Effects of neutrino electromagnetic properties and spin state in elastic neutrino-nucleon scattering.

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Search for electromagnetic neutrino properties

C. Giunti, A. Studenikin, Neutrino electromagnetic interactions: a window to new physics Rev. Mod. Phys. **87**, 531 (2015)
A. Studenikin, Electromagnetic neutrinos: The basic interaction processes and constraints from laboratory experiments and astrophysics. **Nucleus-2024**, 11:30 04.07.2024

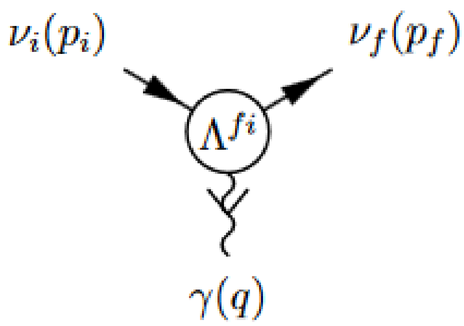
Neutrino electromagnetic properties open a window to the beyond-Standard-Model physics

- Already in SM, neutrinos have a ***charge radius***
  [Bernabeu et al, PRD (2000), PRL (2002), arXiv (2003)]

$$< r_\nu^2 > \sim 10^{-32}\mathrm{cm}^2$$

- Minimally extended SM predicts the neutrino's ***magnetic momer***
  [Fujikawa, Shrock, PRL (1980); Shrock, NPB (1982)]

$$\mu_\nu = 3{,}2 \times 10^{-19} \left(\frac{m_\nu}{1\ \mathrm{eV}}\right) \mu_B$$



- Neutrinos may also have other electromagnetic properties:
  millicharge, electric and anapole moments

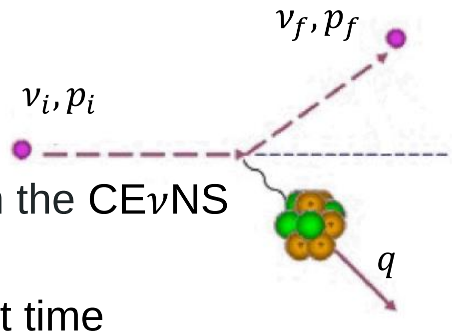
$$(\Lambda_\mu(q))_{jk} = (\gamma_\mu - \frac{q_\mu \not{q}}{q^2})[(f_Q(q^2))_{jk} + \gamma_5 (f_A(q^2))_{jk} q^2] - i\sigma_{\mu\nu} q^\nu (f_M(q^2))_{jk} + \sigma_{\mu\nu} q^\nu \gamma_5 (f_E(q^2))_{jk}$$

$$f_Q^{jk}(0) = e_{jk}, \quad f_M^{jk}(0) = \mu_{jk}, \quad f_E^{jk}(0) = \epsilon_{jk}, \quad f_A^{jk}(0) = a_{jk} \quad \langle r_\nu^2 \rangle = 6 \frac{d f_Q(q^2)}{dq^2}\Big|_{q^2=0}$$

millicharge
magnetic dipole moment
electric dipole moment
anapole moment
charge radius

Coherent elastic neutrino-nucleus scattering

CEνNS



Electromagnetic properties of neutrinos can be probed experimentally with the CEνNS

- In 2017 COHERENT collaboration observed CEνNS process for the first time
- Data of COHERENT and Dresden-II experiments have been already used to obtain limits for neutrino's millicharge, charge radius and magnetic moment

Charge radii ($10^{-32} cm^2$)

	Fixed R_n			Free R_n		
	1σ	2σ	3σ	1σ	2σ	3σ
			CsI + Ar			
$\langle r^2_{\nu_{ee}} \rangle$	$-56 \div -2$	$-68 \div 11$	$-78 \div 22$	$-55 \div -4$	$-67 \div 14$	$-77 \div 25$
$\langle r^2_{\nu_{\mu\mu}} \rangle$	$-64 \div 6$	$-68 \div 12$	$-71 \div 17$	$-64 \div 9$	$-67 \div 15$	$-71 \div 19$
$\langle r^2_{\nu_{e\mu}} \rangle$	<27	<33	<36	<25	<32	<36
$\langle r^2_{\nu_{e\tau}} \rangle$	<27	<40	<50	<26	<40	<50
$\langle r^2_{\nu_{\mu\tau}} \rangle$	<36	<40	<44	<36	<40	<44

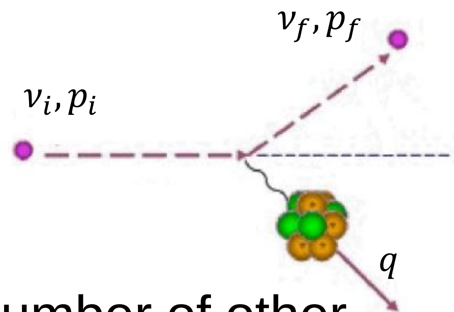
Magnetic moments ($10^{-10}\mu_B$)

	Fixed R_n			Free R_n		
	1σ	2σ	3σ	1σ	2σ	3σ
			CsI + Ar			
$\lvert\mu_{\nu_e}\rvert$	<27	<44	<56	<33	<48	<60
$\lvert\mu_{\nu_\mu}\rvert$	$5 \div 27$	<34	<41	$12 \div 31$	<37	<43

Cadeddu, M., Dordei, F., Giunti, C., Li, Y. F., Picciau, E., & Zhang, Y. Y. (2020). *PRD*, *102*(1), 015030.
Cadeddu, M., Giunti, C., Kouzakov, K. A., Li, Y. F., Studenikin, A. I., & Zhang, Y. Y. (2018). *PRD*, 98(11), 113010.

Coherent elastic neutrino-nucleus scattering

CEνNS

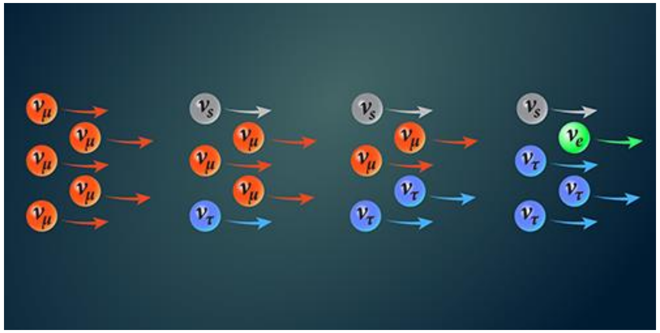


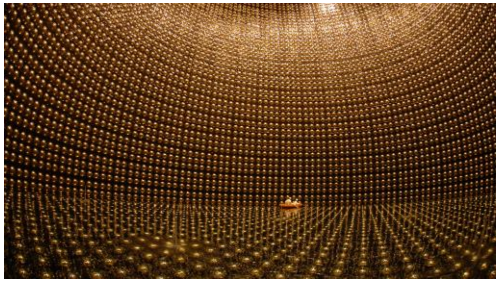
- In addition to COHERENT and Dresden-II there is a number of other CEνNS experiments:
  νGEN, CONUS, CONNIE, NU-CLEUS, MINER, RED-100, CEVENS, Ricochet, TEXONO, ,…

- In order to investigate neutrino electromagnetic properties in CEνNS experiments we need a theoretical apparatus, which takes into account ALL form factors of the neutrino and nucleus

- A proton is the simplest nuclear target. Moreover, elastic neutrino-proton scattering is a promising tool for detecting supernova neutrinos (JUNO yellow book arXiv:1507.05613) Therefore in current investigation we focus on neutrino-nucleon scattering

Astrophysical neutrino's state in the detector on Earth





- source
- oscillation
- detector

Due to interaction of the neutrino magnetic moment with a magnetic field in the astrophysical source and/or with interstellar/intergalactic one the spin-flavor neutrino oscillations arise

Therefore in the most general case the neutrino state in the detector before scattering on a nucleon is described by the spin-flavor density matrix (written in the mass basis)

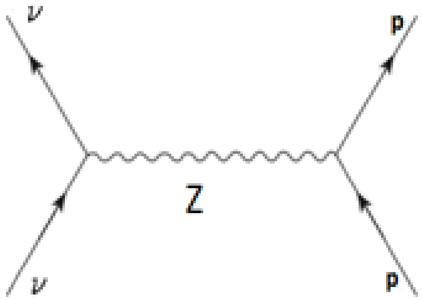
$$\rho_{ij} = \frac{1}{2}\not{k}\left(\tilde{\rho}_{ij} - \zeta_{ij}^{\parallel}\gamma_5 + (\boldsymbol{\zeta}_{ij}^{\perp}\cdot\boldsymbol{\gamma}_{\perp})\gamma_5\right)$$

$\tilde{\rho}_{ij}$ is a reduced density matrix in the neutrino mass space

ζ_{ij} form the matrix of spin polarizations of the neutrino in its rest frame

Neutrino-nucleon scattering

The matrix element of the process with account for all the neutrino and nucleon form factors:



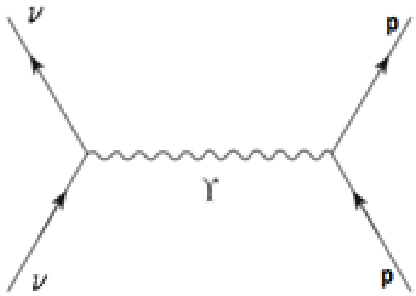
$$= -\frac{G_F}{\sqrt{2}} \bar{u}^{(\nu_n)}_{k',r'} \gamma^\mu (1-\gamma^5) \delta^{ni} u^{(\nu_i)}_{k,r} \, \bar{u}^{(N)}_{p',s'} \Lambda^{(\mathrm{NC};N)}_\mu (-q) u^{(N)}_{p,s}$$

Nucleon neutral weak vertex

+

Neutrino electromagnetic vertex

Nucleon electromagnetic vertex



$$= \frac{4\pi\alpha}{q^2} \bar{u}^{(\nu_n)}_{k',r'} \Lambda^{(\mathrm{EM};\nu)ni}_\mu (q) u^{(\nu_i)}_{k,r} \, \bar{u}^{(N)}_{p',s'} \Lambda^{(\mathrm{NC};N)}_\mu (-q) u^{(N)}_{p,s}$$

Neutrino vertex and form factors

Neutrino electromagnetic vertex:

$$\Lambda_\mu^{(\mathrm{EM};\nu)fi}(q) = (\gamma_\mu - q_\mu \not{q}/q^2)[f_Q^{fi}(q^2) + f_A^{fi}(q^2)q^2\gamma_5] - i\sigma_{\mu\nu}q^\nu[f_M^{fi}(q^2) + i f_E^{fi}(q^2)\gamma_5]$$

$$f_Q^{jk}(0) = e_{jk}, \quad f_M^{jk}(0) = \mu_{jk}, \quad f_E^{jk}(0) = \epsilon_{jk}, \quad f_A^{jk}(0) = a_{jk} \quad \langle r_\nu^2 \rangle = 6 \frac{d f_Q(q^2)}{d q^2}\bigg|_{q^2=0}$$

millicharge

electric dipole moment

charge radius

anapole moment

Nucleon vertexes and form factors

Nucleon electromagnetic vertex:

$$\Lambda_\mu^{(\mathrm{EM};N)}(q) = \gamma_\mu F_Q(q^2) - \frac{i}{2m_N}\,\sigma_{\mu\nu}q^\nu F_M(q^2) + \frac{1}{2m_N}\,\sigma_{\mu\nu}q^\nu\gamma_5 F_E(q^2) - \left(q^2\gamma_\mu - q_\mu \not{q}\right)\gamma_5\frac{F_A(q^2)}{(2m_N)^2}$$

Charge — Magnetic — Electric — Anapole

Nucleon neutral weak vertex:

$$\Lambda_\mu^{(\mathrm{NC};N)}(q) = \gamma_\mu F_1(q^2) - \frac{i}{2m_N}\,\sigma_{\mu\nu}q^\nu F_2(q^2) - \gamma_\mu\gamma_5 G_A(q^2) + \frac{1}{2m_N}\,G_P(q^2)q^\mu\gamma_5$$

Dirac — Pauli — Axial — Pseudoscalar

We omit terms containing the pseudoscalar form factor in the cross section due to a small neutrino mass

The cross section

The full cross section:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^L}{d\Omega} + \frac{d\sigma^R}{d\Omega} + \frac{d\sigma^\perp}{d\Omega} \qquad \frac{d\sigma^K}{d\Omega} = \frac{d\sigma^K_{\rm hp}}{d\Omega} + \frac{d\sigma^K_{\rm hf}}{d\Omega}, \quad K = \{L, R\}$$

$$\frac{d\sigma^K_{\rm hp}}{d\Omega} = \frac{G_F^2 t^2 (s - m_N^2)}{16\pi^2 m_N^2 (s + m_N^2)} \left(1 - \frac{4m_N^2}{t}\right)^{3/2} \left[2\left(1 + \frac{st}{(s-m_N^2)^2}\right)\left(C_V^K + C_A^K - \frac{t}{4m_N^2}(C_M^K + C_E^K)\right) - \right.$$
$$- \frac{4m_N^2 t}{(s-m_N^2)^2}\left(C_A^K - \frac{t}{4m_N^2} C_M^K\right) + \frac{t^2}{(s-m_N^2)^2}\left(C_V^K + C_A^K - 2{\rm Re}\, C^K_{V\&M}\right) \pm$$
$$\left. \pm \frac{2t}{s - m_N^2}\left(2 + \frac{t}{s-m_N^2}\right) {\rm Re}\,(C^K_{V\&A} - C^K_{A\&M})\right],$$

The effects of form factors and oscillations are contained in the following coefficients:

$$C_V^K = Tr\left[\left(-F_1^N \delta_L^K + F_Q^N Q^K\right)^2 \rho^K\right], \quad C_A^K = Tr\left[\left(\delta_L^K G_A^N - \frac{t F_A^N Q^K}{m_N^2}\right)^2 \rho^K\right],$$

$$C^K_{V\&A} = Tr\left[\left(\delta_L^K G_A^N - \frac{t F_A^N Q^K}{m_N^2}\right)\left(-F_1^N \delta_L^K + F_Q^N Q^K\right)\rho^K\right], \quad C_M^K = Tr\left[\left(\delta_L^K F_2^N - F_M^N Q^K\right)^2 \rho^K\right],$$

$$C^K_{V\&M} = Tr\left[\left(\delta_L^K F_2^N - F_M^N Q^K\right)\left(-F_1^N \delta_L^K + F_Q^N Q^K\right)\rho^K\right], \qquad C_E^K = Tr\left[\left(F_E Q^K\right)^2 \rho^K\right],$$

$$C^K_{A\&M} = Tr\left[\left(\delta_L^K F_2^N - F_M^N Q^K\right)\left(\delta_L^K G_A^N - \frac{t F_A^N Q^K}{m_N^2}\right)\rho^K\right],$$

$$\rho^{L,R} = \frac{1}{2}(\tilde\rho \mp \zeta^{\|}), \quad Q^{L,R} = \frac{2\sqrt{2}\pi\alpha}{G_F t}\left(f^Q \mp t f^A\right)$$

Neutrino-helicity-flipping cross section

$$\frac{d\sigma_{\rm hf}^K}{d\Omega} = \frac{\alpha^2 t^2 (s-m_N^2)}{8m_e^2 m_N^2 (s+m_N^2)} \left(1-\frac{4m_N^2}{t}\right)^{3/2} |\mu_\nu^K|^2 \Bigg[-\frac{2m_N}{t}\left(1+\frac{t}{s-m_N^2}\right)(F_Q^N)^2 - $$
$$ -\frac{2t}{m_N^3}\left(1+\frac{st}{(s-m_N^2)^2}\right)(F_A^N)^2 + \frac{m_N t}{(s-m_N^2)^2} F_Q^N F_M^N + \frac{1}{8m_N}\left(4+\frac{4st+t^2}{(s-m_N^2)^2}\right)(F_M^N)^2 + $$
$$ + \frac{1}{8m_N}\left(2+\frac{t}{s-m_N^2}\right)^2 (F_E^N)^2 \Bigg],$$

$$|\mu_\nu^{L,R}|^2 = Tr\left[\left(f^M \pm i f^E\right)\left(f^M \mp i f^E\right)\rho^{L,R}\right]$$

are effective left- and right-handed neutrino magnetic moments

The transverse neutrino polarization part of the cross section

$$\frac{d\sigma^{\perp}}{d\Omega} = \frac{\sqrt{2}G_F\alpha(s-m_N^2)\left(4m_N^2-t\right)^{3/2}}{8\pi m_e m_N^2(s+m_N^2)}\sqrt{1+\frac{st}{(s-m_N^2)^2}}\left\{\frac{2t}{s-m_N^2}\frac{tF_A^N}{m_N^2}(F_Q^N+F_M^N)C_+ +\right.$$

$$+\mu_\nu^{\perp}\left[\left(2+\frac{t}{s-m_N^2}\right)\left(F_1^N F_Q^N+\frac{tF_A^N}{m_N^2}G_A^N - t\frac{F_2^N F_M^N}{4m_N^2}\right)-\frac{t}{s-m_N^2}\left(\frac{tF_A^N}{m_N^2}(F_1^N+F_2^N)+G_A^N(F_Q^N+F_M^N)\right)\right]-$$

$$\left.-\left(2+\frac{t}{s-m_N^2}\right)\left((F_Q^N)^2+\left(\frac{tF_A^N}{m_N^2}\right)^2-t\left(\frac{F_M^N}{2m_N}\right)^2-t\left(\frac{F_E^N}{2m_N}\right)^2\right)C_-\right\},$$

$$\mu_\nu^{\perp} = \mathrm{Re}\,Tr\left[\left(f^M+if^E\right)\rho^{\perp}\right]$$ is effective transverse neutrino magnetic moment

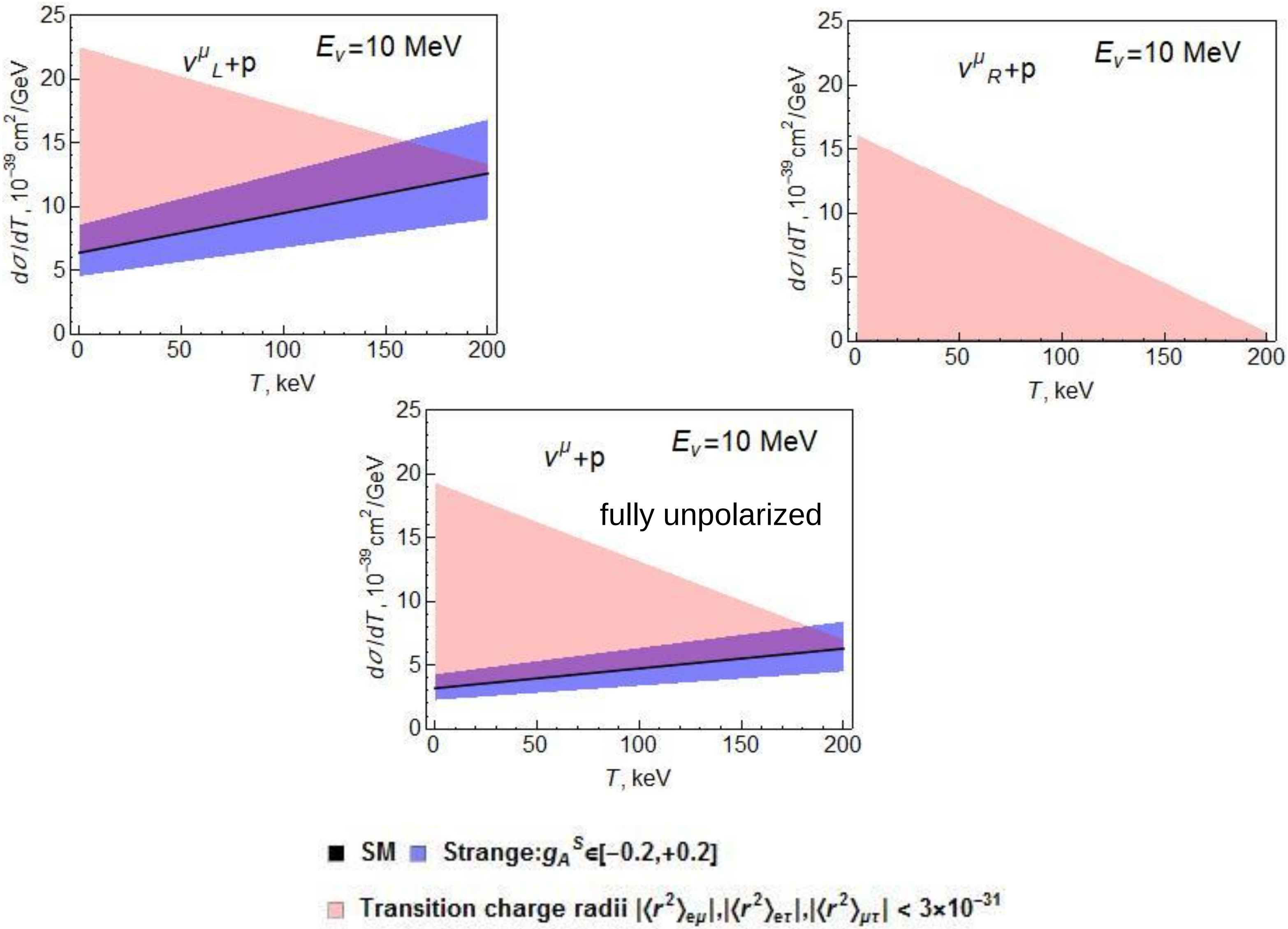
$$C_{\pm} = \mathrm{Re}\,Tr\left[\left[Q^L\left(f^M+if^E\right)\pm\left(f^M+if^E\right)Q^R\right]\rho^{\perp}\right] \qquad Q^{L,R} = \frac{2\sqrt{2}\pi\alpha}{G_F t}\left(f^Q \mp t f^A\right)$$

$$\rho_{ij}^{\perp} = \frac{1}{2}|\zeta_{ij}^{\perp}|e^{-i(\chi_{ij}-\varphi)}$$

length of transverse neutrino polarization vector

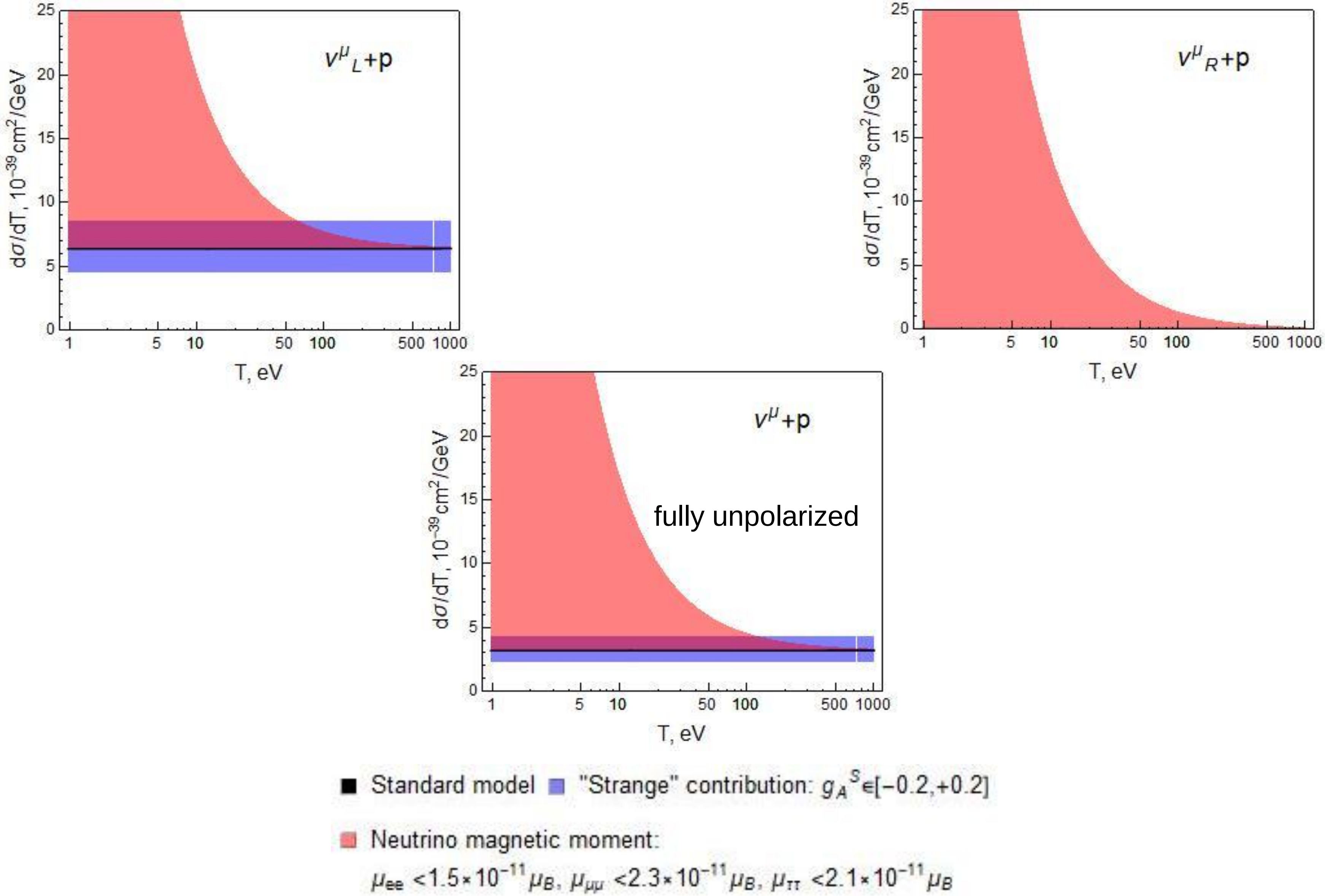
angle between transverse neutrino polarization and transverse recoil momentum

Numerical results for the ν_μ scattering on a nucleon: The effect of the transition charge radii $\langle r_\nu^2\rangle_{e\mu}, \langle r_\nu^2\rangle_{e\tau}, \langle r_\nu^2\rangle_{\mu\tau}$,



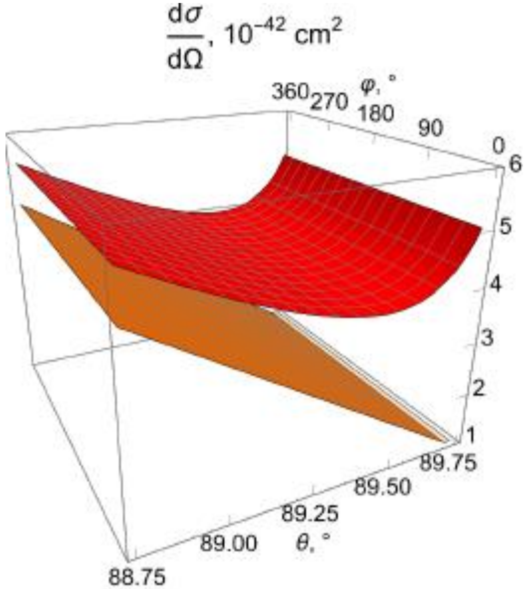
The neutrino energy is typical for supernova neutrinos

Numerical results for the ν_μ scattering on a nucleon: The effect of the diagonal magnetic moment

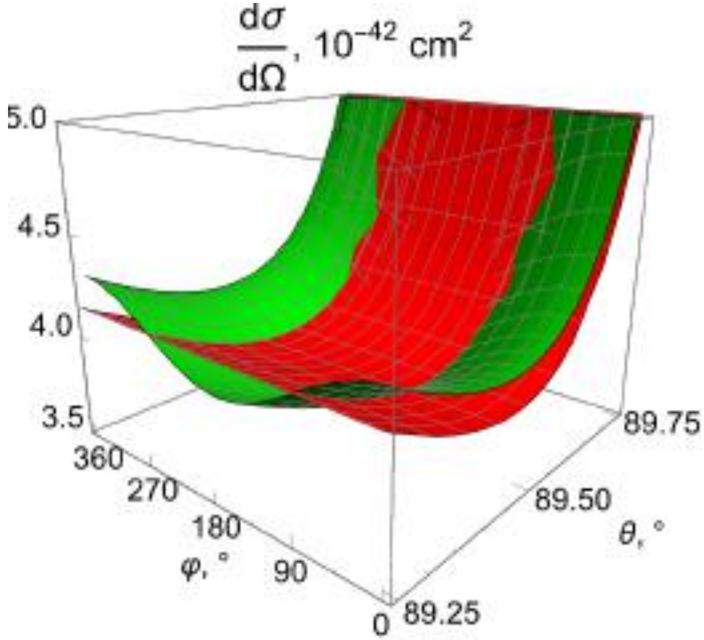


Numerical results for the ν_μ scattering on a nucleon:
The effect of the diagonal magnetic moment and transverse neutrino polarization

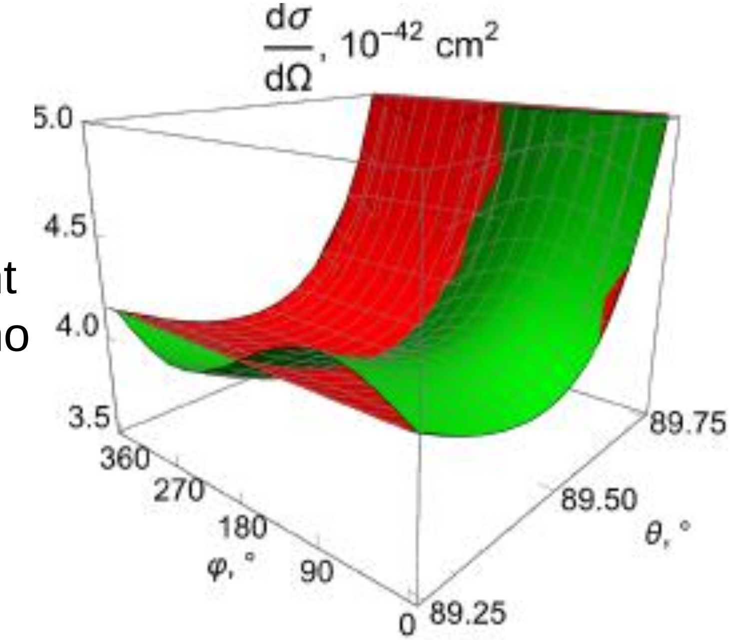
Orange surface is fully unpolarized neutrino in the SM



Red surfaces are fully unpolarized neutrino with magnetic moment



Green surfaces are transversely polarized (2 different orientations) neutrino with magnetic moment



Summary

- Theoretical study of the processes of elastic neutrino-nucleon scattering has been carried out taking into account the electromagnetic form factors of neutrinos and the form factors of a nucleon, as well as the effects of neutrino spin polarizations.

- General expressions have been obtained for the cross sections of elastic neutrino-nucleon scattering. Based on the obtained expressions, numerical calculations have been carried out for elastic neutrino-proton scattering taking into account the charge radii and magnetic moments of neutrinos, taking into account possible effects of supernova neutrino spin polarization. It is shown that the contribution from right-handed neutrinos can be at the same level or even greater than that from left-handed ones. Also there is and of the neutrino transverse spin polarization on the cross section differential with respect to the solid angle of the recoil proton.

Thank you for your attention!

Parametrization of nucleon form factors

For this purpose we use the Sachs form factors →

$$F_Q^N(q^2) = \frac{G_E^N(q^2) - \frac{q^2}{4m_N^2} G_M^N(q^2)}{1 - \frac{q^2}{4m_N^2}},$$

$$F_M^N(q^2) = \frac{G_M^N(q^2) - G_E^N(q^2)}{1 - \frac{q^2}{4m_N^2}},$$

Parametrization of nucleon form factors (see Papoulias D. K., Kosmas T. S. Advances in High Energy Physics 2016 (2016) and references therein)

$$\frac{G_M^N}{\mu_N} = \frac{1 - \frac{q^2}{4m_N^2} a_M^N}{1 - \frac{q^2}{4m_N^2} b_{M1}^N + (\frac{q^2}{4m_N^2})^2 b_{M2}^N - (\frac{q^2}{4m_N^2})^3 b_{M3}^N},$$

$$G_E^p = \frac{1 - \frac{q^2}{4m_N^2} a_E^p}{1 - \frac{q^2}{4m_N^2} b_{E1}^p + (\frac{q^2}{4m_N^2})^2 b_{E2}^p - (\frac{q^2}{4m_N^2})^3 b_{E3}^p},$$

$$G_E^n = \frac{-\frac{q^2}{4m_N^2}\lambda_1}{1 - \frac{q^2}{4m_N^2}\lambda_2}(1 - \frac{q^2}{M_V^2})^{-2},$$

$$G_A^a = g_A(1 - \frac{q^2}{M_A^2})^{-2}$$

$$m_N = 938 MeV, \quad \mu_p = 2.793, \quad \mu_n = -1.913,$$
$$M_V = 843 MeV, \quad g_A = 1.267, \quad M_A = 1049 MeV,$$
$$a_E^p = -0.19, \quad b_{E1}^p = 11.12, \quad b_{E2}^p = 15.16, \quad b_{E3}^p = 21.25,$$
$$a_M^p = 1.09, \quad b_{M1}^p = 12.31, \quad b_{M2}^p = 25.57, \quad b_{M3}^p = 30.61,$$
$$\lambda_1 = 1.68, \quad \lambda_2 = 3.63,$$
$$a_M^n = 8.28, \quad b_{M1}^n = 21.3, \quad b_{M2}^n = 77, \quad b_{M3}^n = 238.$$

Parametrization of the strange form factor (see Butkevich A. V. Phys. Rev. D. 2023. 107, N 7. 073001)

$$F_1^S(q^2) = \frac{\frac{q^2}{6}\langle r_S^2 \rangle}{(1 - \frac{q^2}{4m_N^2})}(1 - \frac{q^2}{M_V^2})^{-2}$$

$$F_2^S(q^2) = \frac{\mu_S}{(1 - \frac{q^2}{4m_N^2})}(1 - \frac{q^2}{M_V^2})^{-2}$$

$$F_A^S(q^2) = g_A^S(1 - \frac{q^2}{M_A^2})^{-2}$$

$g_A^S = \in [-0{,}2, 0{,}2]$, other «strange» parameters equal zero

Connection between neutral weak and electromagnetic form factors of nucleon

According to the hypothesis of vector current conservation, vector neutral weak form factors can be expressed via electromagnetic ones

$$F^{p}_{1,2}(q^2) = (\frac{1}{2} - 2\sin^2\theta_W)F^{p}_{Q,M} - 1\frac{1}{2}F^{n}_{Q,M} - \frac{1}{2}F^{S}_{1,2}$$
$$F^{n}_{1,2}(q^2) = (\frac{1}{2} - 2\sin^2\theta_W)F^{n}_{Q,M} - 1\frac{1}{2}F^{p}_{Q,M} - \frac{1}{2}F^{S}_{1,2}$$

$F^S_{1,2}$ – strange form factors of nucleon

Here we restrict ourselves only with charge and magnetic form factors in the electromagnetic vertex

In the axial case, one can also factorize an axial strange form factor

$$G^{N}_{A,P}(q^2) = \frac{\tau_3}{2}G^{a}_{A,P}(q^2) - \frac{1}{2}G^{S}_{A,P}(q^2)$$