Cumulative production at central rapidities due to interactions involving fluctons

Vechernin V.V., Yurchenko S.V.
Saint-Petersburg State University
v.vechernin@spbu.ru

LXXIV International conference Nucleus-2024: Fundamental problems and applications

1-5 July, 2024

Dubna

Centrality classes in dd collisions

- **Impact parameter *b* is not suitable!**
- **Classes on the number of participants (spectators):**

In a picture without fluctons:

1) **N+N (p+p, p+n, ...)**
2) **2N+N, N+2N**
3) **2N+2N**

The experimental problem of the division of all dd events into centrality classes.

(Can be useful for a wide range of research selecting pn- and nn-collisions.)

Principal possibility of registering a spectator neutron by ZDC??

In a picture with fluctons:

4) **f+N, N+f**
5) **f+2N, 2N+f**
6) **f+f**

Flucton-flucton interaction in dd collisions

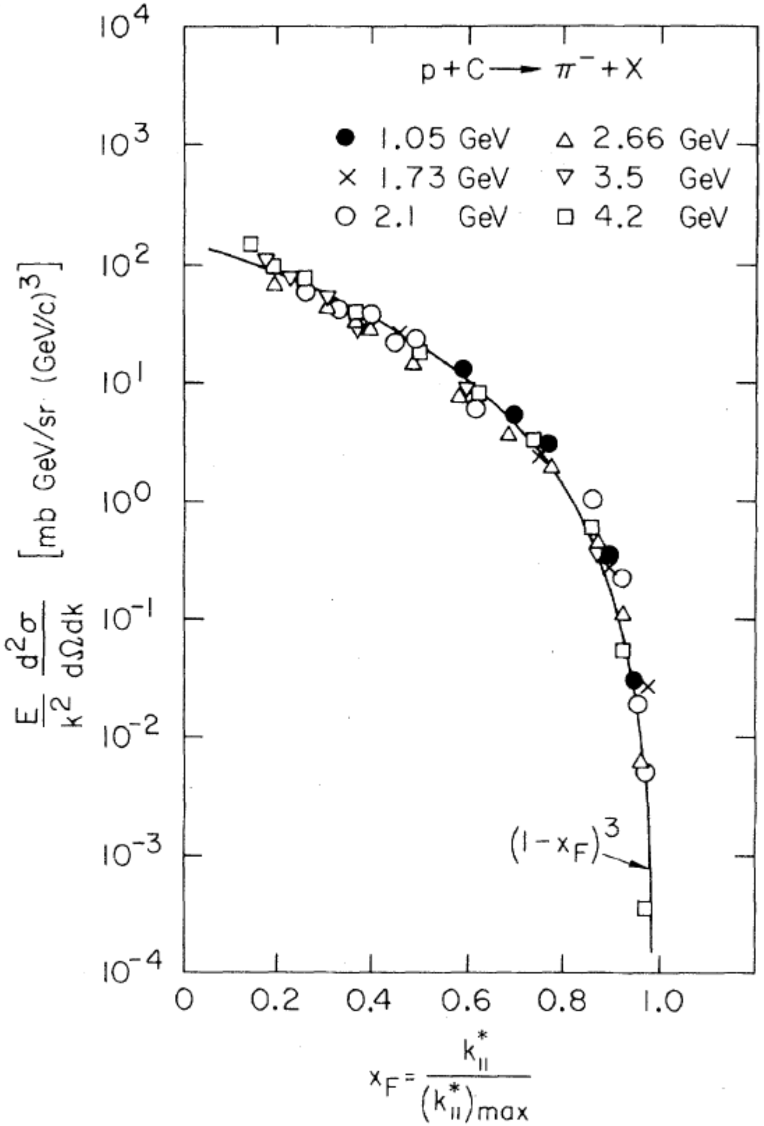
f+f in dd collisions - New and Clear!

- **It can be studied only in new cumulative region of large transverse momenta in mid-rapidity region at NICA (not in the traditional cumulative region of fragmentation of one of the nuclei).**
- **There are no additional interactions in dd collision, compared with collisions of heavier nuclei, if both deuterons are in flucton configuration at the moment of collision. =>**
- **The possibility to register, in addition to the cumulative particle, the particles formed from fragmentation of the flucton residue.**
- **Higher frequency of dd collisions that can be recorded by the SPD, compared to the slower MPD (important for a registration of rare cumulative events).**
- **The studies in new cumulative region becomes possible due to the moderate energy of the NICA collider and is completely impossible at ultrahigh energies of the RHIC and LHC.**

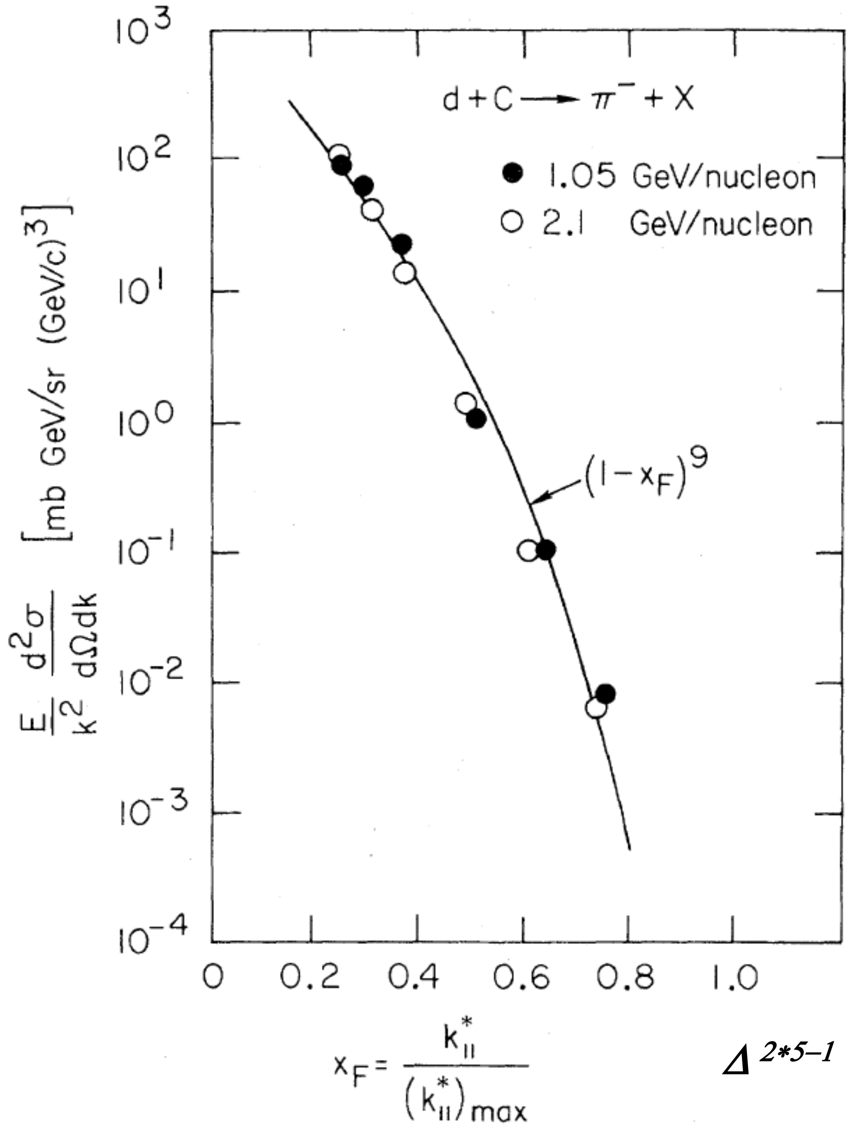
Flucton fragmentation region

Cumulaive production at $|t| << s$

Schmidt I.A., Blankenbecler R. Phys.Rev. ***D15*** *(1977) 3321*



Δ 2*2–1



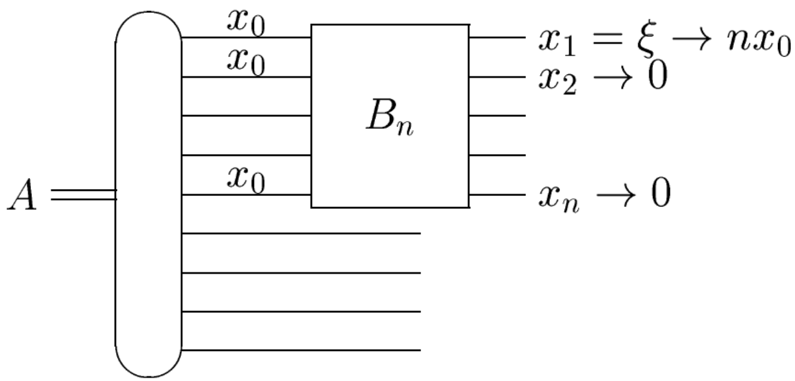
Δ 2*5–1

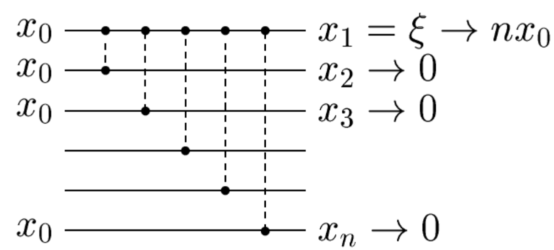
Threshold behaviour of *inclusive cross sections* (quark counting rules) at |t|<<s.

The experimental points from J. Papp et al., Phys.Rev.Lett. 34, 601 (1975).

Description of the hadron asymptotics at x->1
by the intrinsic diagrams of QCD in light-cone gauge
with low-x spectator quarks interact with the target
Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl.Phys. ***B369*** *(1992) 519*

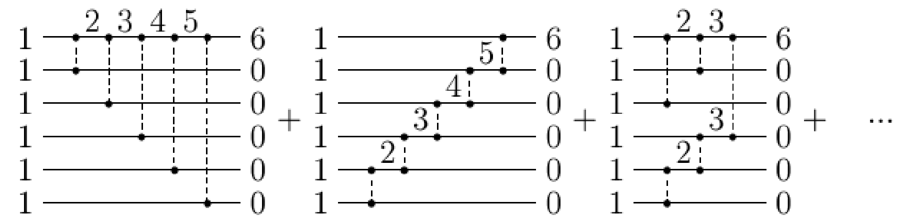
Description of the flucton asymptotic at *x->f*,
f - the number of nucleons in flucton, *n* - the number of quarks in flucton, $x_0 = f / n$ (=1/3).
M.A. Braun, V.V. Vechernin, Nucl.Phys. ***B427*** *(1994) 614.* (DIS in cumulative region)

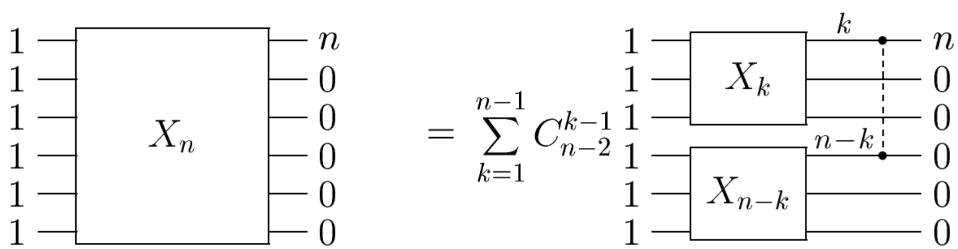




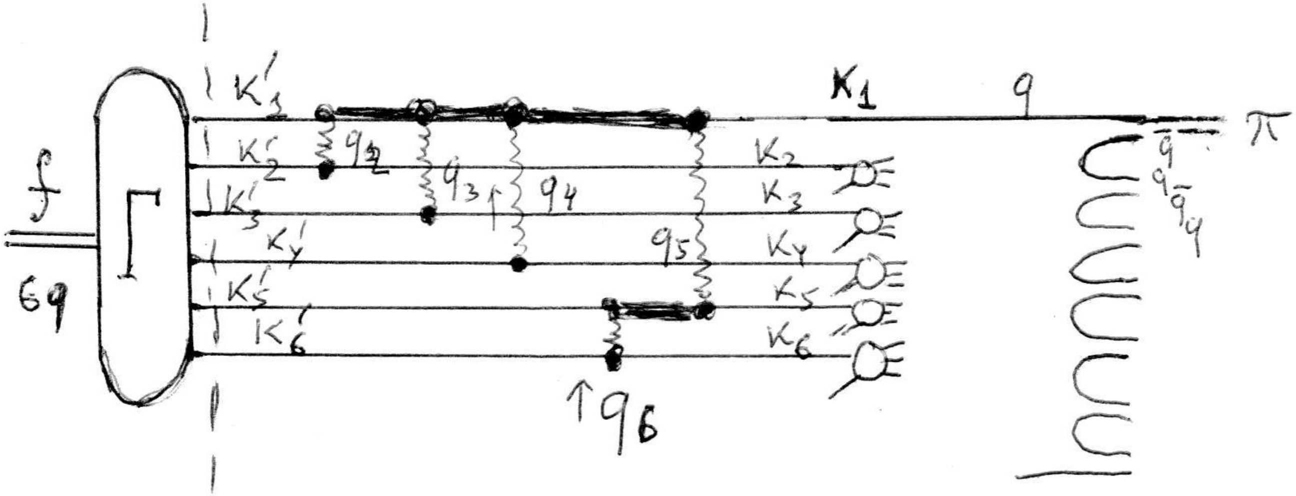
$\sim \Delta^{2p-1}$
$p=n-1$
$\Delta = f - \mathbf{x} = nx_0 - \xi$

At $x_1 \to f$ => all $x_2, \ldots, x_n \to 0$ => all $|q_i| >> m$ =>
pQCD works => min.number of hard exchanges.
Simple instantaneous Coulomb part dominates
in light-cone gauge.





- the recurrence relation



f – number of nucleons which formed flucton

n - number of quarks in flucton

p=n-1 - number of "donors", stopped quarks

$\Gamma = \Gamma(k'_{+i}, k'_{\perp i})$ **then after integration over all** k'_{-i} **we get:**

$\Gamma(k'_{+i}, k'_{\perp i}) \rightarrow \Psi(k'_{+i}, k'_{\perp i})$ – light cone parton wave function of flucton

In all rest parts of the diagram we can put: $k'_{+i} = \dfrac{f\,p_+}{n} = \dfrac{f}{n}p_+ = \dfrac{1}{3}p_+$

Then we get:

$$\int \Psi(k'_{-i}, k'_{\perp i})\;\delta\Big(\sum_{i=1}^{n} k'_{+i} - f\,p_+\Big)\;\delta^2\Big(\sum_{i=1}^{n} k'_{\perp i}\Big)\prod_{i=1}^{n}\frac{dk'_{+i}}{2k'_{+i}}d^2k'_{\perp i} \sim \overline{\Psi}_{cms}(\{r_i - r_j = 0\})$$

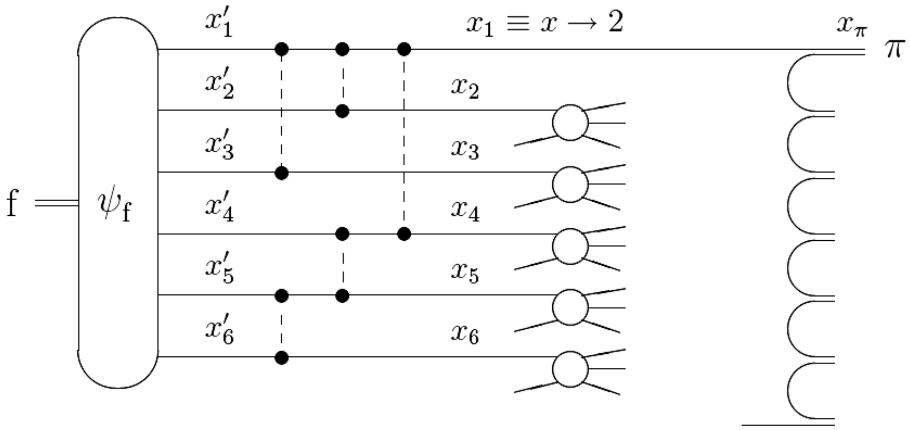
Contribution of (n-1) "Gluon" exchanges and (n-2) internal quark propagators limits to constant, when at $x_1 \Rightarrow f$ **all** $x_2, \ldots, x_n \Rightarrow 0$

The main contribution comes from propagators of stopped quarks $k_{2,\ldots,}\, k_n$ **, which defined the longitudial and transverse momentum dependence.**

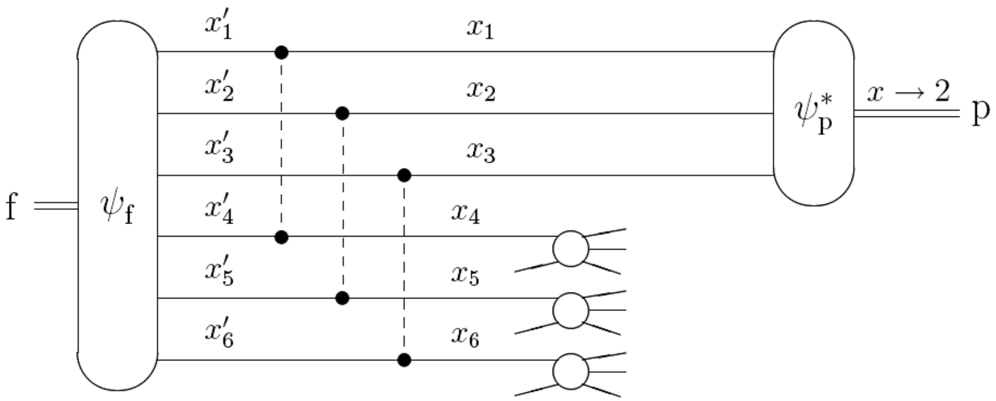
Scaling of cumulative inclusive cross section in the flucton fragmentation region:

$$f_\pi(x, k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C\, s^0\,(f - x)^{2p-1}\,\Phi_p\left(\frac{k_\perp}{m_q}\right)$$

Coherent Quark Coalescence and Production of Cumulative Protons

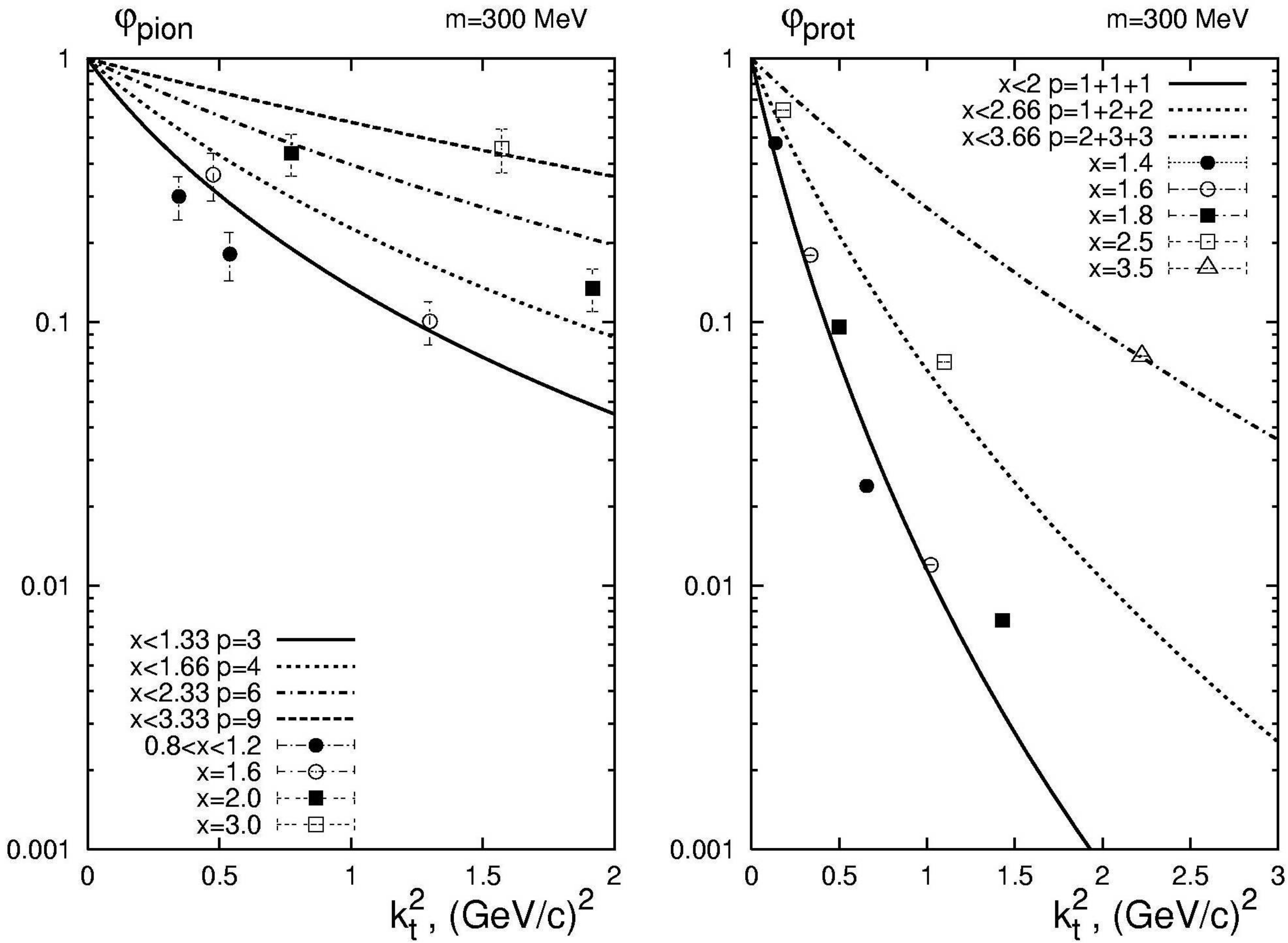


- the cumulative pion production by hadronization of one fast quark
*M.A. Braun, V.V. Vechernin, Nucl.Phys.**B 427**, 614 (1994); Phys.Atom.Nucl. **60**, 432 (1997); **63**, 1831 (2000)*



- the cumulative proton production by **coherent** quark coalescence mechanism:
*M.A. Braun, V.V. Vechernin, Nucl.Phys.**B 92**, 156 (2001); Theor.Math.Phys **139**, 766 (2004); V.Vechernin, AIP Conf.Proc.1701 (2016) 060020.*

The last **recalls** the few nucleon **short-range correlations** in a nucleus
L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).
But instead of using the relativistic generalization of non-relativistic NN wave function
the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge
*Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl. Phys. **B369** (1992) 519.*
was developed and applied.



V.Vechernin,
AIP Conference Proceedings
1701 (2016) 060020.

S.V. Boyarinov et al., Sov.J.Nucl.Phys. ***46****, 871 (1987)*
S.V. Boyarinov et al., Physics of Atomic Nuclei ***57****, 1379 (1994)*
S.V. Boyarinov et al., Sov.J.Nucl.Phys. ***55****, 917 (1992)*

Application of this old approach for higher pT

For AA interaction:

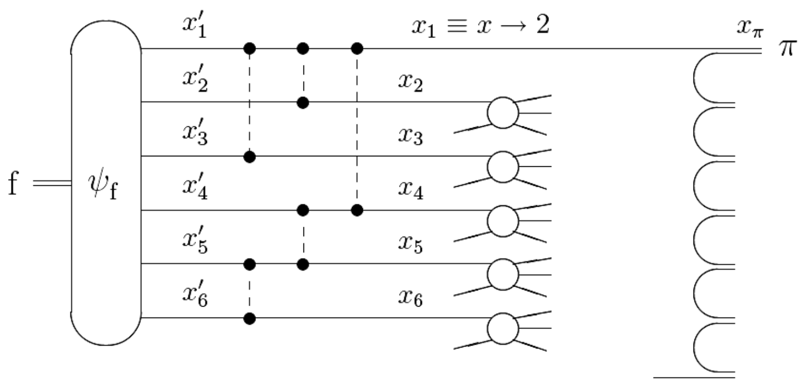
V. Vechernin, S. Belokurova, S. Yurchenko, Dense Cold Quark-Gluon Matter Clusters and Their Studies at the NICA Collider, Symmetry 16 (2024) 79.

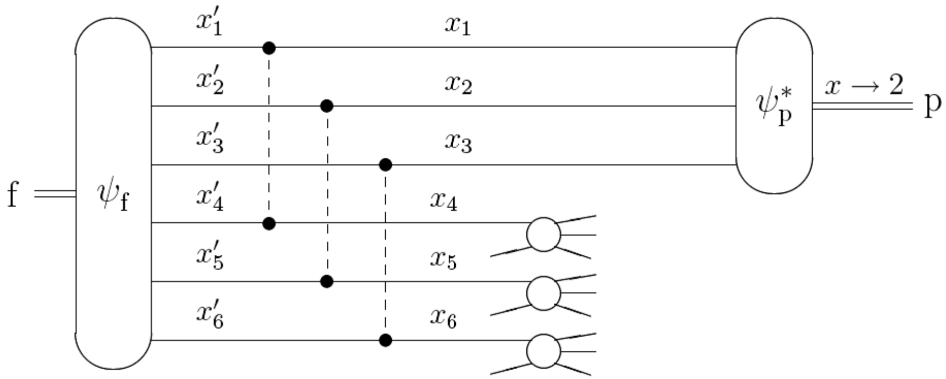
For dd interaction:

V.V. Vechernin, S.N. Belokurova, S.V. Yurchenko, Cumulative Production in the Region of Central Rapidities and Large Transverse Momenta at the NICA Collider Physics of Particles and Nuclei, 2024, Vol. 55, No. 4, pp. 889-894.

V.V. Vechernin, S.V. Yurchenko, Cumulative production at central rapidities and large transverse momenta in the quark model of flucton fragmentation. Moscow University Physics Bulletin, 2024 (in press).

dd collisions





$$f_\pi(x, k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C_\pi (2-x)^9\, \Phi_5\left(\frac{k_\perp}{m_q}\right) / \Phi_5(0)$$

(1)

$$f_p(x, k_\perp) \equiv \frac{k_0\, d^3\sigma_p}{d^3\mathbf{k}} = C_p (2-x)^5\, \Phi_1^3\left(\frac{k_\perp}{3m_q}\right) / \Phi_1^3(0)$$

(2)

$$\Phi_p(t) = 2\pi \int_0^\infty dz\, z J_0(tz)[zK_1(z)]^p$$

$$\Phi_1(t) = \frac{4\pi}{(t^2+1)^2}$$

$$x \equiv 2x_+$$

$$x_+ \equiv \frac{k_+}{k_+^{max}} \, ,$$

$x_+ = 1$
- exact kinematic boundary for dd reaction

$$k_+ \equiv \frac{k_0 + k_z}{\sqrt{2}} \, .$$

$x = \frac{k_+}{p_+}$ - light cone variable

$x_F = \frac{k_z}{k_z^{max}}$ - Feynman variable

$M_f^{min} = X\, m_N$ - cumulative number

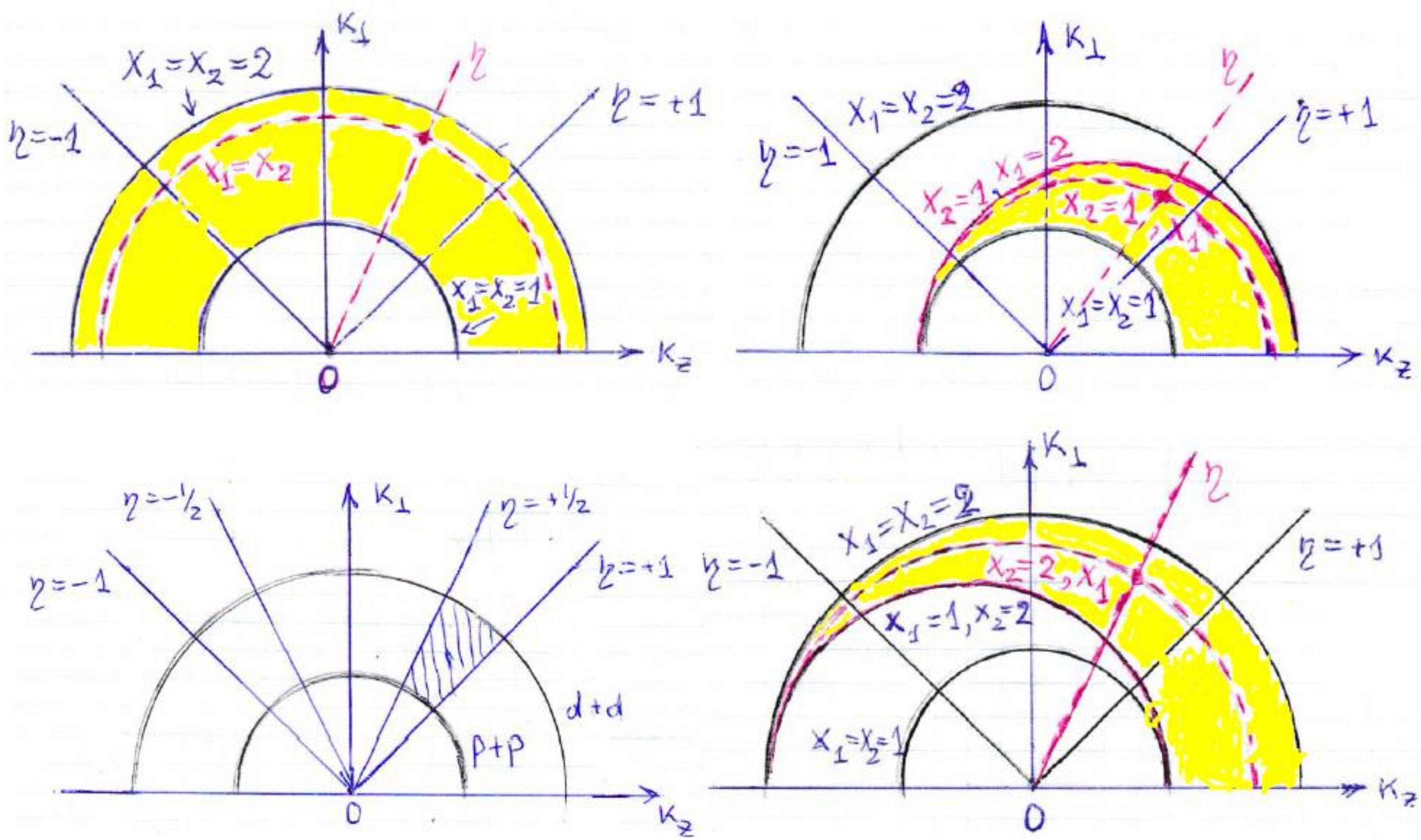
$x \approx x_F \approx X$ at $s \to \infty$

$$\frac{m_N^2}{E^{*2}} = \frac{4m_N^2}{s}$$

Cumulative region in dd collision with different variables

$p \gg m_N$

$$\frac{k_\perp}{p} = \frac{\sqrt{f_1 f_2}}{(f_1+f_2)/2}\sqrt{\left(f_1 - \frac{k_z}{p}\right)\left(f_2 + \frac{k_z}{p}\right)}$$



$f + N$
$x = x_1$
$x_2 = 1$

$f + 2N$
$x = x_1$
$x_2 = 2$

Inclusive cross sections for the production of pions and protons in dd-collisions, integrated over rapidity intervals 0.5 < |y| < 1

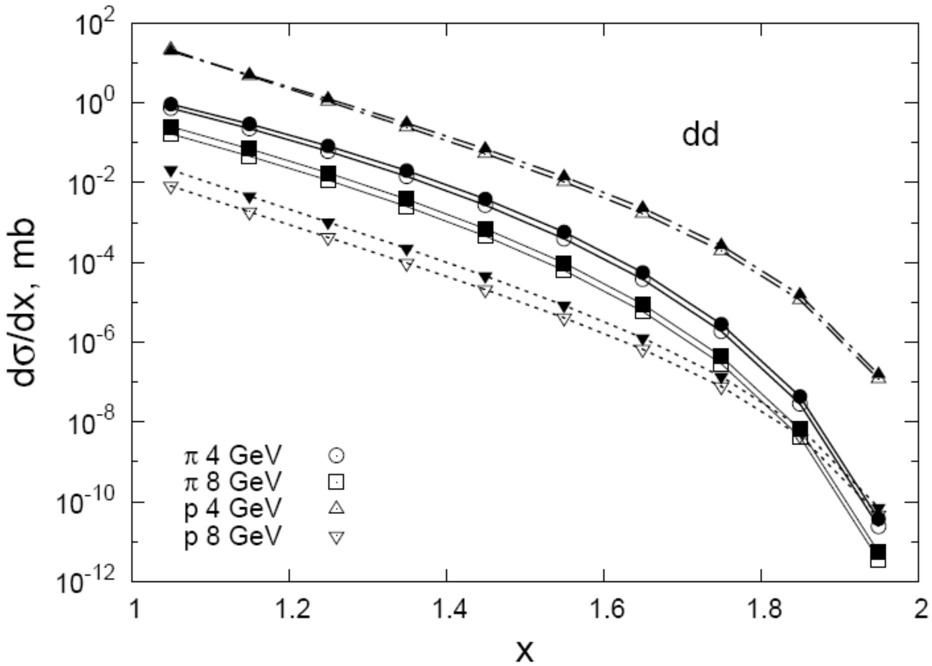


Figure 2. Inclusive cross sections for the production of pions (○, □) and protons (△,▽) in dd collisions, integrated over rapidity intervals $0.5 < |y| < 1$ and available for study with NICA SPD, respectively, for two initial energies $\sqrt{s_{NN}} = 4$ and 8 GeV, as a function of the light-cone cumulative variable $x = 2x_+$ (open simbols) and the cumulative number $x = x_M$ (solid symbols). Model calculations by (9) using (1) and (2). (Curves serve to guide the eye.)

$$\frac{d\sigma}{dx} = \frac{\langle n\rangle_{\rm dd}^{\Delta x}}{\Delta x}\sigma_{\rm dd}^{tot} = \frac{2\pi}{\Delta x}\int_{0.5}^{1} dy \int_{k_\perp^x(y)}^{k_\perp^{x+\Delta x}(y)} dk_\perp\, k_\perp \times$$

$$\times\, f(\,x(y,k_\perp),\, k_\perp)\ , \qquad (9)$$

Vechernin V.V., Yurchenko S.V. Cumulative production at central rapidities and large transverse momenta in the quark model of flucton fragmentation (in press).

f + 2N

$x_1 = x = x_M$

$x_2 = 2$

$\sqrt{s_{NN}}$	4 GeV			8 GeV		
	y	$k_\perp^{min}$	$k_\perp^{max}$	y	$k_\perp^{min}$	$k_\perp^{max}$
dd $\to \pi$	0.5	1.728	2.752	0.5	4.197	6.672
dd $\to \pi$	1.0	1.102	2.002	1.0	2.687	4.86
dd $\to$ p	0.5	1.741	2.999	0.5	4.218	6.803
dd $\to$ p	1.0	0.852	2.089	1.0	2.605	4.915

$\sigma_{\rm dd}^{tot} = 120\ mb.$

	$\sqrt{s_{NN}}$	4 GeV	8 GeV
$\langle n_{\pi^-}\rangle_{\rm dd}$	$x > 1.0$	$9{\cdot}10^{-4}$	$1.9{\cdot}10^{-4}$
	$x > 1.2$	$6.6{\cdot}10^{-5}$	$1.2{\cdot}10^{-5}$
	$x > 1.5$	$3.6{\cdot}10^{-7}$	$5.8{\cdot}10^{-8}$
$\langle n_p\rangle_{\rm dd}$	$x > 1.0$	$2.3{\cdot}10^{-2}$	$9{\cdot}10^{-6}$
	$x > 1.2$	$1.2{\cdot}10^{-3}$	$4.6{\cdot}10^{-7}$
	$x > 1.5$	$1.04{\cdot}10^{-5}$	$4.2{\cdot}10^{-9}$

Estimation of pion and proton yields in the new cumulative region of large transverse momenta and the rapidities 0.5<|y|<1 in dd collisions at SPD for *t* = 1 hour

$$Y_{\rm dd} = 0.1 \cdot L_{dd} \cdot \sigma_{\rm dd}^{tot} \cdot \langle n \rangle_{\rm dd} \cdot t$$

$L_{dd} = 10^{30} cm^{-2} c^{-1}$ at 8 GeV and 100 times lower at 4 GeV

V.M. Abazov, et al. [The SPD collaboration], "Conceptual design of the Spin Physics Detector ArXiv:2102.00442v3 [hep-ex], 2022.

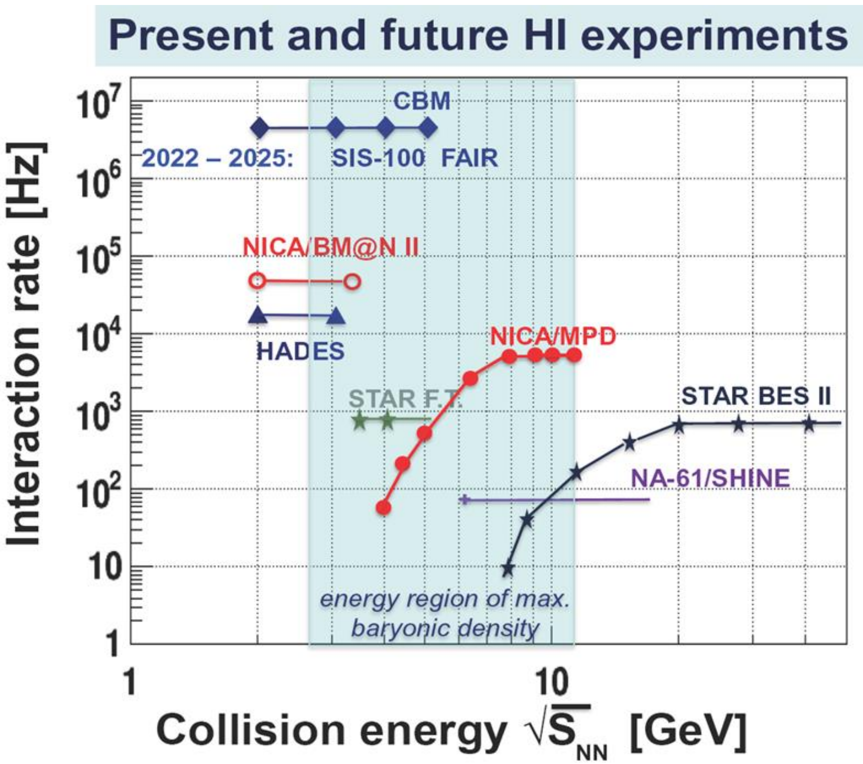
	$\sqrt{s_{NN}}$	4 GeV	8 GeV
$Y_{\rm dd} \to \pi^-$	$x > 1$	400	8 000
	$x > 1.2$	30	500
	$x > 1.5$	0.16	2.5
$Y_{\rm dd} \to {\rm p}$	$x > 1$	10 000	400
	$x > 1.2$	500	20
	$x > 1.5$	4.5	0.18

V.V. Vechernin, S.N. Belokurova, S.V. Yurchenko, Cumulative Production in the Region of Central Rapidities and Large Transverse Momenta at the NICA Collider Physics of Particles and Nuclei, 2024, Vol. 55, No. 4, pp. 889-894.

Comparison of Interaction Rates
in AuAu (BiBi) collisions at MPD and in dd collisions at SPD

$$\mathrm{MPD:}\ L_{AuAu} = 10^{27} cm^{-2} c^{-1}$$

$$\sigma^{tot}_{\mathrm{AuAu}} \cong 7000\ mb$$



$$I_{AuAu} = L_{AuAu}\, \sigma^{tot}_{\mathrm{AuAu}} = 7\,\mathrm{KHz}$$

V. Kekelidze, A. Kovalenko, R. Lednicky, V. Matveev, I. Meshkov, A. Sorin, G.Trubnikov, "Feasibility study of heavy-ion collision physics at NICA", Nuclear Physics A 967 (2017) 884–887.

Higher frequency of dd collisions that can be recorded by the SPD, compared to the slower MPD is important for a registration of rare cumulative events.

$$\sigma^{tot}_{\mathrm{dd}} \cong 120\ mb$$

$$\mathrm{SPD:}\ L_{dd} = 10^{30} cm^{-2} c^{-1}$$

$$I_{dd} = L_{dd}\, \sigma^{tot}_{\mathrm{dd}} = 120\,\mathrm{KHz}$$

V.M. Abazov, et al. [The SPD collaboration], "Conceptual design of the Spin Physics Detector ArXiv:2102.00442v3 [hep-ex], 2022.

Conclusions (nucleon – flucton interaction)

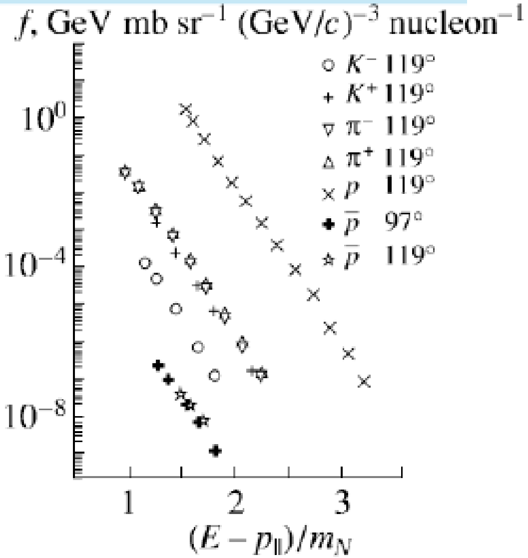
- We have made estimates of pion and proton production in a new cumulative region of central rapidities and large transverse momenta in dd collisions using the theoretical results for the transverse momentum dependence of cumulative particle with given *x*, obtained in the microscopic (at the quark level) model of the nucleon – flucton interaction.

- It is shown that the observation of particle yields in this new cumulative region is accessible for study in dd collisions at the SPD, due to higher frequency of dd collisions that can be recorded by the SPD, compared to the slower MPD, which is important for registering of rare cumulative processes.

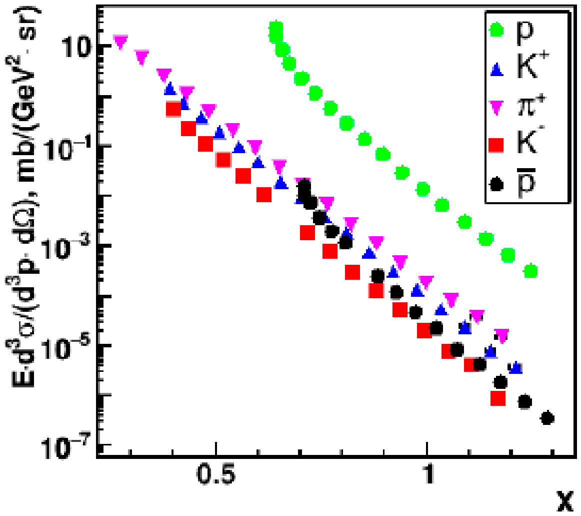
- The multiplicities of cumulative particles drop with increase of initial energy due to general increase of transverse momenta. That in region $\sqrt{s_{NN}}$ from 4 and 8 GeV can be partially compensated by the increase of luminosity. In this new cumulative region studies are not possible for colliders with large initial energy.

- It is shown also that in this new cumulative region the yields of pions in comparison with the yields of protons are not suppressed so strongly as in the nuclear fragmentation region, what can be explained by the different mechanisms of the formation of these cumulative particles. (However, it should be noted that the possible contribution of rescattering processes at large distances to cumulative protons has not been taken into account.)

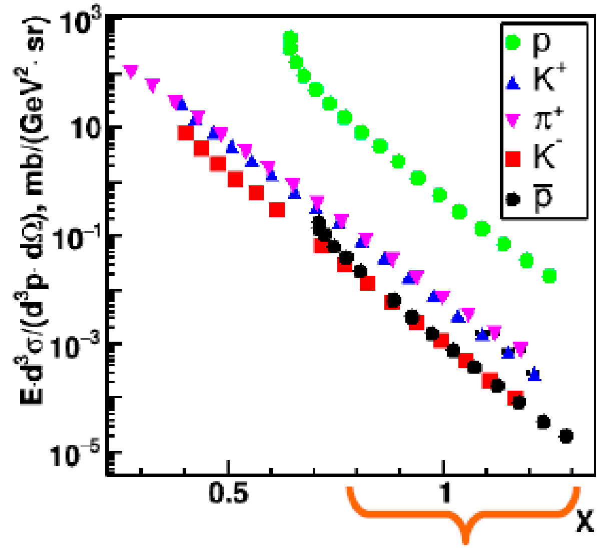
Г.А. Лексин, ЯФ, т. 65, № 11, 2002, стр. 2042 — 2051



C target



W target



Large fraction of cumulative processes

$\sqrt{s_{NN}}$ = 9.8 GeV

N. Antonov, V. Gapienko, G. Gapienko, M. Ilushin, A. Prudkoglyad, V. Romanovskiy, A. Semak, I. Solodovnikov, M. Ukhanov, V. Viktorov “High pt anti-proton and meson production in cumulative pA reaction at 50 GeV/c”
(National Research Center Kurchatov Institute - Institute for High Energy Physics, Protvino)
LXX International Conference “NUCLEUS – 2020. Nuclear physics and elementary particle physics. Nuclear physics technologies”, St Petersburg, October 11-17, 2020.

Flucton-flucton interaction

Cumulaive production at $|t| \sim s$

Quark counting rules for *elastic and quasi elastic* reactions with nuclei

Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett. Nuovo Cimento 7 (1973) 719
Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153; Phys.Rev. D11 (1975) 1309
Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269

$s \to \infty, \quad t/s$ fixed

$$(d\sigma/dt)_{\pi p\to\pi p} \sim s^{-8},\ (d\sigma/dt)_{pp\to pp} \sim s^{-10},\ (d\sigma/dt)_{\gamma p\to\pi p} \sim s^{-7},\ (d\sigma/dt)_{\gamma p\to\gamma p} \sim s^{-6}$$

$\sim s^{-n}$ *A+B->C+D* $n=n_A+n_B+n_C+n_D-2$ $n_p=3\ \ n_\pi=2\ \ n_\gamma=1$

$$\frac{d\sigma}{dt}(A+B\to C+D)\to\frac{1}{t^{N-2}}f(t/s) \qquad N=n_A+n_B+n_C+n_D$$

Yu.L. Dokshitzer, QCD Phenomenology,
Lectures at the CERN–Dubna School, Pylos, August 2002

the deuteron break-up by a photon, $\gamma + D \to p + n$

$$\frac{d\sigma}{dt} = \frac{f(\Theta)}{s^{K-2}}; \qquad \frac{t}{s} = \text{const}, \qquad K-2=1+6+3+3-2=11$$

For light nuclei:

Yu.N. Uzikov, Indication of Asymptotic Scaling in the Reactions
dd->p^{3}H, dd->n^{3}He and pd->pd, JETP Letters 81 (2005) 303.

$\sim s^{-22}$ (6+6+3+9-2=22) and $\sim s^{-16}$ (3+6+3+6-2=16)

The same is valid for formfactors:

Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003

$$F_n(q^2) \sim \left(\frac{1}{q^2}\right)^{n-1}$$

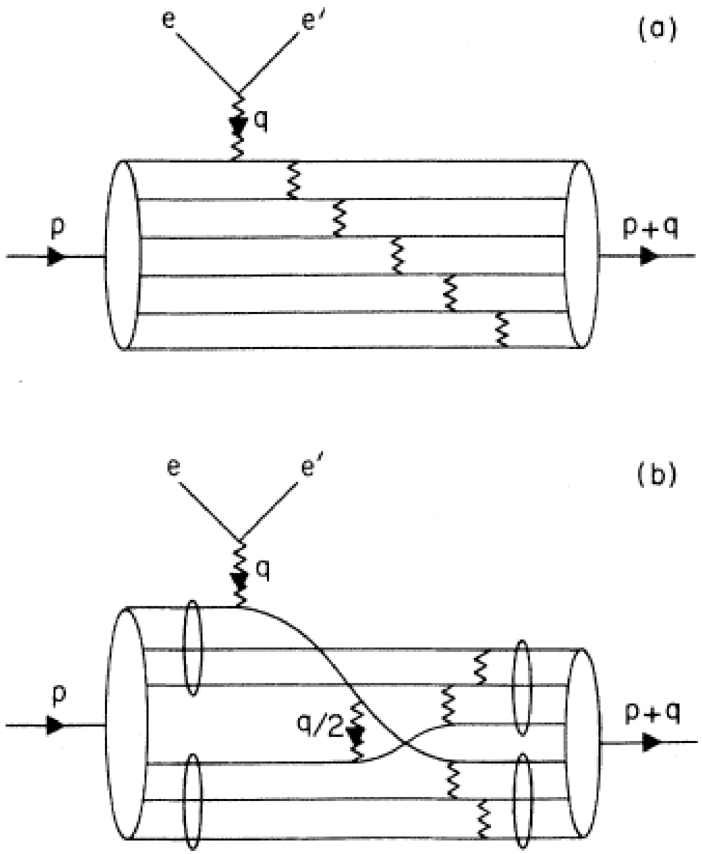


FIG. 2. Two possible quark-constituent views of e–D elastic scattering are (a) the democratic chain (cascade) model and (b) the quark-interchange model.

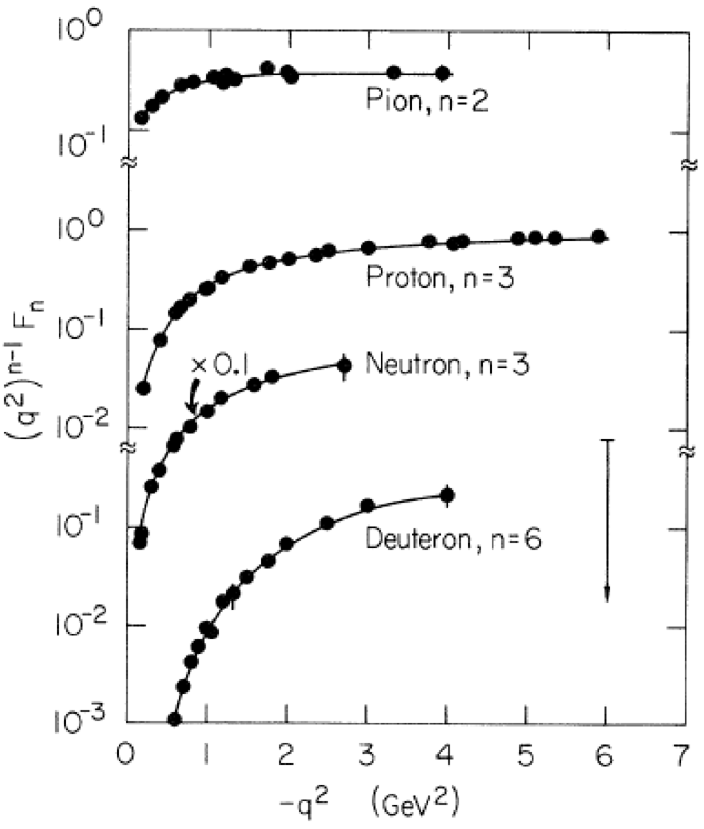


FIG. 1. Elastic electromagnetic form factors of hadrons for large spacelike q^2 in terms of the dimensional-scaling quark model. The curves simply connect the data points. (The neutron data have been multiplied by 0.1.)

Some details of formfactor calculations (compare to our slide 9)

Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153; Phys.Rev. D11 (1975) 1309
Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269

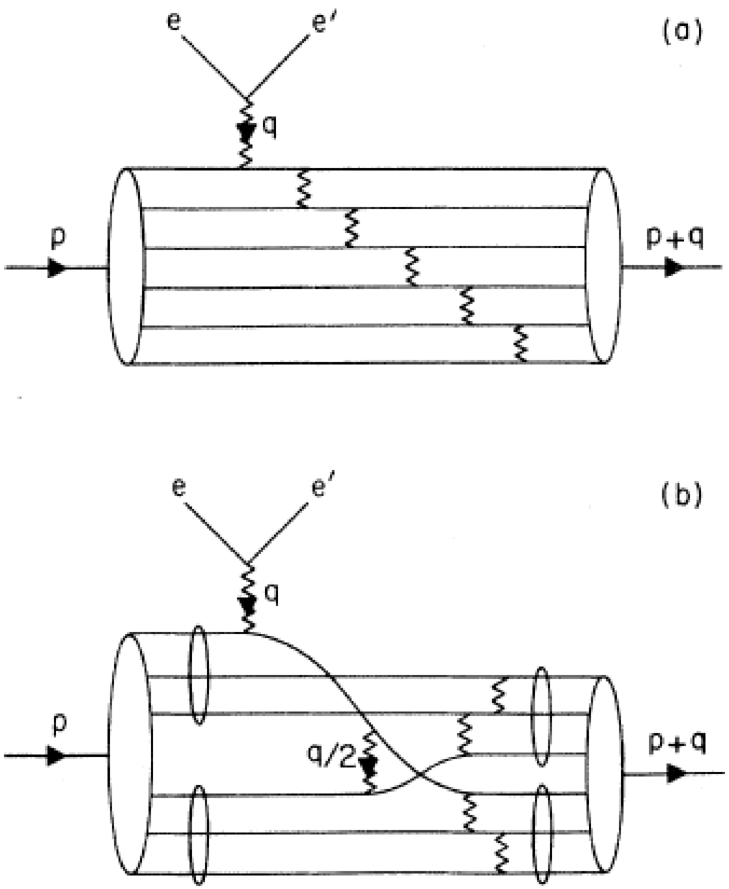


FIG. 2. Two possible quark-constituent views of e–D elastic scattering are (a) the democratic chain (cascade) model and (b) the quark-interchange model.

$$\psi_n(0) \equiv \int \prod_{j=1}^{n-1} d^3\vec{k}_j \psi(\vec{k}_j)$$

Hence, e interacts with d, when d is in the flucton configuration.

$$F_n(\vec{q}^2) \sim \left[\frac{2m}{\vec{q}^2} V(\vec{q}^2)\right]^{n-1} \psi_n^2(0)$$

In the case of quantum electrodynamics, and in fact any renormalizable theory, we have effectively (modulo powers of $\log q^2$ from finite orders in perturbation theory)

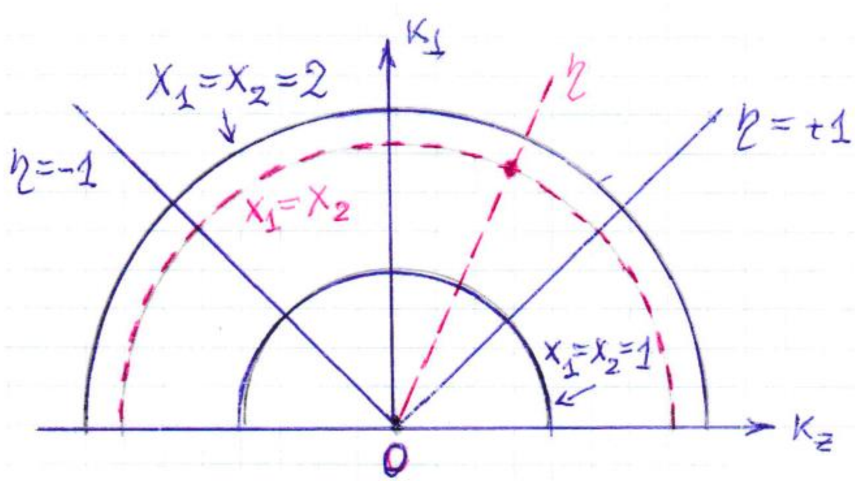
$$V(q^2) \sim \frac{e^2}{q^2}\left[1 + O\left(\frac{q^2}{m^2}\right)\right],$$

i.e., $V(q^2)$ becomes constant in the relativistic domain and

for large q^2 the gluon propagator is always compensated by its couplings to the quark currents

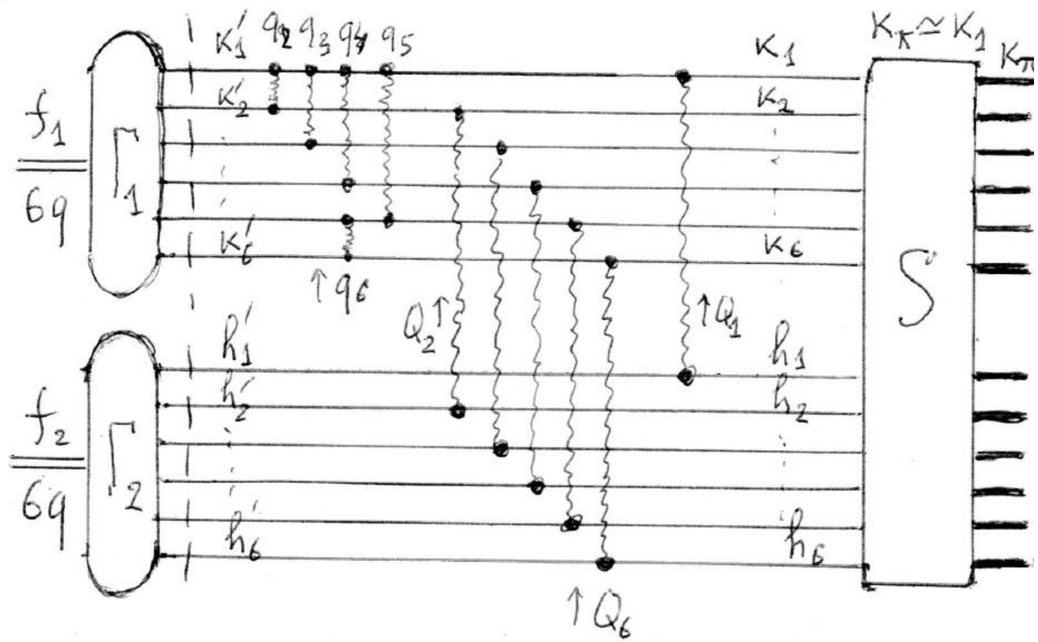
$$F_n(q^2) \sim \left(\frac{1}{q^2}\right)^{n-1}$$

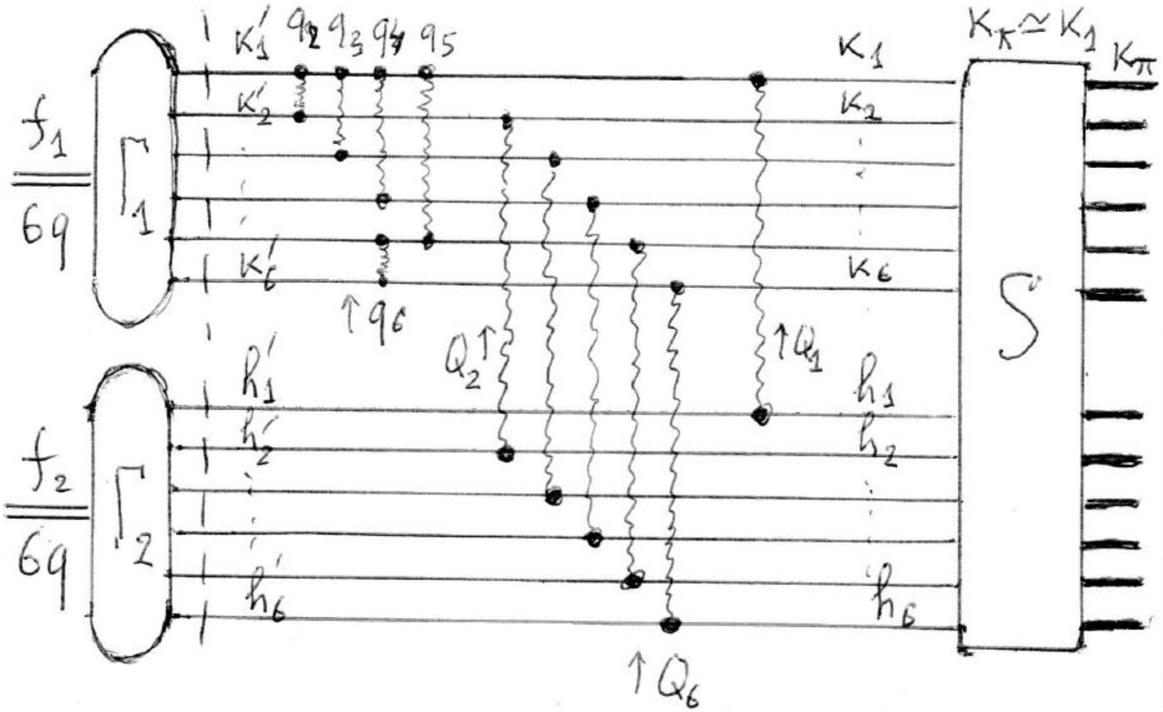
Quark counting rules for *inclusive cross sections* at |t| ~ s



$$\Gamma_1(k'_{+i}, k'_{\perp i}) \to \Psi(k'_{+i}, k'_{\perp i})$$

$$\Gamma_2(k'_{-i}, k'_{\perp i}) \to \Psi(k'_{-i}, k'_{\perp i})$$





$$f(\mathbf{k}) \equiv \frac{k_0\, d^3\sigma}{d^3\mathbf{k}} = f(x,\eta) = C\, s^{-m}\, (2-x)^{\frac{3}{2}(p-1)-1}\, F(\eta)$$

$x = x_1 = x_2$ - cumulative number

$\eta = -\ln \mathrm{tg}\, \frac{\theta^*}{2}$ - pseudorapidity

p – number of recoil quarks (p=11)

Incorporating diquarks

V.T. Kim, Diquarks and Dynamics of Large P(T) Baryon Production, Mod.Phys.Lett.A 3 (1988) 909.

p/π^+ - ratio explanation, using that the diquark distribution function is harder: $(1\text{-}x)^1$ vs $(1\text{-}x)^3$ for quarks [$(1\text{-}x)^{2p\text{-}1}$].

Yu.L. Dokshitzer, QCD Phenomenology, Lectures at the CERN–Dubna School, Pylos, August 2002

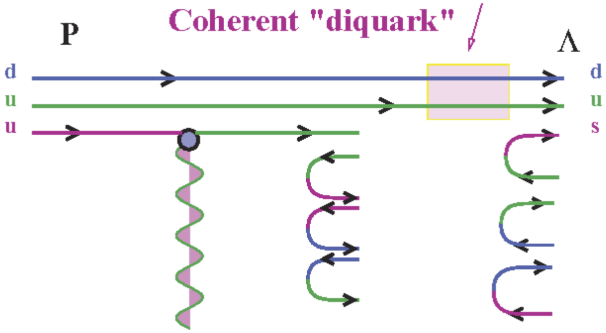


Fig. 4a: Gluon exchange produces a leading baryon.

M.A. Braun, V.V. Vechernin, Nuclear Structure Functions and Particle Production in the Cumulative Region in the Parton Model, Nucl.Phys. B427 (1994) 614

Can string junction carries the baryon number?

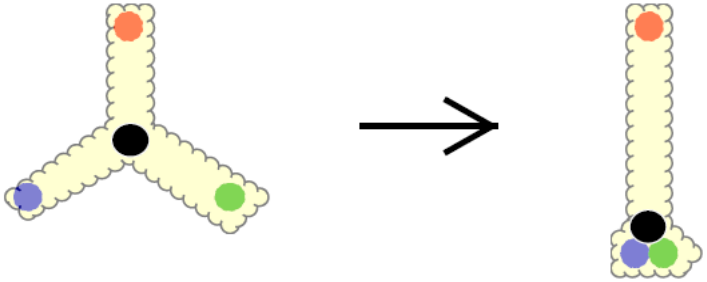
L. Montanet, G. C. Rossi, and G. Veneziano, “Baryonium Physics,”
Phys. Rept. 63, 149–222 (1980).

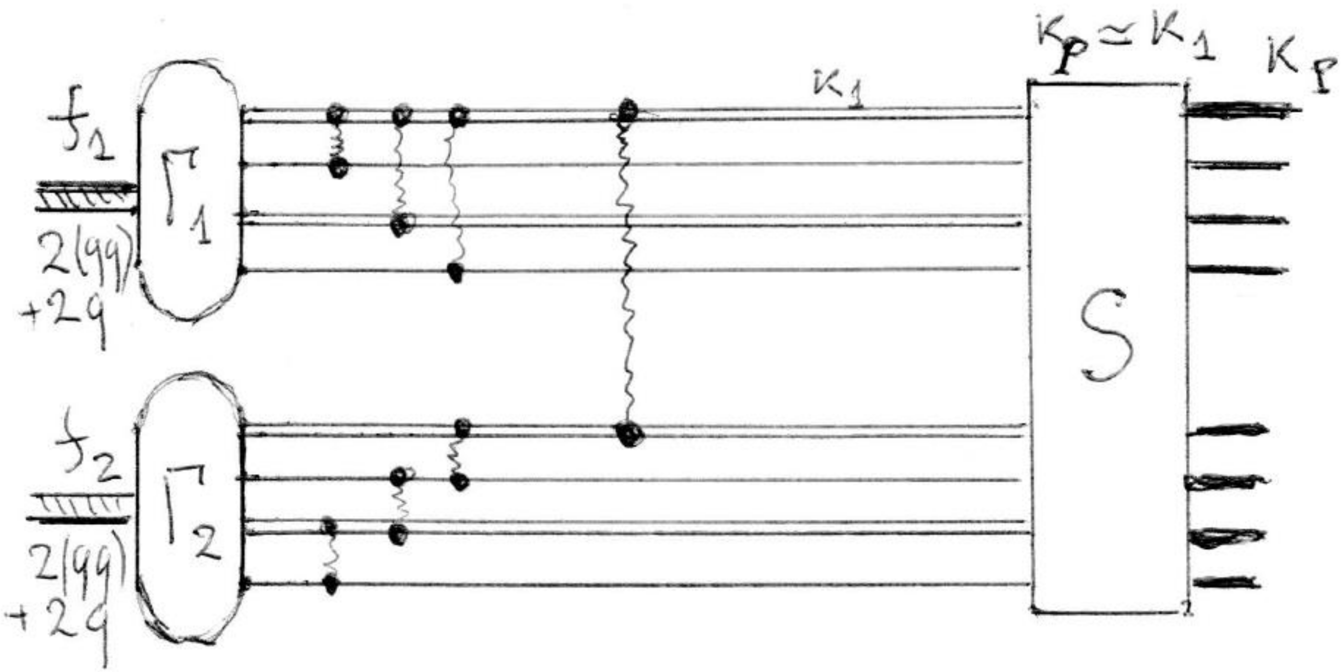
D. Kharzeev, “Can gluons trace baryon number?”
Phys.Lett. B 378, 238–246 (1996), arXiv:nucl-th/9602027.
Can be verified experimentally by studing of
baryon stopping in central pp and AA collisions.

Yu.M. Shabelski,
String Junction and Diffusion of Baryon Charge in Multiparticle Production Processes,
arXiv: 0705.0947 [hep-ph], (2007).
F. Bopp, Yu.M. Shabelski,
String junction effects for forward and central baryon production in hadron-nucleus collisions
Eur.Phys.J.A 28 (2006) 237-243

G.Pihan, A.Monnai, B.Schenke, Chun Shen,
Unveiling baryon charge carriers through charge stopping in isobar collisions
arXiv:2405.19439v1 [nucl-th] (2024).

Connection with diquarks:
Now B=1 corresponds to diquark





Possible mechanism for the production of cumulative protons by fragmentation of a diquark into a proton (along with the mechanism of coherent quark coalescence described above).

In this case for the number of recoil quarks and diquarks we have: *p=7*

Conclusions (flucton – flucton interaction)

- The study of multiquark fluctons in dd collisions at SPD has a number of advantages compared with MPD (<u>see slide 2</u>).

- The inclusive cross sections for particle production in the new cumulative region of large transverse momenta at mid-rapidities will decrease with both the initial energy s and the cumulative number $x=x_1=x_2$.

- To evaluate this behaviour and find asymptotes at s>>m and (2-x)<<1 we need to generalize the quark counting rules, known now only for
1) the inclusive cross sections in the fragmentation region (|t|<<s) and
2) the elastic and quasielastic cross sections in the high pT region (|t|~s),
to the case of inclusive cross sections in the high pT region (|t|~s).

The work was supported by the Russian Science Foundation grant 23-12-00042.

Backup slides

Modeling of the dd scattering within the framework of the Glauber approach

Both analytical and MC modeling without fluctons (Belokurova S.N.)

$$T_A(a_1,\ldots,a_A)=\prod_{j=1}^{A}T_A(a_j).$$

=>

$$T_{d_1}(a_1,a_2)=T_{d_1}(a_1)T_{d_1}(a_2)\delta(a_1-a_2).$$

$$T(a)=\int|\Psi(a,\,z)|^2\,dz$$

$$\Psi(r)=C(e^{-\gamma r}-e^{-\mu r})/r,\;\; C^2=\frac{\gamma(\gamma+\mu)\mu}{2\pi(\mu-\gamma)^2},$$

$$\gamma=45,8\ \text{МэВ},\ \mu=140\ \text{МэВ}.$$

$$\sigma(a)=exp\left(-\frac{a^2}{r_N^2}\right)$$

$$\sigma_{NN}\equiv\int db\,\sigma(b).\qquad \sigma_{NN}=\pi r_N^2.$$

$$\langle N_{coll}(\beta)\rangle=4\chi(\beta)$$

$$\chi(\beta)\equiv c^{-1}\int\sigma(a-b+\beta)\ \ (T_{d_1}(a))^2\ \ da\ \ (T_{d_2}(b))^2\ \ db,$$

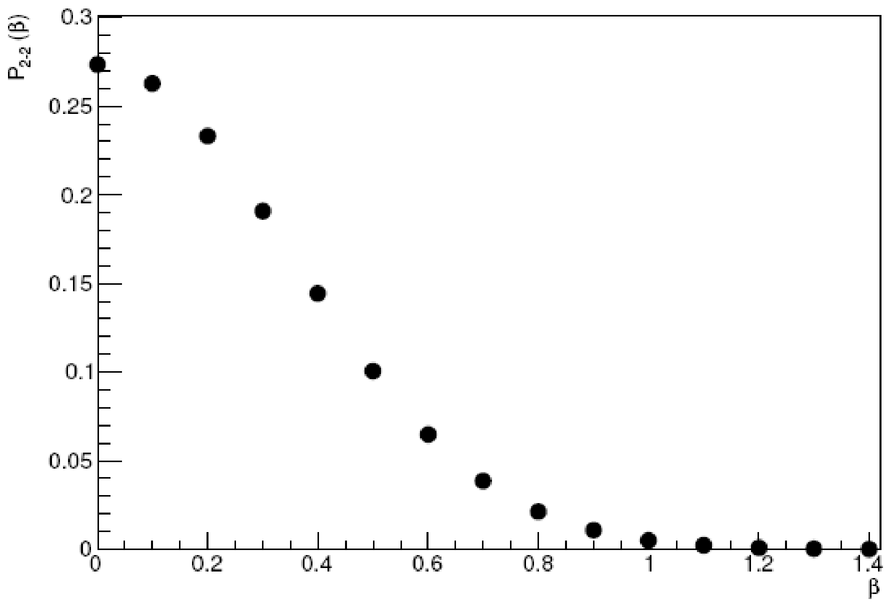
$$V\left[N_{coll}(\beta)\right]=$$ ...

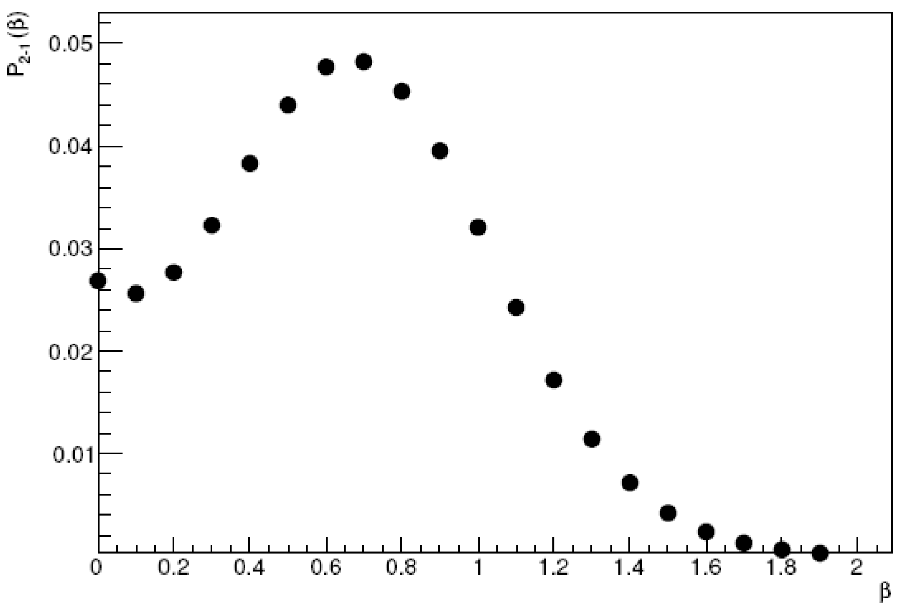
$$\langle N_w^{d_1}(\beta)\rangle+\langle N_w^{d_2}(\beta)\rangle=$$ ...

$$V[N_w^{d_1}(\beta)+N_w^{d_2}(\beta)]=$$ ...

$$P_{2-2}(\beta) = c^{-1} \int \sigma(a-b+\beta)\sigma(a+b+\beta)\sigma(-a+b+\beta)\sigma(-a-b+\beta) \ \ (T_{d_1}(a))^2 \ da \ \ (T_{d_2}(b))^2 \ db$$

$$P_{2-1}(\beta) = 2c^{-1} \int \sigma(a-b+\beta)\,[1-\sigma(a+b+\beta)]\,[1-\sigma(-a+b+\beta)]\,\sigma(-a-b+\beta) \ \ (T_{d_1}(a))^2 \ da \ \ (T_{d_2}(b))^2 \ db$$



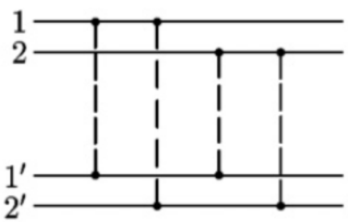


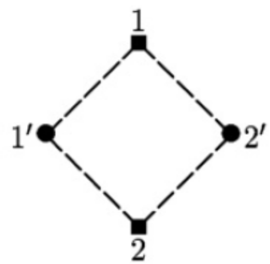
min.bias:

$$P_{2-1}^{min.bias} = 0.067 \qquad P_{2-2}^{min.bias} = 0.0046$$

Variation of the number of participant and NN collisions in AA (dd) interactions

Vechernin, V.V. and Nguyen, H.S. Phys. Rev. C 84 (2011) 054909





$$\chi(\beta) \equiv \int da\, db\, T_A(a) T_B(b) \sigma(a - b + \beta)$$

$$\approx \sigma_{NN} \int da\, T_A(a) T_B(a + \beta)$$

$$\wp_{\text{opt}}(N_{\text{coll}}) = C_{AB}^{N_{\text{coll}}}\, \chi(\beta)^{N_{\text{coll}}}\, [1 - \chi(\beta)]^{AB - N_{\text{coll}}}$$

$$\sigma_{NN} \equiv \int db\, \sigma(b),$$ **=>** $$I(a) \equiv \int db\, \sigma(b)\, \sigma(b + a).$$ **!!!**

So for variation of the number of participant and NN collisions in AA (dd) interactions the general analytical formulas from textbooks (“the optical approximation”):

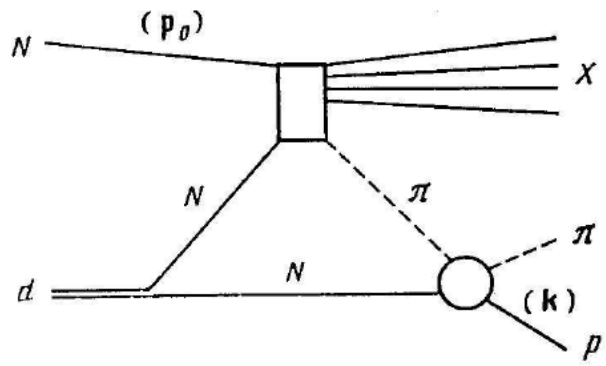
C.-Y. Wong, Introduction to High-Energy Heavy-Ion Collisions (World Scientific, Singapore, 1994).

R. Vogt, Ultrarelativistic Heavy-Ion Collisions (Elsevier, Amsterdam, 2007).

are not correct and are not supported by MC simulations.

Contribution of pion rescattering to cumulative proton production from deuteron

(long distance contribution !)



Prediction:

Braun M.A.,Vechernin V.V., Yad.Fiz. 28 (1978) 1466.

Experiment:

Ableev V.G. et al., Nucl.Phys.A 393 (1983) 491.

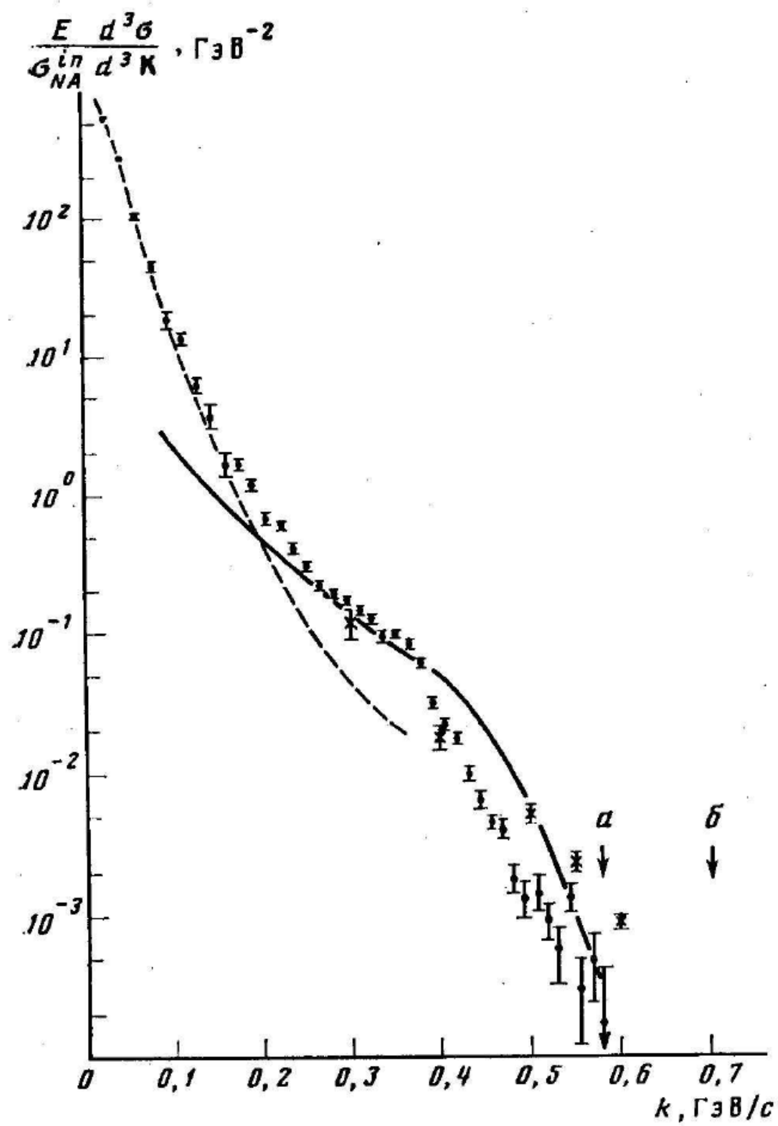
Preprint JINR El-82-377, Dubna, 1982.

Confirmation:

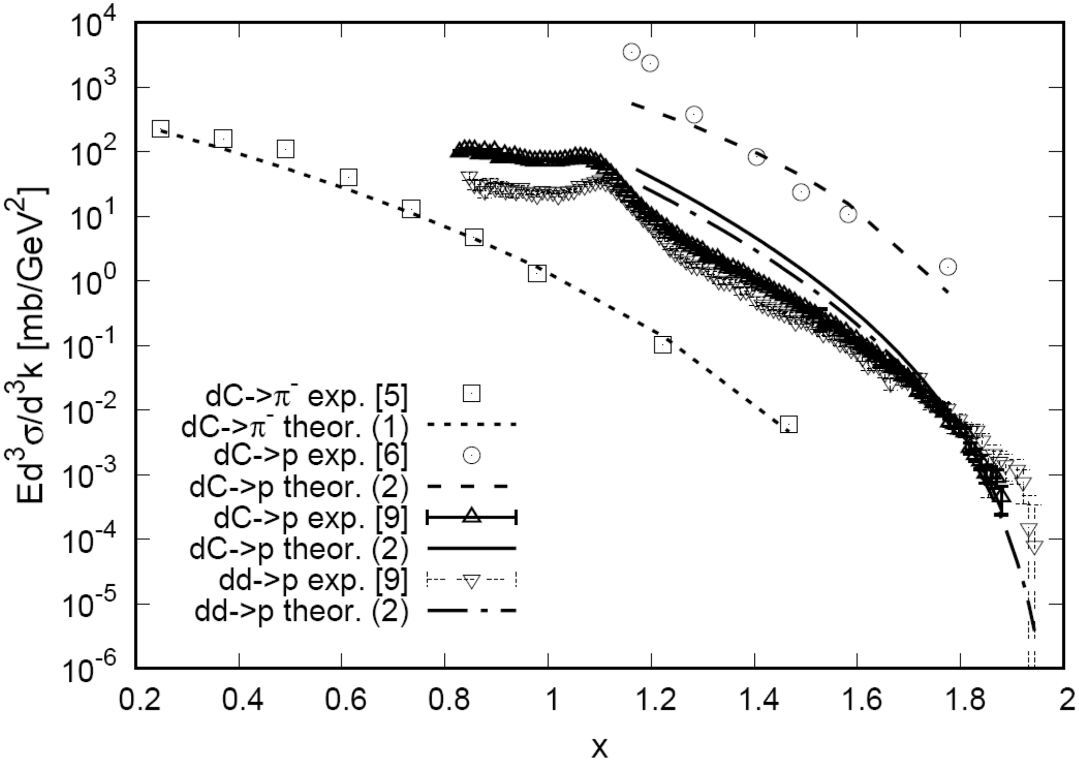
Braun M.A.,Vechernin V.V., Yad.Fiz. 40 (1984) 1588.

Braun M.A.,Vechernin V.V., Yad.Fiz. 43 (1986) 1579.

The shoulder in the spectrum is due to the contribution of the Δ-resonance to elastic πN scattering amplitude



Fixation of normalization constants



$$C_\pi^{dC} = 1.4\ mb/GeV^2\ ,$$

$$C_{\rm p}^{dC} = 1500\ mb/GeV^2$$

[9] L.S. Azhgirei et al., Sov. J. Nucl. Phys. 46, 661 (1987)

d+C=>p+X , d+d=>p+X

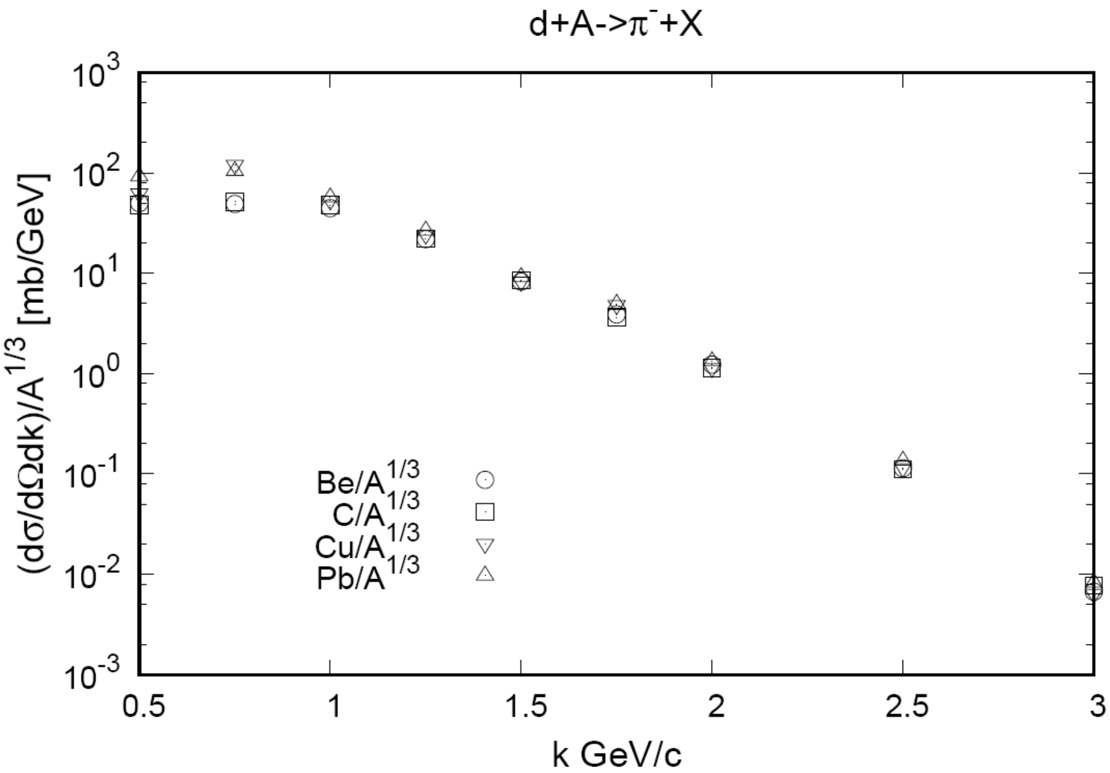
$p_{lab}^d = 9.0\ GeV\ (\sqrt{s_{NN}} = 3.2\ GeV)\qquad 0.139\, rad \doteq 8^\circ$

d+C=>π⁻+X , d+C=>p+X

[5] J. Papp et al., Phys. Rev. Lett. 34, 601 (1975)
[6] J. Papp, Ph. D. thesis, Univ. of California, Berkeley, Report No. LBL-3633, 1975
[I.A. Schmidt and R. Blankenbecler, Phys. Rev. D 15, 3321-1326 (1977)]

$E_{lab}^{kin} = 2.1\ GeV\qquad \sqrt{s_{NN}} = 2.7\ GeV\qquad 2.5^\circ$

A-dependence of the deuteron fragmentation

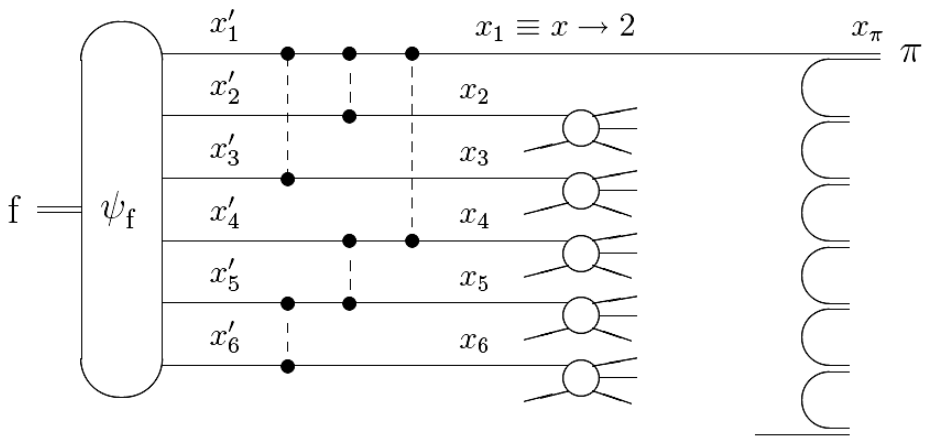


[5] ~ $A^{1/3}$ =>

$$C_\pi = C_\pi^{dd} = 0.77\ mb/GeV^2$$

$$C_{\mathrm{p}} = C_{\mathrm{p}}^{dd} = 825\ mb/GeV^2$$

Transverse momentum spectra of cumulative pions



- the cumulative pion production

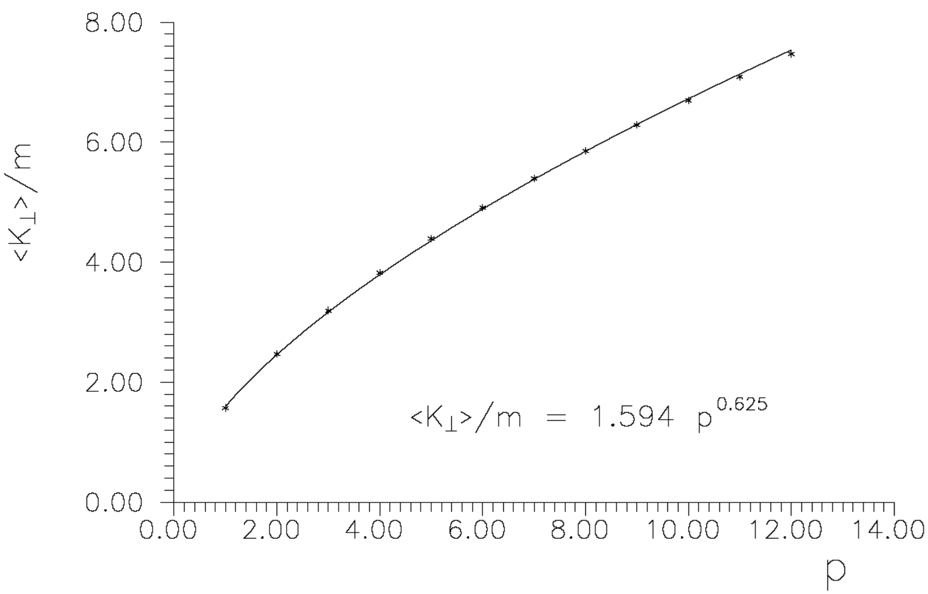
k_T – dependence:
*M.A. Braun, V.V. Vechernin, Phys.Atom.Nucl. **63**, 1831 (2000)*

$$\sigma_{pion}(x, k_\perp; p) = C(p)\ (x_{frag} - x)^{2p-1}\ f_p\left(\frac{k_\perp}{m}\right)$$

$$x < x_{frag}(p) = 1/3 + p/3$$

p – the number of "donors", stopped quarks

m – the constituent quark mass



$$f_p(t) = \frac{1}{\pi^p} \int \prod_{i=1}^{p} \frac{d^2 t_i}{(t_i^2+1)^2} (2\pi)^2 \delta^{(2)}(\sum_{i=1}^{p} t_i + t)$$

$$t = k_\perp/m, \quad t_i = k_{i\perp}/m$$

$$f_p(t) = 2\pi \int_0^\infty dz\, z J_0(tz)[zK_1(z)]^p$$

$$\langle |K_\perp| \rangle = pm \int_0^\infty dz K_0(z)(zK_1(z))^{p-1}$$