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Numerical study of the influence of model parameters on the effect of magnetic moment reversal in systems of φ_0 Josephson junctions with pulsed and inductive current sources

Bashashin M.V.[1,2], **Rahmonov I.R.**[1,2,3], **Rahmonova A.R.**[1,2], **Zemlyanaya E.V.**[1,2]

[1]*Joint Institute for Nuclear Research, Dubna, Russia*
[2]*Dubna State University, Dubna, Russia*
[3]*Moscow Institute of Physics and Technology, Dolgoprudny, Russia*

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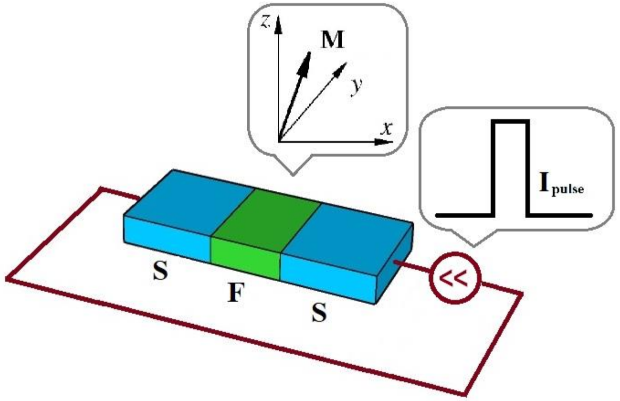
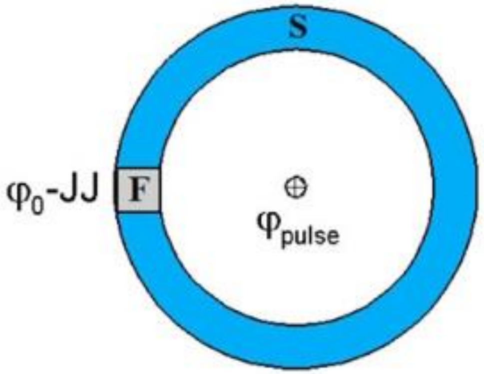
Introduction





- Superconducting spintronics is one of the most intensively developing areas of condensed matter physics. An important place in this area is occupied by the study of Josephson junctions associated with magnetic systems.
- In the superconductor–ferromagnetic–superconductor (SFS) structures, the spin-orbit coupling in ferromagnetic layer without inversion symmetry provides a mechanism for a direct (linear) coupling between the magnetic moment and the superconducting current. Such Josephson junctions are called φ_0-junction. The possibility to control magnetization by the Josephson current and vice versa Josephson current by magnetization, has attracted much recent attention.
- Variants of a φ_0-junction system with pulsed and inductive current sources (we have investigated the peculiarities of the MR under the pulse of external magnetic field in the single junction superconducting quantum interference device (SQUID) with φ_0-junction).
- Both variants are described by the Cauchy problem for a system of nonlinear ordinary differential equations. They are solved numerically in the first case by the two-step Gauss-Legendre method, in the second case - by the four-step Runge-Kutta method.
- Computer simulation in a wide range of varying model parameters was organized using parallel programming technologies.

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Theoretical model (pulse current model)

The dynamics of the magnetization in ferromagnetic layer in the φ_0-Josephson junctions is described by the Landau-Lifshitz-Gilbert equation.

$$\frac{d\vec{m}}{dt} = -\frac{\omega_F}{1+\vec{m}^2\alpha^2}\left\{[\vec{m}\times\vec{H}] + \alpha\left[\vec{m}\left(\vec{m}\vec{H}\right) - \vec{H}\vec{m}^2\right]\right\}, \qquad (1)$$

where $\boldsymbol{\alpha}$ is damping parameter, $\boldsymbol{\omega_F}$ is normalized frequency of ferromagnetic resonance. Here $\vec{\boldsymbol{H}}$ is effective magnetic field with the components

$$\begin{cases} H_x = 0 \\ H_y = Gr\sin\left(\varphi(t) - rm_y(t)\right) \\ H_z = m_z(t) \end{cases} \qquad (2)$$

where $\boldsymbol{G}$ – relation of Josephson energy to energy of magnetic anisotropy, $\boldsymbol{r}$ – the spin-orbit coupling parameter, $\boldsymbol{m_{x,y,z}}$ is $\boldsymbol{x,y,z}$-component of magnetic moment $\vec{\boldsymbol{m}}$. Initial conditions:

$m_x(0)=0$, $m_y(0)=0$, $m_z(0)=1$.

Theoretical model (pulse current model)

The Josephson phase difference φ can be found using equation

$$\frac{d\varphi}{dt} = I_{pulse}(t) - \sin(\varphi - r m_y), \tag{3}$$

where the pulse current is given by

$$I_{pulse} = \begin{cases} A_S, t \in [t_0 - 1/2\Delta t, t_0 + 1/2\Delta t] \\ 0, \quad \text{otherwise} \end{cases}. \tag{4}$$

Here A_s is the amplitude of the pulse current, and Δt is the time interval, in which the pulse current is applied, t_0 is the time point the maximal amplitude.

Thus, the system of equations (1) with effective field (2),(3) and with the pulse current (4) describes the dynamics of the φ_0-junction.

Theoretical model (SQUID model)

The dynamics of the magnetization ***M*** described by the Landau-Lifshitz-Gilbert equation, with the corresponding effective magnetic field $\boldsymbol{H_{eff}}$.

$$\frac{dM}{dt} = -\frac{\Omega_F}{1+(M\alpha)^2}\{[M\,H_{eff}] + \alpha[M(MH_{eff}) - H_{eff}M^2]\}$$

$$H_{eff} = \frac{K}{M_0}[G_r \sin\left(\varphi - r\frac{M_y}{M_0}\right)e_y + \frac{M_z}{M_0}e_z] \qquad (1)$$

where $\boldsymbol{\Omega_F}$ is ferromagnetic resonance frequency, $\boldsymbol{\alpha}$ is Gilbert damping, ***K*** is anisotropic constant, $\boldsymbol{G=E_J / K_V}$, $\boldsymbol{E_J}$ is Josephson energy, ***V*** is the volume of the ferromagnetic layer, $\boldsymbol{r= l\ (V_{SO} / V_{SF})}$ -spin orbit coupling parameter, $\boldsymbol{M_0}$ is magnetization saturation.
According to the SQUID theory and well known resistively shunted junction model expressions for total flux through the system can be written as

$$\frac{2\pi}{\Phi_0}\left[\Phi_{pulse} - L\left(\frac{I_c}{\omega_c}\frac{d\varphi}{dt} + I_C sin(\varphi - rm_y)\right)\right] = \varphi - rm_y, \qquad (2)$$

where $\boldsymbol{\Phi_0=h/2e}$ is the flux quanta, $\boldsymbol{\Phi_{pulse}}$ is the flux created by the external magnetic field pulse, ***L*** is the inductance of the superconducting loop, and ***I*** is the current through $\boldsymbol{\varphi_0}$-junction, $\boldsymbol{\omega_c= 2\pi I_c R/\Phi_0}$.

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Theoretical model (SQUID model)

Coupled system of equations in normalized variables takes form

$$\frac{dm_x}{dt} = -\frac{\omega_F}{1+\alpha^2}\{m_y m_z - Grm_z sin(\varphi - rm_y) + \alpha[Grm_x m_y sin(\varphi - rm_y) + m_x m_z^2]\} \quad (3)$$

$$\frac{dm_y}{dt} = -\frac{\omega_F}{1+\alpha^2}\{-m_x m_z + \alpha[Gr(m_y^2 - 1)sin(\varphi - rm_y) + m_y m_z^2]\} \quad (4)$$

$$\frac{dm_z}{dt} = -\frac{\omega_F}{1+\alpha^2}\{Grm_x sin(\varphi - rm_y) + \alpha[Grm_y m_z sin(\varphi - rm_y) + m_z(m_z^2 - 1)]\} \quad (5)$$

$$\frac{d\varphi}{dt} = \frac{\varphi_{pulse} - \varphi + rm_y}{L} - \sin(\varphi - rm_y)$$

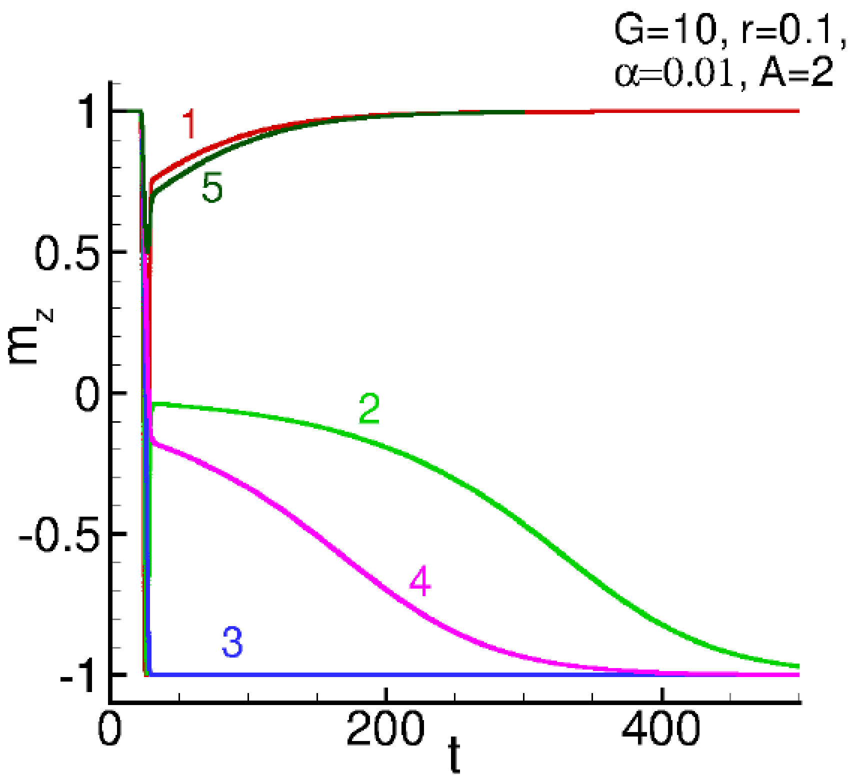
where m_i is magnetization components (***i = x, y, z***) normalised to $\boldsymbol{M_0}$, $\boldsymbol{\omega_F}$ is frequency of ferromagnetic resonance normalized to $\boldsymbol{\omega_c}$, $\boldsymbol{\varphi_{pulse}=(2\pi/\Phi_0)\Phi_{pulse}}$ is normalized external magnetic flux. Here ***L*** is normalized to $\boldsymbol{L_0= \Phi_0/2\pi I_c}$ and time to $\boldsymbol{\omega_c}$. The external flux pulse $\boldsymbol{\varphi_{pulse}}$ has rectangular form

$$\varphi_{pulse}(t) = \begin{cases} A, & t \in [t_0 - \Delta t/2, t_0 + \Delta t/2]; \\ 0 & \text{otherwise} \end{cases}. \quad (6)$$

where ***A*** and $\boldsymbol{\Delta_t}$ are the pulse amplitude and width, respectively. The initial conditions for the LLG equation are the $\boldsymbol{m_x(0)=0}$; $\boldsymbol{m_y(0)=0}$; $\boldsymbol{m_z(0)=1}$; $\boldsymbol{\varphi(0)=0}$.

Magnetic reversal

Magnetic reversal is an effect when m_z–component of the magnetic field changes the sign and takes the value ***-1*** for a given initial value of ***+1***.



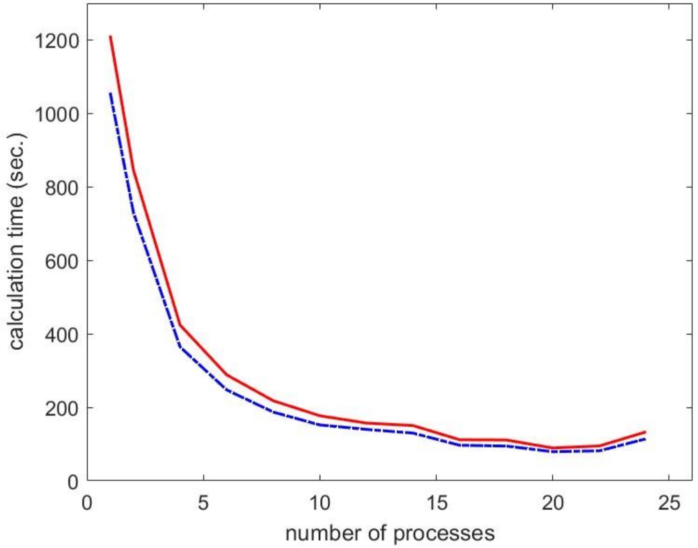
As an example, we analyze effect of the SQUID inductance ***L*** on MR. In figure the time dependence of the m_z for values of the ***L = 1, 2, 3, 4, 5*** for the pulse parameters ***A=3*** and Δ_t***=6***.

The numbers in figure show the value of corresponding ***L***.

- In case of ***L = 1*** there no reversal, at ***L = 2*** MR is realized and complete reversal realized at ***t = 500***.
- At ***L = 3*** the fast MR is realized, i.e. already during the acting of pulse.
- In case of the ***L = 4***, like the case of ***L = 2***, the MR takes long time (about ***t = 400***).
- At ***L = 5*** again we can see that MR is not realized.

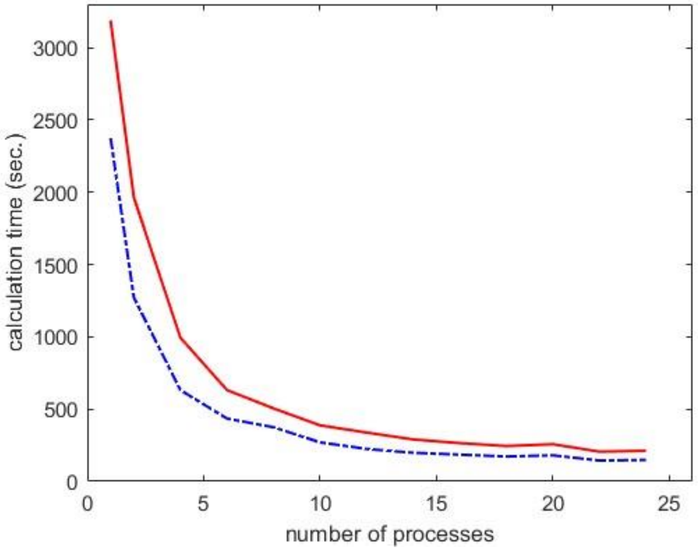
Parallel implementation

- For the numerical solution of the system of equations, the implicit two-step Gauss – Legendre method was used for system with pulsed current and 4th order Runge-Kutta method was used for system with inductive current.
- The parallelization process is based on the distribution of the points of the (***G***, ***α***)-plane between parallel threads. The values of ***G***, ***α*** where the condition $|m_z(T_{max})+1|<\varepsilon$ is satisfied, are saved in output structure and writing to the output file. The plane parameters can be changed to others.
- Also, using a parallel implementation, the influence of the AVX-512 vector instructions built into the latest versions of Intel server processors was tested. These processors are available on the Govorun supercomputer.



Inductive current system further speed up by 1.1-1.15 times.

Time of calculations depending on the number of MPI-processes.
Red lines: MPI realization with basic compiler options;
Blue lines: MPI realization with AVX-512 options.



Inductive current system further speed up by 1.3-1.6 times.

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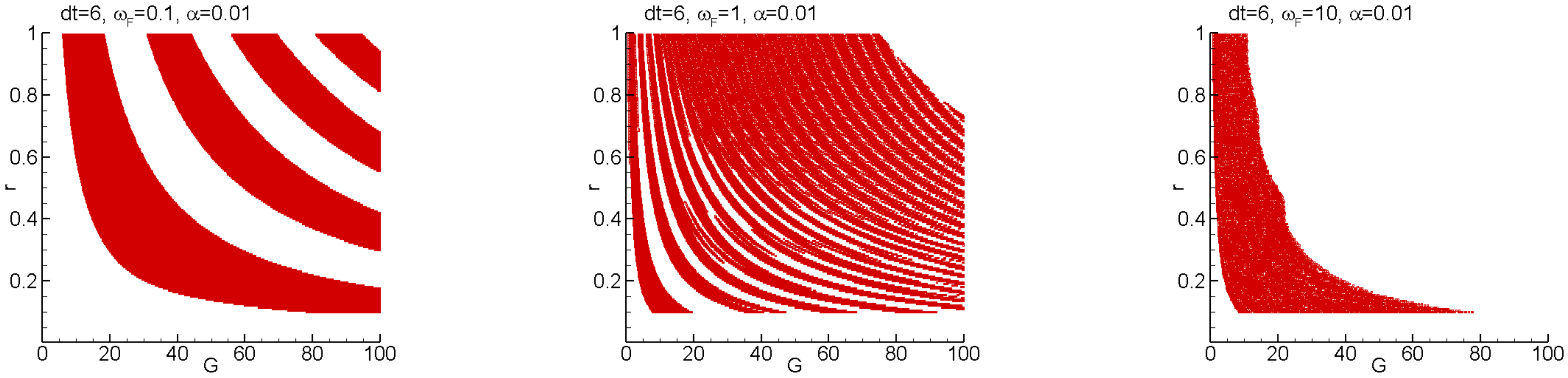
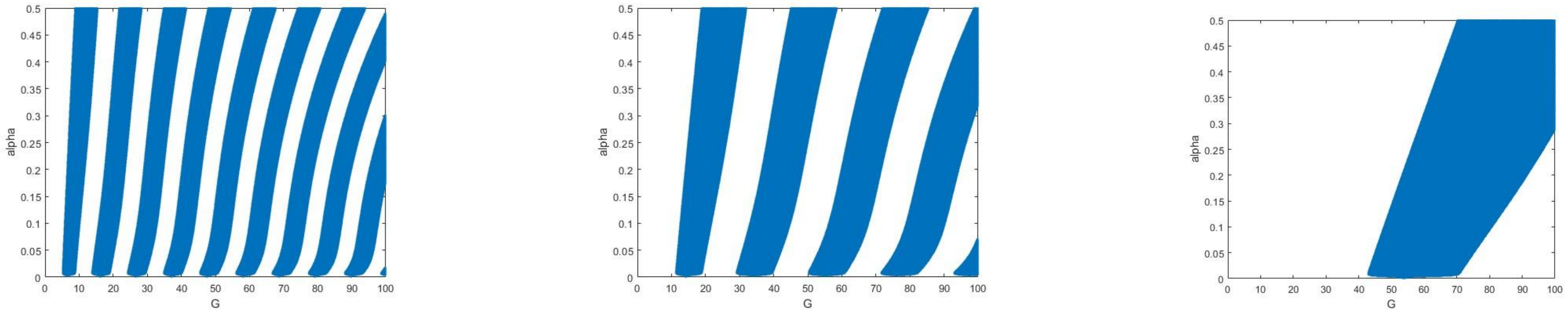
Magnetic reversal

The influence of different parameters was considered for different systems:

For pulse current system we present the (***G, r***)-plane diagram for different values of the normalized frequency of the magnetic resonance $\boldsymbol{\omega_F}$. The results are obtained with ***G***-stepsize $\boldsymbol{\Delta G}$=0.1, ***r*** -stepsize $\boldsymbol{\Delta r}$=0.005 at $\boldsymbol{A_s}$ = 1.5; $\boldsymbol{t_0}$ =25.



For the inductive current system we present the (***G, a***)-plane diagram for different values of the inductance of the superconducting loop L=0.1, L=2, L=10. G-stepsize ΔG=0.1, α-stepsize $\Delta\alpha$=0.001 at A_s = 1.5; r = 0.1; t_0 =25; Δ_t = 6; ω_F = 1.



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Conclusions

For pulse current system:

- The influence of the normalized frequency of the magnetic resonance $\boldsymbol{\omega_F}$ on the width of magnetization reversal domains was revealed. Shown that an increase in the inductance parameter leads to an increase in the width of the MR bands.
- Using AVX-512 instructions allows us to further speed up the program by 1.1-1.15 times in comparison with the standard MPI-version.
- Maximal speedup of MPI + AVX-512 implementation is about 16 times compared to the single-thread calculation.

For inductive current system:

- The influence of the inductance parameter $\boldsymbol{L}$ on the width of magnetization reversal domains was revealed. Shown that an increase in the inductance parameter leads to an increase in the width of the MR bands.
- Using AVX-512 instructions allows us to further speed up the program by 1.3-1.6 times in comparison with the standard MPI-version.
- Maximal speedup of MPI + AVX-512 implementation is about 22 times compared to the single-thread calculation.