The Fayans energy-density functional in applications to neutrons stars

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TOWARDS A UNIVERSAL NUCLEAR DENSITY FUNCTIONAL

S.A.Fayans

… a large disagreement between …… approaches may indicate that some important physical ingredients are missing in the EDF construction, and perhaps the form of the EDF used so far in the microscopic calculations is not flexible enough to effectively incorporate them. Searches for a better parameter set for these "old" functionals are still continuing.

it would be of great advantage if a new EDF could be used not only for nuclei throughout the nuclear chart but also for describing such objects as neutron stars, with the crystal structure in their crust.

Fayans functional

$$\varepsilon = \varepsilon_{\rm kin} + \varepsilon_V + \varepsilon_{\rm surf} + \varepsilon_{\rm Coul} + \varepsilon_{sl} + \varepsilon_{\rm pair}$$

Energy-density functional for the volume part (we consider infinite matter)

$$E(n,x) = m_N\, n + \frac{3}{5}\epsilon_{\rm F,0} n \left(\frac{n}{n_0}\right)^{2/3} 2^{2/3}\left[x^{5/3} + (1-x)^{5/3}\right]$$ *kinetic energy*

$$+ \frac{1}{3}\epsilon_{\rm F,0}\frac{n^2}{n_0}\left[a_+ \frac{1-h_{1,+}\left(\frac{n}{n_0}\right)^\sigma}{1+h_{2,+}\left(\frac{n}{n_0}\right)^\sigma} + a_- \frac{1-h_{1,-}\left(\frac{n}{n_0}\right)}{1+h_{2,-}\left(\frac{n}{n_0}\right)}(1-2x)^2\right],$$

$$n = n_p + n_n \qquad x_p = n_p/n$$

density expansion (e.g. Skyrme) → Re-summed density series

(one can use it at high densities)

$a_\pm,\ h_{1,\pm},\ h_{2,\pm}$ dimensionless parameters

n_0 — nuclear saturation density

$$\epsilon_{\rm F,0} = \frac{(3\pi^2 n_0/2)^{2/3}}{2\, m_N} \qquad m_N = 939\,{\rm MeV} \text{ — free nucleon mass}$$

$$\frac{1}{3}\epsilon_{\rm F,0}\frac{n^2}{n_0} = C_0\frac{n^2}{4}\,, \qquad C_0 = \frac{\pi^2}{p_{\rm F} m_N}$$ *inverse density of sates at the Fermi surface*

for $n_0 = 0.16\,{\rm fm}^{-3}$ → $C_0 = 307\,{\rm MeV{\cdot}fm^3}$

NB! In some works, C_0 is fixed to 300 MeV fm^3

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S V Tolokonnikov[1,2], I N Borzov[1,3], M Kortelainen[4,5], Yu S Lutostansky[1] and E E Saperstein[1,6]

Fayans functional for nuclei

PHYSICAL REVIEW C **95**, 064328 (2017)

Toward a global description of nuclear charge radii: Exploring the Fayans energy density functional

P.-G. Reinhard[1] and W. Nazarewicz[2,3]

Conclusion: The Fayans paring functional, with its generalized density dependence, significantly improves the description of charge radii in odd and even nuclei. Adding differential charge radii to the set of fit observables

Arxiv: 2402.15380

Paul-Gerhard Reinhard[1], Jared O'Neal[2], Stefan M Wild[3,4], Witold Nazarewicz[5,6]

Extended Fayans energy density functional: optimization and analysis

Abstract. The Fayans energy density functional (EDF) has been very successful in describing global nuclear properties (binding energies, charge radii, and especially differences of radii) within nuclear density functional theory. In a recent study,

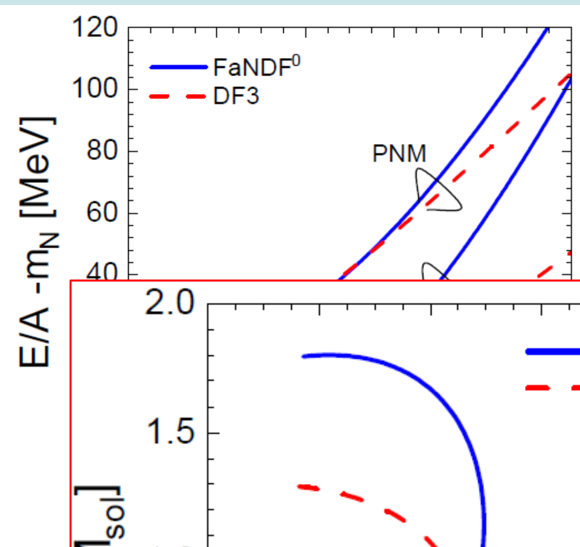
Fayans functional for neutron stars

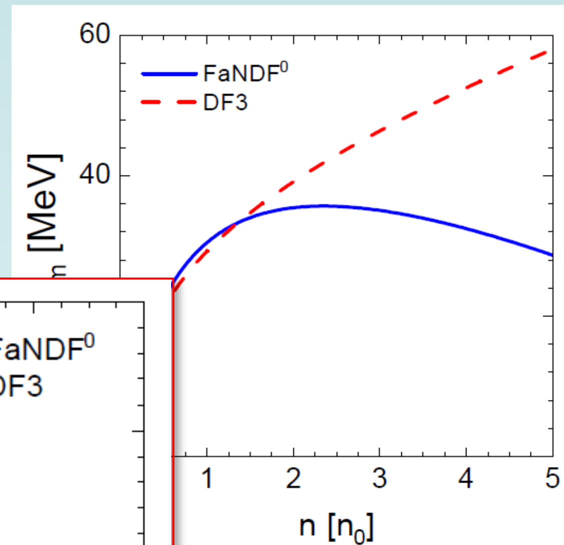
Two popular set of parameters:

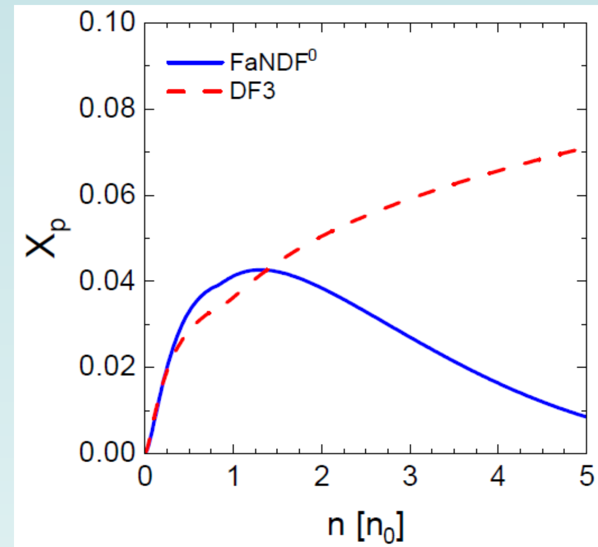
FaNDF^0 [Fayans, JETP Lett 68, 169 (1998)]

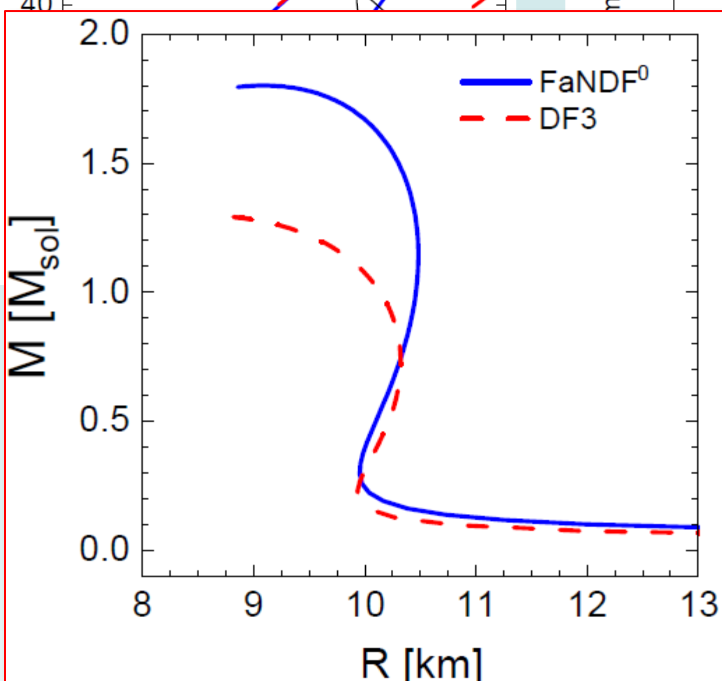
DF3 [Borzov, Fayans, Kroemer, Zawischa, Z. Phys. A 355, 117 (1996)]

FANDF^0 was fitted to reproduce energy density calculated by
Fridman, Pandharipande, NPA 361(1981)502 and Wiringa, Fiks, Fabrocini, PRC38(1988)1010









Parameters must be refitted

Neutron stars with masses > 2 M_{sol} are found

Fayans functional for at saturation density

$$\varepsilon(n,x) = \frac{1}{n}E(n,x) - m_N\,,$$ *binding energy per nucleon*

$$\varepsilon(n,x) = \frac{3}{5}\epsilon_{\mathrm{F},0}\left(\frac{n}{n_0}\right)^{2/3} 2^{2/3}\left[x^{5/3} + (1-x)^{5/3}\right]$$
$$+\frac{1}{3}\epsilon_{\mathrm{F},0}\frac{n}{n_0}\left[a_+\frac{1-h_{1,+}\left(\frac{n}{n_0}\right)^{\sigma}}{1+h_{2,+}\left(\frac{n}{n_0}\right)^{\sigma}} + a_-\frac{1-h_{1,-}\left(\frac{n}{n_0}\right)}{1+h_{2,-}\left(\frac{n}{n_0}\right)}(1-2x)^2\right]$$

binding energy for the symmetric nuclear matter (SNM) and the pure neutron matter (PNM)

$$\varepsilon_{\mathrm{SNM}}(n) = \varepsilon(n,1/2) \qquad \varepsilon_{\mathrm{PNM}}(n) = \varepsilon(n,1)$$

symmetry energy can be defined in two different ways

$$\varepsilon_{\mathrm{sym}}(n) = \varepsilon_{\mathrm{PNM}}(n) - \varepsilon_{\mathrm{SNM}}(n) = \frac{3}{5}\left(2^{2/3}-1\right)\epsilon_{\mathrm{F},0}\left(\frac{n}{n_0}\right)^{2/3} + \frac{1}{3}\epsilon_{\mathrm{F},0}\frac{n}{n_0}a_-\frac{1-h_{1,-}\left(\frac{n}{n_0}\right)}{1+h_{2,-}\left(\frac{n}{n_0}\right)}$$

$$\varepsilon_{\mathrm{sym}}(n) = \frac{1}{8}\frac{\partial^2}{\partial x^2}\varepsilon(n,x)\Big|_{x=\frac{1}{2}} = \frac{1}{3}\epsilon_{\mathrm{F},0}\left(\frac{n}{n_0}\right)^{2/3} + \frac{1}{3}\epsilon_{\mathrm{F},0}\frac{n}{n_0}a_-\frac{1-h_{1,-}\left(\frac{n}{n_0}\right)}{1+h_{2,-}\left(\frac{n}{n_0}\right)}$$

$$\frac{3}{5}\left(2^{2/3}-1\right) = 0.352441\,, \qquad \frac{1}{3} = 0.333333$$ 5% difference

For $n \sim n_0$ one characterize the EoS by expansion coefficients

$$\varepsilon_{\rm SNM}(n) = \mathcal{E}_0 + \frac{0}{3}\frac{(n-n_0)}{n_0} + \frac{K}{18}\frac{(n-n_0)^2}{n_0^2} + \dots$$

$$\varepsilon_{\rm sym}(n) = J + \frac{L}{3}\frac{n-n_0}{n_0} + \frac{K_{\rm sym}}{18}\frac{(n-n_0)^2}{n_0^2} + \dots$$

$a_+,\ h_{1,+},\ h_{2,+} \Leftrightarrow \mathcal{E}_0, K, \text{min. cond.}$ $\qquad$ $a_-,\ h_{1,-},\ h_{2,-} \Leftrightarrow J, L, K_{\rm sym}$

$$a_+ = \frac{3}{\epsilon_{\rm F,0}}\left(\frac{3}{5}\epsilon_{\rm F,0} - \mathcal{E}_0\right)\frac{(h_{2,+}+1)}{(h_{1,+}-1)},$$

$$h_{1,+} = \frac{h_{2,+}\sigma(3\epsilon_{\rm F,0} - 5\mathcal{E}_0) + (h_{2,+}+1)(5\mathcal{E}_0 - \epsilon_{\rm F,0})}{5\mathcal{E}_0(h_{2,+}+\sigma+1) - \epsilon_{\rm F,0}(h_{2,+}+3\sigma+1)},$$

← only 2 parameters σ and K

$$h_{2,+} = \frac{45\mathcal{E}_0(\sigma+1) - 3\epsilon_{\rm F,0}(3\sigma+1) + 5K}{45\mathcal{E}_0(\sigma-1) + \epsilon_{\rm F,0}(3-9\sigma) - 5K},$$

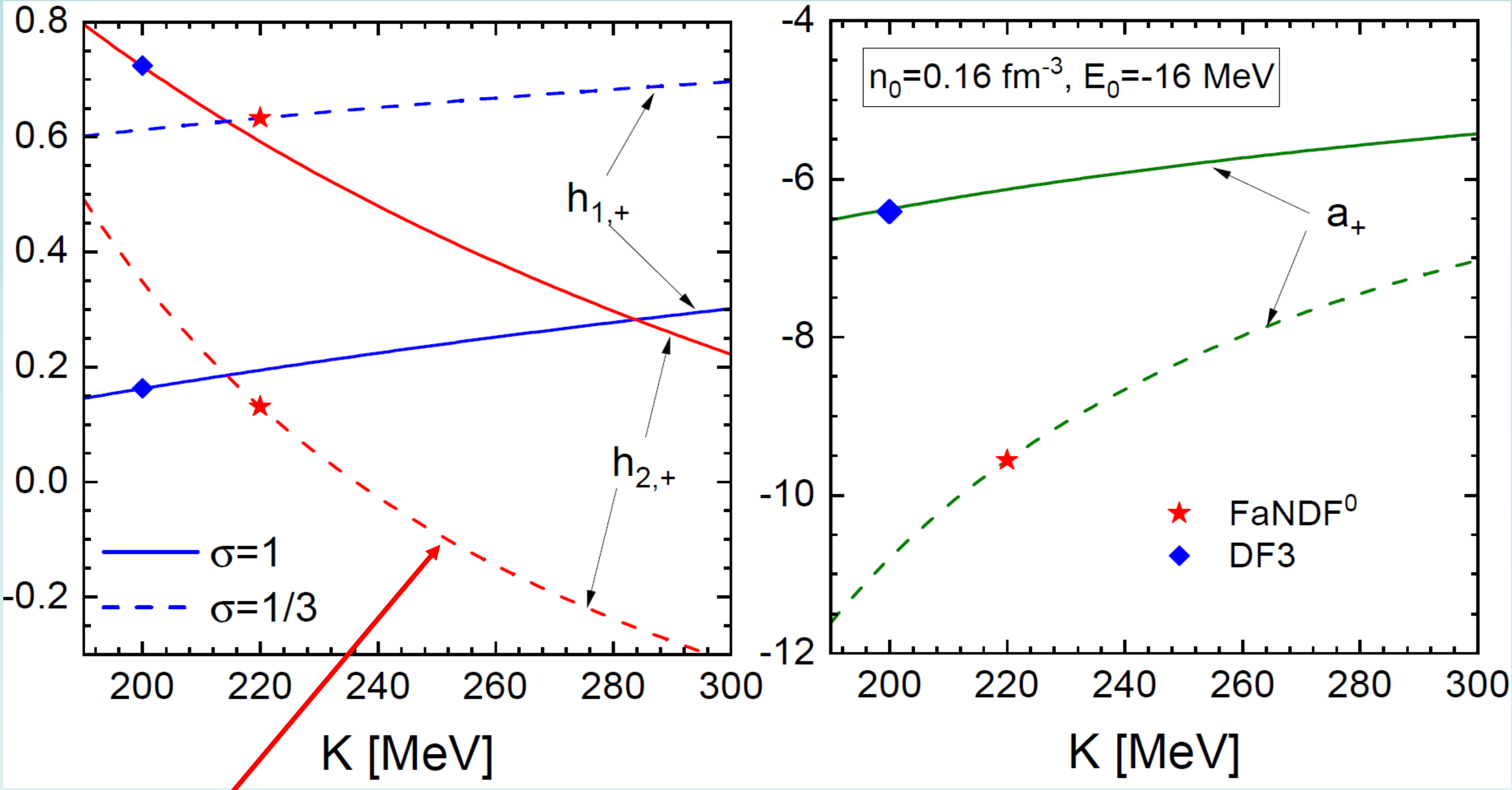
$$a_- = \frac{3}{\epsilon_{\rm F,0}}\left(\frac{1}{3}\epsilon_{\rm F,0} - J\right)\frac{(h_{2,-}+1)}{(h_{1,-}-1)},$$

3 parameters →

$$h_{1,-} = \frac{\frac{1}{3}\epsilon_{\rm F,0}(1-2h_{2,-}) + L(1+h_{2,-}) - 3J}{\frac{1}{3}\epsilon_{\rm F,0}(4+h_{2,-}) + h_{2,-}(L-3J) + (L-6J)},$$

$$h_{2,-} = \frac{4\epsilon_{\rm F,0} - 54J - 3K_{\rm sym} + 18L}{2\epsilon_{\rm F,0} + 3K_{\rm sym}}$$

- *symmetric nuclear matter*



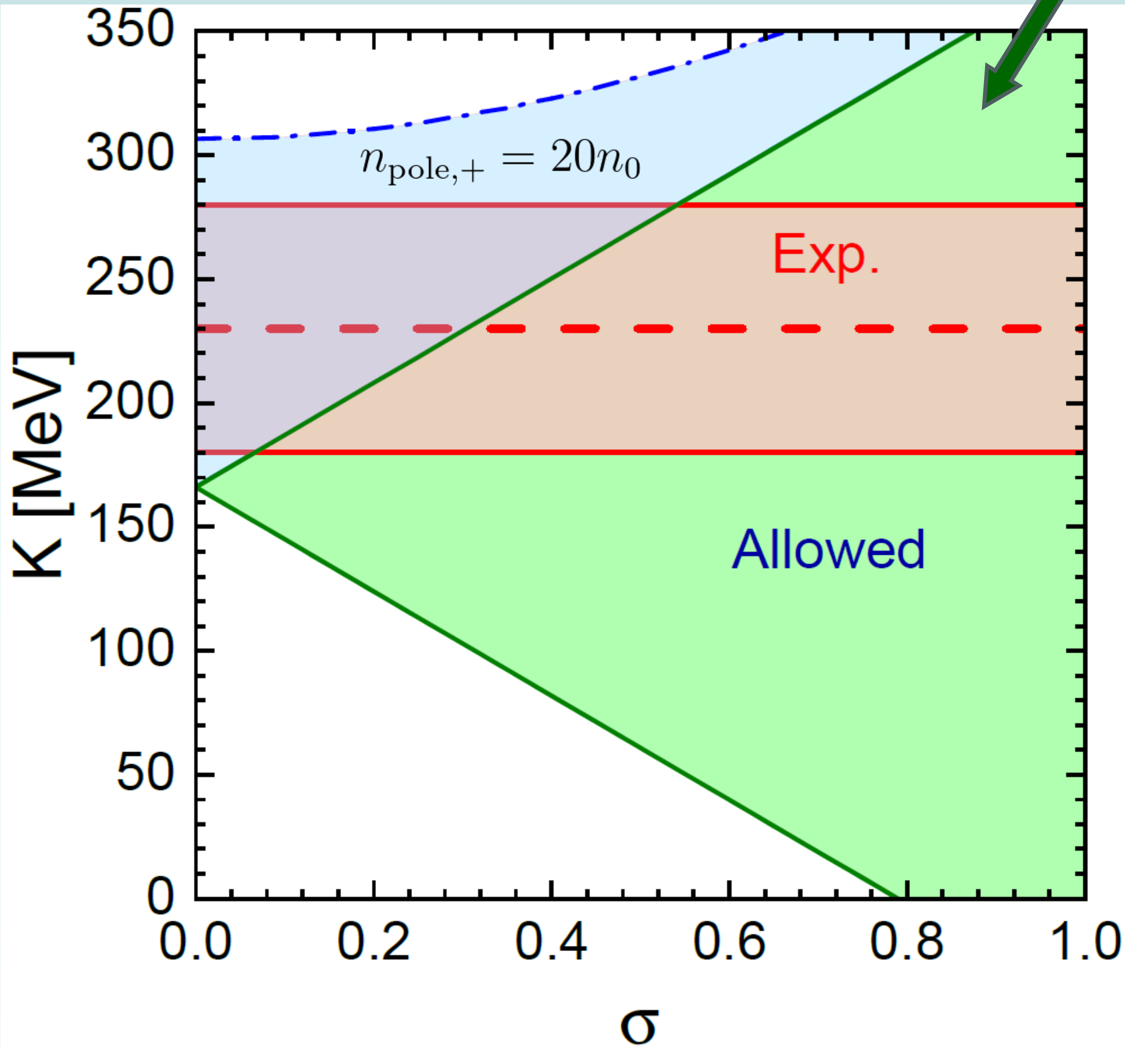
$\boldsymbol{h_{2,+} < 0}$

$$\frac{1 - h_{1,+}\left(\frac{n}{n_0}\right)^\sigma}{1 + h_{2,+}\left(\frac{n}{n_0}\right)^\sigma}$$ diverges at $n_{\text{pole},+} = |h_{2,+}|^{-1/\sigma} n_0$

$$h_{2,+} = \frac{45\mathcal{E}_0(\sigma+1) - 3\epsilon_{\mathrm{F},0}(3\sigma+1) + 5K}{45\mathcal{E}_0(\sigma-1) + \epsilon_{\mathrm{F},0}(3-9\sigma) - 5K} > 0$$

if $\frac{3}{5}\epsilon_{\mathrm{F},0} - 9\mathcal{E}_0 + 9\left(\mathcal{E}_0 - \frac{1}{5}\epsilon_{\mathrm{F},0}\right)\sigma < K < \frac{3}{5}\epsilon_{\mathrm{F},0} - 9\mathcal{E}_0 - 9\left(\mathcal{E}_0 - \frac{1}{5}\epsilon_{\mathrm{F},0}\right)\sigma$

$$(166.1 - 210.3\sigma)\,\mathrm{MeV} < K \leq (166.1 + 210.3\sigma)\,\mathrm{MeV}$$



If we allow for a pole at some high density n_{pole}

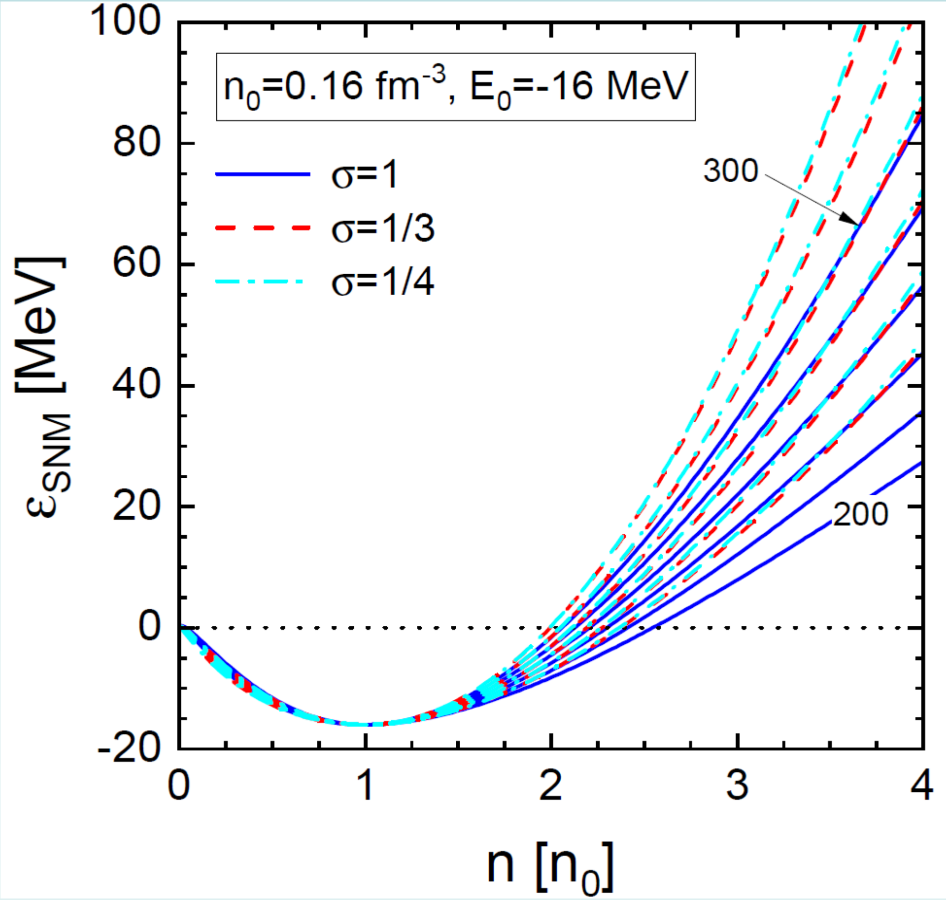
$$K < (166.1 + 210.3\sigma)\,\rho, MeV + \frac{420.6\,\mathrm{MeV}\sigma}{\left(\frac{n_{\mathrm{pole},+}}{n_0}\right)^{\sigma} - 1}$$

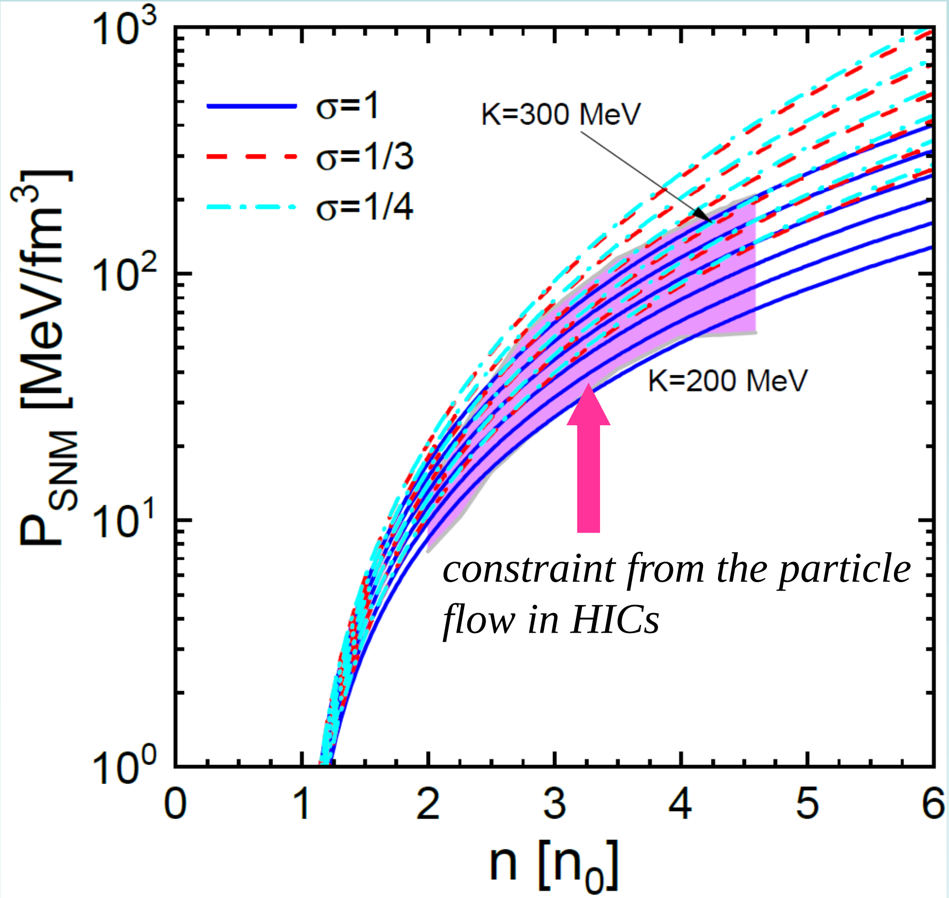
$$\sigma \to 0$$

$$166.1\,\mathrm{MeV} < K < 166.1\,\mathrm{MeV} + \frac{420.6\,\mathrm{MeV}}{\log\frac{n_{\mathrm{pole},+}}{n_0}}$$

- *Equation of state for symmetric nuclear matter*

$$P = n\frac{\mathrm{d}E}{\mathrm{d}n} - E$$





decrease of σ leads to a stiffening of the EoS but this effect saturates for σ < 1/3

$$K \leq 300\,\mathrm{MeV} \quad \text{for} \quad \sigma = 1$$

$$K \leq 240\,\mathrm{MeV} \quad \text{for} \quad \sigma \leq \frac{1}{3}$$

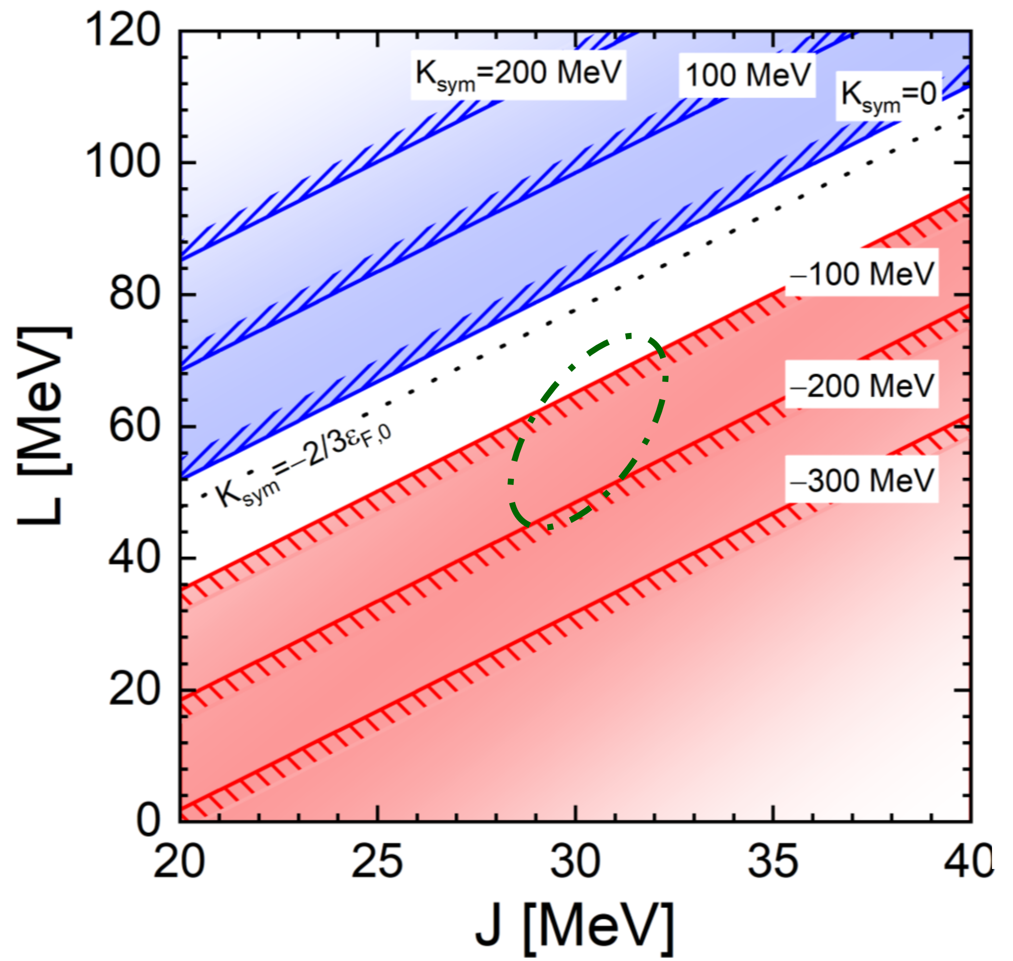
- *symmetry energy*

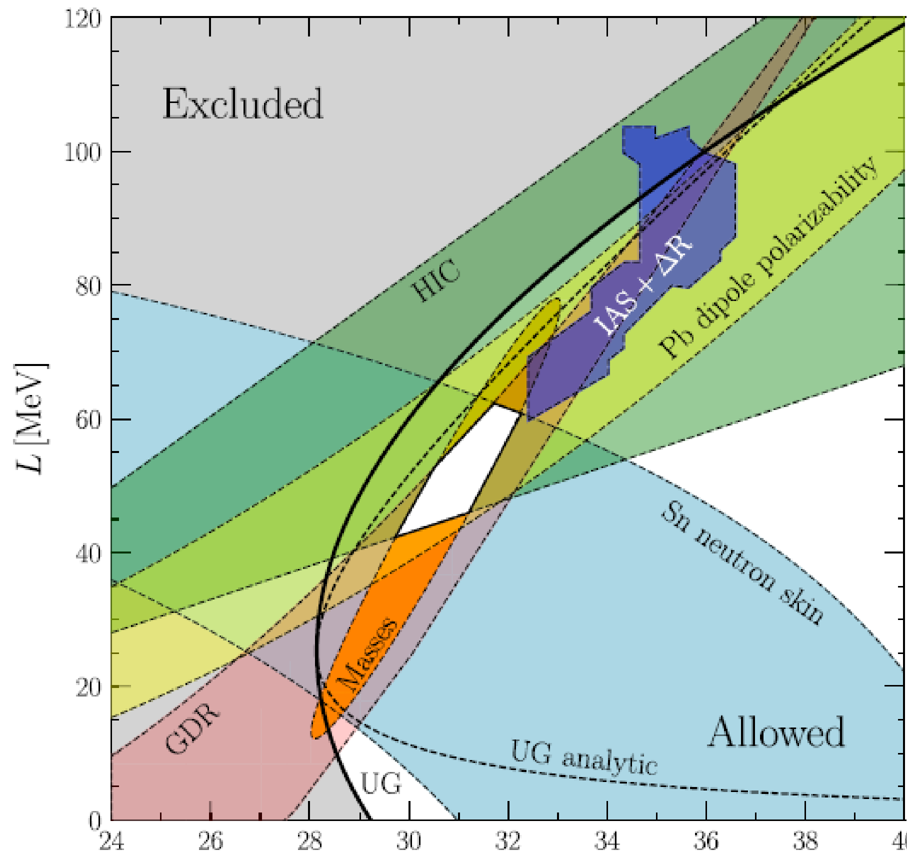
$$\frac{1 - h_{1,-}\left(\frac{n}{n_0}\right)}{1 + h_{2,-}\left(\frac{n}{n_0}\right)}$$ will have a pole if $h_{2,-} < 0$

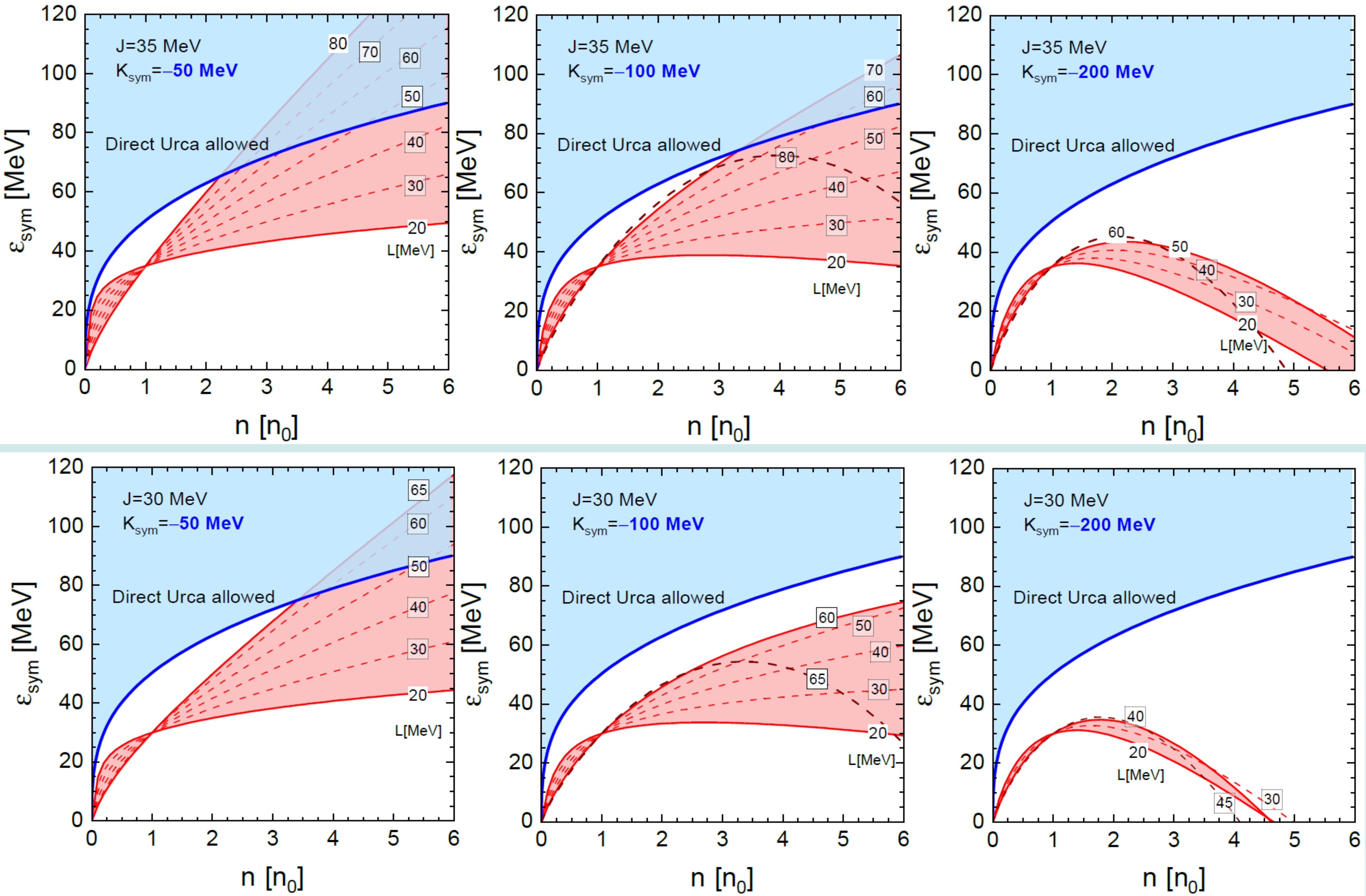
$$h_{2,-} = \frac{4\epsilon_{\mathrm{F},0} - 54J - 3K_{\mathrm{sym}} + 18L}{2\epsilon_{\mathrm{F},0} + 3K_{\mathrm{sym}}} > 0$$

$$L > -\tfrac{2}{9}\epsilon_{\mathrm{F},0} + 3J + \tfrac{1}{6}K_{\mathrm{sym}} \text{ if } K_{\mathrm{sym}} > -\tfrac{2}{3}\epsilon_{\mathrm{F},0}$$

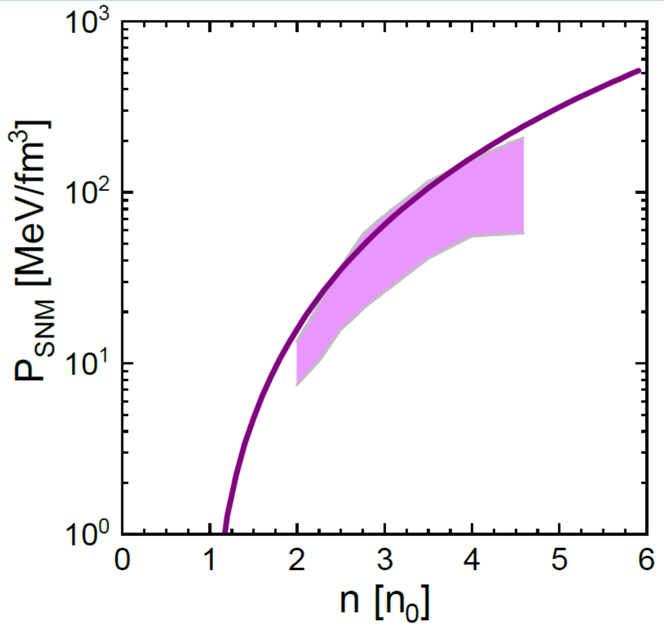
$$L < -\tfrac{2}{9}\epsilon_{\mathrm{F},0} + 3J + \tfrac{1}{6}K_{\mathrm{sym}} \text{ if } K_{\mathrm{sym}} < -\tfrac{2}{3}\epsilon_{\mathrm{F},0}$$

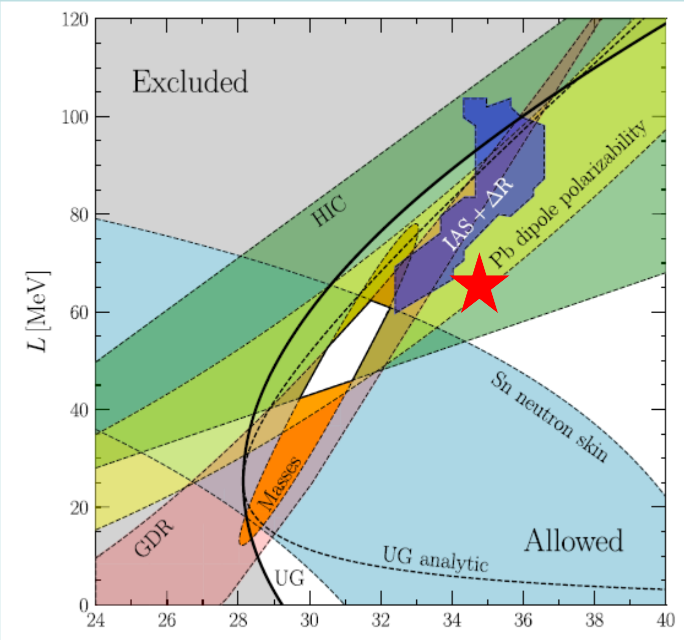


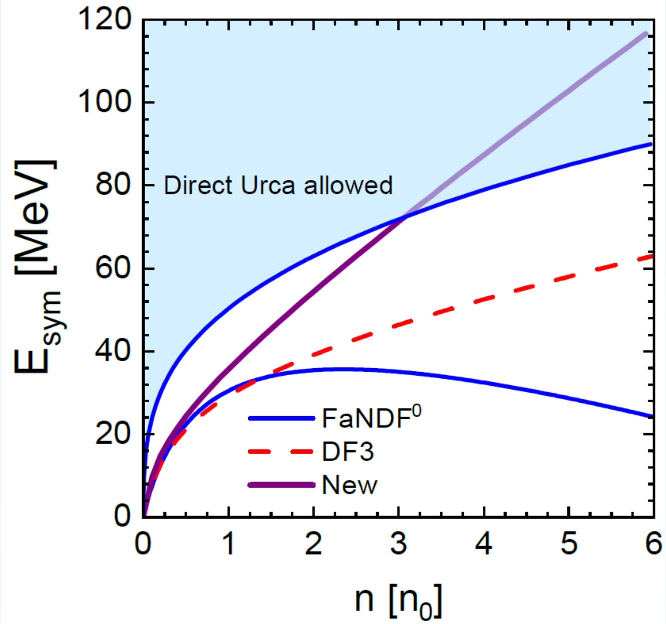


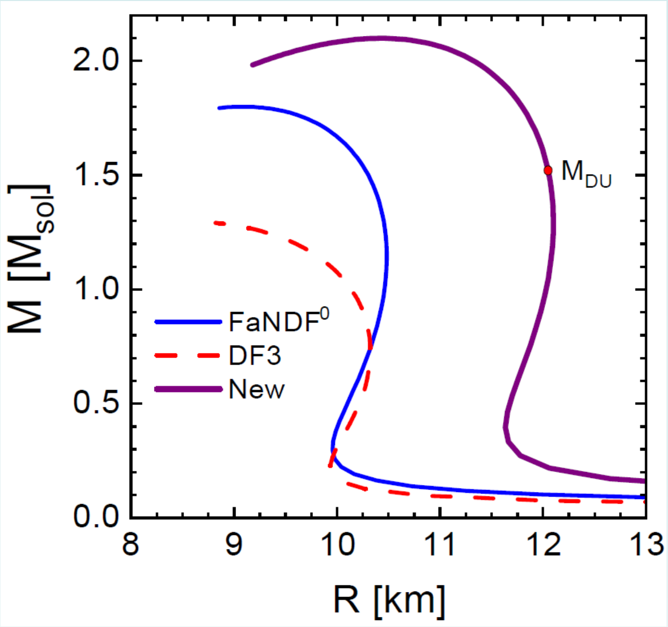

J=35 MeV
K_sym=–50 MeV
Direct Urca allowed
80
70
60
50
40
30
20
L[MeV]
J=35 MeV
K_sym=–100 MeV
Direct Urca allowed
70
60
50
80
40
30
20
L[MeV]
J=35 MeV
K_sym=–200 MeV
Direct Urca allowed
60
50
40
30
20
L[MeV]
J=30 MeV
K_sym=–50 MeV
Direct Urca allowed
65
60
50
40
30
20
L[MeV]
J=30 MeV
K_sym=–100 MeV
Direct Urca allowed
60
50
40
65
30
20
L[MeV]
J=30 MeV
K_sym=–200 MeV
Direct Urca allowed
40
20
L[MeV]
45
30
ε_sym [MeV]
n [n_0]

σ=1⁄3; K=260 MeV; J=35 MeV; L=60 MeV ; K_{sym}=−50 MeV;









Conclusion

Fayns ED functional with traditionally-used sets of parameters
are not compatible with neutron star observations

We use theone-to-one correspondence between the EDF parameters $a_{\pm},\ h_{1,\pm},\ h_{2,\pm}, \sigma$
and nuclear matter parameters at the saturation $\mathcal{E}_0, n_0, K, J, L, K_{\rm sym}$

The form of the EDF imposes constraints on the allowed values of K, L, and J

We demonstrated that it is possible to find a set of parameters
that allows to describe neutrons stars with $M \simeq 2.1\, M_\odot$ and $M_{\rm DU} > 1.5\, M_\odot$