Transition form-factor of $\pi\gamma \to \pi\pi$ in nonlocal quark model.

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Outline

- Chiral anomalies
- Nonlocal quark model
- Transition form-factor
- Results

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CONSEQUENCES OF ANOMALOUS WARD IDENTITIES

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The anomalies of Ward identities are shown to satisfy consistency or integrability relations, which restrict their possible form. For the case of SU(3) × SU(3) we verify that the anomalies given by Bardeen satisfy the consistency relations. A solution of the anomalous Ward identities is also given which describes concisely all anomalous contributions to low energy theorems. The contributions to strong five pseudoscalar interactions, to K_{l4}, to one- and two-photon interactions with three pseudoscalars are explicitly exhibited.

The one photon-three pseudoscalar interaction is given by

$$\frac{e}{i 24\pi^2 F_\pi^3} \epsilon_{\mu\nu\sigma\tau} F_{\mu\nu} \times$$

$$\times \left[\left(\partial_\sigma \Pi^+ \partial_\tau \Pi^- + \partial_\sigma K^+ \partial_\tau K^-\right)\left(\Pi^0 + \frac{1}{\sqrt{3}}\eta\right) + \partial_\sigma K^0 \partial_\tau \bar{K}^0 \left(\Pi^0 - \sqrt{3}\,\eta\right)\right]$$

$$A(\pi^0 \to \gamma\gamma) = F_{\gamma\gamma}(M_{\pi^0}^2)\epsilon^{\mu\nu\alpha\beta}\epsilon^{\mu}k_1^{\nu}\epsilon^{\alpha}k_2^{\beta}, \quad (1)$$

where ϵ_j^i and k_j^i - polarizations and momenta of photons and

$$F_{\gamma\gamma}(0) = \frac{e^2}{4\pi^2 f_\pi}. \quad (2)$$

Where $f_\pi = f_0[1 + O(m_q)] = 92.4\,\mathrm{MeV}$ is pion decay constant.
Other processes $\gamma\pi^\pm \to \pi^\pm\pi^0$ or $e^+e^- \to \gamma^* \to \pi^0\pi^-\pi^+$ which also are connect to WZW anomalous effective action and amplitude of reaction has a form

$$A(\gamma\pi^- \to \pi^-\pi^0) = -iF(s,t,u)_{3\pi}\epsilon^{\mu\nu\alpha\beta}\epsilon^{\mu}p_0^{\nu}p_1^{\alpha}p_2^{\beta}, \quad (3)$$

where ϵ^μ is polarization of incident photon and p_i – momenta of pions. In chiral limit in low-order by quark-loops form-factor of this amplitude is independent from the Mandelstam variables s, t, u and have a simple form

$$F_{3\pi}(0,0,0) = \frac{e}{4\pi^2 f_\pi^3} = 9.72\,\mathrm{GeV}^{-3}. \quad (4)$$

Estimation from experiment (IHEP accelerator (Serpukhov)) estimate of the value $F_{3\pi}$ was given

$$F_{3\pi}^{\mathrm{exp}} = (12.9 \pm 0.9 \pm 0.5)\,\mathrm{GeV}^{-3}, \quad (5)$$

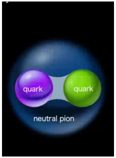
The experiment was based on pion pair production by pions in the nuclear Coulomb field via the Primakoff reaction

$$\pi^- + (Z,A) \to \pi^{-\prime} + (Z,A) + \pi^0. \quad (6)$$

The Lagrangian of the $SU(2) \times SU(2)$ nonlocal chiral quark model has the form [1]

$$\mathcal{L}_{N\chi QM} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J^a_S(x)J^a_S(x) + J^a_P(x)J^a_P(x)]$$

where $q\,(x)$ are the quark fields, m_c is the diagonal matrix of the quark current masses G is the four-quark coupling constant.



The nonlocal structure of the model is introduced via the nonlocal quark currents

$$J^a_{S,P}(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x-x_1)\,\Gamma^a_{S,P}q(x+x_2),$$
$$\Gamma^a_S = \tau^a, \Gamma_P = i\gamma^5\tau^a, \qquad (7)$$

where $f(x)$ is a form factor reflecting the nonlocal properties of the QCD vacuum and τ^a is matrix of Pauli.

[1] Anikin:2000, Scarpettini:2003

Integrating out the quark fields produced functional have a form:

$$Z = \int D\vec{\pi} D\sigma \, \exp[-S_E^{(\sigma,\pi)}], \tag{8}$$

where bosonisated action

$$S_E^{(\sigma,\pi)} = -\ln\det(\boldsymbol{D}) + \frac{1}{2G}\int \frac{d^4p}{(2\pi)^4}(\sigma^2 + \vec{\pi}^2). \tag{9}$$

The operator $\boldsymbol{D}$ in momentum space can be written as

$$\boldsymbol{D} = (-\hat{p} - m_c)(2\pi)^4\delta(p - p') + f(p^2)f(p'^2)(\sigma + \tau^a\pi^a),$$

where $f(p)$ is Fourier transform from form factor $f(x)$.

Bosonized effective action is

$$S_E^{(\sigma,\pi)} = S_E^{MF} + S_E^{quad} + \dots \tag{10}$$

where

$$\frac{S^M F_E}{V^{(4)}} = -4N_c \int \frac{d^4p}{(2\pi)^4} \ln[p^2 + m^2(p^2)] + \frac{m_d^2}{2G}, \tag{11}$$
$$S_E^{quad} = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} G^-(p^2)\vec{\pi}(p) \cdot \vec{\pi}(-p).$$

$$m(p^2) = m_c + m_d f^2(p^2), \tag{12}$$
$$G^-(p^2) = \frac{1}{G} - 8N_c\Pi_a(p^2), \tag{13}$$

where the mass of quark received a dependence on momentum and $\Pi_a(p^2)$ is polarization operaton

$$\Pi_a(p^2) = \int \frac{d_E^4k}{(2\pi)^4} \frac{f_{k_+}^2 f_{k_-}^2 \left[(k_+ \cdot k_-) + m(k_+^2)m(k_-^2)\right]}{\left[k_+^2 + m^2(k_+^2)\right]\left[k_-^2 + m^2(k_-^2)\right]}, \tag{14}$$

with $k_\pm = k \pm p/2$ and $f_{k_i} = f(k_i^2)$. The integration here and later is gone on Euclid space d_E^4k.

The nonlocal vertex of interaction quark-antiquark with external field can be written:

$$\Gamma_\mu(q) = \gamma_\mu - (p_2 + p_1)_\mu m(p_1, p_2), \tag{15}$$

where p_1 and $p_2 = p_1 + q$ are momentums of quark, q - momentum of external field [2]. For interaction quark-antiquark with scalar or pseudoscalar mesons:

$$\Gamma^a_\sigma = g_\sigma(q^2)\tau^a f(p_1^2) f(p_2^2), \tag{16}$$
$$\Gamma^a_\pi = g_\pi(q^2)\gamma_5\tau^a f(p_1^2) f(p_2^2). \tag{17}$$

where p_1 and $p_2 = p_1 + q$ are momentums of quarks, q - momentum of meson, $g_\sigma(q^2)$ and $g_\pi(q^2)$ are constants which described a renormalization of scalar or pseudoscalar meson fields accordingly. The constants $g_{\sigma,\pi}(q^2)$ can be found from expression on propagator of meson:

$$\frac{1}{-G + \Pi_{\sigma,\pi}(p^2)} = \frac{g^2_{\sigma,\pi}(p^2)}{p^2 - m^2_{\sigma,\pi}}, \tag{18}$$

and in case of mass-shell of pion

$$\frac{1}{g^2_{\sigma,\pi}(m^2_{\sigma,\pi})} = \left.\frac{\partial\Pi_{\sigma,\pi}(p^2)}{\partial p^2}\right|_{p^2=m^2_{\sigma,\pi}} \tag{19}$$

where $\Pi_{\sigma,\pi}(p^2)$ is polarization operator

[2] Anikin:2000rq,Dorokhov:2015psa, Dorokhov:2011zf

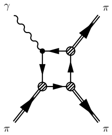


Figure: Feynman diagram which described transition form-factor $\gamma^* \pi^- \to \pi^0 \pi^-$. All vertexes are nonlocal.

The amplitude of transition gamma in three pions can be written as

$$A(\gamma \to \pi^+ \pi^0 \pi^-) = -i F_{3\pi}(s,t,u) \epsilon^{\mu\nu\alpha\beta} \epsilon^{\mu} p_0^{\nu} p_1^{\alpha} p_2^{\beta}, \quad (20)$$

where p_i are momenta of pions, ϵ^{μ} is polarization of photon and $F_{3\pi}(s,t,u)$ is a Lorentz scalar function of the Mandelstam variables which is defined from three types of diagrams in different kinematics:

$$F_{3\pi}(s,t,u) = F_1(s,t,u) + F_2(t,s,u) + F_3(u,t,s). \quad (21)$$

where s, t, u - are Mandelstam invariance variables.

$$F_1(s,t,u) = 4eN_c \int \frac{d_E^4 k}{(2\pi)^4} \frac{g_\pi(p_0^2) g_\pi(p_1^2) g_\pi(p_2^2) f_k f_{k+p_1}^2 f_{k-p_0}^2 f_{k-p_0-p_2}}{D(k)D(k+p_1)D(k-p_0)D(k-p_0-p_2)} \times \\ \times \mathrm{Tr}_f [Q(\pi^-\pi^0\pi^+ + \pi^+\pi^0\pi^-)]\{m(k^2)[A+1-B] - m((k-p_0)^2)[C+A] \\ + m((k+p_1)^2)C + m((k-p_0-p_2)^2)B\} \quad (22)$$

where $D(k) = k^2 + m^2(k^2)$, Q is a charge matrix of quark, and $\pi^i = \pi^a \tau^a / \sqrt{2}$ where π^a is matrix of pion fields.

$$F_2(t,s,u) = F_1(s,t,u)(p_0 \leftrightarrows -p_1, \pi^0 \leftrightarrow \pi^+) \quad (23)$$

$$F_3(u,t,s) = F_1(s,t,u)(p_0 \leftrightarrow -p_2, \pi^0 \leftrightarrow \pi^-). \quad (24)$$

In low energy limit when kinematic invariants $s = t = u = 0$ the transition form factor have a form

$$F_{3\pi}(0,0,0) = eN_cN_f g_\pi^3 \int \frac{d_E^4 k}{(2\pi)^4} f^6(k) \left\{ 4 \left[\frac{(m(k^2) - m'(k^2)k^2)}{D(k)^4} \right] \right.$$
$$\left. -32\, m_c \left[\frac{(m^2(k^2) - m(k^2)m'(k^2)k^2)}{D(k)^5} - \frac{1}{8}\frac{1}{D(k)^4} \right] \right\}, \quad (25)$$

here $m(k^2) = m_d f^2(k^2)$, $g_\pi = g_\pi(0)$ and second term here gives a dependence from current mass of quark. In chiral limit when current mass of quark m_c is zero this form factor takes a form

$$F_{3\pi}(0,0,0) = \frac{eN_cN_f}{f_\pi^3} \int \frac{d_E^4 k}{(2\pi)^4} \left[\frac{4m^4(k^2) - 4m'(k^2)m^3(k^2)k^2}{D(k)^4} \right], \quad (26)$$

where $f_\pi = g_\pi/m_d$ and $m'(k^2) = \dfrac{\partial m(k^2)}{\partial k^2}$.

In local limit of model when parameter of nonlocality $\Lambda \to \infty$, $f(k^2) \to 1$ and $m'(k) = 0$, $m(k^2) = m_d$. And integral can be solved analytically:

$$\int_0^\infty dk^2 \frac{k^2 m^4}{(k^2 + m^2)^4} = \frac{1}{6} \quad (27)$$

Transition form factor in local limit reproduces the WZW form-factor :

$$F_{3\pi} = \frac{eN_cN_f}{24\pi^2 f_\pi^3} = \frac{e}{4\pi^2 f_\pi^3} \simeq 9.72\,(0.09)\,\mathrm{GeV}^{-3}. \quad (28)$$

For physical masses of pions, transition form factor should be calculated on the physical threshold for $q^2 = 0$ and $s + t + u = 3m_\pi^2$. In this case, kinematics variables take the form of $s^{thr} = (m_{\pi^-} + m_{\pi^0})^2$, $t^{thr} = -m_{\pi^-} m_{\pi^0}^2/(m_{\pi^-} + m_{\pi^0})$ and $u^{thr} = m_{\pi^-}(m_{\pi^-}^2 - m_{\pi^-} m_{\pi^0} - m_{\pi^0}^2)/(m_{\pi^-} + m_{\pi^0})$. In this case, in low order of perturbation by m_π^2 transition form factor $F_{3\pi}^{thr}$ will be have similar form as in chiral limit:

$$F_{3\pi}^{thr}(s^{thr}, t^{thr}, u^{thr}) = eN_c N_f g_\pi^3(m_\pi^2) \int \frac{d_E^4 k}{(2\pi)^4} f^6(k^2) \times \left[\frac{4m(k^2) - 4m'(k^2)k^2}{D(k)^4}\right] + \mathcal{O}(m_\pi^2), \tag{29}$$

The correction of pion mass is suppressed. Dependence of current quark changes a quantity of transition form factor:

$$F_{3\pi}^{thr}(s^{thr}, t^{thr}, u^{thr}) = 10.3\,(0.52)\,\mathrm{GeV}^{-3}. \tag{30}$$

Results

group/ approach	data GeV^{-3}
exp. $O(p^4)(e=0)$	12.9 ± 1.4
exp. $O(p^6)(e=0)$	11.9 ± 1.3
exp. $O(p^8)(e=0)$	11.4 ± 1.3
NPCR	11.4 ± 1.3
Holstein	11.9 ± 1.4
Ametller ($e \neq 0$)	10.7 ± 1.2
This calc.[3]	10.3 ± 0.52
chiral anomaly (WZW)	9.72 ± 0.3

Model/theory	Cross-section [nb]	$\mathcal{F}_{3\pi}^{\text{thr}}$ [GeV^{-3}]	$\mathcal{F}_{3\pi}^{(0)\text{extr}}$ [GeV^{-3}]
1) $\mathcal{F}_{3\pi} = \frac{e}{4\pi^2 F_\pi^3} = 9.72\,\text{GeV}^{-3}$	1.92	9.7	10.2 ± 1.1
2) Terent'ev, eq. (35) with $\Delta_\rho = 0.5$ and $\Delta_\omega = 0$	2.80	10.3	8.4 ± 0.9
3) Terent'ev, eq. (35) with $\Delta_\rho = 0.5$ and $\Delta_\omega = 1.5$	2.62	10.3	8.7 ± 1.0
4) Terent'ev, eq. (35) with $\Delta_\rho = 0.35$ and $\Delta_\omega = 0$	2.51	10.1	8.9 ± 1.0
5) Terent'ev, eq. (35) with $\Delta_\rho = 0.35$ and $\Delta_\omega = 3.2$	2.18	10.1	9.6 ± 1.1
6) Rudaz, eq. (36)	2.36	10.0	9.2 ± 1.0
7) ChPT at $\mathcal{O}(p^6)$ (eq. (29)) without q^2-dependence	2.33	10.4	9.2 ± 1.0
8) ChPT at $\mathcal{O}(p^6)$ (eq. (29)) with q^2-dependence	2.05	10.4	9.9 ± 1.1
9) ChPT at $\mathcal{O}(p^6)$ (eq. (29)) with q^2-dependence plus electromagnetic correction of eq. (34)	2.17	12.1	9.6 ± 1.1
10) ChPT at $\mathcal{O}(p^6)$ with modified dependence of eq. (33)	2.83	10.5	8.4 ± 0.9
11) Holstein, eq. (37)	3.05	10.4	8.1 ± 0.9

Figure: from Giller Eur. Phys. J. A 25, 229-240 (2005)

[3] this work is supported by RSF grant

Thank you for attention!

Figure: TSU