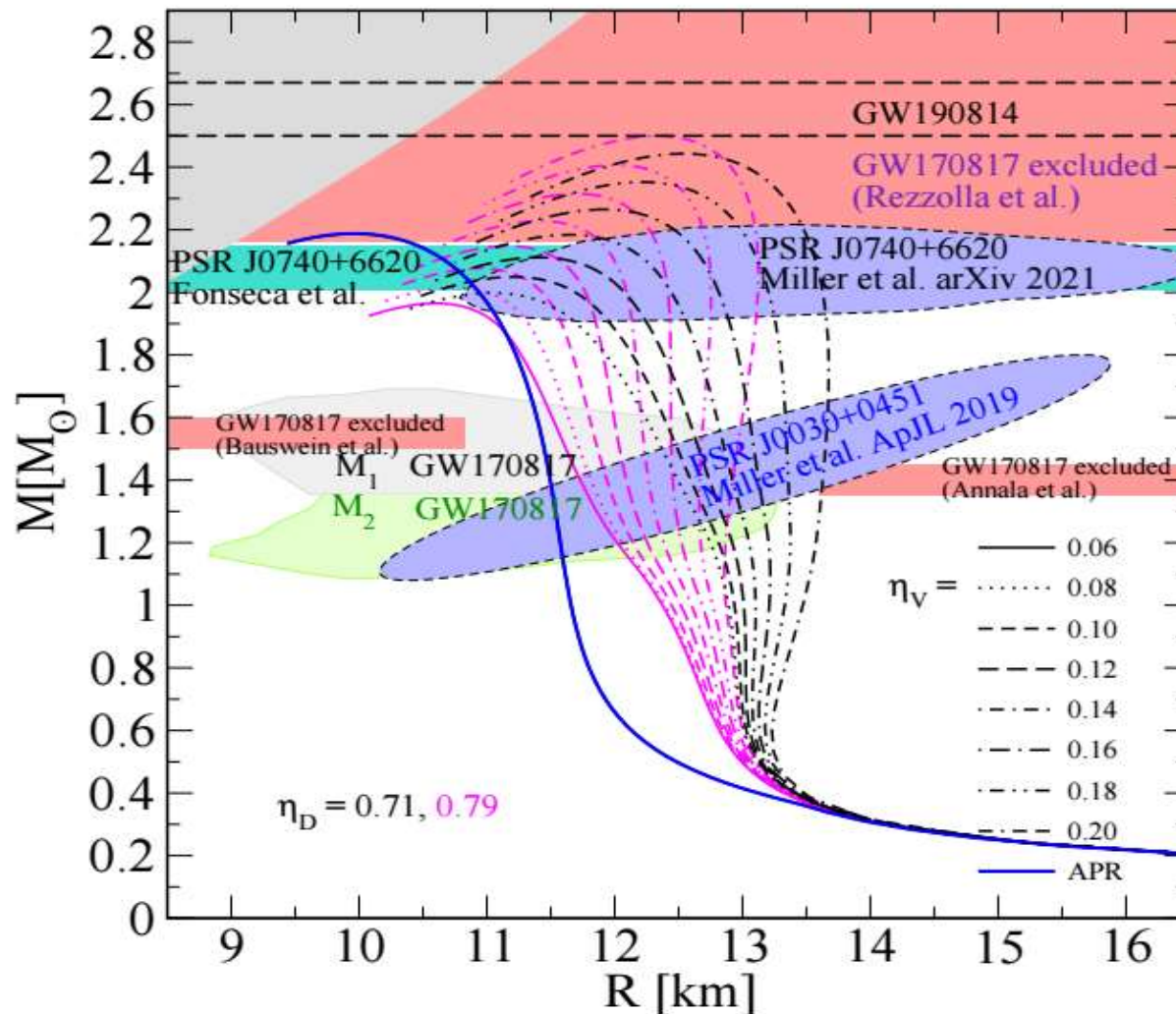
## HDD, DD2 & DDvex-NJL EoS models



# Different Configurations with the same NS mass

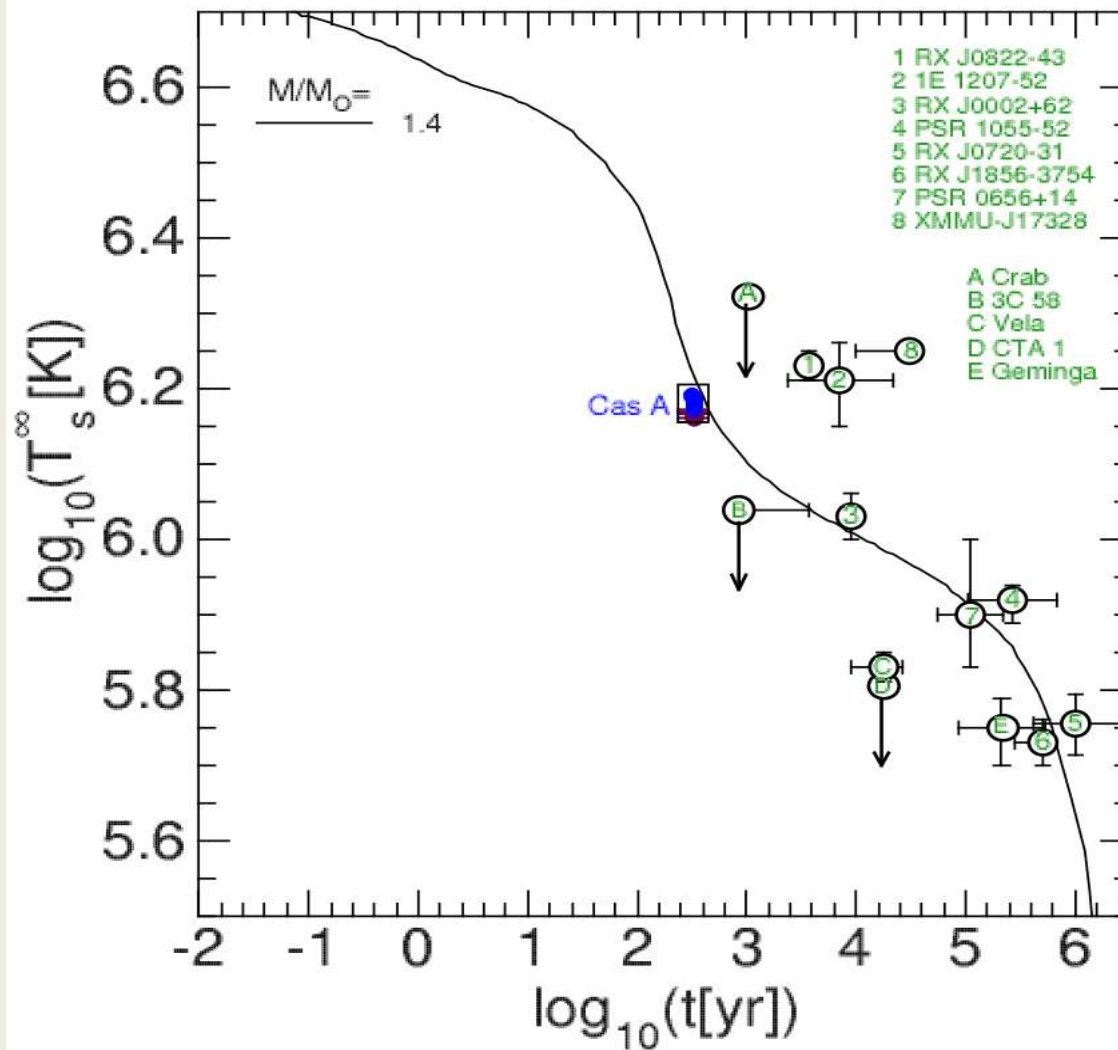


# Modern MR Data and Models





# Surface Temperature & Age Data



# Cooling Mechanism

$$\frac{dU}{dt} = \sum_i C_i \frac{dT}{dt} = -\varepsilon_\gamma - \sum_j \varepsilon_\nu^j$$

## Cooling Processes

- ➔ Direct Urca:  $n \rightarrow p + e + \bar{\nu}_e$
- ➔ Modified Urca:  $n + n \rightarrow n + p + e + \bar{\nu}_e$
- ➔ Photons:  $\rightarrow \gamma$
- ➔ Bremsstrahlung:  $n + n \rightarrow n + n + \nu + \bar{\nu}$

# Cooling Evolution

The energy flux per unit time  $l(r)$  through a spherical slice at distance  $r$  from the center is:

$$l(r) = -4\pi r^2 k(r) \frac{\partial(Te^\Phi)}{\partial r} e^{-\Phi} \sqrt{1 - \frac{2M}{r}}.$$

The equations for energy balance and thermal energy transport are:

$$\frac{\partial}{\partial N_B}(le^{2\Phi}) = -\frac{1}{n}(\epsilon_\nu e^{2\Phi} + c_V \frac{\partial}{\partial t}(Te^\Phi))$$

$$\frac{\partial}{\partial N_B}(Te^\Phi) = -\frac{1}{k} \frac{le^\Phi}{16\pi^2 r^4 n}$$

where  $n = n(r)$  is the baryon number density,  $N_B = N_B(r)$  is the total baryon number in the sphere with radius  $r$

$$\frac{\partial N_B}{\partial r} = 4\pi r^2 n \left(1 - \frac{2M}{r}\right)^{-1/2}$$

F.Weber: Pulsars as Astro. Labs ... (1999);

D. Blaschke Grigorian, Voskresensky, A& A 368 (2001)561.

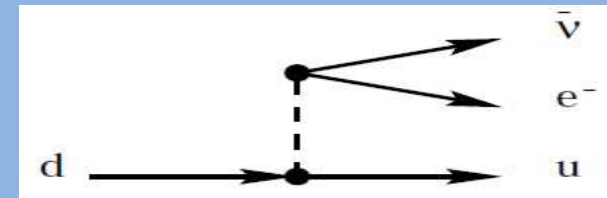
# Neutrino emissivities in quark matter:

- Quark direct Urca (QDU) the most efficient processes

$$d \rightarrow u + e + \bar{\nu} \text{ and } u + e \rightarrow d + \nu$$

$$\epsilon_{\nu}^{\text{QDU}} \simeq 9.4 \times 10^{26} \alpha_s u Y_e^{1/3} \zeta_{\text{QDU}} T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1},$$

Compression  $n/n_0 \simeq 2$ , strong coupling  $\alpha_s \approx 1$



- Quark Modified Urca (QMU) and Quark Bremsstrahlung

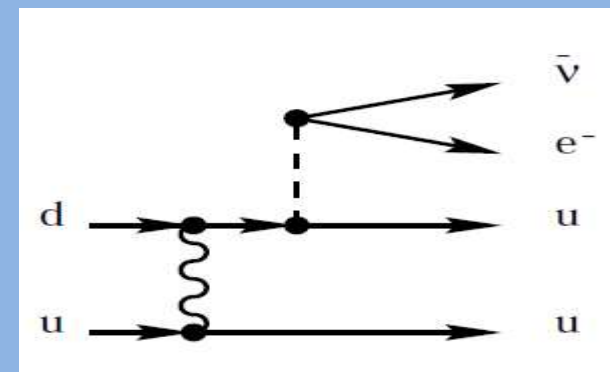
$$d + q \rightarrow u + q + e + \bar{\nu} \text{ and } q_1 + q_2 \rightarrow q_1 + q_2 + \nu + \bar{\nu}$$

$$\epsilon_{\nu}^{\text{QMU}} \sim \epsilon_{\nu}^{\text{QB}} \simeq 9.0 \times 10^{19} \zeta_{\text{QMU}} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}.$$

- Suppression due to the pairing

$$\text{QDU} : \zeta_{\text{QDU}} \sim \exp(-\Delta_q/T)$$

$$\text{QMU and QB} : \zeta_{\text{QMU}} \sim \exp(-2\Delta_q/T) \text{ for } T < T_{\text{crit},q} \simeq 0.57 \Delta_q$$



- Enhanced cooling due to the pairing

- $e + e \rightarrow e + e + \nu + \bar{\nu}$  (becomes important for  $\Delta_q/T \gg 1$ )

$$\epsilon_{\nu}^{ee} = 2.8 \times 10^{12} Y_e^{1/3} u^{1/3} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1},$$

Quark PBF

# Neutrino emissivities in hadronic matter:

- Direct Urca (DU) the most efficient processes

$$\epsilon_{DU} = M_{DU} * (m_p^*)(m_n^*) * \Gamma_{wN}^2 * (n_e)^{1/3} (T_9)^6 * R_D;$$

$$M_{DU} = 4 \times 10^{27} \text{ erg/s/cm}^3$$

- Modified Urca (MU) and Bremsstrahlung

$$\epsilon_{MUP} = F_M * M_p * (m_p)^3 (m_n^*) (T_9)^8 (n_e)^{1/3} * R_{MUP}(v_n, v_p);$$

$$\epsilon_{nnBS} = P_{nnBS} * R_{BS}^{nn}(v_n) * \Gamma_w^2 \Gamma_s^4 (n_b)^{4/3} (T_9)^8 (m_n^*)^4 / (\omega)^3;$$

- Suppression due to the pairing

$$v_N = \Delta_N(T)/T = \sqrt{1 - \tau_N} \left( 1.456 - \frac{0.157}{\sqrt{\tau_N}} + \frac{1.766}{\tau_N} \right)$$

- Enhanced cooling due to the pairing

$$\epsilon_{\nu}^{\text{NPBF}} = 6.6 \times 10^{28} (m_n^*/m_n) (\Delta_n(T)/\text{MeV})^7 u^{1/3} \\ \times \xi I(\Delta_n(T)/T) \text{ erg cm}^{-3} \text{ s}^{-1},$$

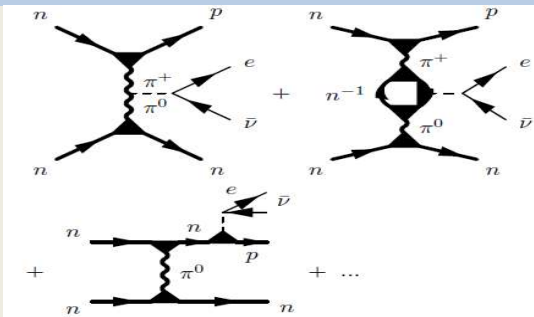
$$\epsilon_{\nu}^{\text{PPBF}} = 0.8 \times 10^{28} (m_p^*/m_p) (\Delta_p(T)/\text{MeV})^7 u^{2/3} \\ \times I(\Delta_p(T)/T) \text{ erg cm}^{-3} \text{ s}^{-1},$$

# Medium Effects In Cooling Of Neutron Stars

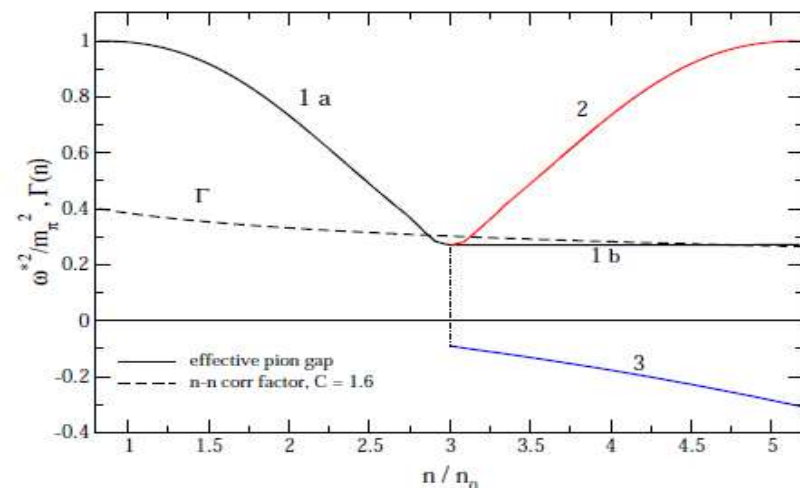
- Based on Fermi liquid theory ( Landau (1956), Migdal (1967), Migdal et al. (1990))
- MMU – insted of MU

- Main regulator in Minimal Cooling

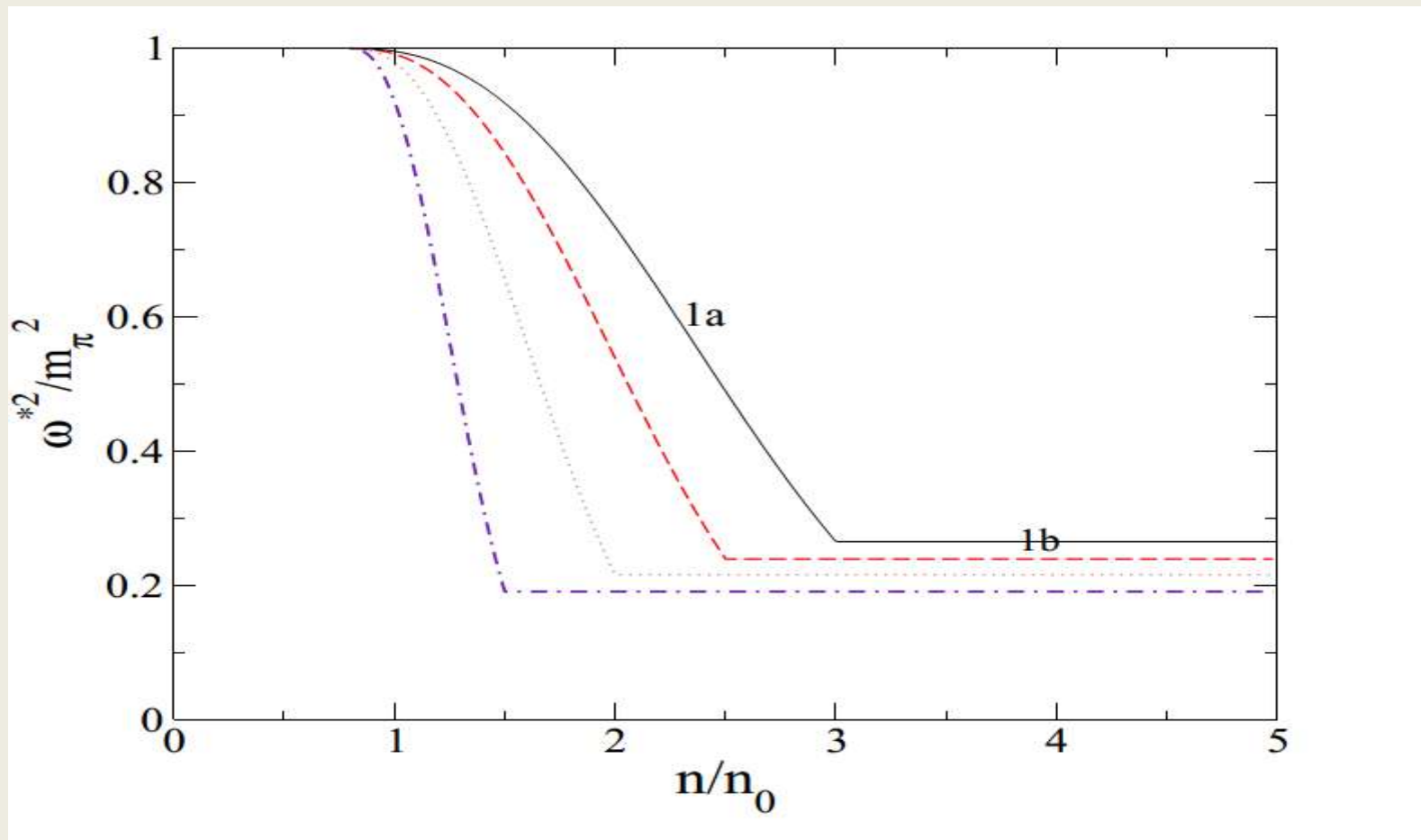
$$\varepsilon_\nu [\text{MpPBF}] \sim 10^{29} \frac{m_N^*}{m_N} \left[ \frac{p_{Fp}}{p_{Fn}(n_0)} \right] \left[ \frac{\Delta_{pp}}{\text{MeV}} \right]^7 \times \left[ \frac{T}{\Delta_{pp}} \right]^{1/2} \xi_{pp}^2 \frac{\text{erg}}{\text{cm}^3 \text{ sec}}, \quad T < T_{cp}.$$



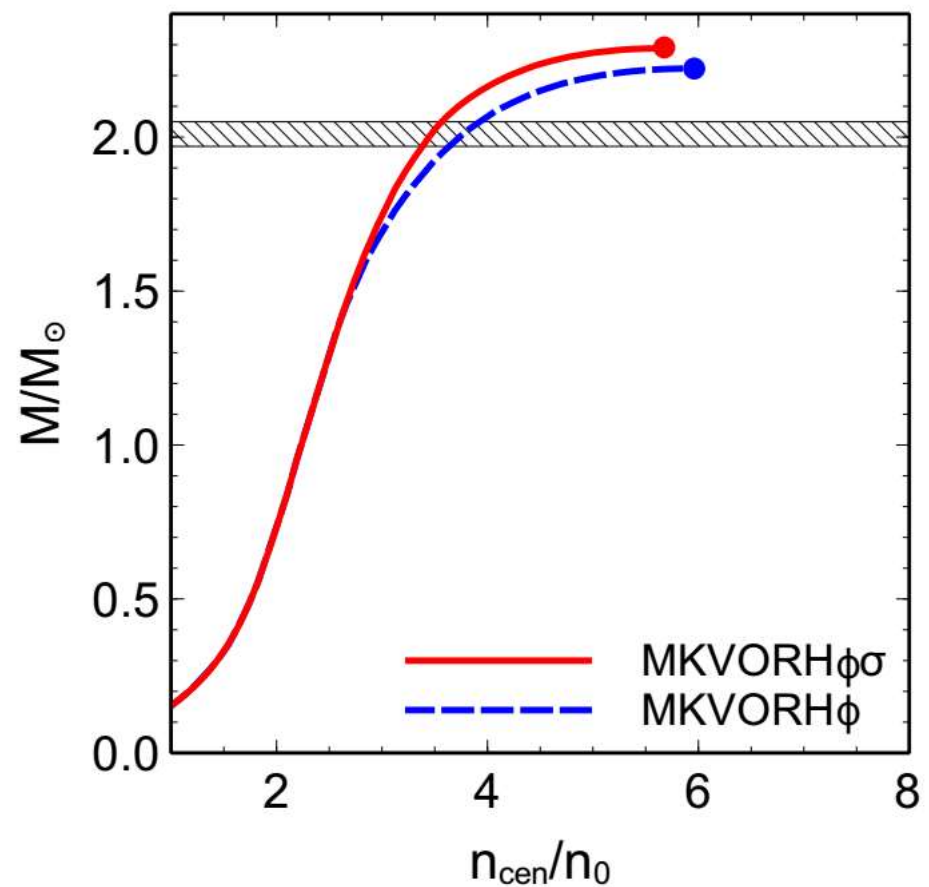
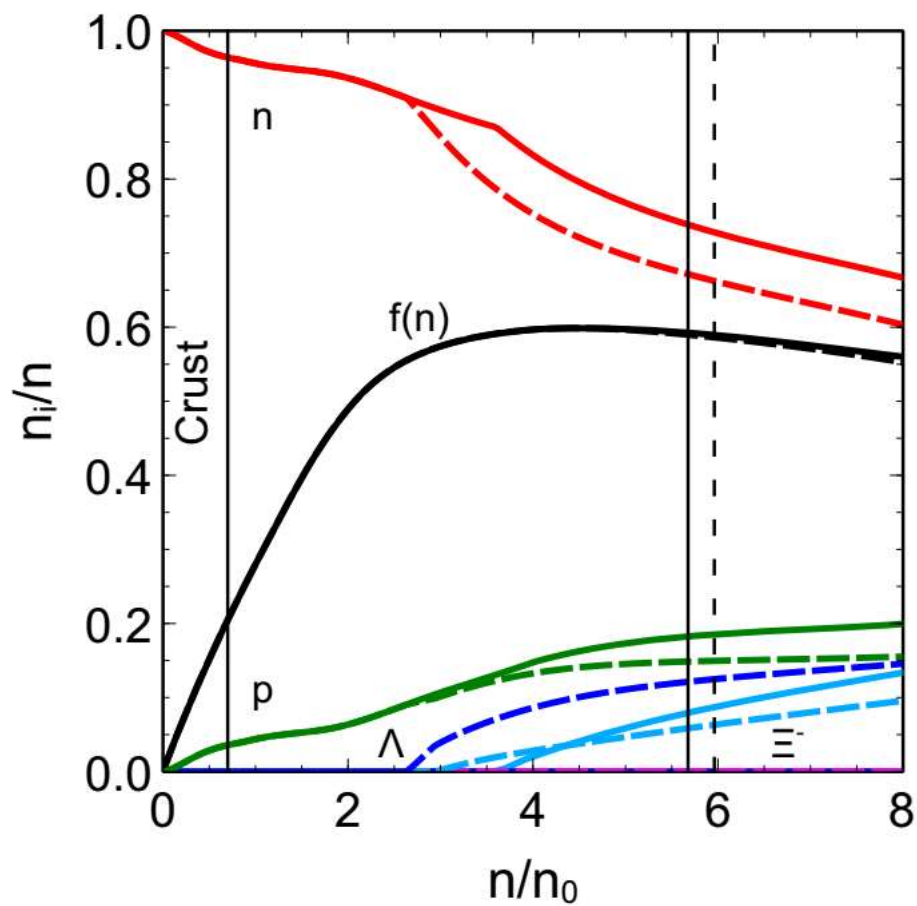
$$\frac{\varepsilon_\nu [\text{MMU}]}{\varepsilon_\nu [\text{MU}]} \sim 10^3 \left( \frac{n}{n_0} \right)^{10/3} \frac{\Gamma^6(n)}{[\omega^*(n)/m_\pi]^8},$$



# Medium Effects In Cooling Of Neutron Stars

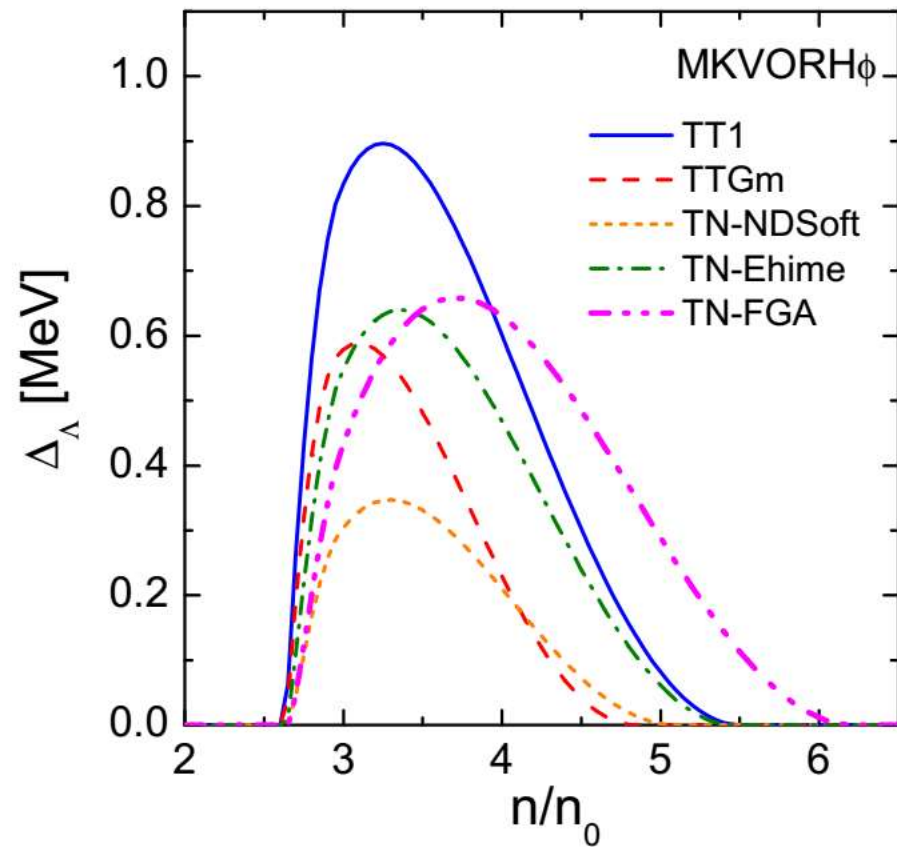
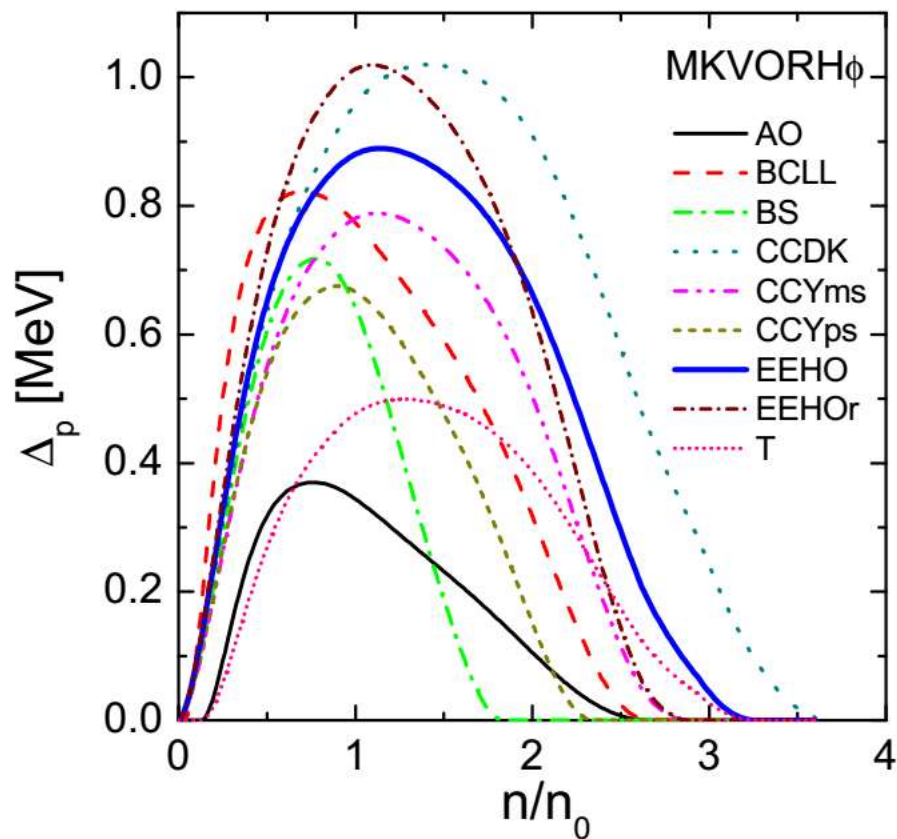


# MKVOR – EoS model





# MKVORHp - Gap models



# Crust Model

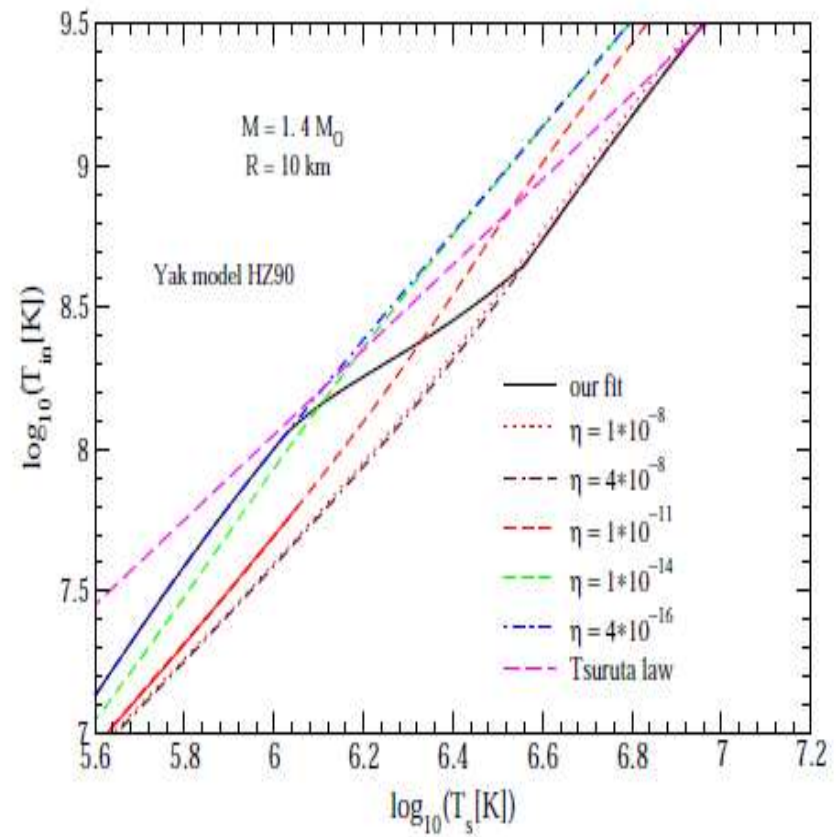
Time dependence of the light element contents in the crust

$$\Delta M_L(t) = e^{-t/\tau} \Delta M_L(0)$$

**Blaschke, Grigorian, Voskresensky,**  
*A&A* 368 (2001)561.

Page, Lattimer, Prakash & Steiner,  
*Astrophys.J.* 155, 623 (2004)

Yakovlev, Levenfish, Potekhin,  
Gnedin & Chabrier, *Astron. Astrophys*  
, 417, 169 (2004)

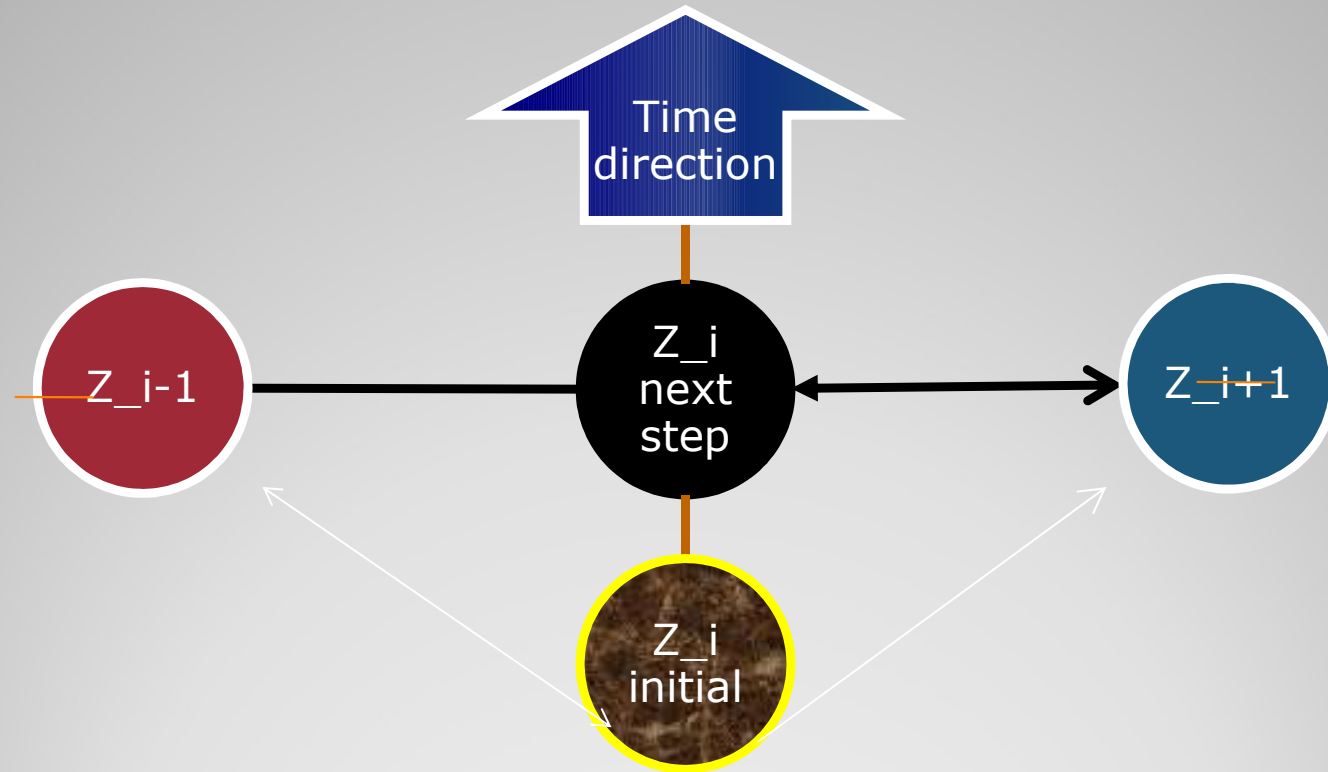


# Equations for Cooling Evolution

$$\left\{ \begin{array}{l} \frac{\partial z(\tau, a)}{\partial \tau} = A(z, a) \frac{\partial L(\tau, a)}{\partial a} + B(z, a) \\ L(\tau, a) = C(z, a) \frac{\partial z(\tau, a)}{\partial a} \end{array} \right. \quad z(\tau, a) = \log T(\tau, a)$$

$$L_{i\pm 1/2} = \pm \frac{C_i + C_{i\pm 1}}{2} \frac{z_{i\pm 1} - z_i}{\Delta a_{i-1/2(1\mp 1)}} \quad \frac{\partial L_i}{\partial a} = 2 \frac{L_{i+1/2} - L_{i-1/2}}{\Delta a_i + \Delta a_{i-1}}$$

# Finite difference scheme



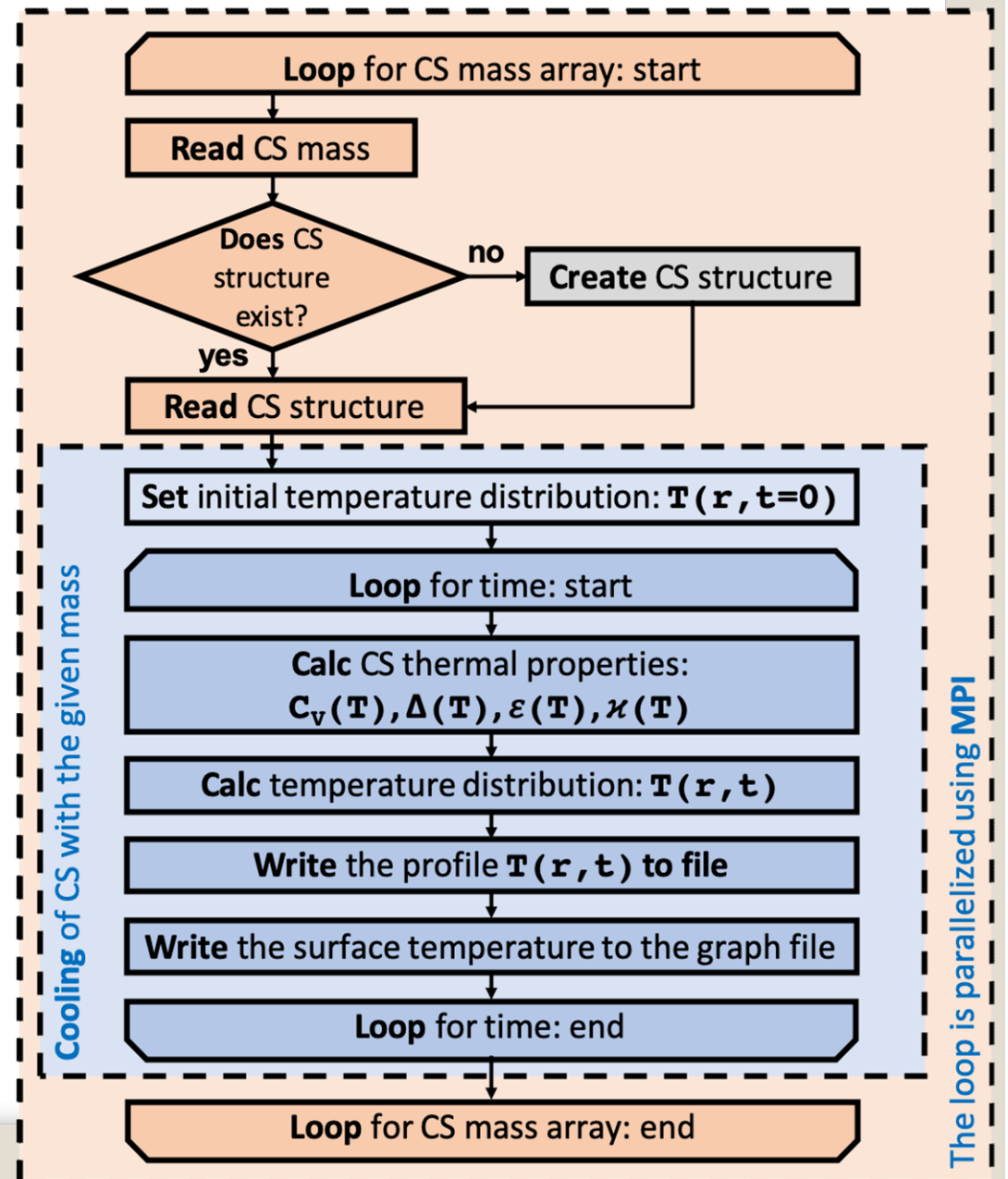
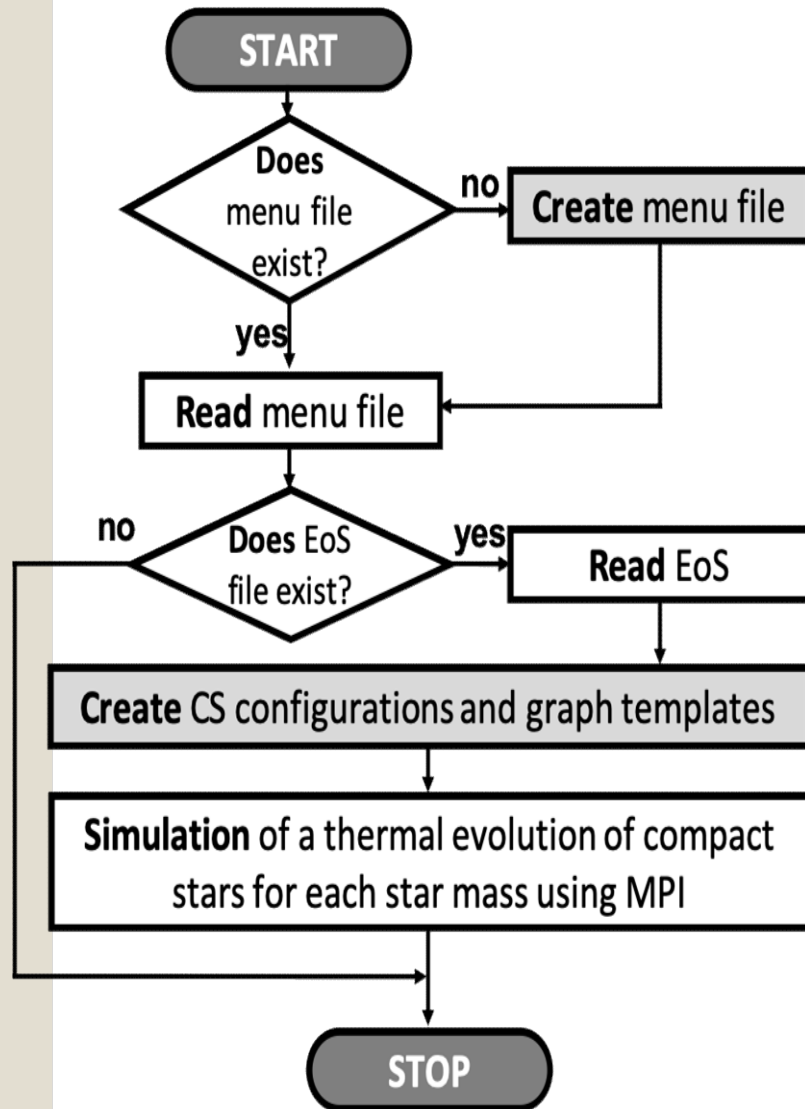
$$\alpha_{i,j-1} z_{i+1,j} + \beta_{i,j-1} z_{i,j} + \gamma_{i,j-1} z_{i-1,i} = \delta_{i,j-1}$$

# Finite difference scheme

$$\begin{pmatrix}
 \beta_{0,j-1} & \alpha_{0,j-1} & & & 0 \\
 \gamma_{1,j-1} & * & & * & \\
 & * & & * & * \\
 & & * & * & \alpha_{N-1,j-1} \\
 0 & & & \gamma_{N,j-1} & \beta_{N,j-1}
 \end{pmatrix}
 \begin{pmatrix}
 z_{0,j} \\
 z_{1,j} \\
 * \\
 * \\
 z_{N,j}
 \end{pmatrix}
 =
 \begin{pmatrix}
 \delta_{0,j-1} \\
 \delta_{1,j-1} \\
 * \\
 * \\
 \delta_{N,j-1}
 \end{pmatrix}$$

$$\alpha_{i,j-1} z_{i+1,j} + \beta_{i,j-1} z_{i,j} + \gamma_{i,j-1} z_{i-1,j} = \delta_{i,j-1}$$

# Program Algorithm



# Model parameters – DD2

```
Menu_dd2_2017n.dat

Model Parametrs

The HOME directory is : .\Data\DD2\Configs-2
The EV UOTPUT directory : .\Data\DD2\17-12-2019\EV-DD2-pi-F4-o3-D
Make EoS file : 0
Make new config. file : 0
Read full EoS from a file : 1
Read from : .\EoS\DD2_HG
Hadronic EoS
LWalecka (0) NLW (1) HDD (3) BSk20 (4): 3
Normal Shell : 0
Quark EoS SM model (1) Bag model (0) : 0
In case of SM GF (0) GL(1) NJL (2) : 0
with Quark core : 1
without Mixed phase : 1
Superconducting Quark core : 1
Quark Star : 0
Medium effects : 1
Pion condensate : 1
Crust Model (Yakovlev - Y Tsuruta - T our - G) : G
Gaps in Hadrons Model (Yakovlev - Y AV18 - A Schwenk - U Armen-fit - F) : F
for F-fit p-Gap
1-A0
2-BCLL
3-BS
4-CCDK
5-CCYms
6-CCYps
7-EEHO
8-EEHOr
9-T
: 4
for F-fit n-Gap
2-AWP2
3 - AWP3
4 - CCDK
5 - CLS
6 - GIPSF
7 - MSH
8 - SCLBL
9 - SFB
0 - WAP : 0
XGaps in 2SC QModel constant 0 - 0
constant 0.1 MeV - 1
constant 0.05 MeV - 5
constant 0.03 MeV - 3
rising 0.03 + MeV - A
incrising 0.03 - MeV - B
constant 0.03 ++ MeV - C
constant 0.03 -- MeV - D
: C
```

```
Menu_dd2_2017n.dat

Gap factors in HM
Protons 1S0p : 1
Neutrons 1S0n : 1
Neutrons 3P2n : 0.1

End time point log10(t/yr) : 8

initial temperatur in MeV : 0.5

minimal value of log Temperature : 5.5

Print output files for LogN-LogS : 0

Print profiles for the time points : 0

Number of points : 7
0 0 0 0 0 0 0

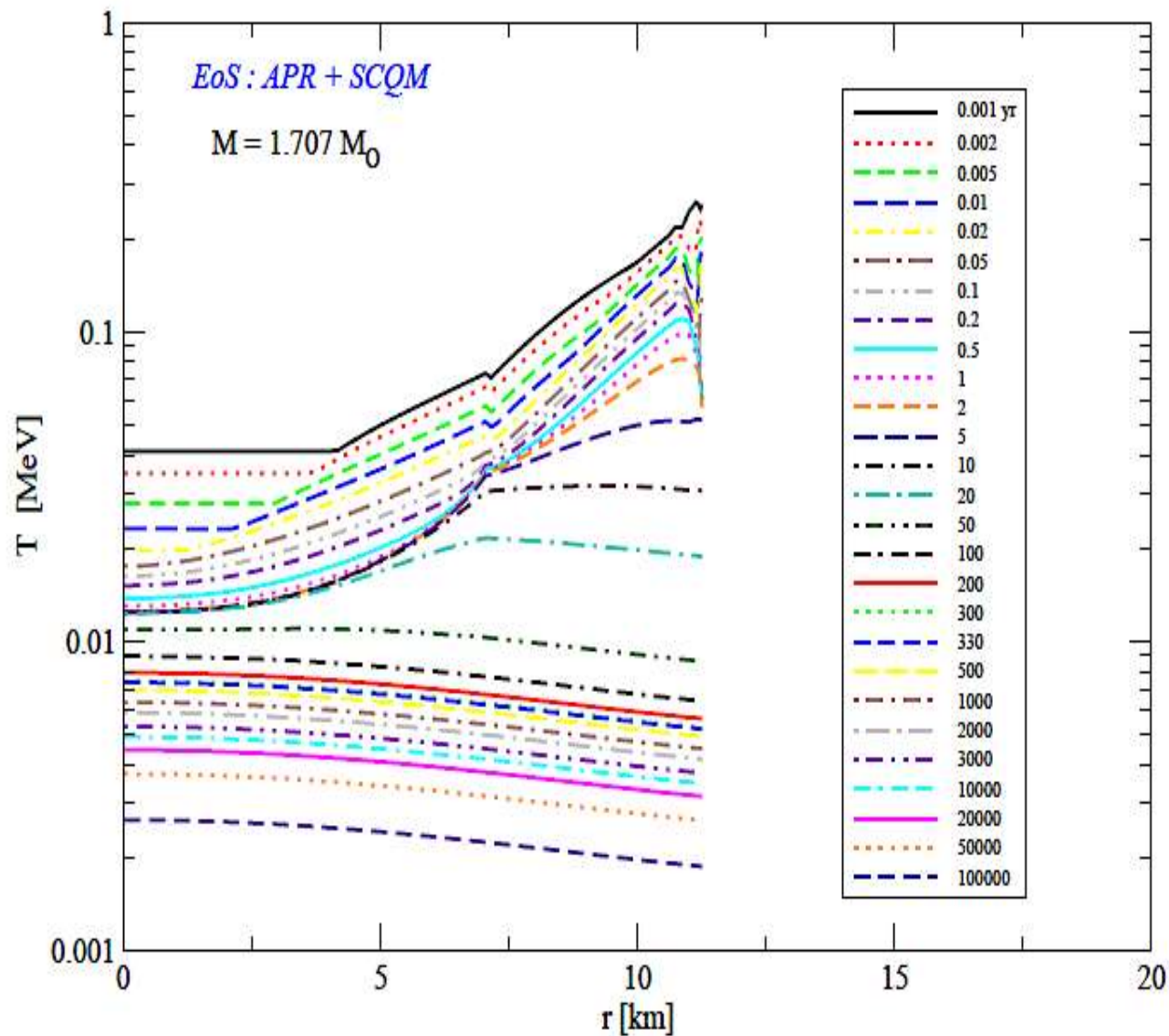
The Masses [Mo] of Configurations to be Cooled

Number of points : 51
1.450
0.5 0.51 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.59
0.6 0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.69
0.7 0.71 0.72 0.73 0.74 0.75 0.76 0.77 0.78 0.79
0.8 0.81 0.82 0.83 0.84 0.85 0.86 0.87 0.88 0.89
0.9 0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99

1.0 1.01 1.02 1.03 1.04 1.05 1.06 1.07 1.08 1.09
1.10 1.11 1.12 1.13 1.14 1.15 1.16 1.17 1.18 1.19
1.20 1.21 1.22 1.23 1.24 1.25 1.26 1.27 1.28 1.29
1.30 1.31 1.32 1.33 1.34 1.35 1.36 1.37 1.38 1.39
1.40 1.41 1.42 1.43 1.44 1.45 1.46 1.47 1.48 1.49
1.50 1.51 1.52 1.53 1.54 1.55 1.56 1.57 1.58 1.59
1.60 1.61 1.62 1.63 1.64 1.65 1.66 1.67 1.68 1.69
1.70 1.71 1.72 1.73 1.74 1.75 1.76 1.77 1.78 1.79
1.80 1.81 1.82 1.83 1.84 1.85 1.86 1.87 1.88 1.89
1.90 1.91 1.92 1.93 1.94 1.95 1.96 1.97 1.98 1.99
2.00 2.01 2.02 2.03 2.04
2.05 2.06 2.07 2.08 2.09
2.10 2.11 2.12 2.13 2.14 2.15 2.16 2.17 2.18 2.19
2.20
2.21 2.22 2.23 2.24 2.25 2.26 2.27 2.28 2.29
2.30 2.31 2.32 2.33 2.34 2.35 2.36 2.37 2.38 2.39
2.40 2.41

1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2
```

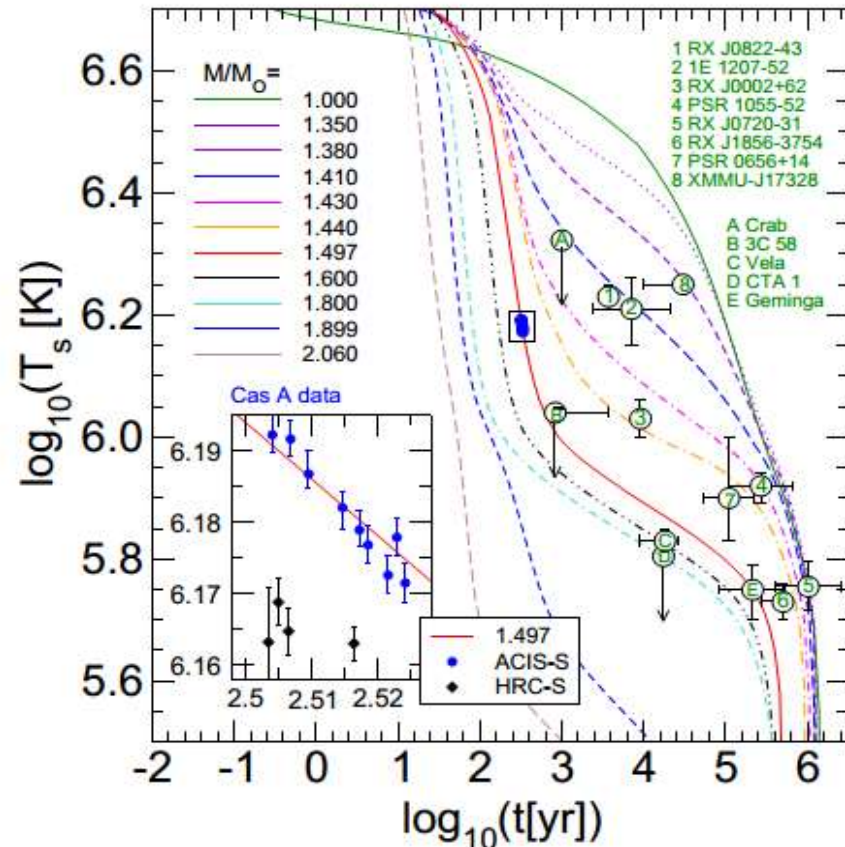
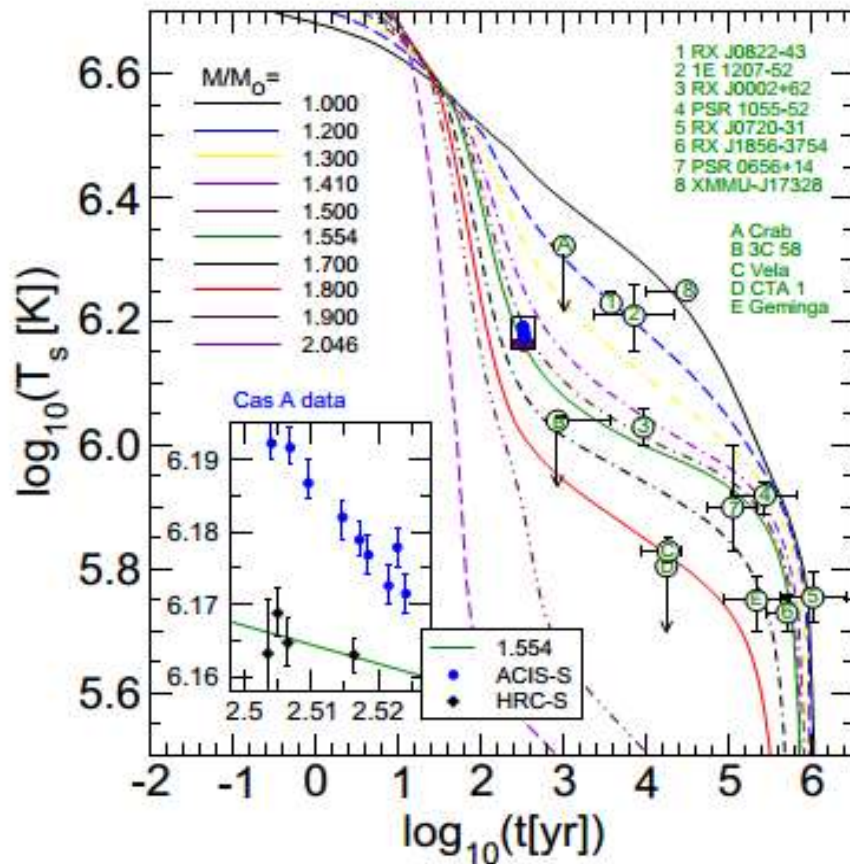
# Temperature in the Hybrid Star Interior





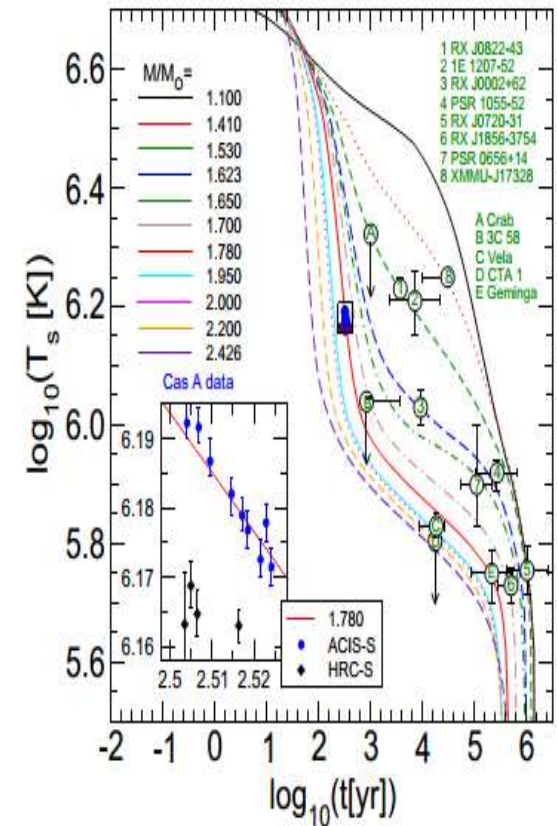
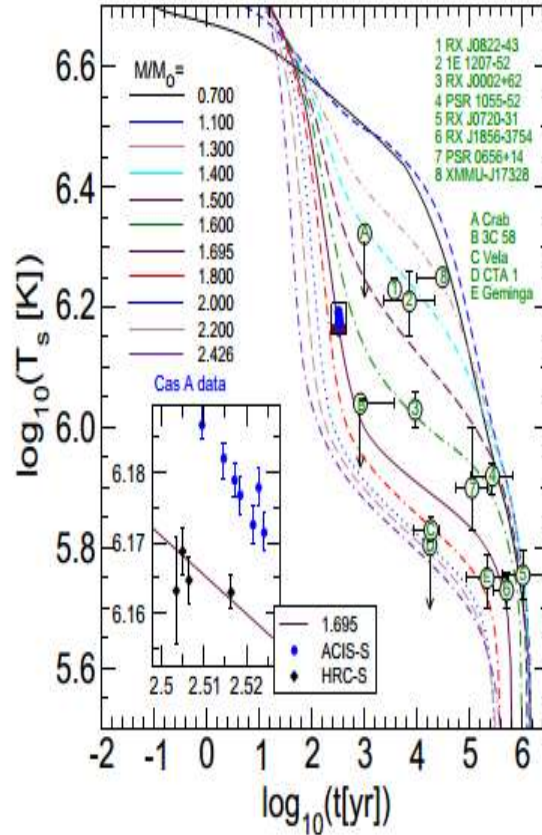
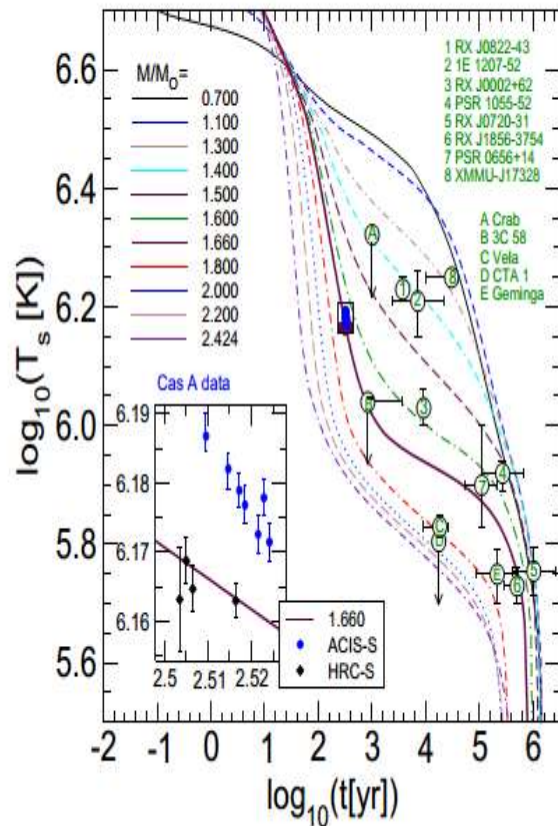
# HDD - AV18 , Yak.

## ME nc = 3 n0



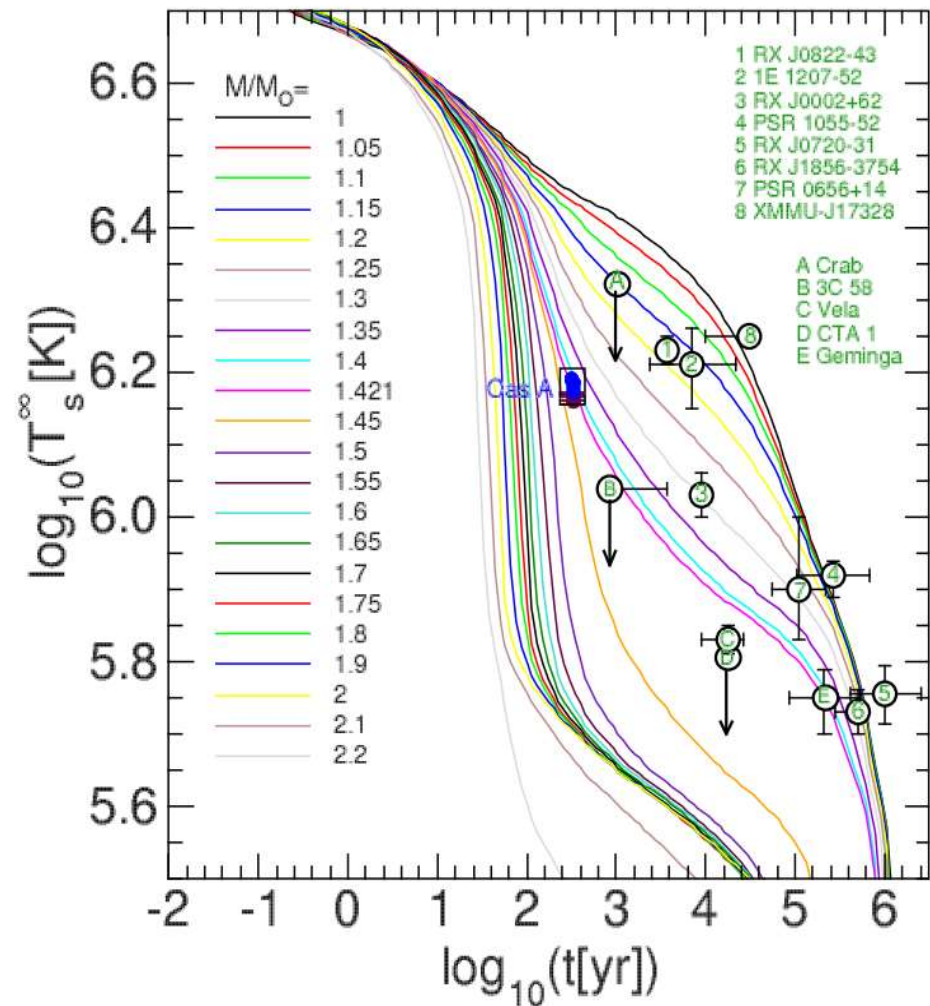
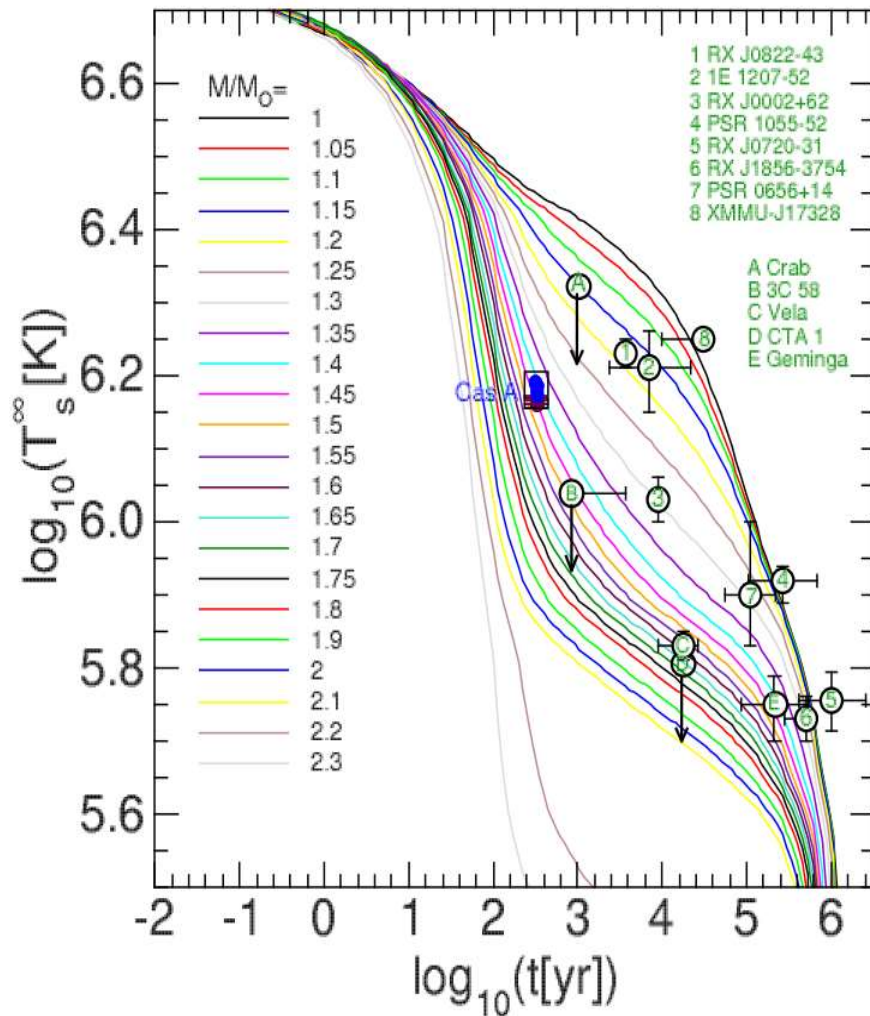
# DD2 – EEH0r

## ME-nc=1.5, 2.0, 2.5n0



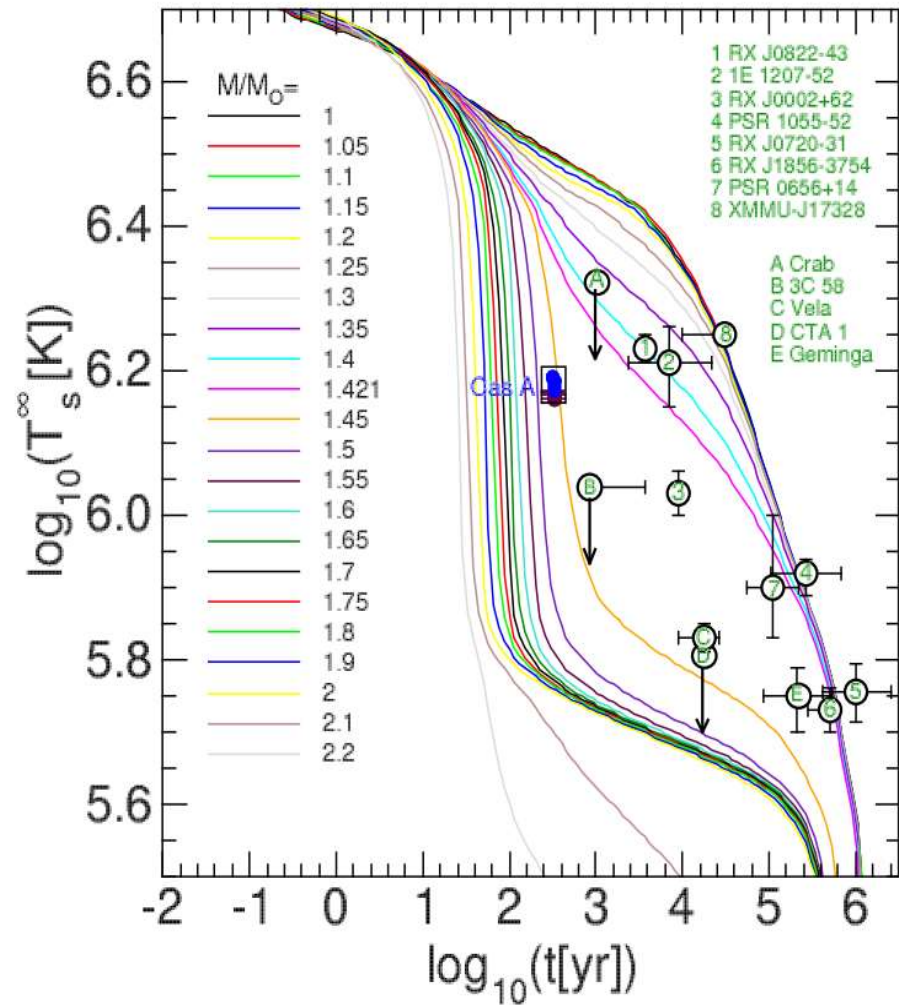
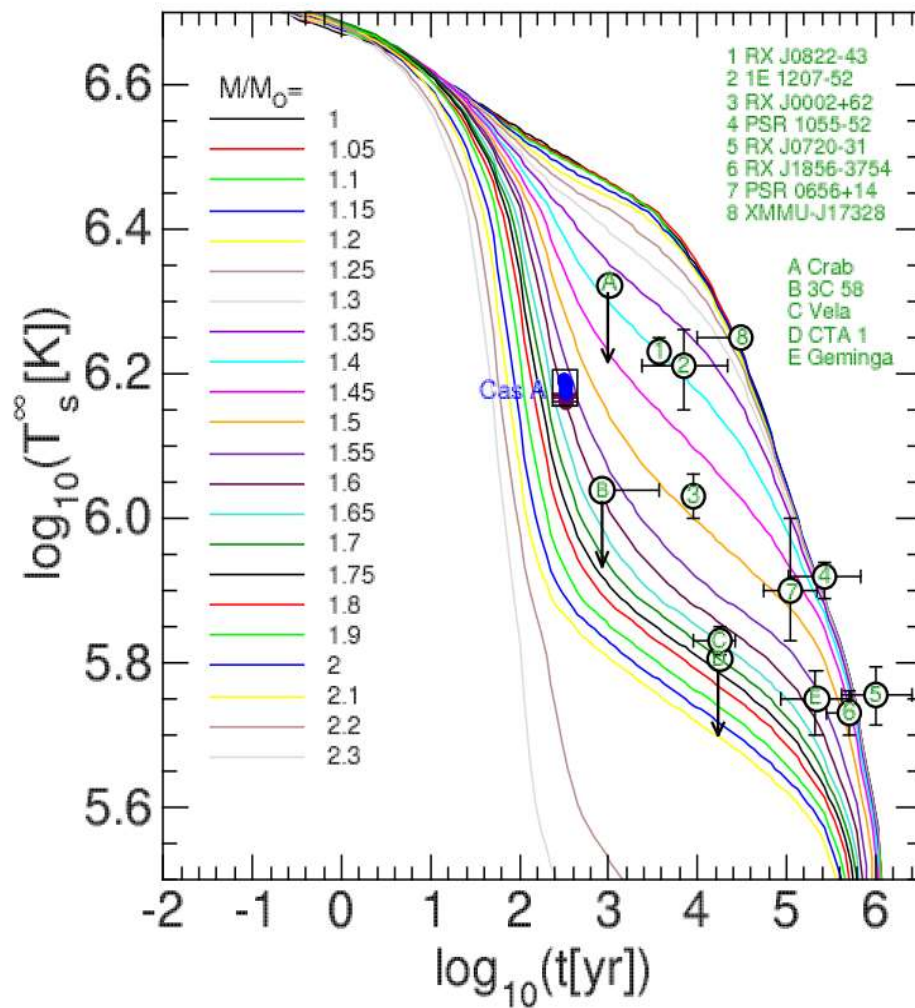
# MKVOR – BCLL, TN-FGA

## ME-nc=3.0n0

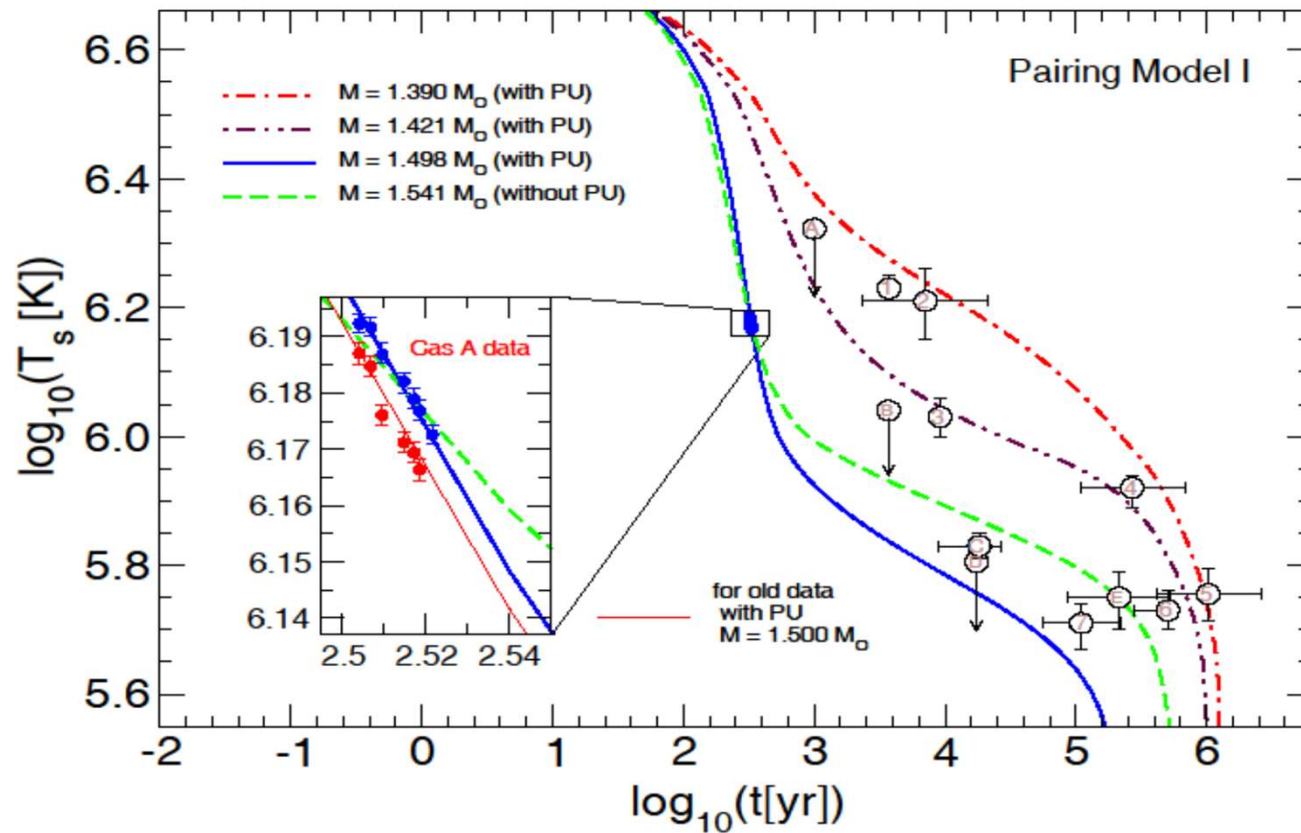


# MKVOR Hyp - EEH0r, TN-FGA

## ME-nc=3.0n0

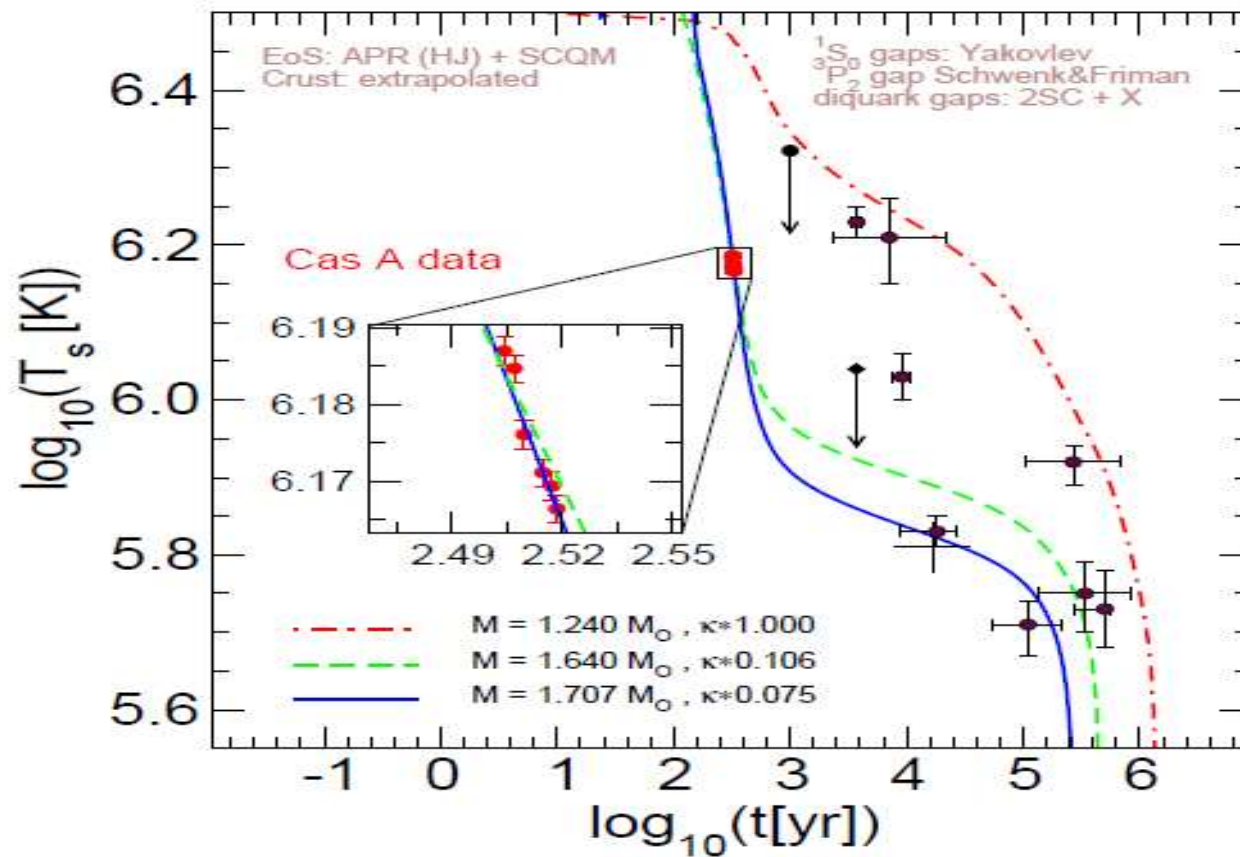


# Cas A as an Hadronic Star

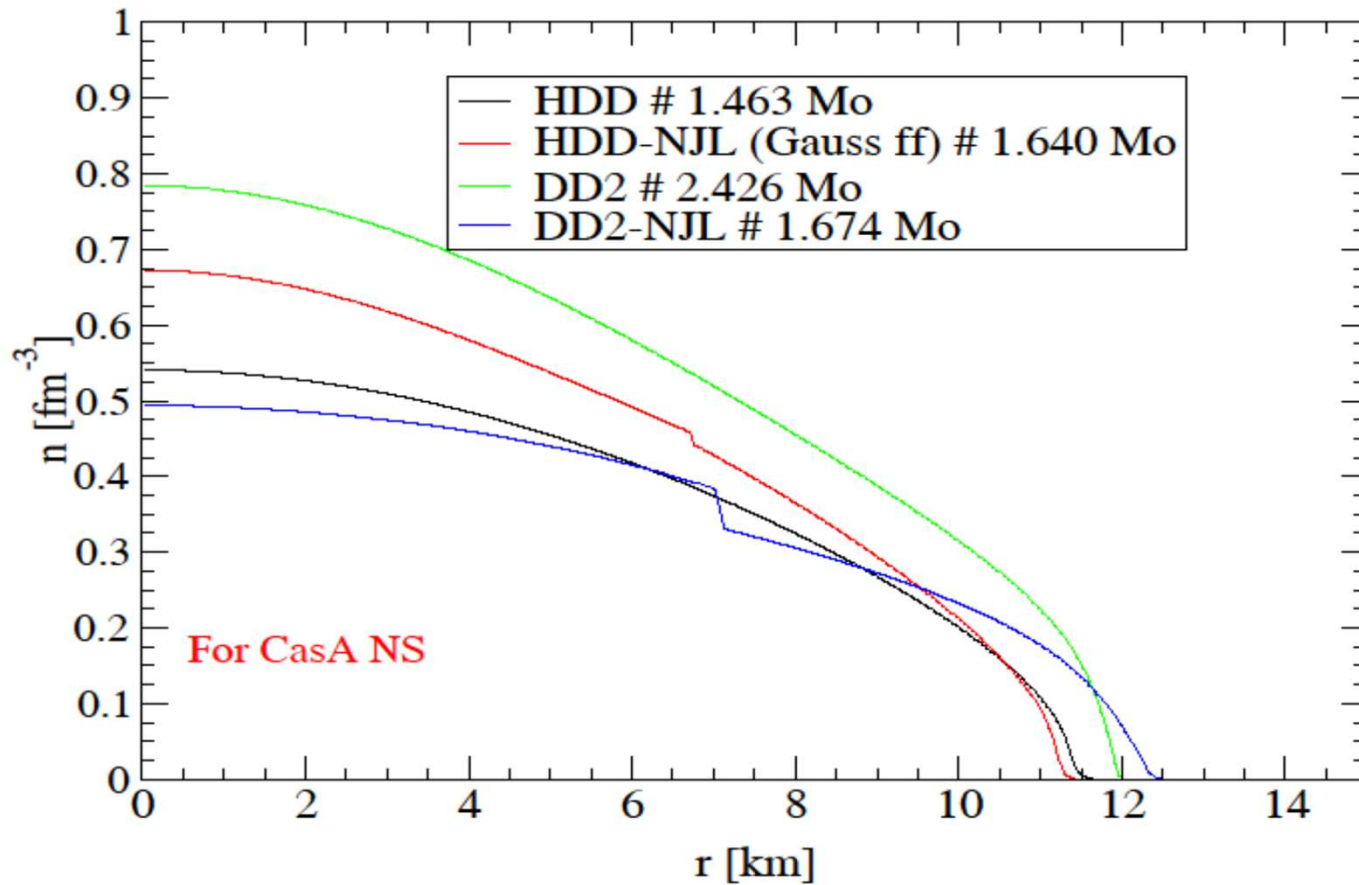


# Cas A As An Hybrid Star

H. Grigorian, D. Blaschke, D.N. Voskresensky, Phys. Rev. C 71, 045801 (2005)



# Possible internal structure of CasA



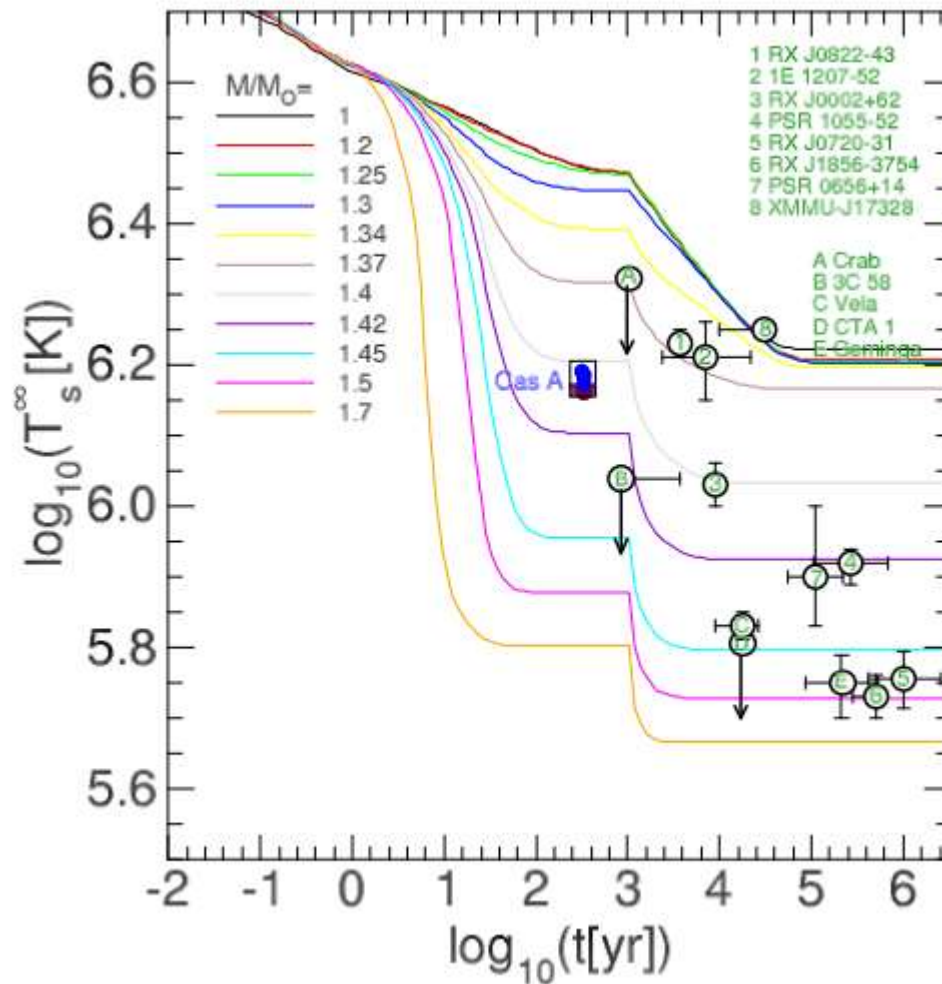


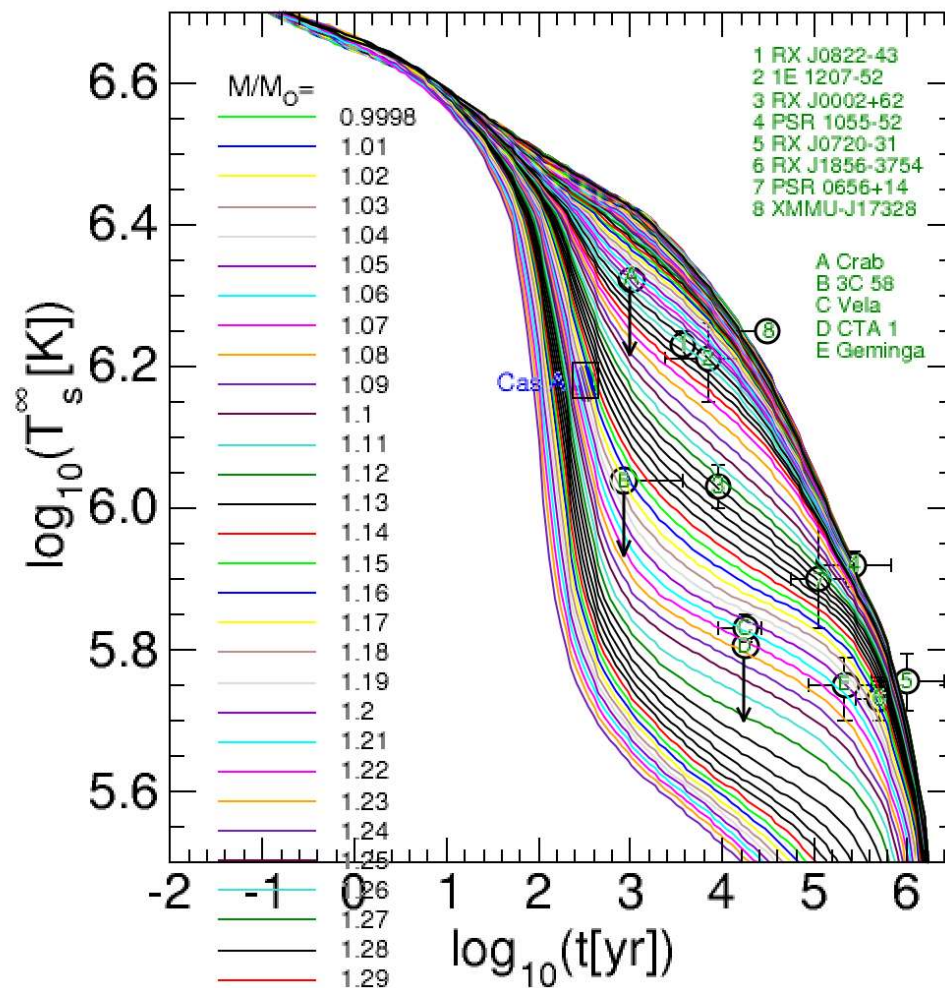
FIG. 2. Surface temperature as a function of NS age with masses  $M \in [1, 1.7]M_{\odot}$  including self-annihilating conducting DM ( $m_{\chi} = 0.1$  GeV).  $\chi$  emissivity has been enhanced a factor 5 larger than in Figure 1. LDM enhanced processes are active up to  $\tau \sim 10^3$  yr, followed by a period of decline, and again for  $t \gtrsim 1.5 \times 10^3$  yr. See text for details.

M. Ángeles Pérez-García  
H. Grigorian, C. Albertus,  
D. Barba, J. Silk

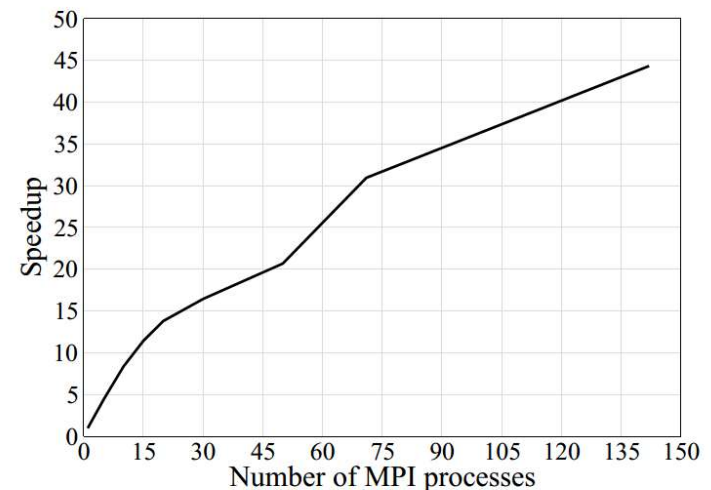
Physics Letters B  
 827(2022)136937



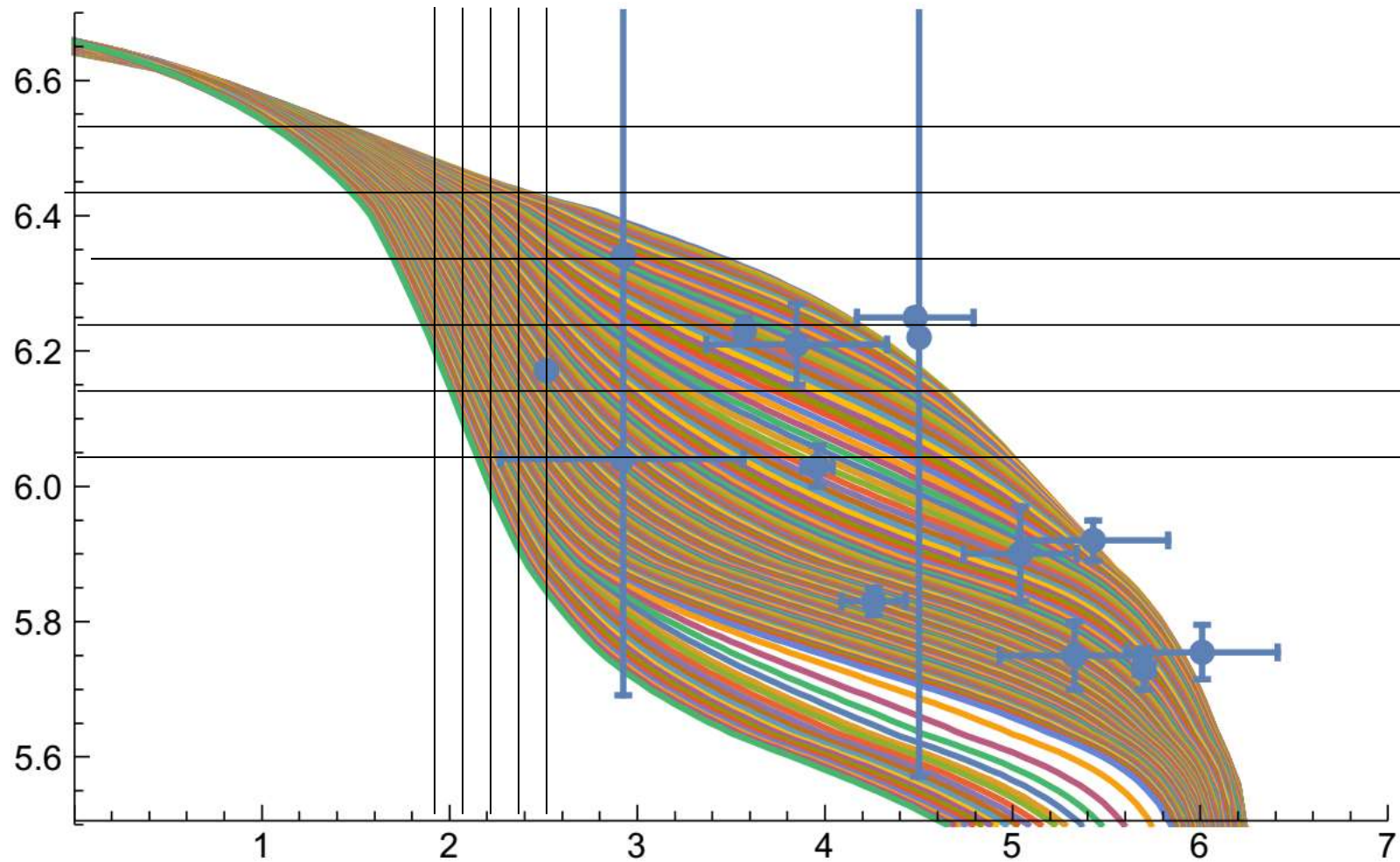
# Results produced with use of MPI Technology



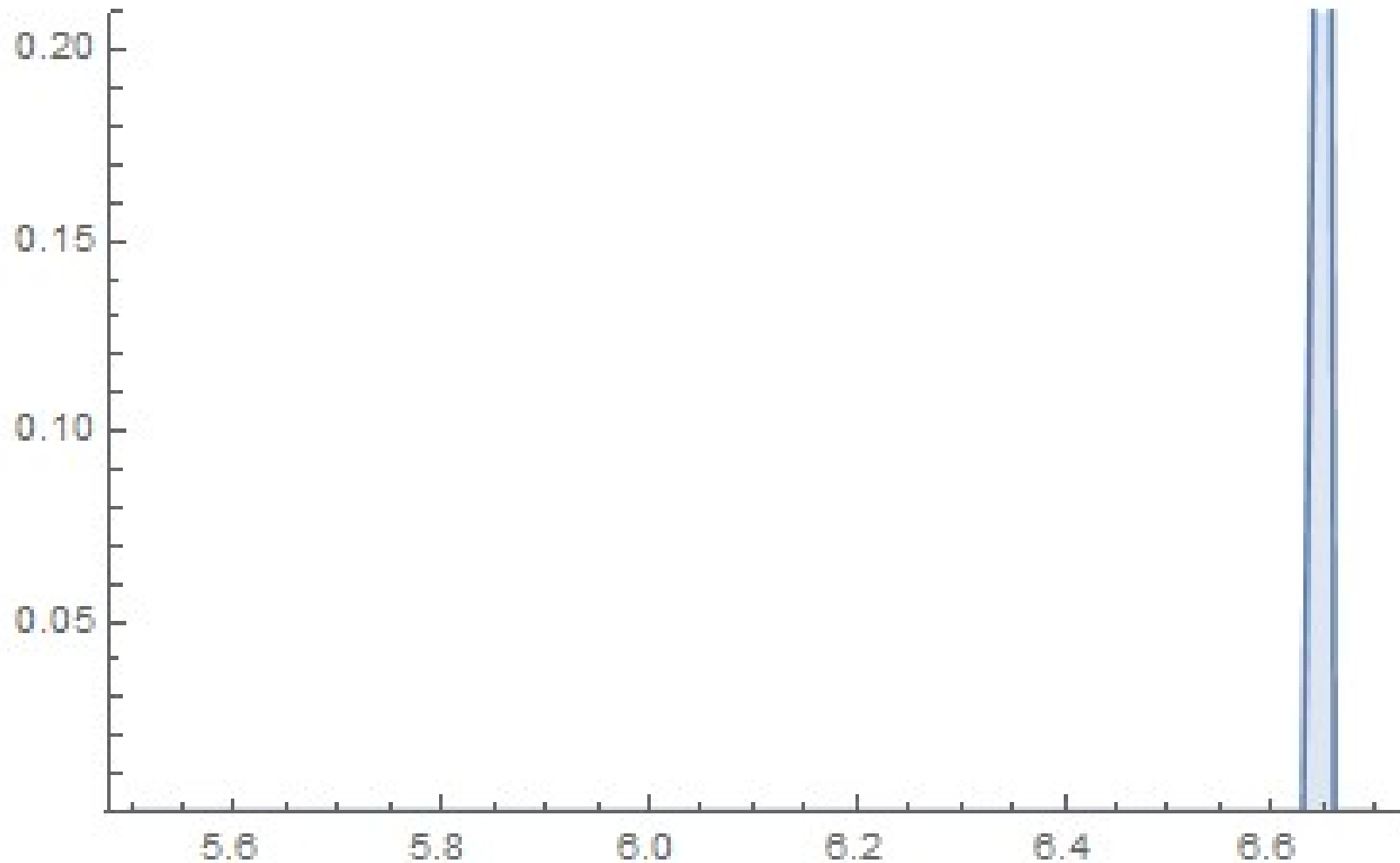
142 configurations has been calculated in **0m49s** on the 142 processes. On 1 process it takes **36m14s** – acc is **~ 44 times**



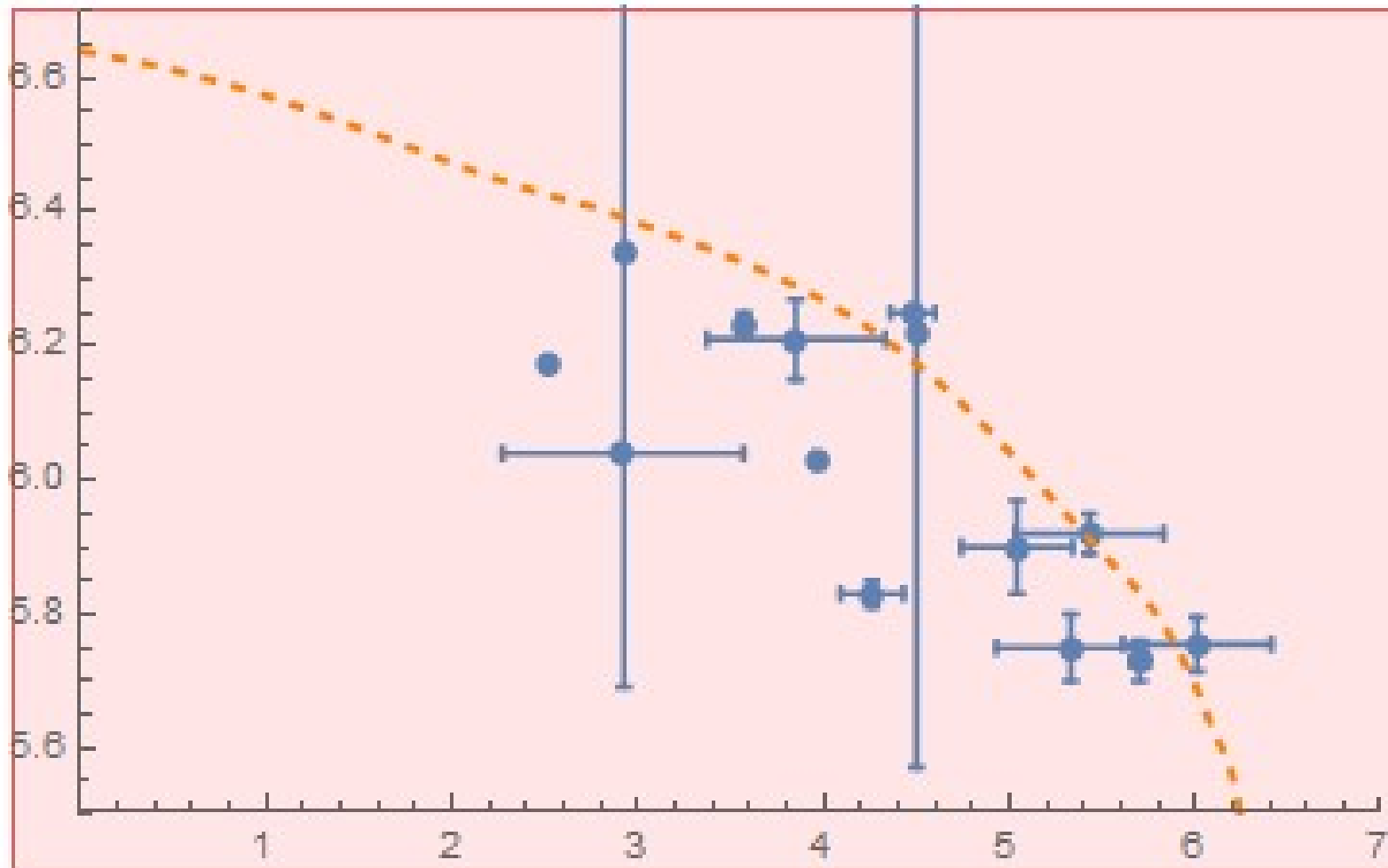
# Distribution of Evolution tracks via Temperature at given Time



## Distribution of Evolution tracks via Temperature at given Time

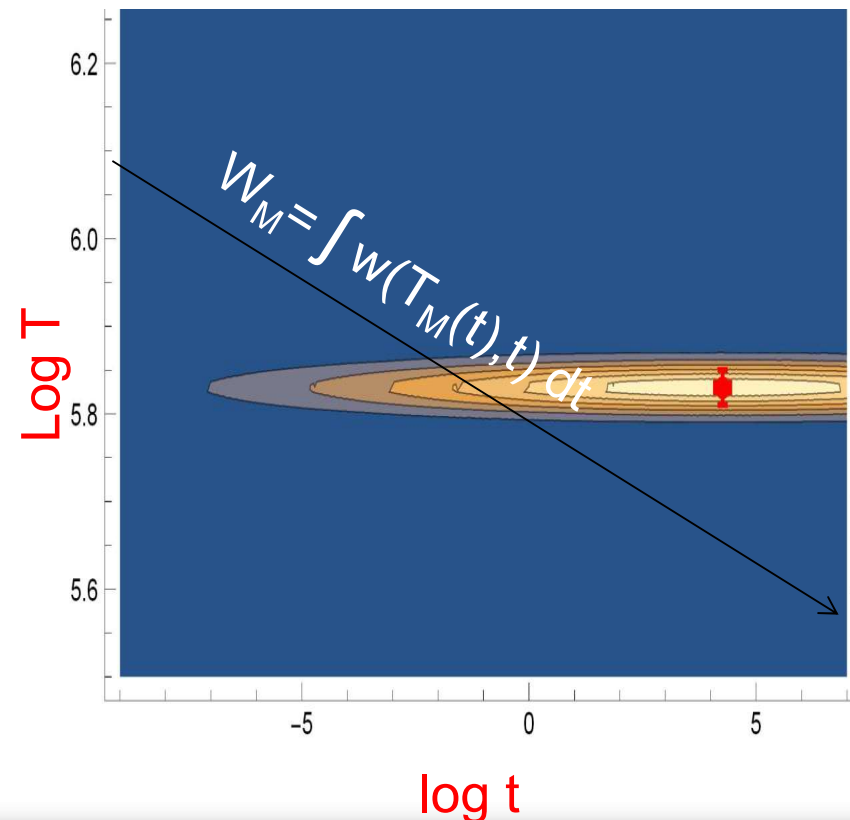
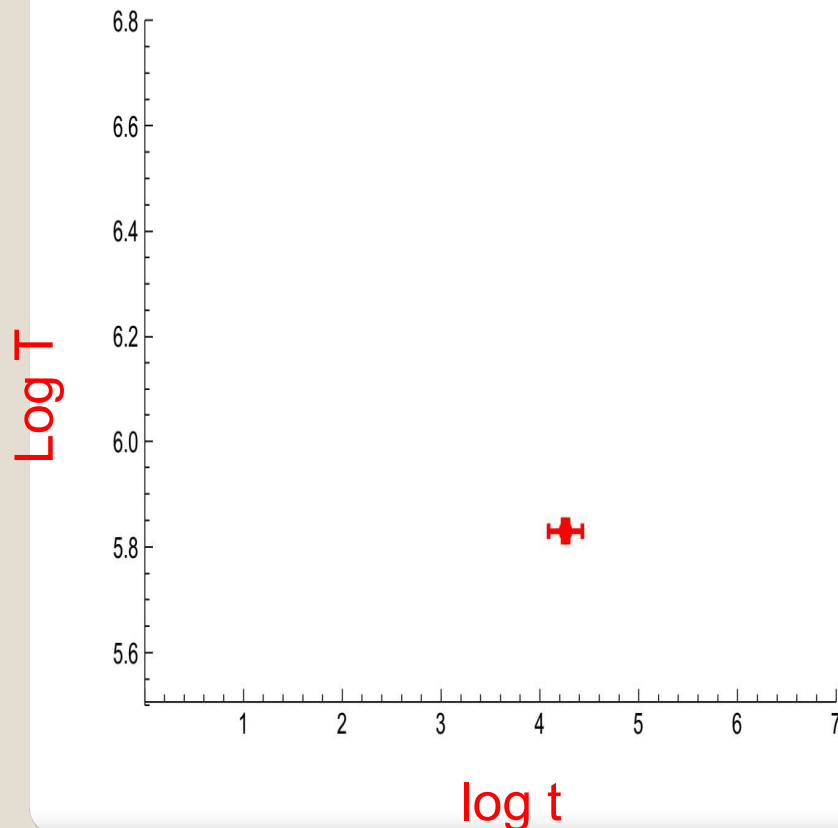


# Evolution tracks for different NS Masses

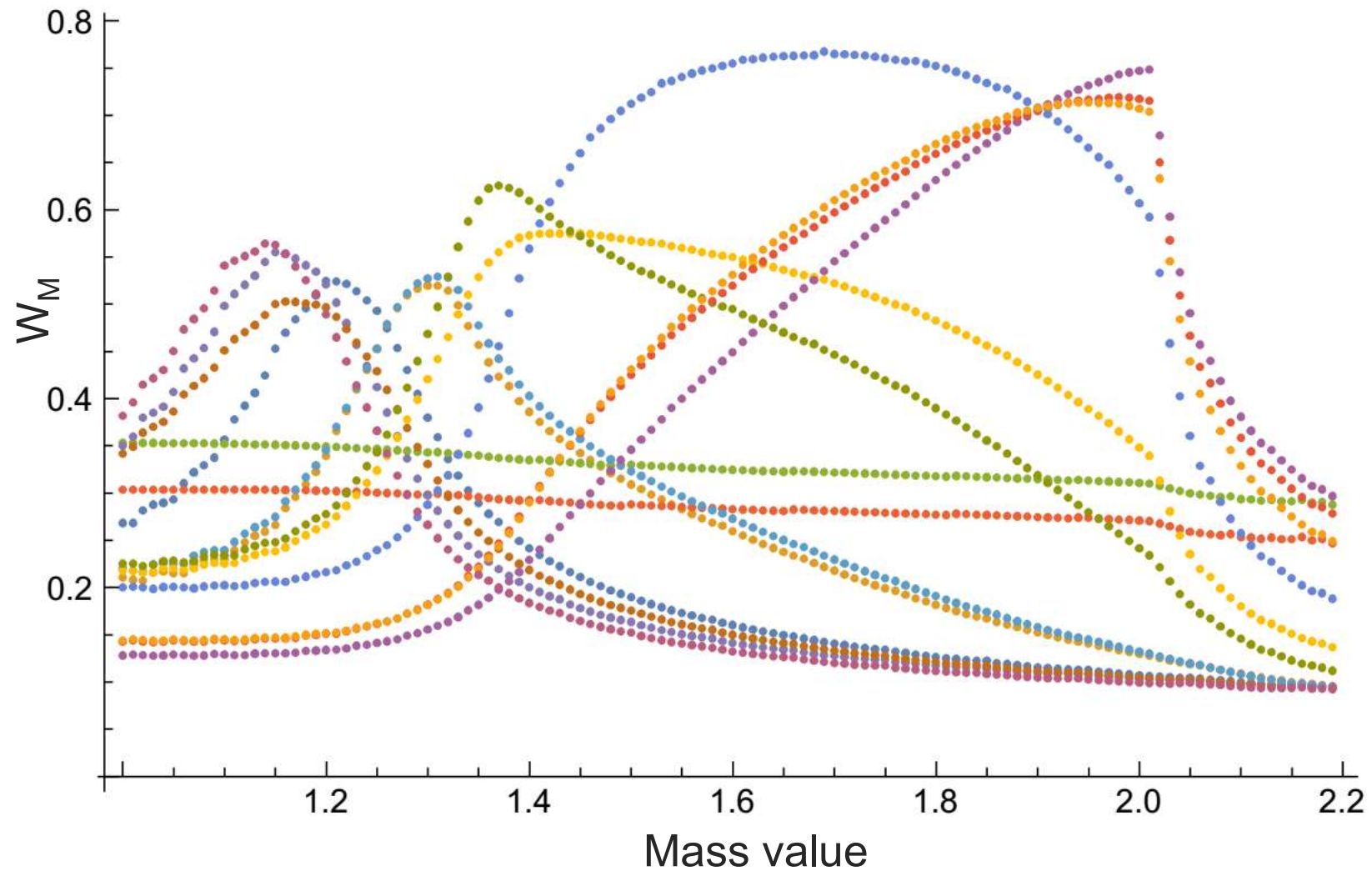


# Weighting of Data point on the Temperature - Age Diagram

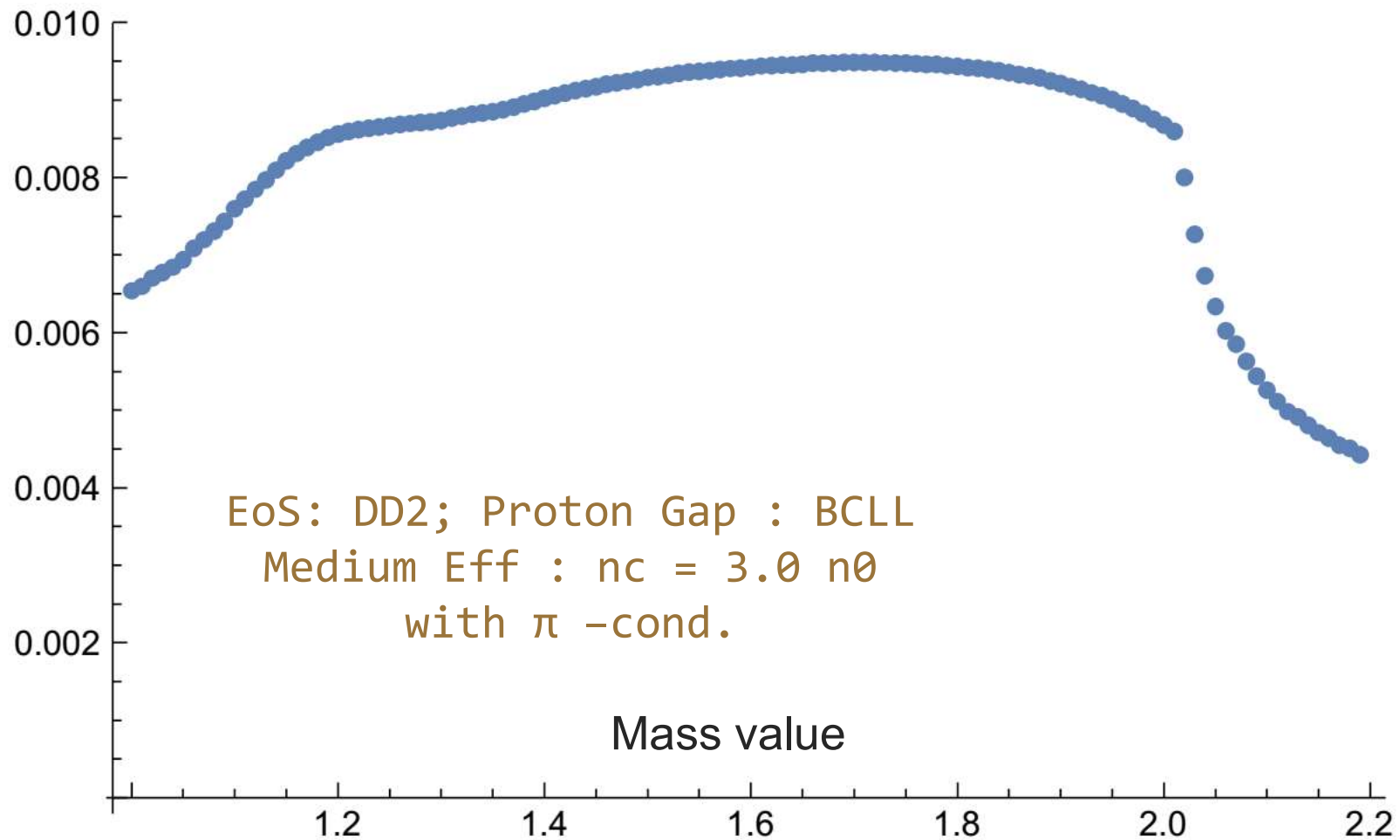
$$w(T,t) = \text{Exp}\left\{(\log T - \log T_D)^2 / \sigma_T + (\log t - \log t_D)^2 / \sigma_t\right\}$$



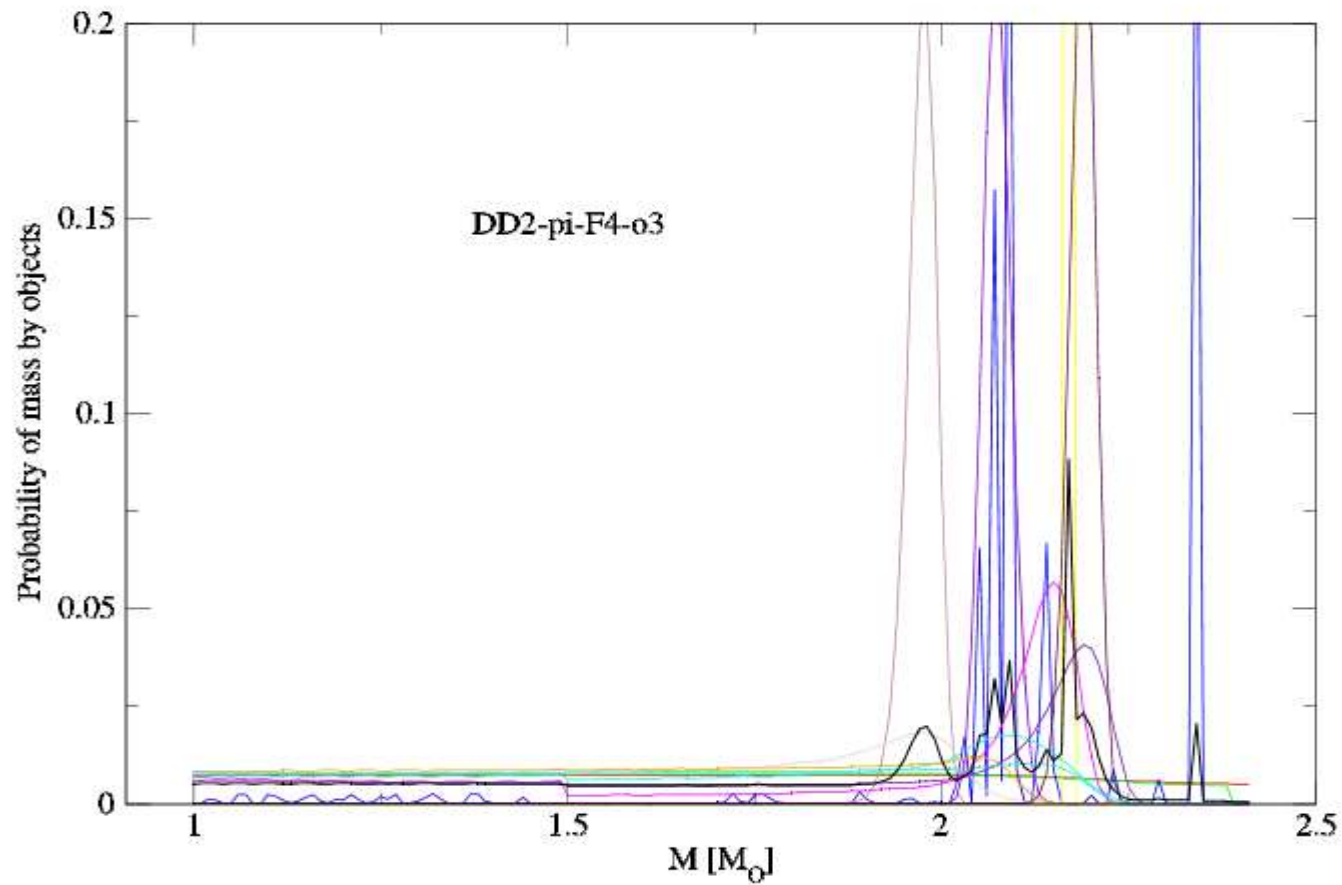
# Expected Mass value for the Data points on the T - t Diagram



# Expected Mass value for the Data points on the T - t Diagram

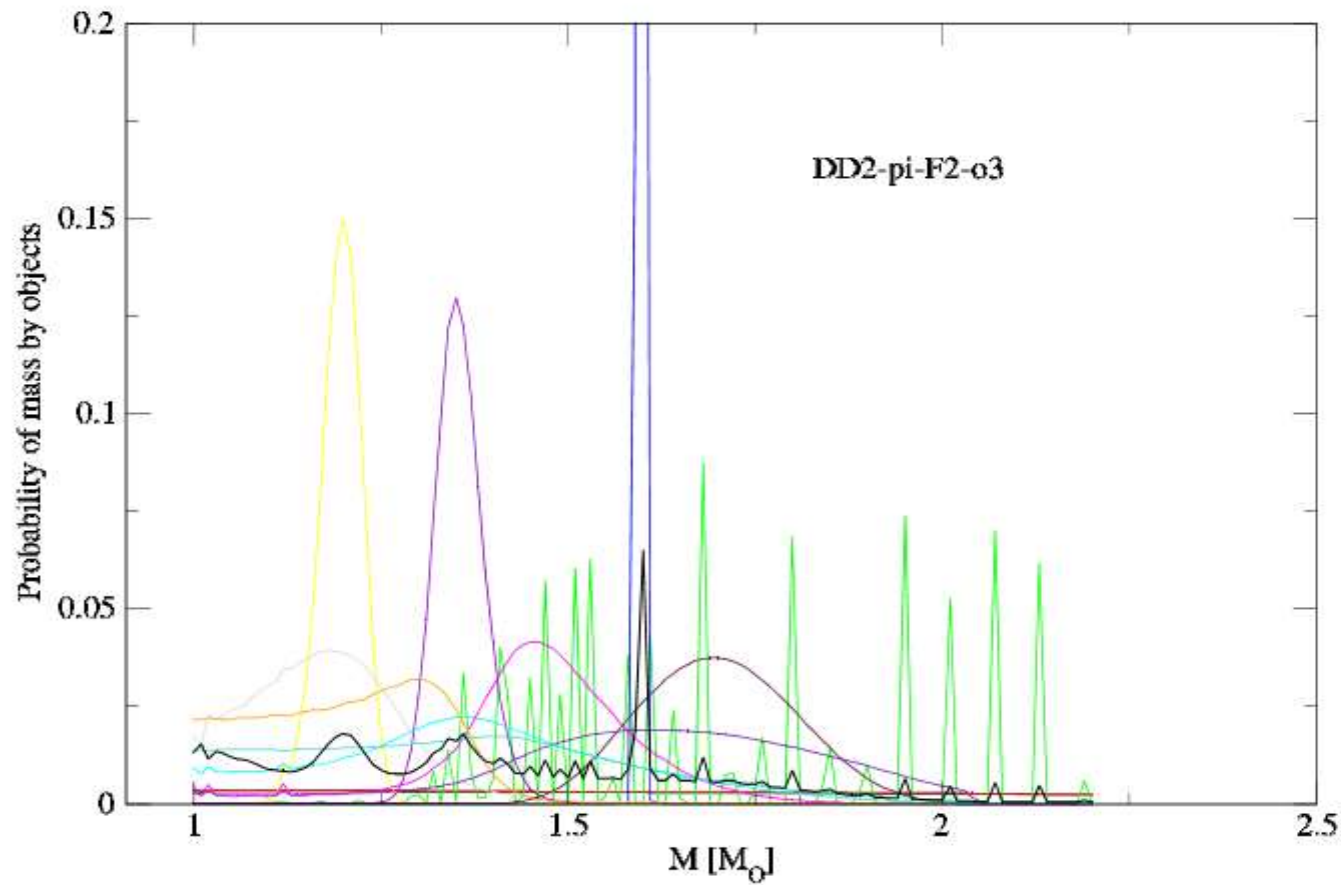


# Expected Mass value for the Data points on the T - t Diagram

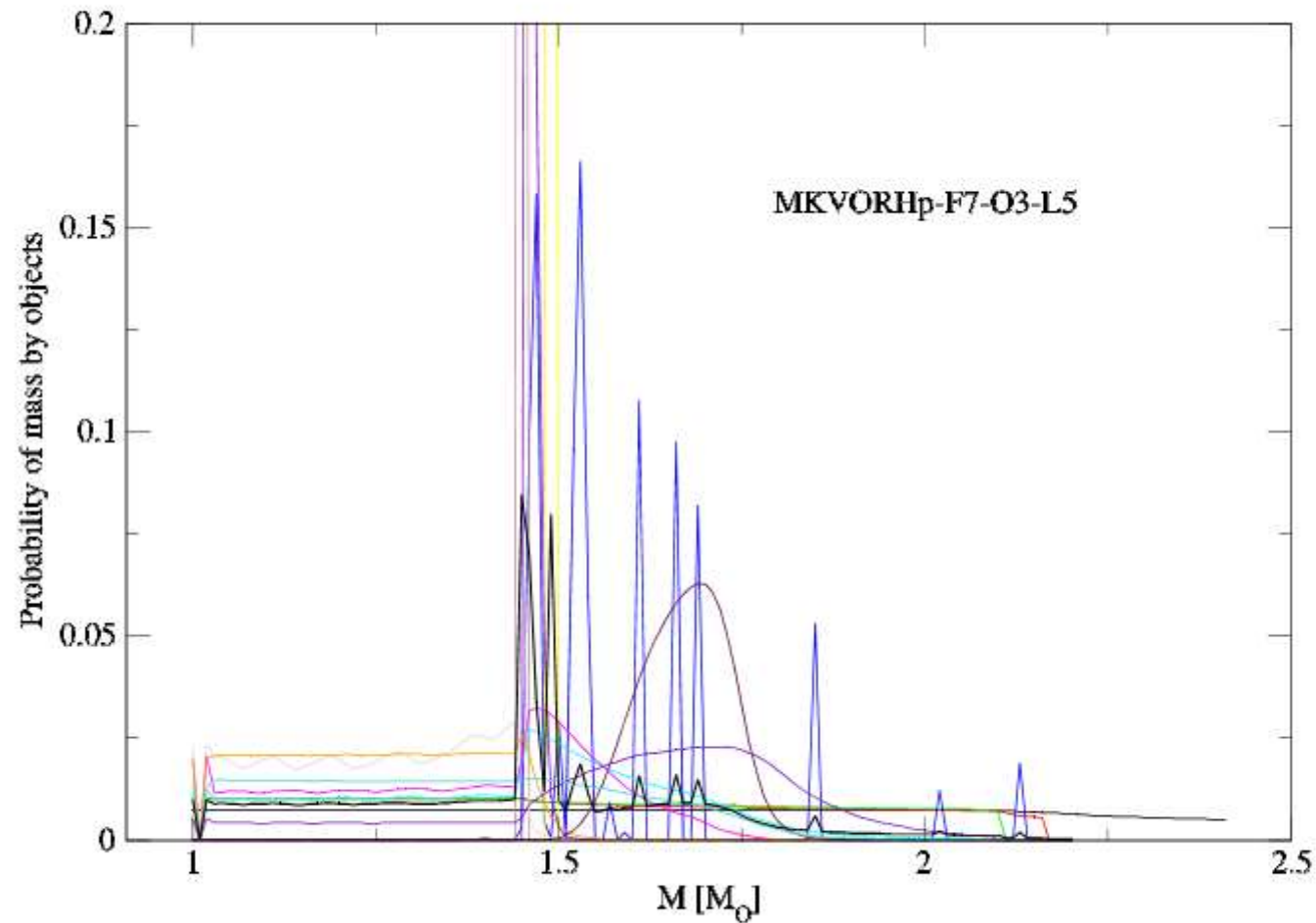




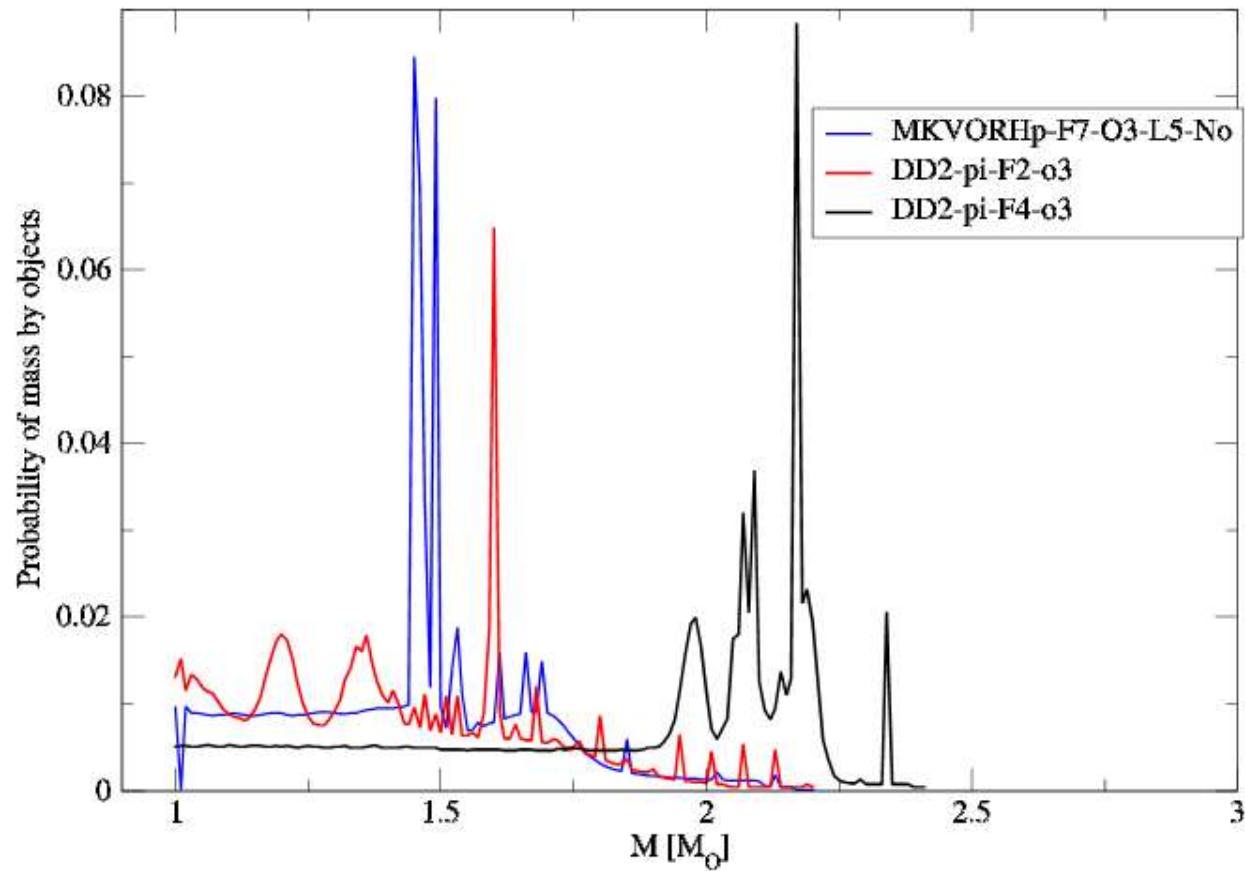
# Expected Mass value for the Data points on the T - t Diagram



# Expected Mass value for the Data points on the T - t Diagram



# Expected Mass value for the Data points on the T - t Diagram

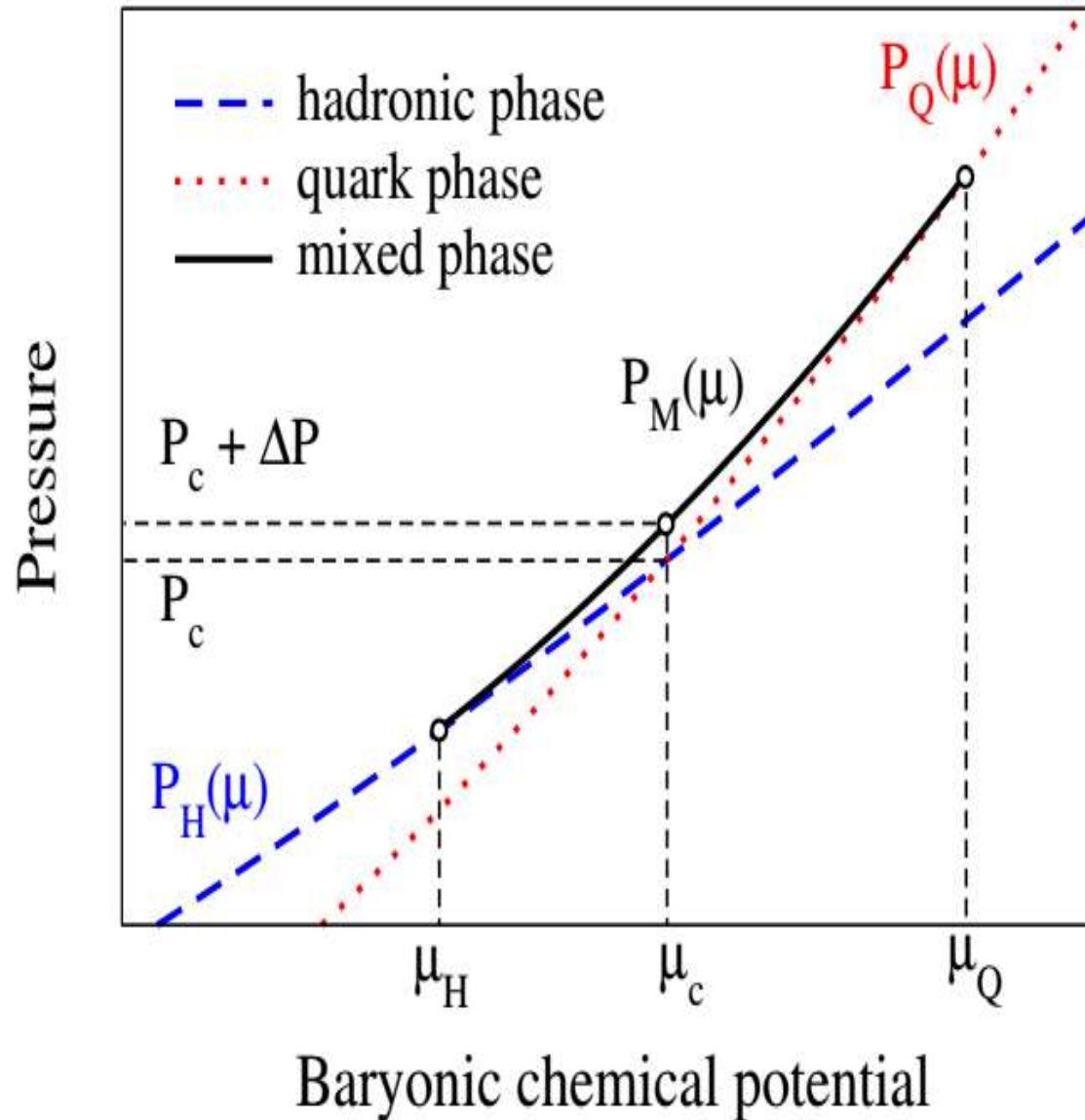


# Conclusions

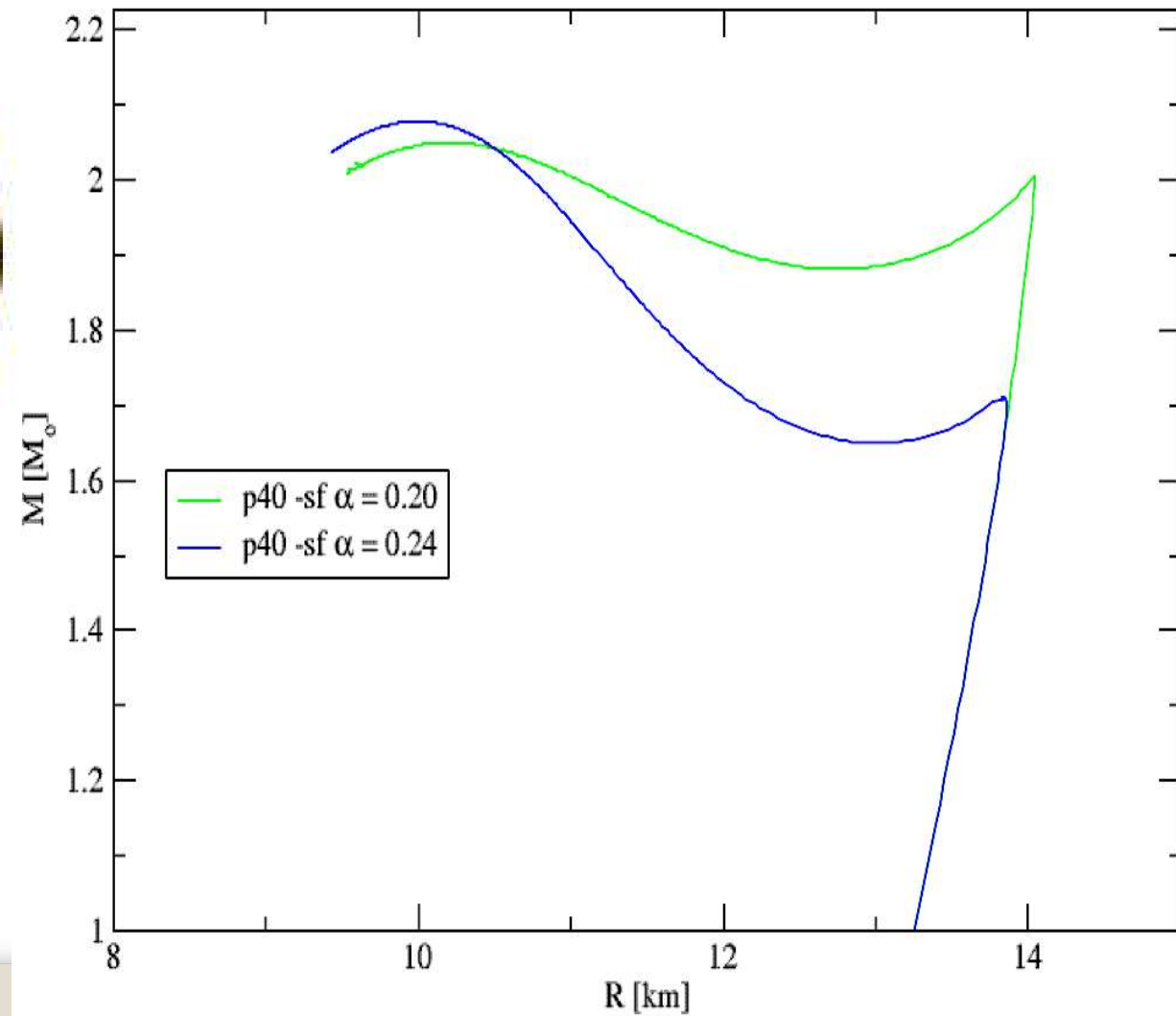
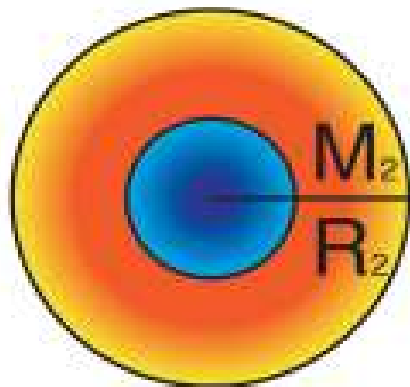
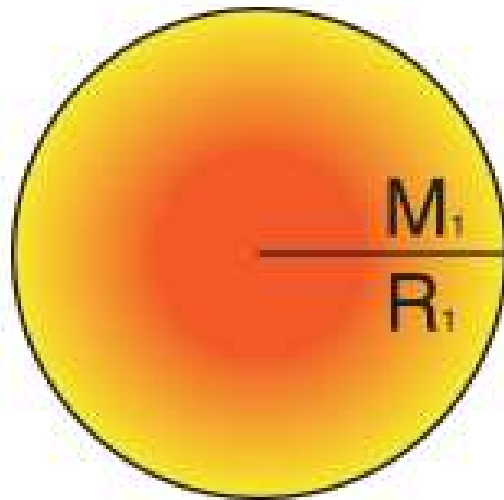
- All known cooling data including the Cas A rapid cooling consistently described by the “nuclear medium cooling” scenario
- Influence of stiffness on EoS and cooling can be balanced by the choice of corresponding gap model.
- Parallelization allowed to make the calculations for statistical analyses of models in reasonable time,
- it allows to estimate the masses of observed objects.
- The cases of existence of Hyperons and/or Quarks or Dark-Matter in high-mass stars could be discussed for extraction of stars masses.

Thank YOU!!!!

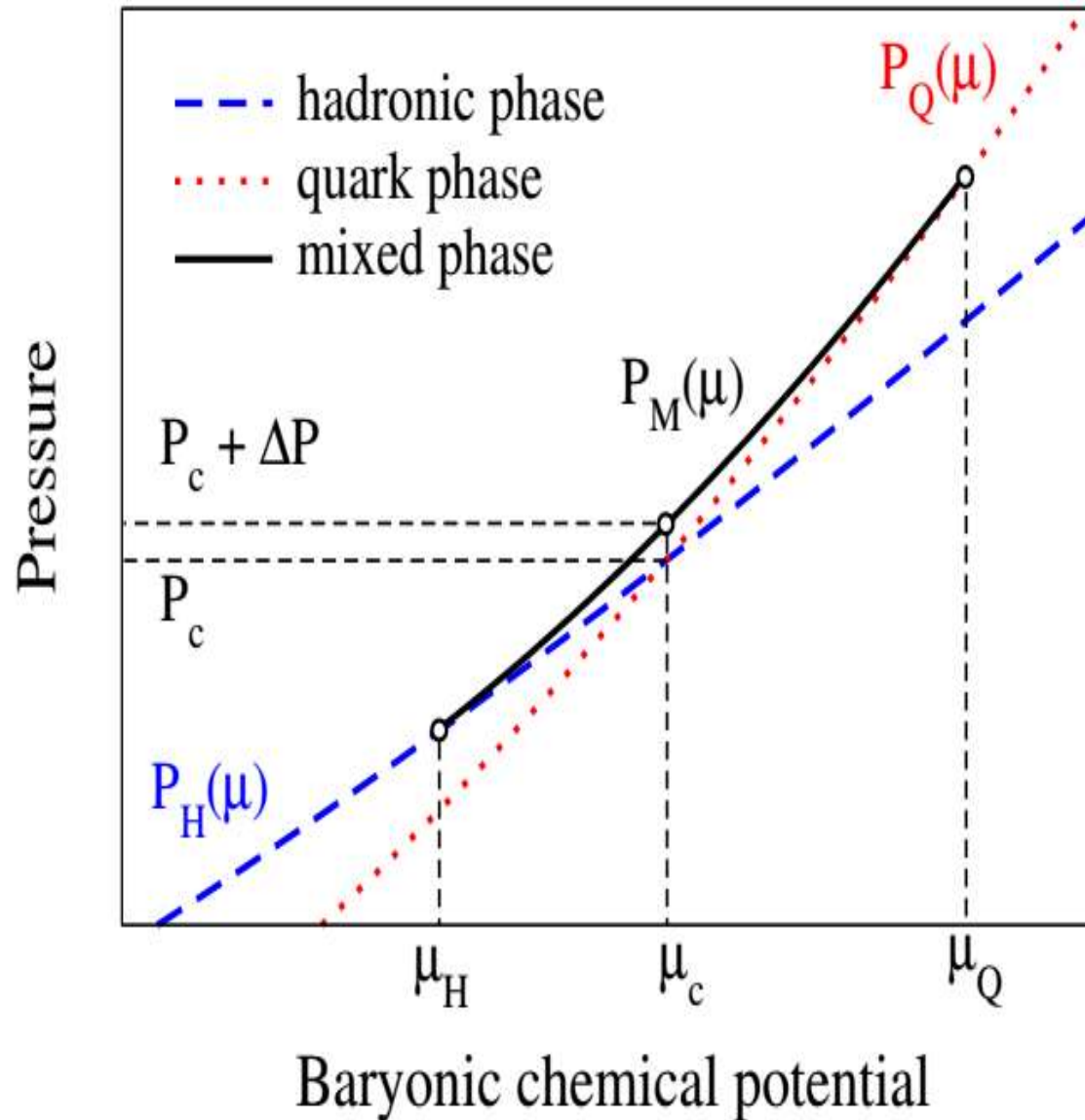
# Mixed Phase in Quark-Hadron Phase Transition



# High Mass Twin CS

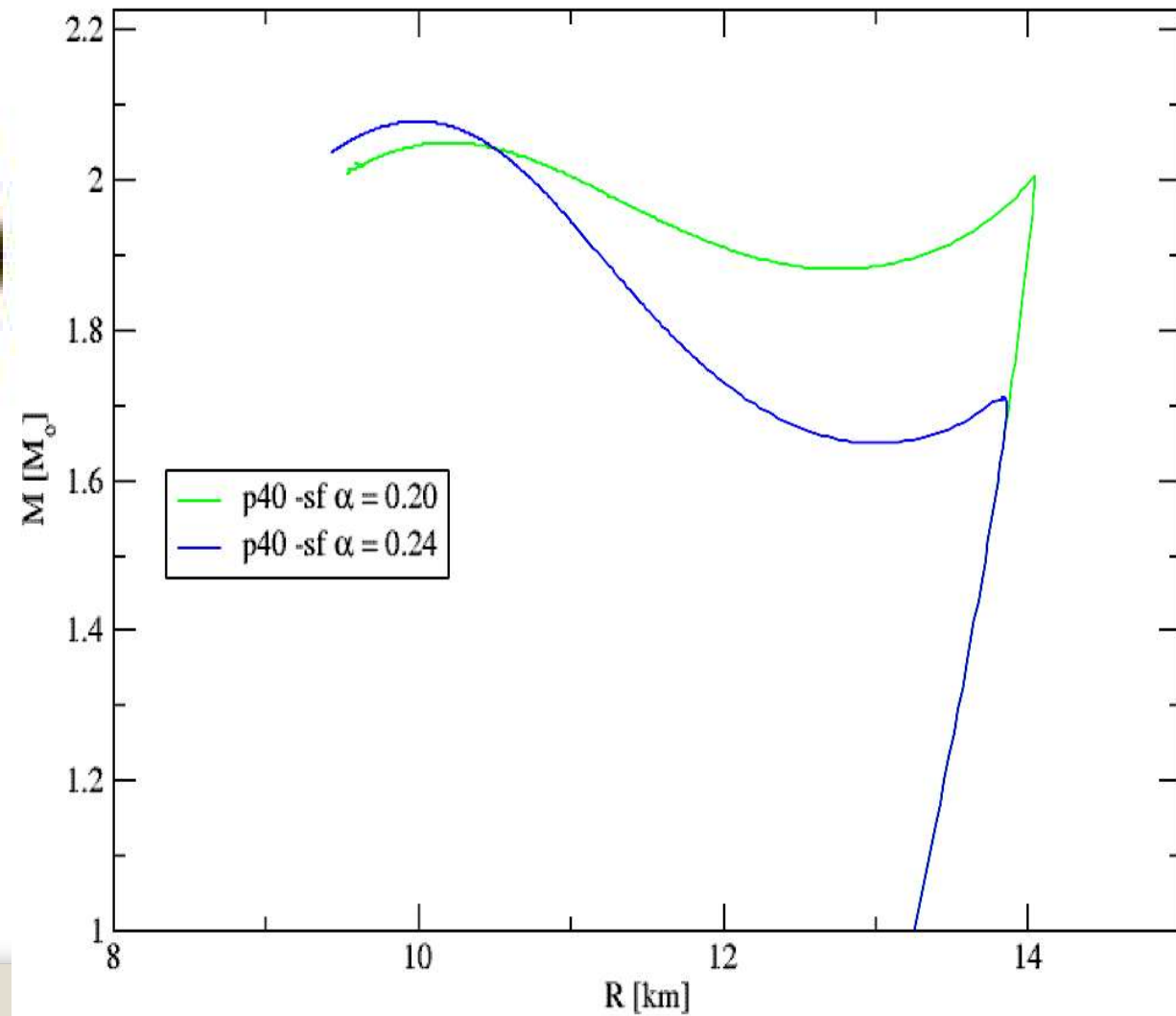
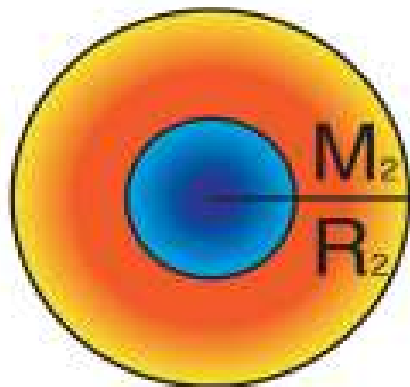
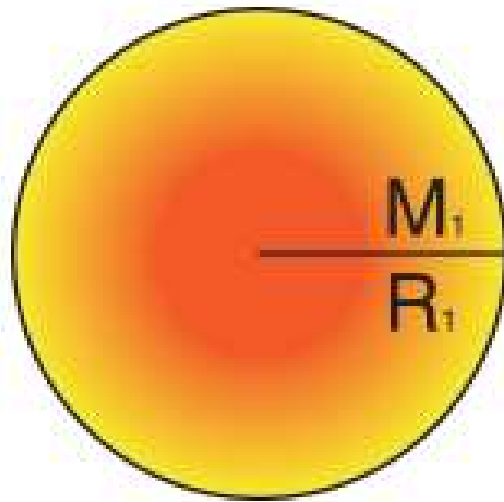


# Mixed Phase in Quark-Hadron Phase Transition



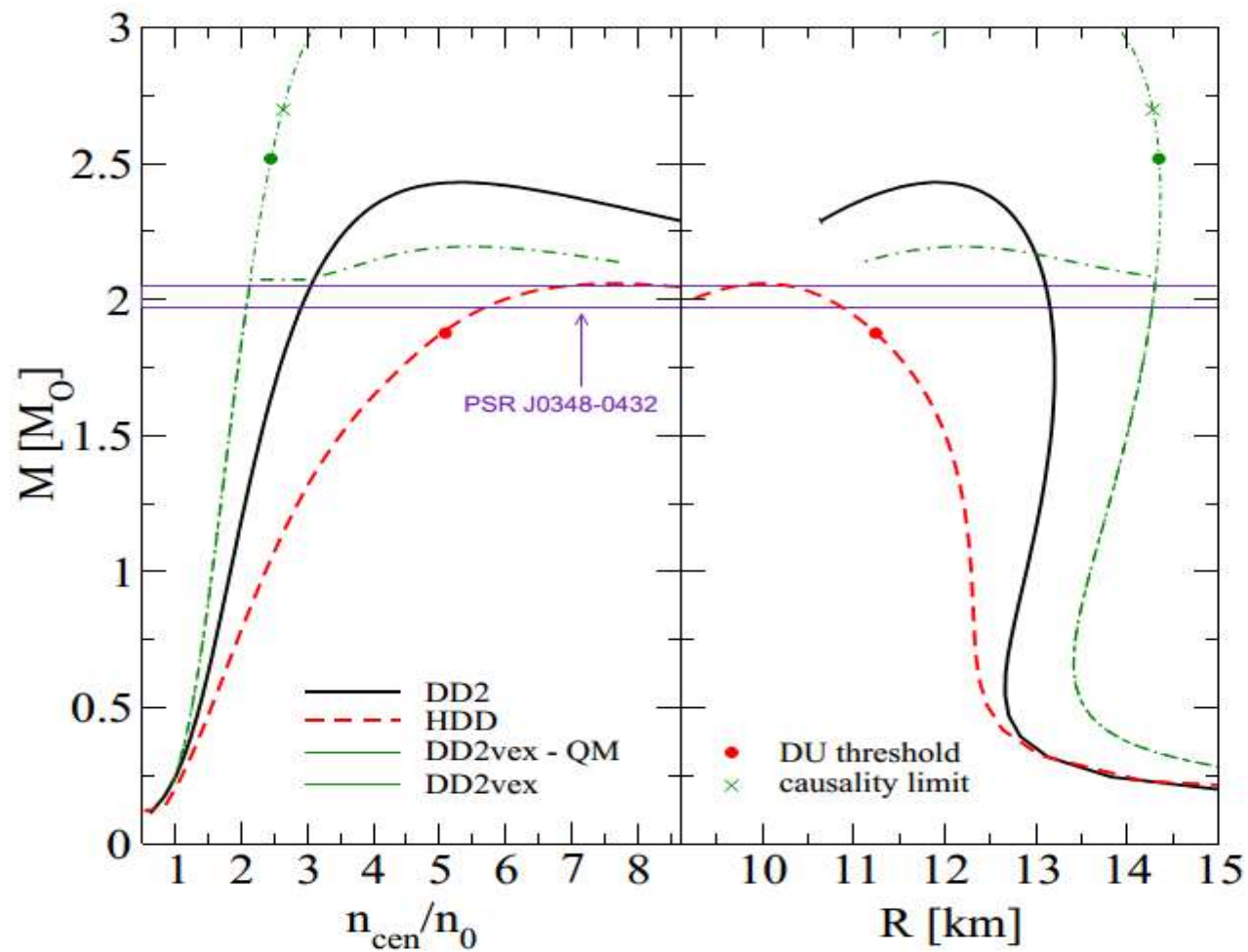


# High Mass Twin CS

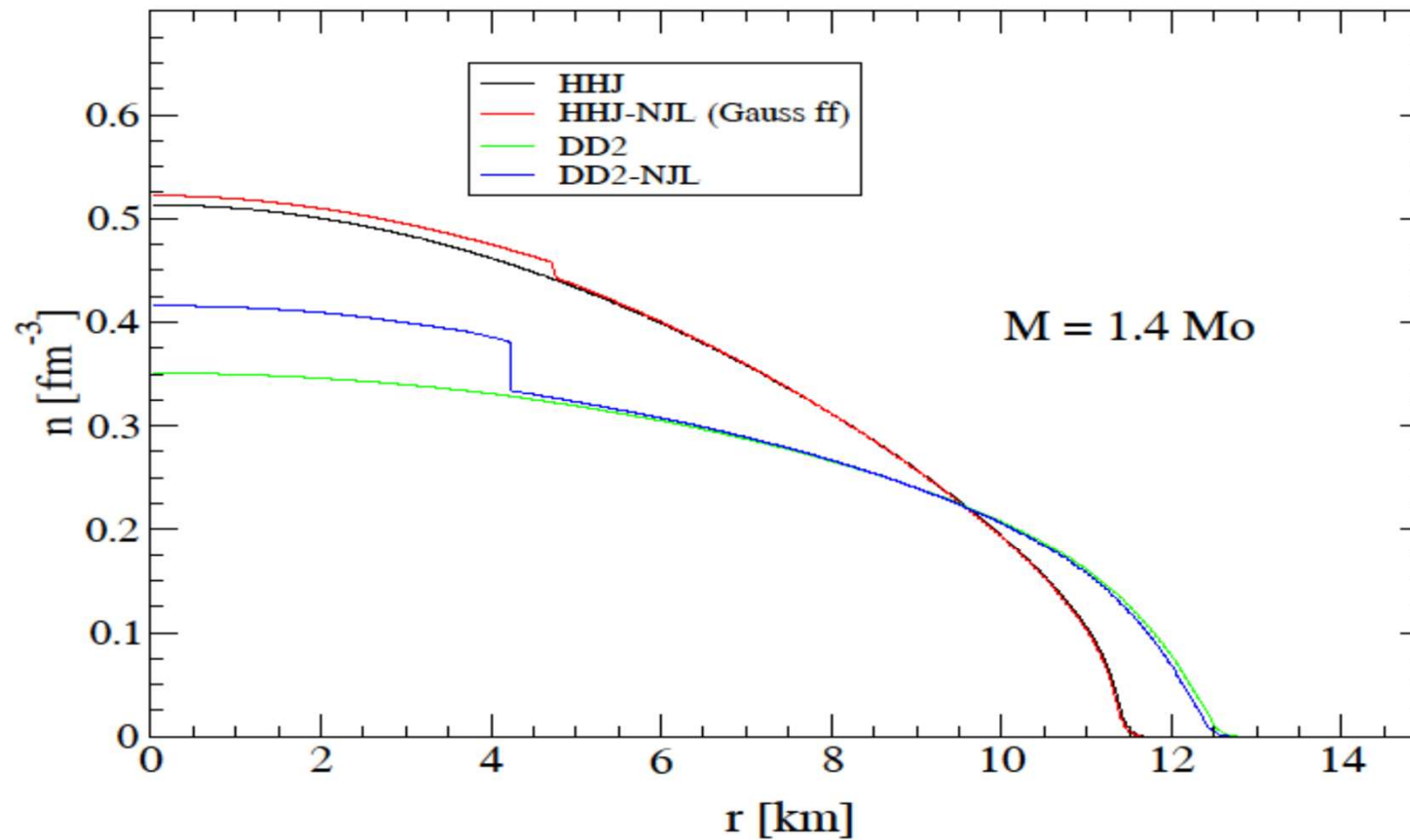


# Stability of stars

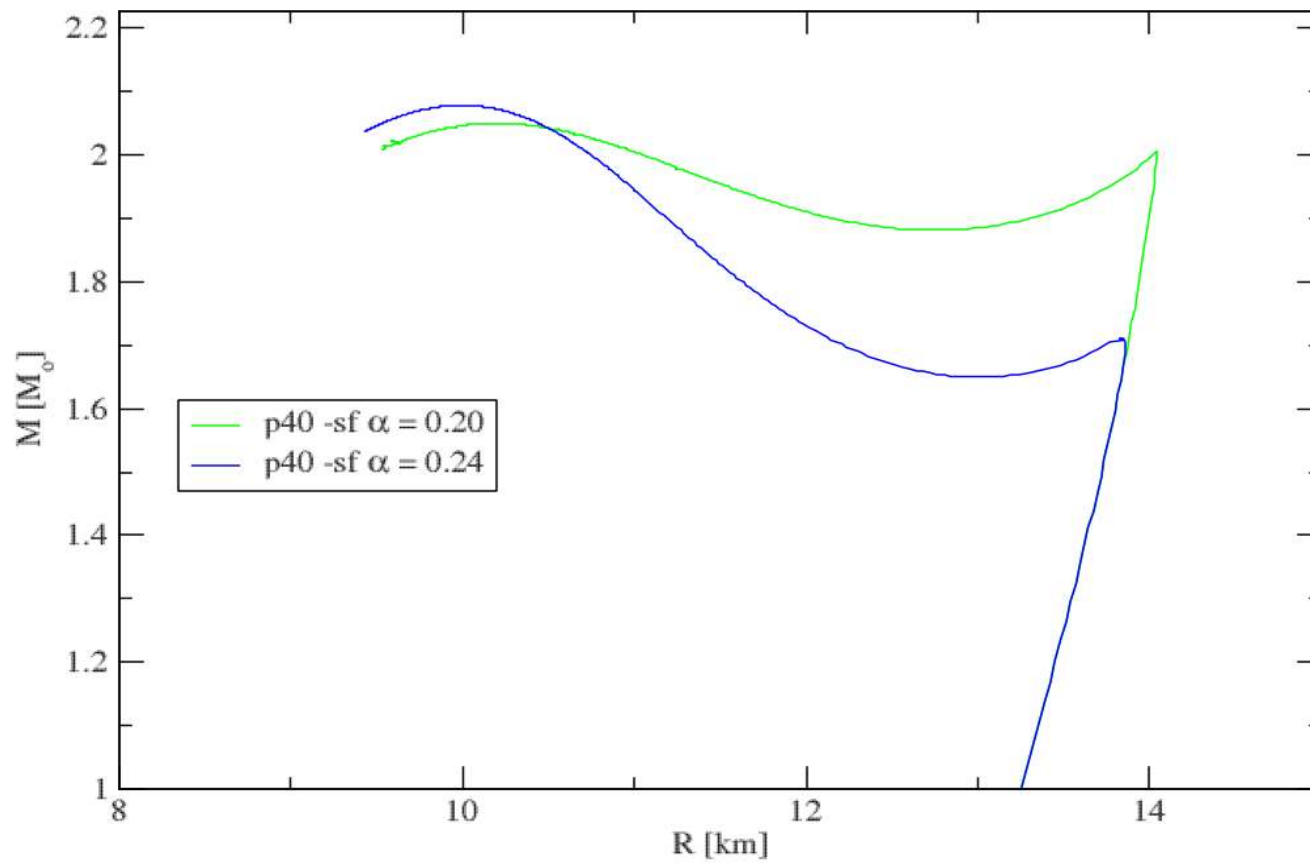
## HDD, DD2 & DDvex-NJL EoS model



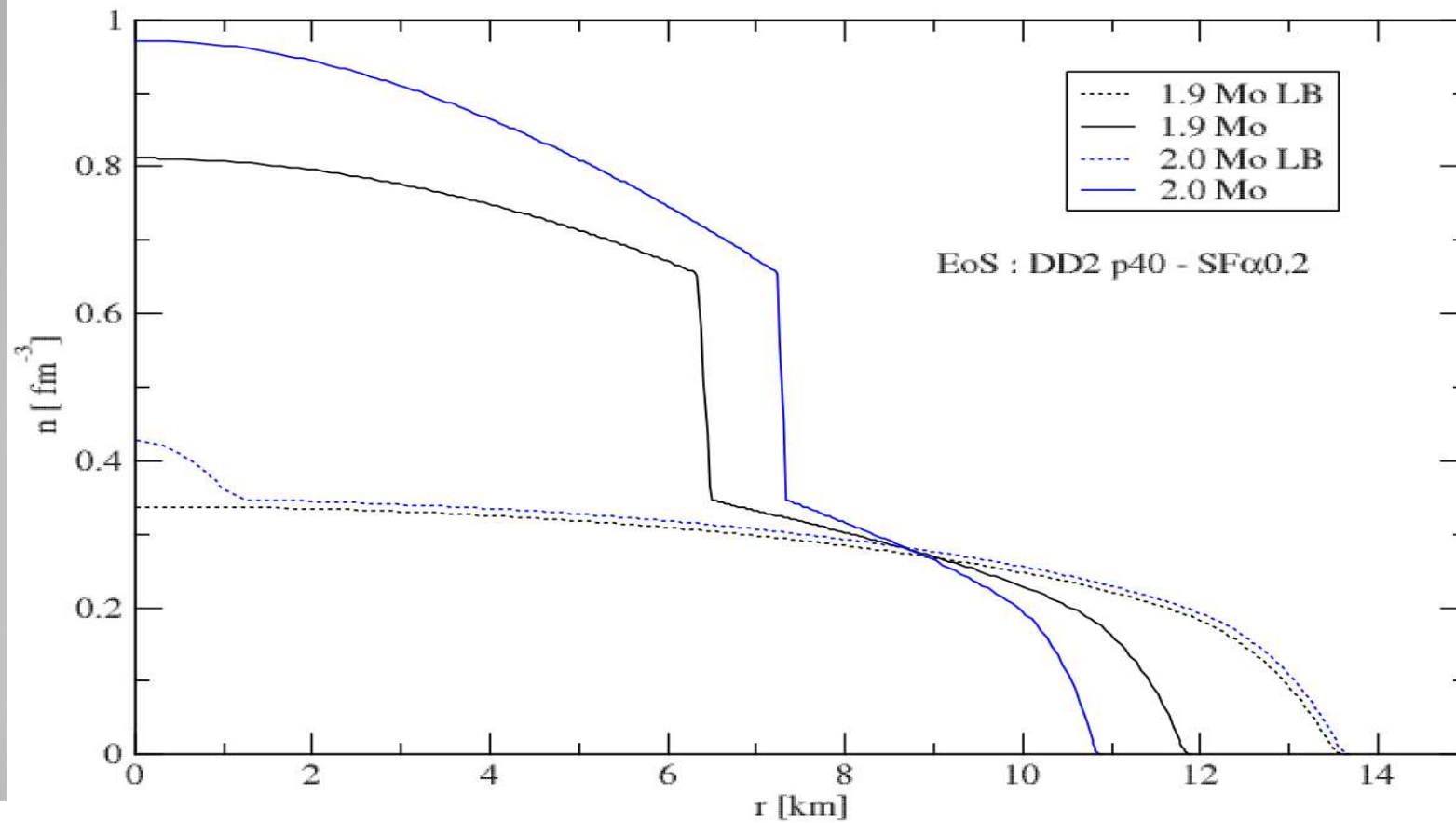
# Different Configurations with the same NS mass



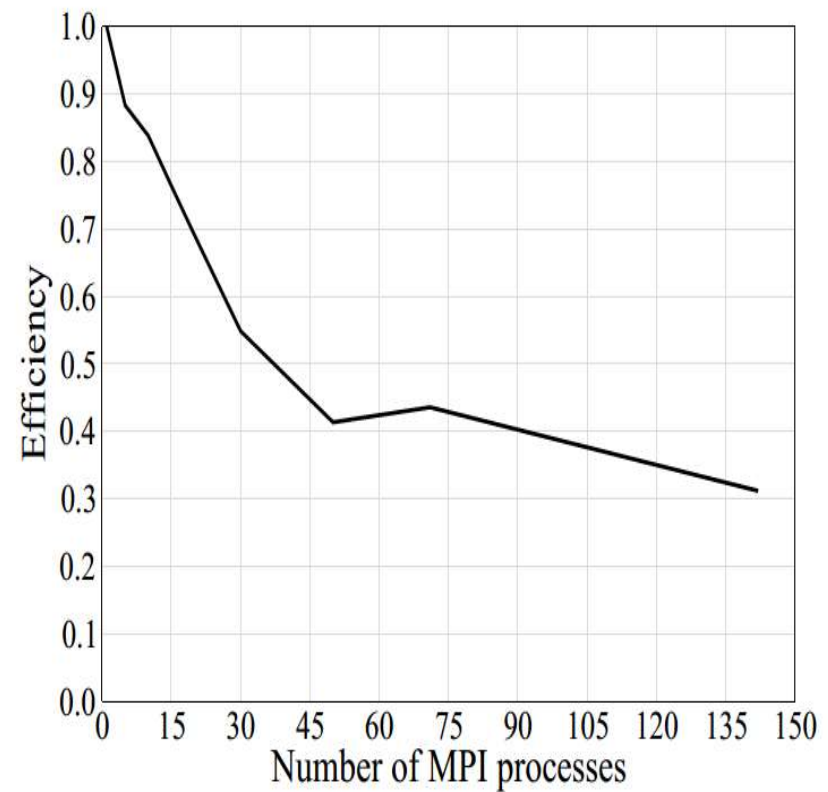
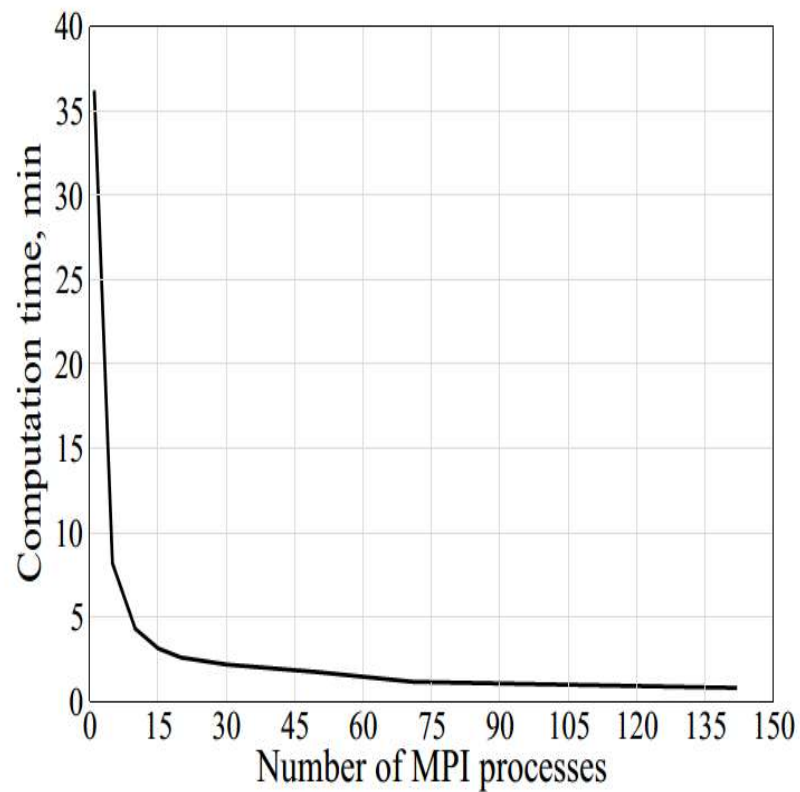
# High Mass Twin CS



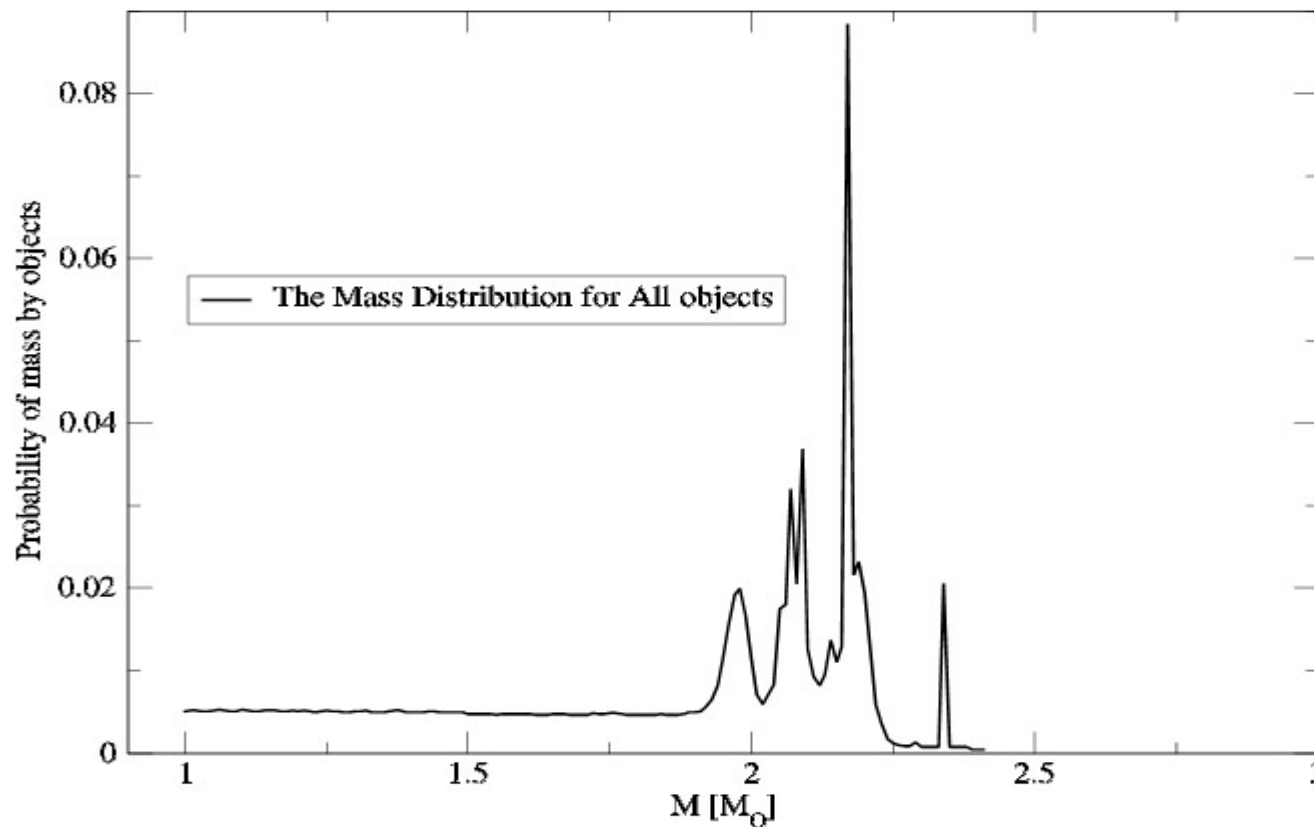
# Different Configurations with the same NS mass



# Calculation Time and efficiency

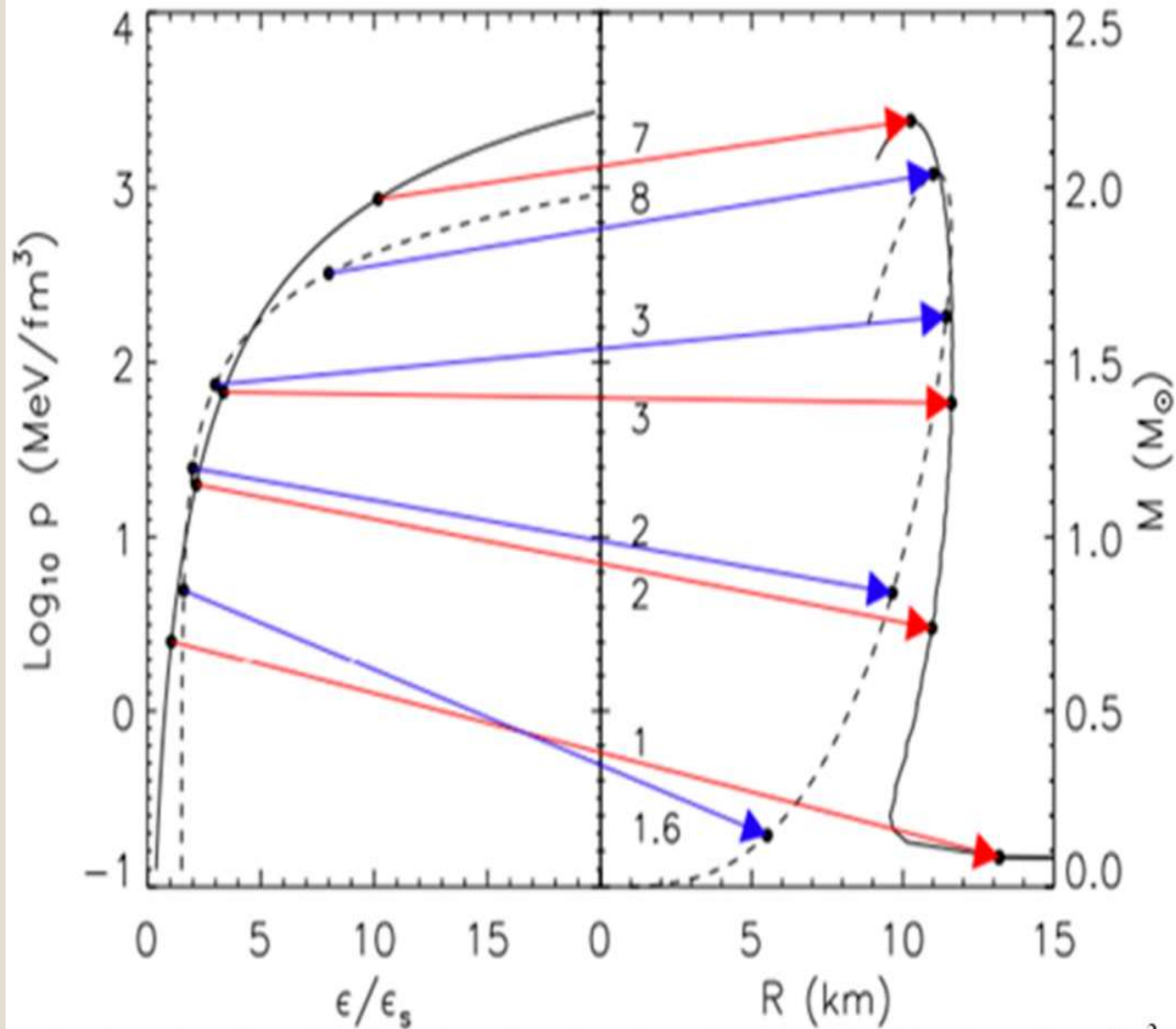


# Expected Mass value for the Data points on the T - t Diagram



EoS:  
DD2;  
Proton  
Gap :  
BCLL  
Medium  
Eff :  $n_c$   
 $= 3.0 n_0$   
with  $\pi$  -  
cond.

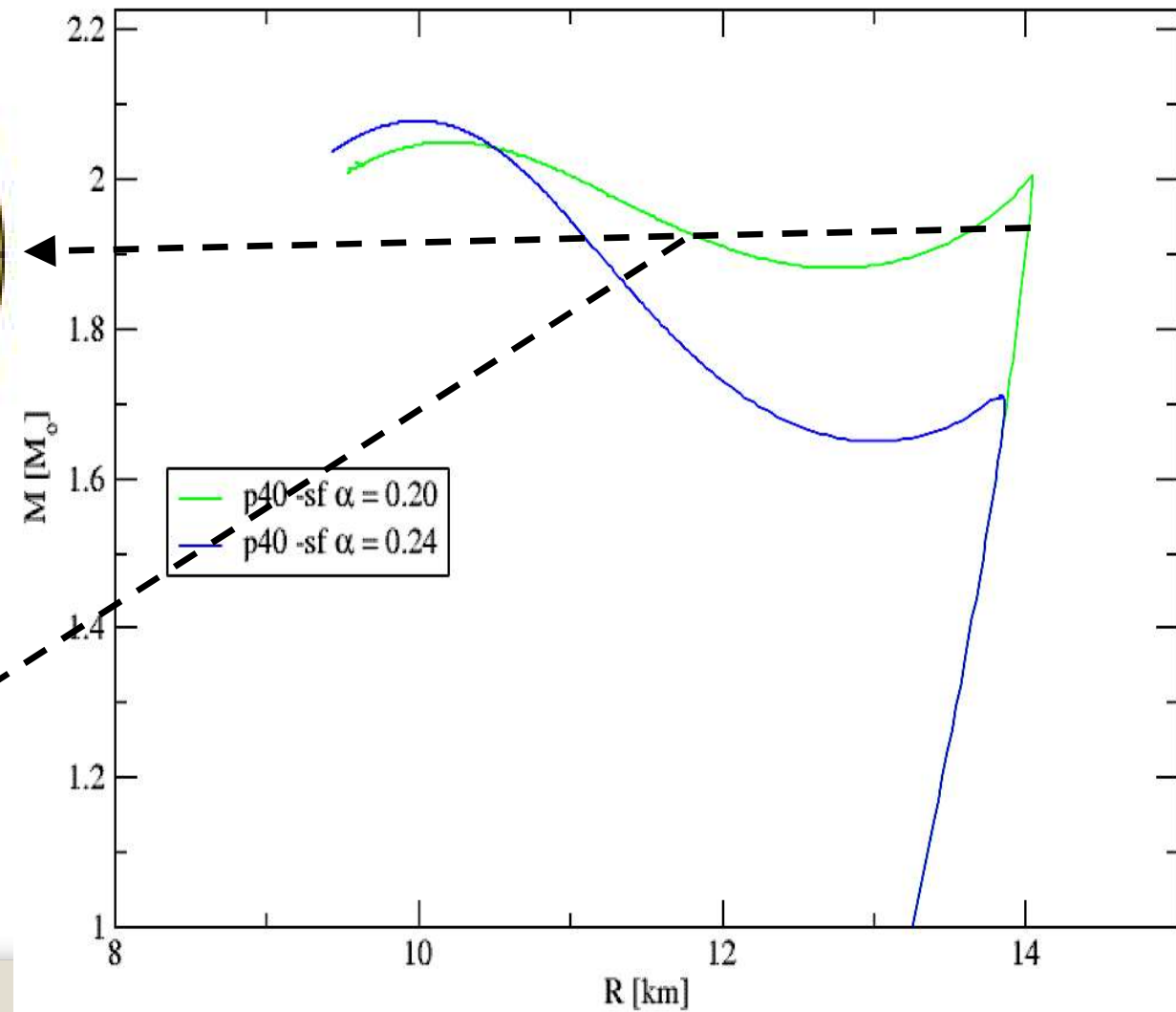
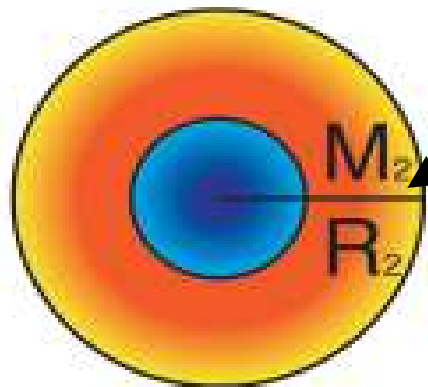
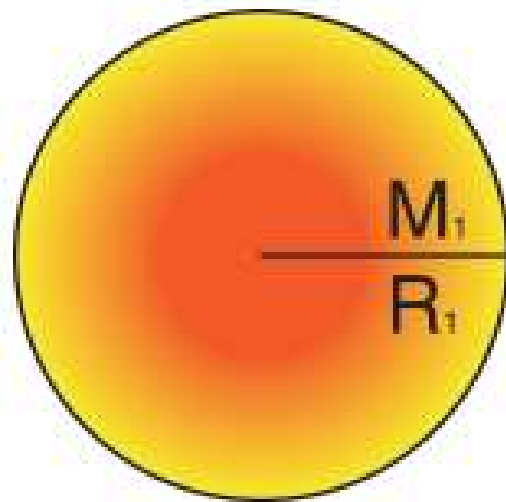
# EoS vs. Mass Radius of NS



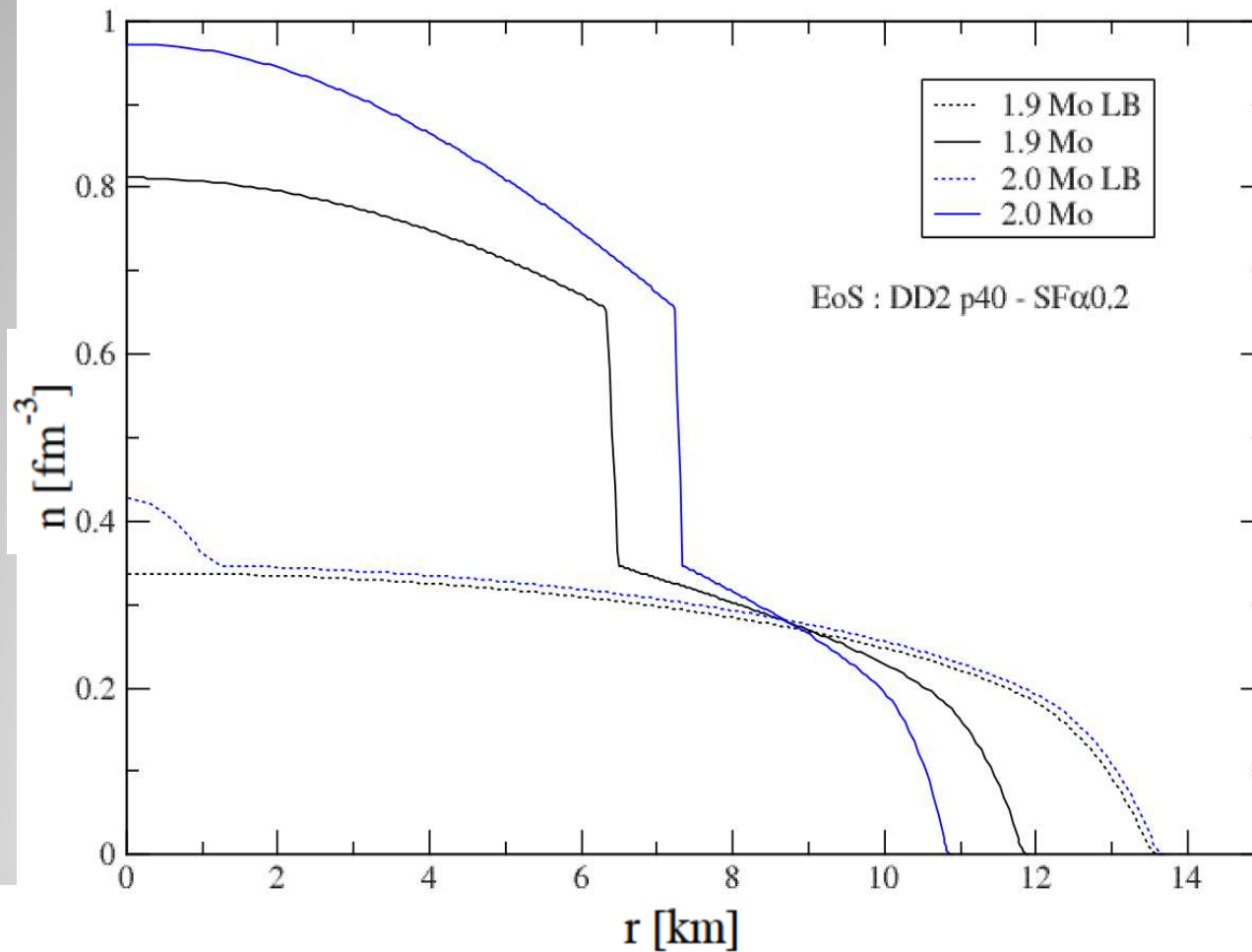
Lattimer,  
Annu. Rev. Nucl. Part. Sci. 62,  
485 (2012)  
arXiv: 1305.3510



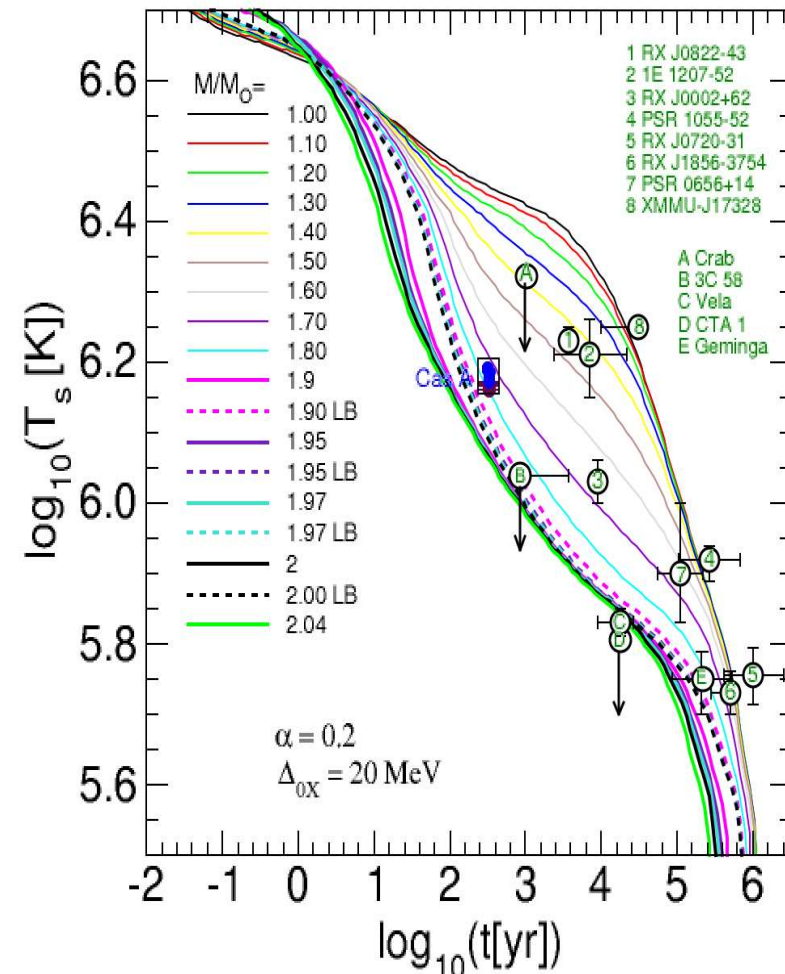
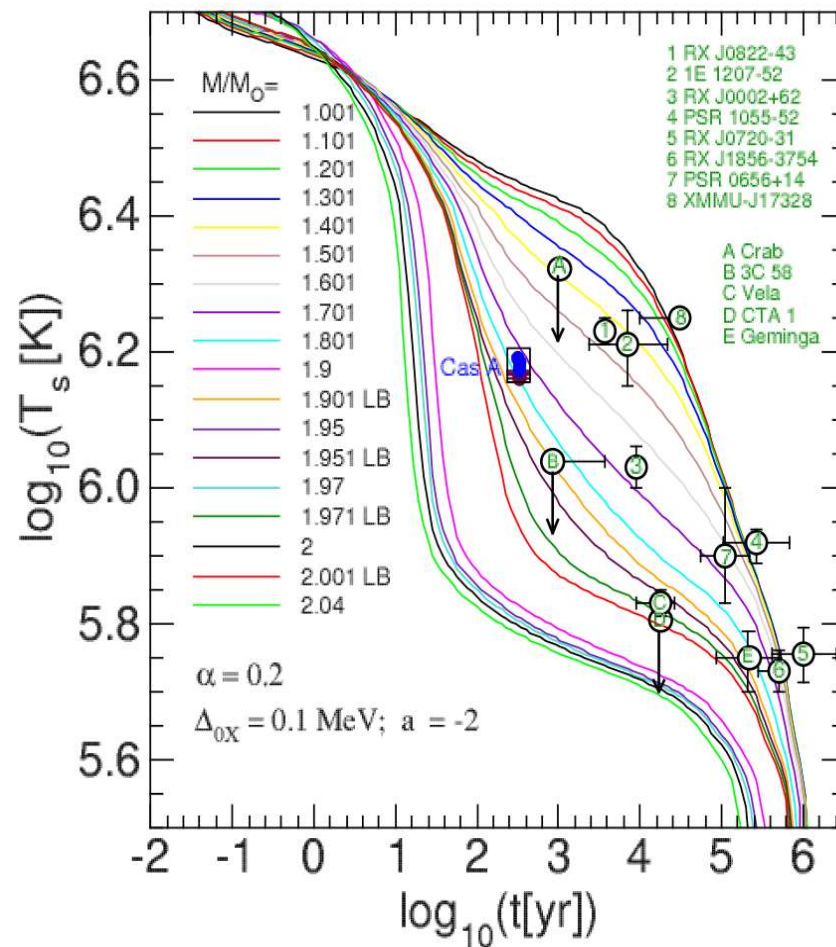
# High Mass Twin CS



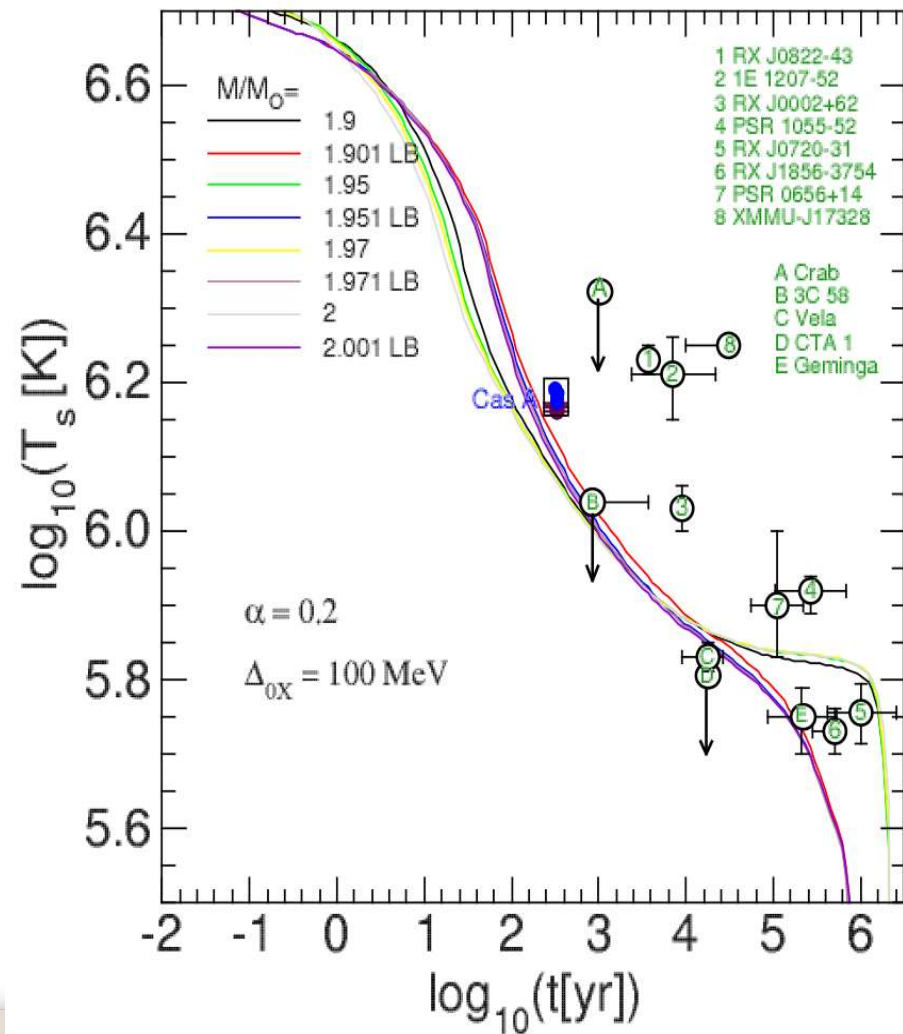
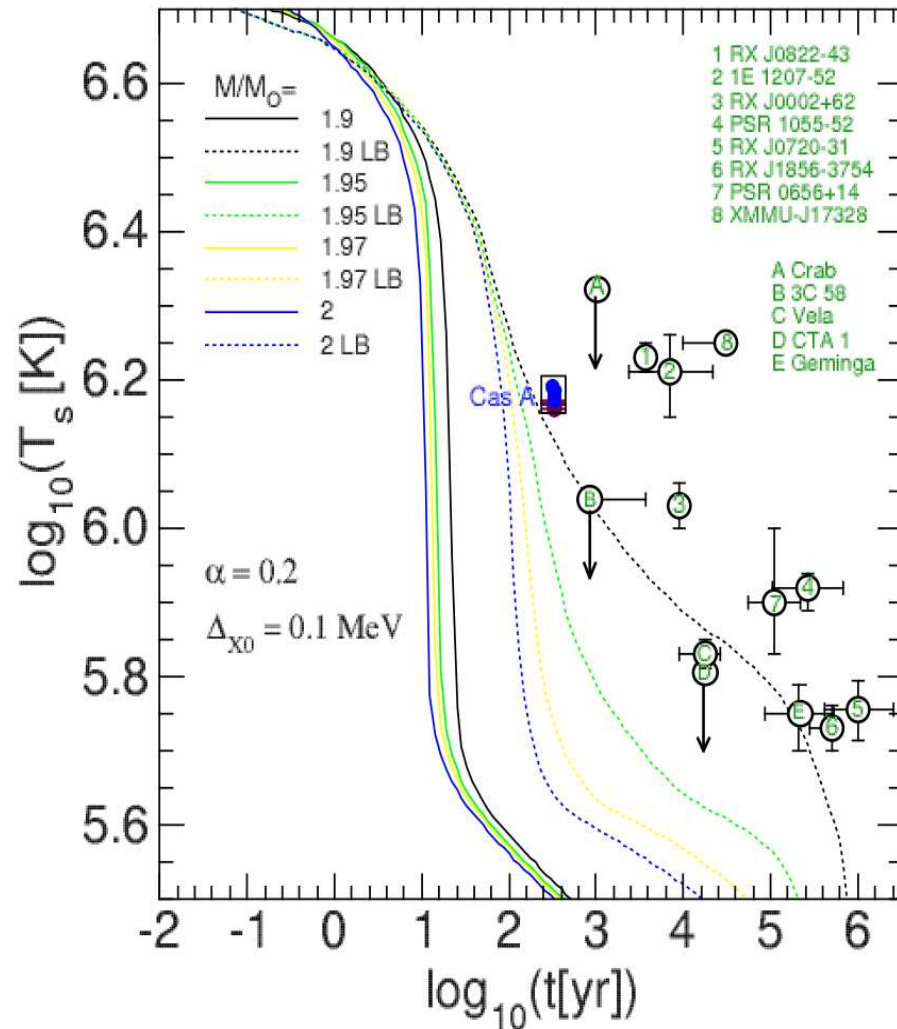
# Different Configurations with the same NS mass

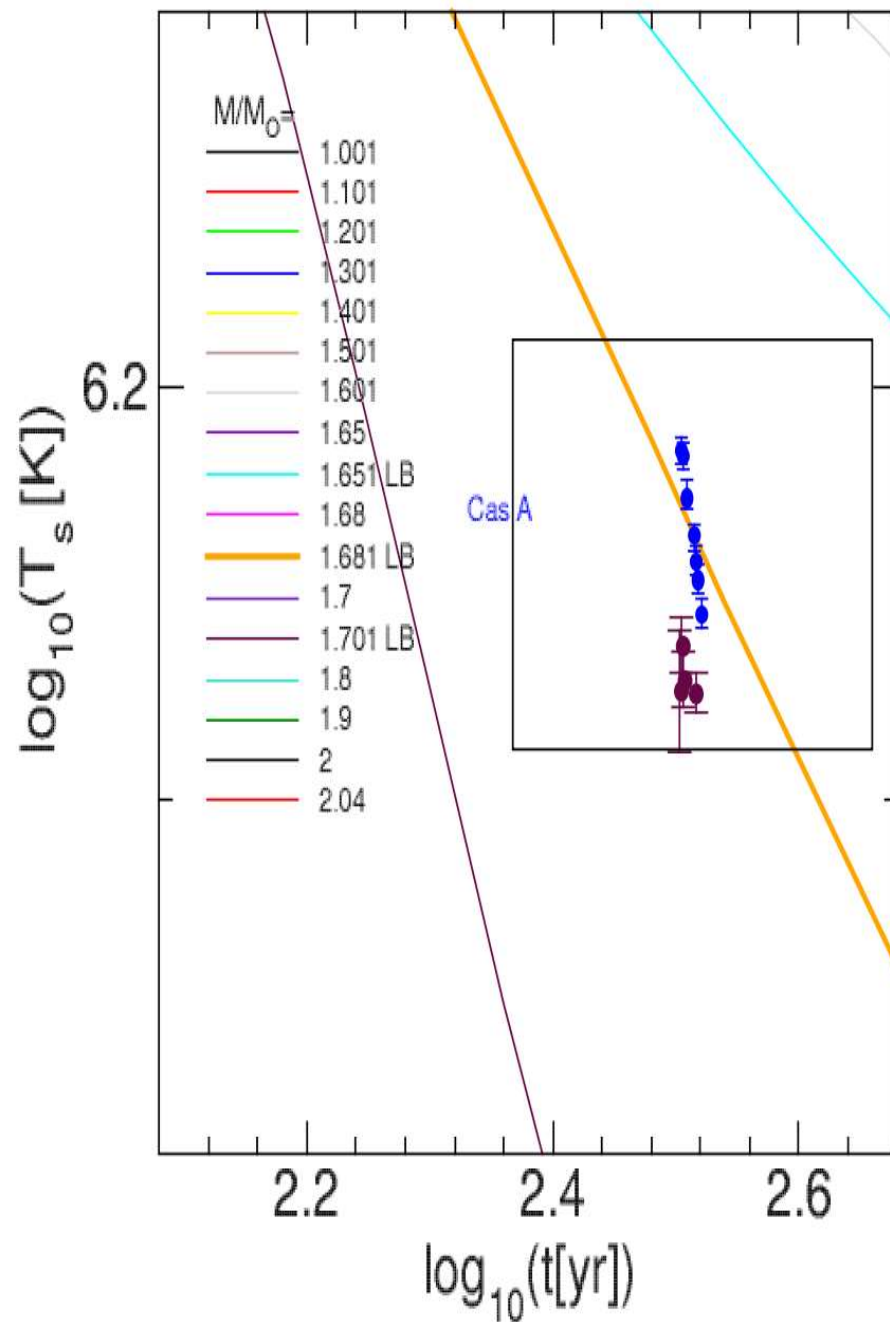
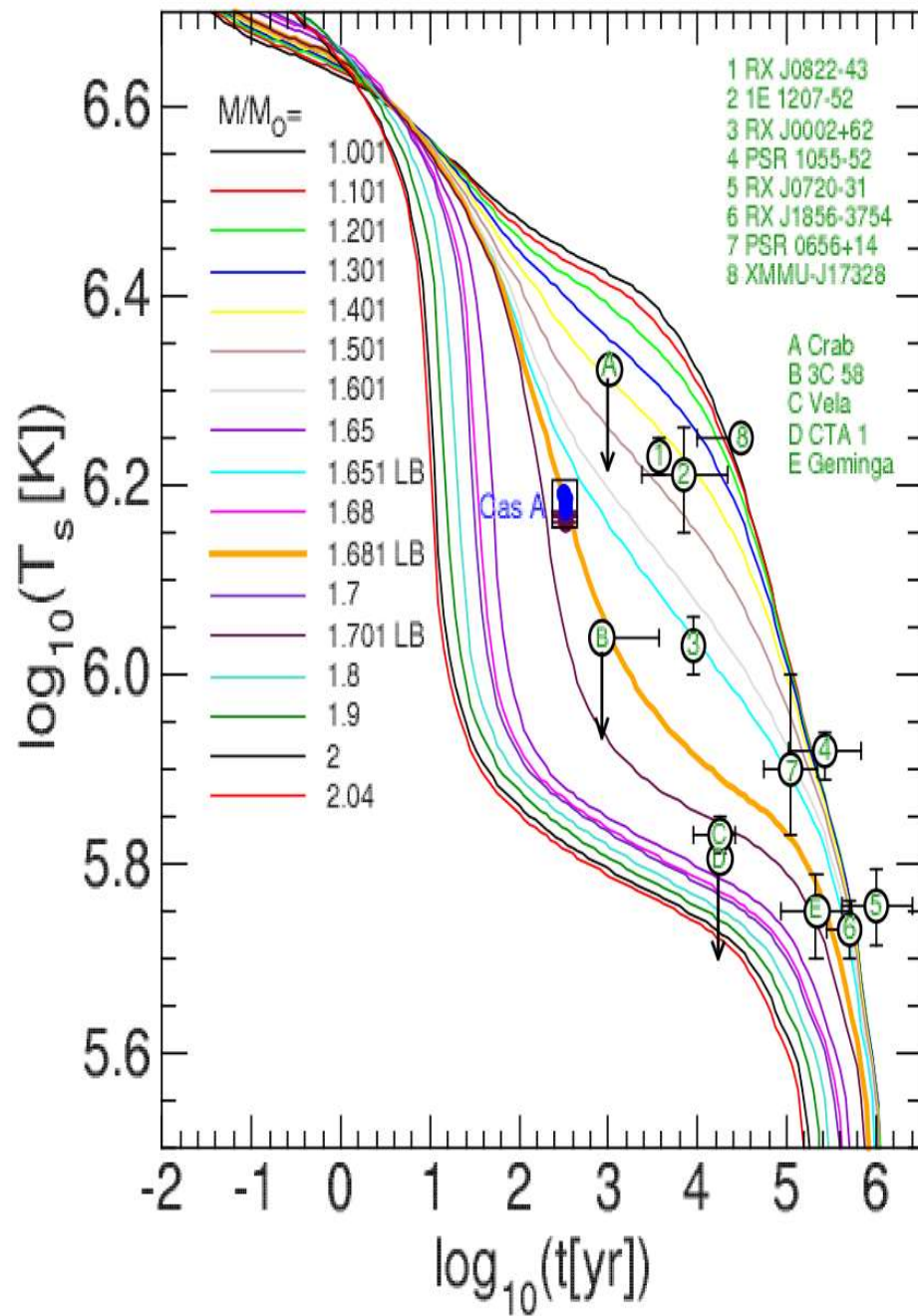


# Cooling of Twin CS



# Highmass Twins: QM SC Effect





# Cooling of Neutron Stars admixed with Light Dark Matter

$$\frac{e^{-\lambda-2\Phi}}{4\pi r^2} \frac{\partial}{\partial r} (e^{2\Phi} L) = -Q + Q_h - \frac{c_V}{e^\Phi} \frac{\partial T}{\partial t},$$

$$\frac{L}{4\pi \kappa r^2} = e^{-\lambda-\Phi} \frac{\partial}{\partial r} (T e^\Phi)$$

$$N_\chi(t) \simeq N_{\chi,0} + \frac{dN_\chi}{dt} (t - t_0),$$

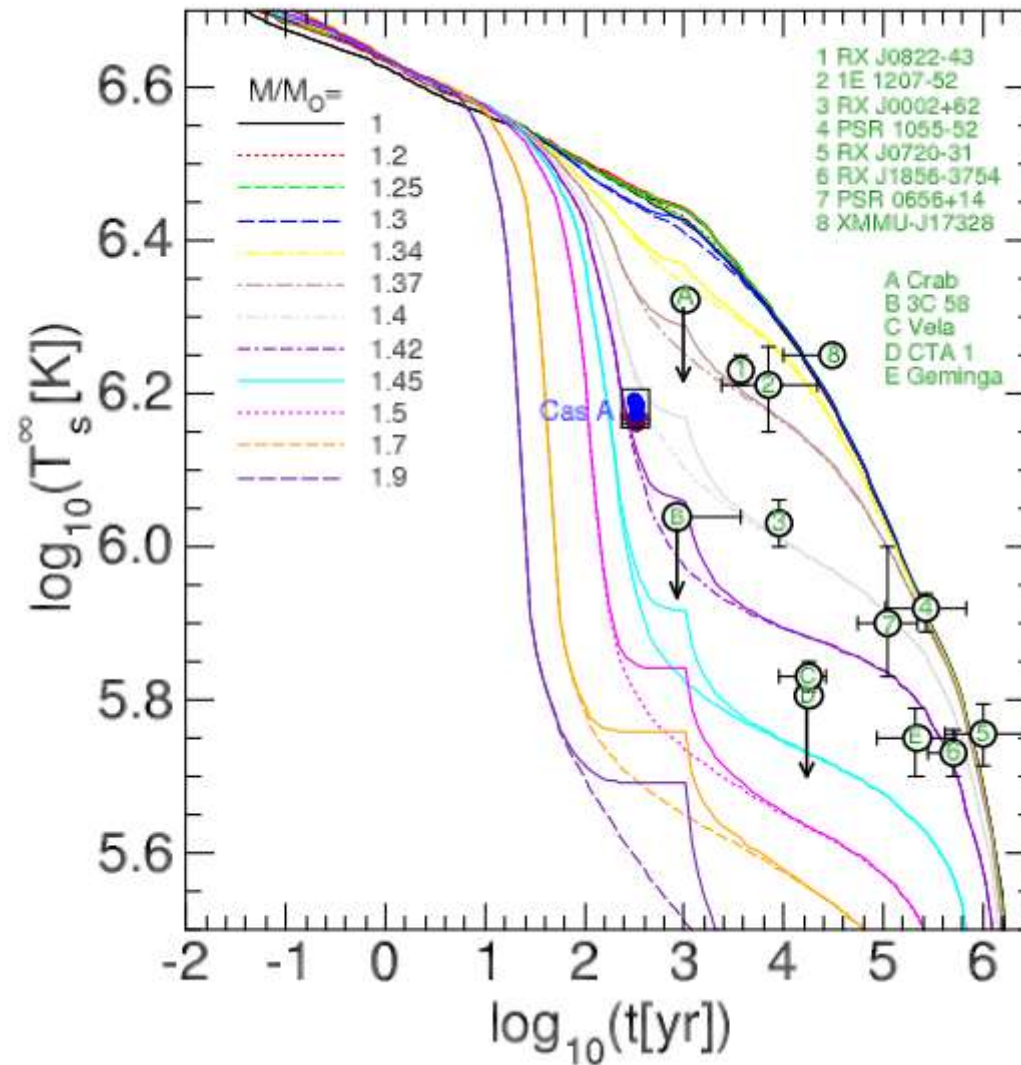
$$\frac{dN_\chi}{dt} = C_\chi - C_a N_\chi^2.$$

The DM capture rate can be approximated by

$$C_\chi \simeq 5.6 \times 10^{26} \left( \frac{M}{1.5 M_\odot} \right) \left( \frac{R}{14 \text{ km}} \right) \left( \frac{0.1 \text{ GeV}}{m_\chi} \right) \left( \frac{\rho_\chi}{0.4 \frac{\text{GeV}}{\text{cm}^3}} \right) \text{ s}^{-1}$$

the thermally-averaged self-annihilation rate  $\langle \sigma v \rangle \sim 5 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ ,

$$C_a \simeq 2 \times 10^{-42} \left( \frac{0.1 \text{ GeV}}{m_\chi} \frac{2\rho_0}{\rho_N} \frac{T}{0.5 \text{ MeV}} \right)^{-3/2} \text{ s}^{-1}.$$



NSs with masses  $M \in [1, 1.9]M_\odot$  with the effect of self-annihilating LDM ( $m_\chi = 0.1 \text{ GeV}$ ) originating a plateau or without LDM (continuous decline). Existing series of cooling