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Semi-Inclusive Deep Inelastic Scattering in Wandzura-Wilczek-type Approximation[1]

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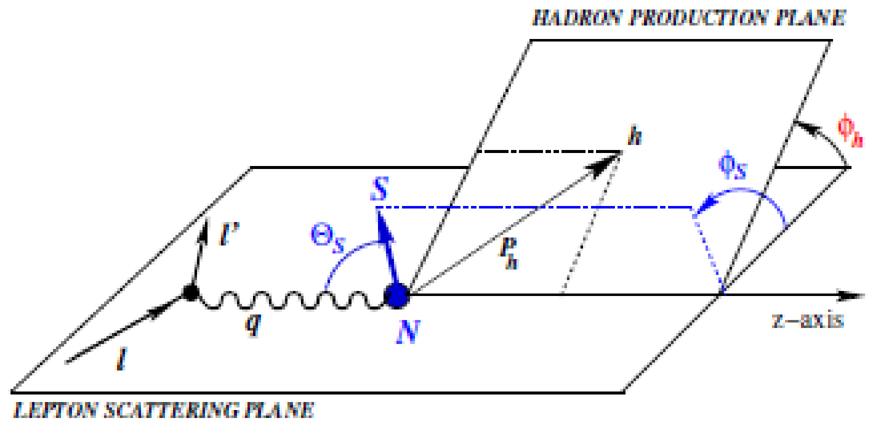
[1]The full text (70 pages) published in arXiv:1807.10606 [hep-ph] 70 pages, 200 references.

The SIDIS process

Nucleon / Quark	U	L	T
U	$f_1^q(x,k_T^2)$ Number density		$f_{1T}^{q\perp}(x,k_T^2)$ Sivers
L		$g_1^q(x,k_T^2)$ Helicity	$g_{1T}^{q\perp}(x,k_T^2)$ Worm-gear T
T	$h_1^{q\perp}(x,k_T^2)$ Boer-Mulders	$h_{1L}^{q\perp}(x,k_T^2)$ Kotzinian- Mulders Worm-gear L	$h_1^q(x,k_T^2)$ Transversity $h_{1T}^{q\perp}(x,k_T^2)$ Pretzelosity

+ two FFs: $D_{1q}^h(z,P_\perp^2)$ and $H_{1q}^h(z,P_\perp^2)$



$$x = \frac{Q^2}{2\,P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad Q^2 = -q^2.$$

At the leading order

$$\frac{d^6\sigma_{\text{leading}}}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2} = \frac{1}{4\pi}\frac{d\hat\sigma}{dy}F_{UU}(x,z,P_{hT}^2)\Big\{1+\cos(2\phi_h)\,p_1\,A_{UU}^{\cos(2\phi_h)}$$
$$+S_L\sin(2\phi_h)\,p_1\,A_{UL}^{\sin(2\phi_h)} + \lambda\,S_L\,p_2\,A_{LL}$$
$$+S_T\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)} + S_T\sin(\phi_h+\phi_S)\,p_1\,A_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+S_T\sin(3\phi_h-\phi_S)\,p_1\,A_{UT}^{\sin(3\phi_h-\phi_S)} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2\,A_{LT}^{\cos(\phi_h-\phi_S)}\Big\}. \quad (2.3a)$$

At the subleading order

$$\frac{d^6\sigma_{\text{subleading}}}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2} = \frac{1}{4\pi}\,\frac{d\hat{\sigma}}{dy}\,F_{UU}(x,z,P_{hT}^2)\Bigg\{\cos(\phi_h)\,p_3\,A_{UU}^{\cos(\phi_h)}$$

$$+\lambda\sin(\phi_h)\,p_4\,A_{LU}^{\sin(\phi_h)} + S_L\sin(\phi_h)\,p_3\,A_{UL}^{\sin(\phi_h)} + \lambda\,S_L\cos(\phi_h)\,p_4\,A_{LL}^{\cos(\phi_h)}$$

$$+S_T\sin(2\phi_h-\phi_S)\,p_3\,A_{UT}^{\sin(2\phi_h-\phi_S)} + S_T\sin(\phi_S)\,p_3\,A_{UT}^{\sin(\phi_S)}$$

$$+\lambda\,S_T\cos(\phi_S)\,p_4\,A_{LT}^{\cos(\phi_S)} + \lambda\,S_T\cos(2\phi_h-\phi_S)\,p_4\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg\} \qquad (2.3\text{b})$$

$$A_{XY}^{\text{weight}} \equiv A_{XY}^{\text{weight}}(x,z,P_{hT}) = \frac{F_{XY}^{\text{weight}}(x,z,P_{hT})}{F_{UU}(x,z,P_{hT})}$$

Depolarizaton factors (neglecting M/Q correction)

$$p_1 = \frac{1-y}{1-y+\frac{1}{2}y^2}, \quad p_2 = \frac{y(1-\frac{1}{2}y)}{1-y+\frac{1}{2}y^2}, \quad p_3 = \frac{(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2}, \quad p_4 = \frac{y\sqrt{1-y}}{1-y+\frac{1}{2}y^2};$$

Bacchetta et al., JHEP 02 (2007) 093

Wandzura-Wilczek-type Approximation

$$g_T^a(x) = \int_x^1 \frac{dy}{y}\, g_1^a(y) + \tilde{g}_T^a(x) \overset{\text{WW}}{\approx} \int_x^1 \frac{dy}{y}\, g_1^a(y)\,, \tag{3.2a}$$

$$h_L^a(x) = 2x \int_x^1 \frac{dy}{y^2}\, h_1^a(y) + \tilde{h}_L^a(x) \overset{\text{WW}}{\approx} 2x \int_x^1 \frac{dy}{y^2}\, h_1^a(y)\,, \tag{3.2b}$$

$$x\, \tilde{e}^a(x) \overset{\text{WW}}{\approx} 0\,, \tag{3.2c}$$

The relations (3.2a–3.2c) have been derived basically using operator product expansion Techniques.

Wandzura, Wilczek PL B72 (1977) 195; Jaffe, Ji, NPB 375 (1992) 527; Kotzinian, Nucl. Phys. B441 (1995) 234–248; Kotzinian, Mulders, PRD54 1229 (1996); Mulders, Tangerman, NPB 461 (1996);

One uses QCD equations of motion to separate contributions from $\bar{q}q$ –terms and $\bar{q}gq$–terms (denoted with a tilde) and assumes that the latter can be neglected with respect to the leading $\bar{q}q$–terms with a useful accuracy.

$$\left| \frac{\langle \bar{q} g q \rangle}{\langle \bar{q} q \rangle} \right| \ll 1\,.$$

In the T-even case one obtains the following approximations

$$xe^q(x,k_\perp^2) \overset{\text{WW-type}}{\approx} 0, \tag{3.3a}$$

$$xf^{\perp q}(x,k_\perp^2) \overset{\text{WW-type}}{\approx} f_1^q(x,k_\perp^2), \tag{3.3b}$$

$$xg_L^{\perp q}(x,k_\perp^2) \overset{\text{WW-type}}{\approx} g_1^q(x,k_\perp^2), \tag{3.3c}$$

$$xg_T^{\perp q}(x,k_\perp^2) \overset{\text{WW-type}}{\approx} g_{1T}^{\perp q}(x,k_\perp^2), \tag{3.3d}$$

$$xg_T^q(x,k_\perp^2) \overset{\text{WW-type}}{\approx} g_{1T}^{\perp(1)q}(x,k_\perp^2), \tag{3.3e}$$

$$xh_L^q(x,k_\perp^2) \overset{\text{WW-type}}{\approx} -2\,h_{1L}^{\perp(1)q}(x,k_\perp^2), \tag{3.3f}$$

$$xh_T^q(x,k_\perp^2) \overset{\text{WW-type}}{\approx} -h_1^q(x,k_\perp^2) - h_{1T}^{\perp(1)}(x,k_\perp^2), \tag{3.3g}$$

$$xh_T^{\perp q}(x,k_\perp^2) \overset{\text{WW-type}}{\approx} h_1^q(x,k_\perp^2) - h_{1T}^{\perp(1)}(x,k_\perp^2). \tag{3.3h}$$

In the T-odd case one obtains the approximations

$$xf_T^{\perp q}(x,k_\perp^2) \overset{\text{WW-type}}{\approx} f_{1T}^{\perp q}(x,k_\perp^2), \tag{3.4f}$$

$$xf_T^q(x,k_\perp^2) \overset{\text{WW-type}}{\approx} -f_{1T}^{\perp(1)q}(x,k_\perp^2), \tag{3.4g}$$

$$xh^q(x,k_\perp^2) \overset{\text{WW-type}}{\approx} -2\,h_1^{\perp(1)}(x,k_\perp^2). \tag{3.4h}$$

Other T-odd TMDs = 0 in WW-approximation

Two very useful WW-type approximations follow from combining the WW approximations (3.2a, 3.2b) with the WW-type approximations (3.3e, 3.3f):

$$g_{1T}^{\perp(1)a}(x) \overset{\text{WW-type}}{\approx} x \int_x^1 \frac{dy}{y}\, g_1^a(y)\,, \qquad (3.6a)$$

$$h_{1L}^{\perp(1)a}(x) \overset{\text{WW-type}}{\approx} -x^2 \int_x^1 \frac{dy}{y^2}\, h_1^a(y)\,. \qquad (3.6b)$$

Mulders, Tangerman, NPB 461 (1996);
Kotzinian, Mulders, PRD54 1229 (1996)

for FFs:

$$D^{\perp}(z, P_\perp^2) \overset{\text{WW-type}}{\approx} z\, D_1(z, P_\perp^2)\,, \qquad (3.7c)$$

$$H(z, P_\perp^2) \overset{\text{WW-type}}{\approx} -\frac{P_\perp^2}{z M_h^2}\, H_1^{\perp}(z, P_\perp^2)\,. \qquad (3.7d)$$

A. Bacchetta et al., JHEP 02 (2007) 093, [hep-ph/0611265].

Predictions from instanton vacuum model

Instantons form dilute medium characterized by a non-trivial small parameter $\rho/R=1/3$, where ρ and R denote average instanton size and separation.

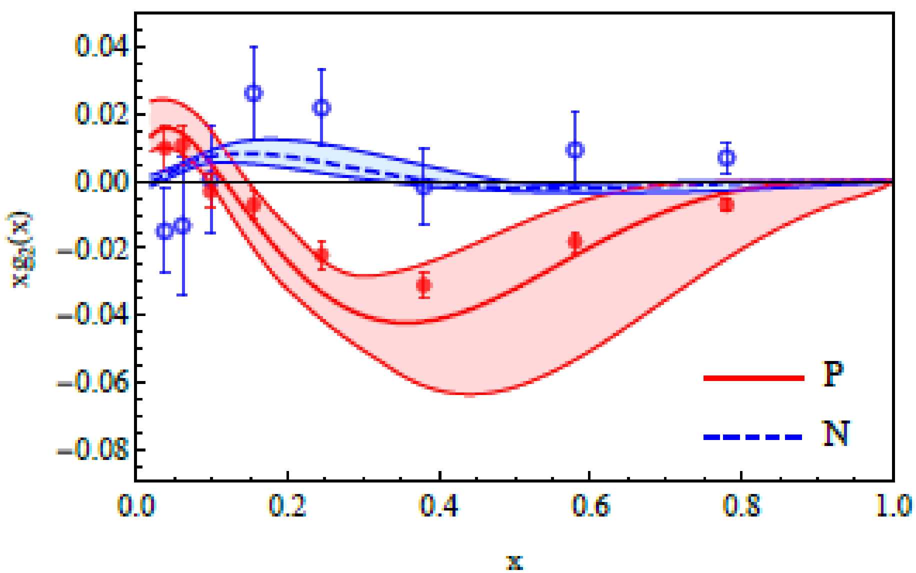
Diakonov, Polyakov, Weiss, NPB 461 (1996) 539

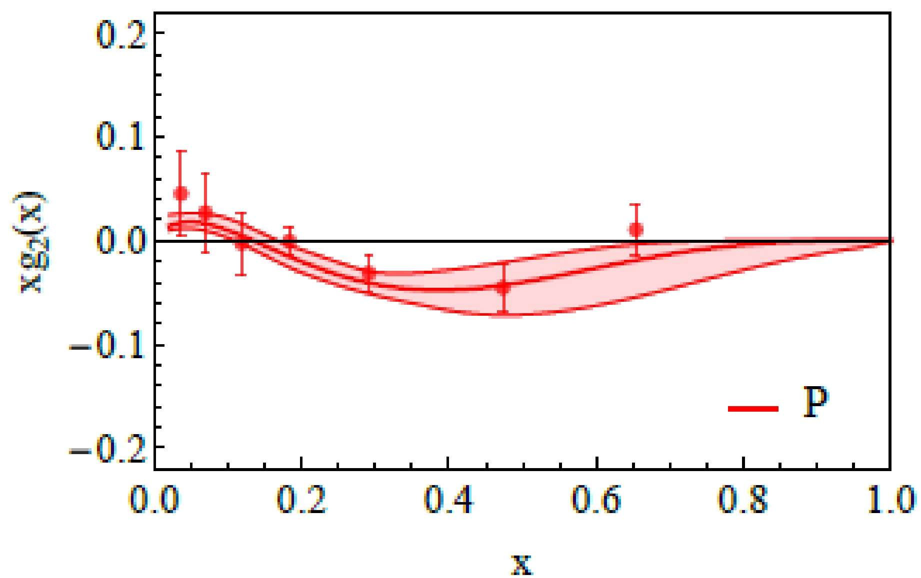
$$\frac{\tilde{g}_T^q}{g_T^q} \sim \frac{\tilde{h}_L^q}{h_L^q} \sim \frac{\langle \bar{q} g q \rangle}{\langle \bar{q} q \rangle} \sim \left(\frac{\rho}{R}\right)^4 \log\left(\frac{\rho}{R}\right) \sim 10^{-2}$$

Tests of WW approximation in DIS experiments

$$g_2(x) \stackrel{\mathrm{WW}}{\approx} g_2(x)_{\mathrm{WW}} \equiv \frac{d}{dx}\left[x \int_x^1 \frac{dy}{y}\, g_1(y)\right]$$

Gluck, Reya, A. Vogt, EPJC5 (1998) 461–470





Left panel: data from E144 and E155 experiments at $\langle Q^2 \rangle = 7.1$ GeV^2.
Right panel: HERMES data for $Q^2 > 1$ GeV^2 with $\langle Q^2 \rangle = 2.4$ GeV^2.

Tests in lattice QCD

$$\underbrace{\int dx\, g_{1T}^{\perp(1)u}(x)}_{=0.1041(85)} \overset{!}{\approx} \underbrace{\frac{1}{2}\int dx\, x\, g_1^u(x)}_{=0.104(9)}\,, \qquad \underbrace{\int dx\, g_{1T}^{\perp(1)d}(x)}_{0.0232(42)} \overset{!}{\approx} \underbrace{\frac{1}{2}\int dx\, x\, g_1^d(x)}_{=-0.025(9)}$$

$$\underbrace{\int dx\, h_{1L}^{\perp(1)u}(x)}_{=-0.0881(72)} \overset{!}{\approx} \underbrace{-\frac{1}{3}\int dx\, x\, h_1^u(x)}_{=-0.093(3)}\,,$$

Musch, Hägler, Negele, Schäfer, PRD 83 (2011) 094507
Green, Jansen and Steffens, PRL 121 (2018) 022004, [1707.07152]

Tests in models

Effective approaches and models such as bag , spectator, chiral quark-soliton, or light-cone constituent models support the approximations (3.2a, 3.2b) for PDFs within an accuracy of (10 -- 30)% at low hadronic scale below 1 GeV.
Applicability of WW-type approximations to FFs remains the least tested point in our approach.

Basis functions for the WW-type approximations

6 leading–twist TMDs $f_1^a,\ f_{1T}^{\perp a},\ g_1^a,\ h_1^a,\ h_1^{\perp a},\ h_{1T}^{\perp a}$ and 2 leading–twist FFs $D_1^a,\ H_1^{\perp a}$ provide a basis in the sense that in WW-type approximation all other TMDs and FFs can either be expressed in terms of these basis functions or vanish. The experiment will tell us how well the approximations work.

WW-type approximations are useful for the following two **leading-twist** structure functions:

$$F_{LT}^{\cos(\phi_h-\phi_S)} \stackrel{\text{WW}}{=} \mathcal{C}\left[\omega_{\text{B}}^{\{1\}}\, g_{1T}^{\perp} D_1\right]\Bigg|_{\substack{g_{1T}^{\perp a} \to g_1^a \\ \text{Eq. (3.6a)}}}, \tag{4.1a}$$

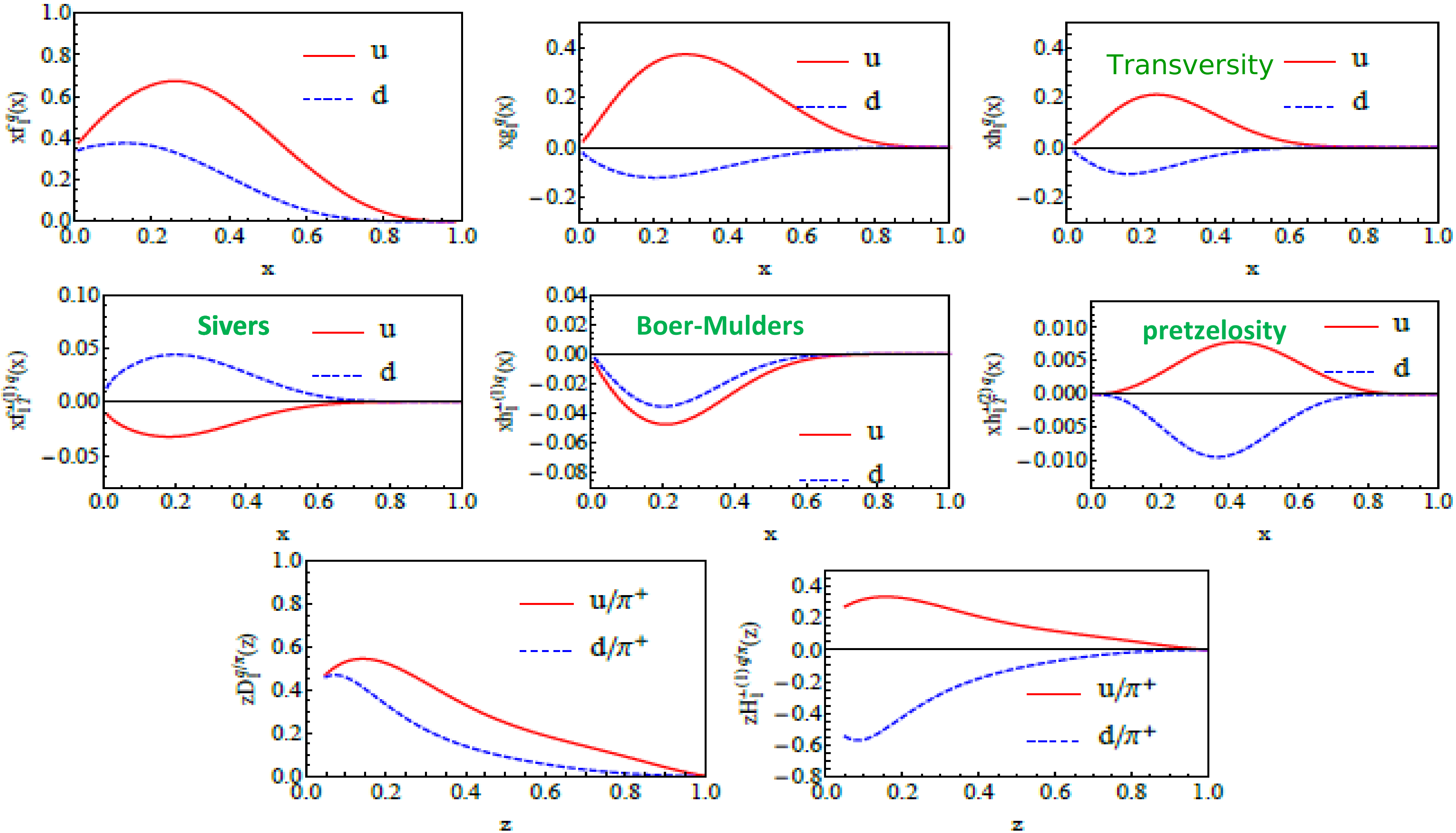
$$F_{UL}^{\sin 2\phi_h} \stackrel{\text{WW}}{=} \mathcal{C}\left[\omega_{\text{AB}}^{\{2\}}\, h_{1L}^{\perp} H_1^{\perp}\right]\Bigg|_{\substack{h_{1L}^{\perp a} \to h_1^a \\ \text{Eq. (3.6b)}}}. \tag{4.1b}$$

$$\mathcal{C}\left[\omega\, f\, D\right] = x \sum_a e_a^2 \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{P}_\perp\, \delta^{(2)}(z\boldsymbol{k}_\perp + \boldsymbol{P}_\perp - \boldsymbol{P}_{hT})\, \omega\, f^a(x, \boldsymbol{k}_\perp^2)\, D^a(z, \boldsymbol{P}_\perp^2)$$

where ω is a weight function which in general depends on $\boldsymbol{k}_\perp$ and $\boldsymbol{P}_\perp$

We will use the so-called Gaussian Ansatz for the TMDs and FFs. This Ansatz, which for a generic TMD or FF is given by

$$f(x, k_\perp^2) = f(x)\, \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}, \quad D(z, P_\perp^2) = D(z)\, \frac{e^{-P_\perp^2/\langle P_\perp^2\rangle}}{\pi\langle P_\perp^2\rangle}, \tag{4.3}$$



The basis functions $f_1^a,\ g_1^a,\ h_1^a, f_{1T}^{\perp a},\ h_1^{\perp a},\ h_{1T}^{\perp a};\ D_1^a,\ H_1^{\perp a}$ ..

D. de Florian, R. Sassot and M. Stratmann, PR D75 (2007) 114010, [hep-ph/0703242].
M. Gluck, E. Reya, M. Stratmann and W. Vogelsang,PRD63 (2001) 094005, [hep-ph/0011215].
M. Anselmino et al., Nucl. Phys. Proc. Suppl. 191 (2009) 98–107, [0812.4366].

Subleading twist structure functions in WW-type approximations

$$F_{UU}^{\cos\phi_h} \overset{\text{WW}}{=} \frac{2M_N}{Q}\,\mathcal{C}\Big[\ \omega_{\text{A}}^{\{1\}}\,x\,h\,H_1^{\perp} - \omega_{\text{B}}^{\{1\}}\,x\,f^{\perp}D_1\Big]\Bigg|_{\substack{f^{\perp a}\to f_1^a,\ h^a\to h_1^{\perp a}\\ \text{with Eqs. (3.3b, 3.4h)}}} \tag{4.2a}$$

$$F_{UL}^{\sin\phi_h} \overset{\text{WW}}{=} \frac{2M_N}{Q}\,\mathcal{C}\Big[\ \omega_{\text{A}}^{\{1\}}\,x\,h_L H_1^{\perp}\Big]\Bigg|_{\substack{h_L^a\to h_{1L}^{\perp a}\\ \text{with Eq. (3.3f)}}} \tag{4.2b}$$

$$F_{UT}^{\sin\phi_S} \overset{\text{WW}}{=} \frac{2M_N}{Q}\,\mathcal{C}\Big[\ \omega^{\{0\}}\,x\,f_T D_1 - \frac{\omega_{\text{B}}^{\{2\}}}{2}\,(xh_T - xh_T^{\perp})\,H_1^{\perp}\Big]\Bigg|_{\substack{f_T^a\to f_{1T}^{\perp a},\\ h_T^a - h_T^{\perp a}\to h_1^a\\ \text{(3.4g, 3.3g, 3.3h)}}} \tag{4.2c}$$

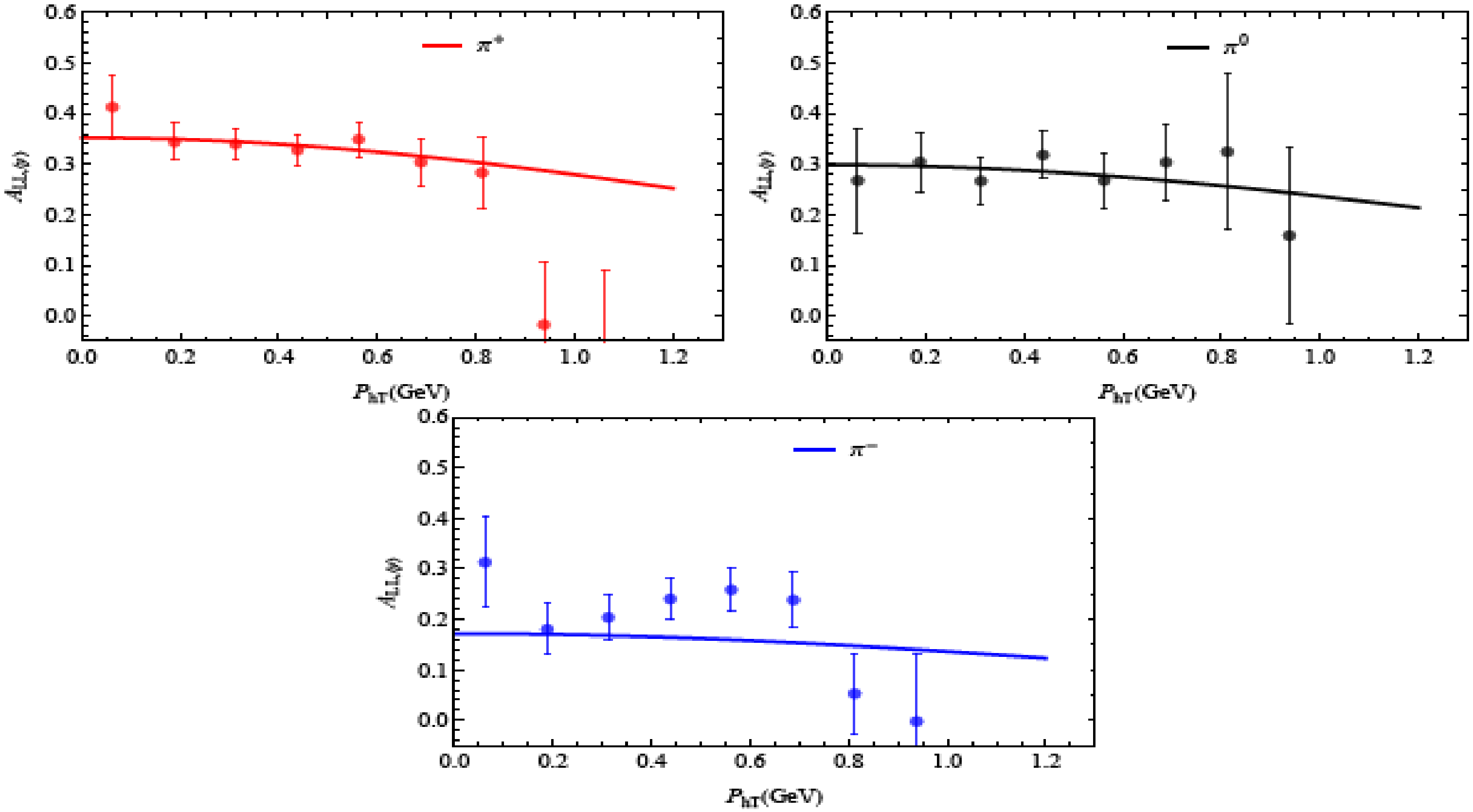
$$F_{UT}^{\sin(2\phi_h-\phi_S)} \overset{\text{WW}}{=} \frac{2M_N}{Q}\,\mathcal{C}\Big[\omega_{\text{C}}^{\{2\}}\,x\,f_T^{\perp}D_1 + \frac{\omega_{\text{AB}}^{\{2\}}}{2}x(h_T + h_T^{\perp})H_1^{\perp}\Big]\Bigg|_{\substack{f_T^{\perp a}\to f_{1T}^{\perp a},\\ (h_T^a + h_T^{\perp a})\to h_{1T}^{\perp a}\\ \text{with (3.4f, 3.3g, 3.3h)}}} \tag{4.2d}$$

$$F_{LT}^{\cos\phi_S} \overset{\text{WW}}{=} \frac{2M_N}{Q}\,\mathcal{C}\Big[-\omega^{\{0\}}\,x\,g_T D_1\Big]\Bigg|_{\substack{g_T^a\to g_1^a\\ \text{Eq. (3.2a)}}} \tag{4.2f}$$

$$F_{LL}^{\cos\phi_h} \overset{\text{WW}}{=} \frac{2M_N}{Q}\,\mathcal{C}\Big[-\omega_{\text{B}}^{\{1\}}\,xg_L^{\perp}D_1\Big]\Bigg|_{\substack{g_L^{\perp a}\to g_1^a\\ \text{Eq. (3.3c)}}} \tag{4.2g}$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} \overset{\text{WW}}{=} \frac{2M_N}{Q}\,\mathcal{C}\Big[-\omega_{\text{C}}^{\{2\}}\,xg_T^{\perp}D_1\Big]\Bigg|_{\substack{g_T^{\perp a}\to g_1^a\\ \text{Eqs. (3.3d, 3.6a)}}} \tag{4.2h}$$

Leading-twist A_{LL} and test of Gaussian Ansatz in polarized scattering

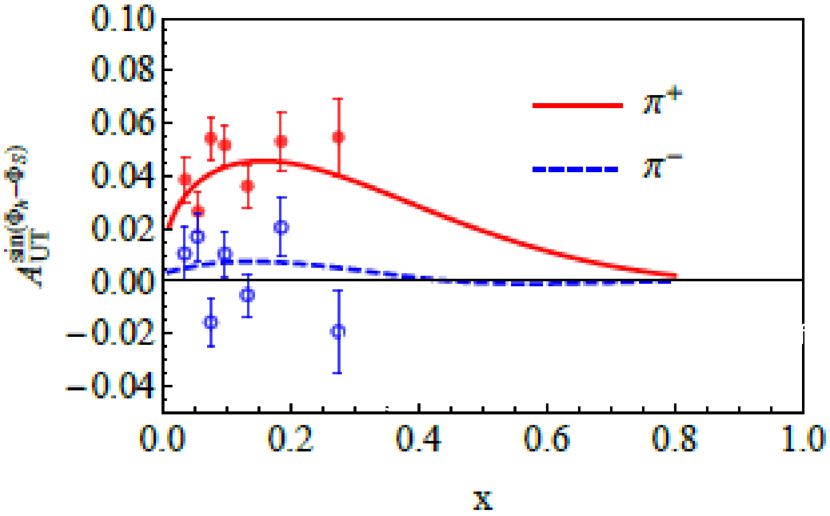


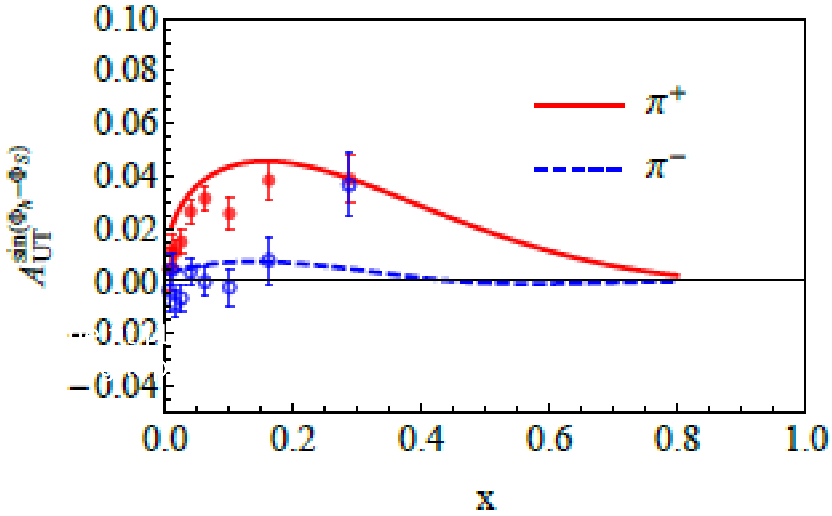
$A_{LL,<y>}$ as function of P^h_T vs JLab data for π^+, π^0, π^-. The solid lines are our results for the mean values of kinematical variables <x> = 0,25, <z> = 0.5, <Q^2 >= 1,67 GeV^2.

CLAS collaboration, H. Avakian et al., PRL 105 (2010) 262002, [1003.4549].

Leading-twist $A_{UT}^{\sin(\phi_h-\phi_S)}$ Sivers asymmetry

$$F_{UT}^{\sin(\phi_h-\phi_S)}(x,z,\langle P_{hT}\rangle) = -x\sum_q e_q^2\, f_{1T}^{\perp(1)q}(x)\, D_1^q(z)\, c_{\rm B}^{(1)}\left(\frac{z}{\lambda^{1/2}}\right), \quad c_{\rm B}^{(1)} = \sqrt{\pi}\, M_N$$





Sivers asymmetry for a proton target as function of x in comparison to (left panel) HERMES and (right panel) COMPASS data. (left palel

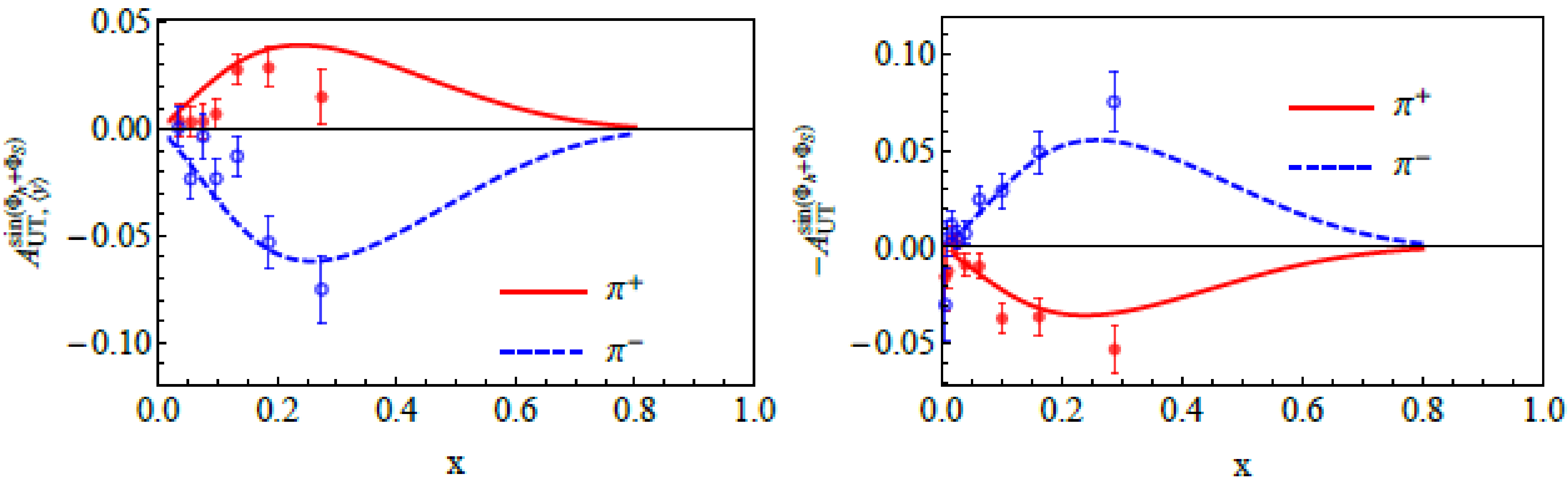
D. W. Sivers, PR D41 (1990) 83

HERMES collaboration, A. Airapetian et al., Phys. Rev. Lett. 103 (2009) 152002,

COMPASS collaboration, C. Adolph et al., Phys. Lett. B717 (2012) 383–389,

Leading-twist $A_{UT}^{\sin(\phi_h+\phi_S)}$ Collins asymmetry

$$F_{UT}^{\sin(\phi_h+\phi_S)}(x,z,\langle P_{hT}\rangle) = x\sum_q e_q^2\, h_1^q(x)\, H_1^{\perp(1)q}(z)\, c_{\mathrm{A}}^{(1)}\left(\frac{z}{\lambda^{1/2}}\right), \quad c_{\mathrm{A}}^{(1)} = \sqrt{\pi}\, m_h,$$



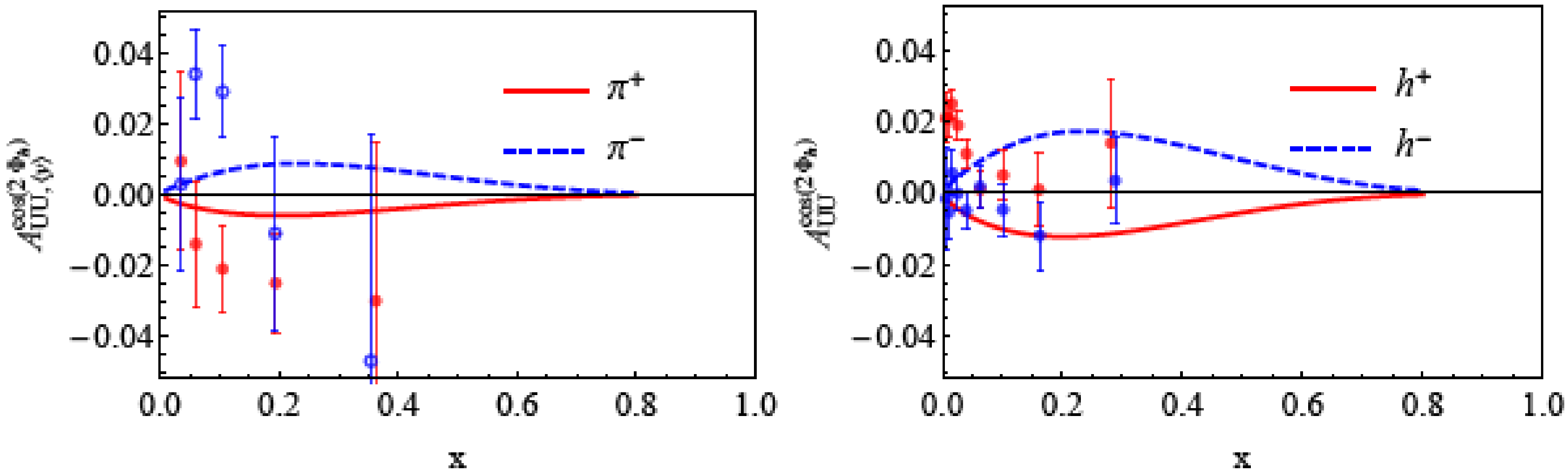
Collins asymmetry for a proton target as function of x based on the fit in comparison to (left panel) HERMES and (right panel) COMPASS data (different agreement in signes)

J. C. Collins, Nucl. Phys. B396 (1993) 161–182.
HERMES collaboration, A. Airapetian et al., PLB693 (2010) 11–16,
COMPASS collaboration, C. Adolph et al., PLB744 (2015) 250–259

Leading-twist $A_{UU}^{\cos(2\phi_h)}$ Boer–Mulders asymmetry

Arises from a convolution of the Collins fragmentation function and Boer-Mulders TMD

$$F_{UU}^{\cos 2\phi_h}(x, z, \langle P_{hT}\rangle) = x \sum_q e_q^2\, h_1^{\perp(1)q}(x)\, H_1^{\perp(1)q}(z)\, c_{\rm AB}^{(2)} \left(\frac{z}{\lambda^{1/2}}\right)^2, \qquad c_{\rm AB}^{(2)} = 4M_N m_h,$$



The asymmetry for a proton target as function of x based on the fit in comparison to (left panel) HERMES and (right panel) COMPASS data.

HERMES collaboration, A. Airapetian et al., PRD87 (2013), [1204.4161]
COMPASS Collaboration, C. Adolf at al., NPB886 (2014) 1046-1077, [14014161]
Barone, Melis and Prokudin, PRD81 (2010) 114026, [0912.5194]

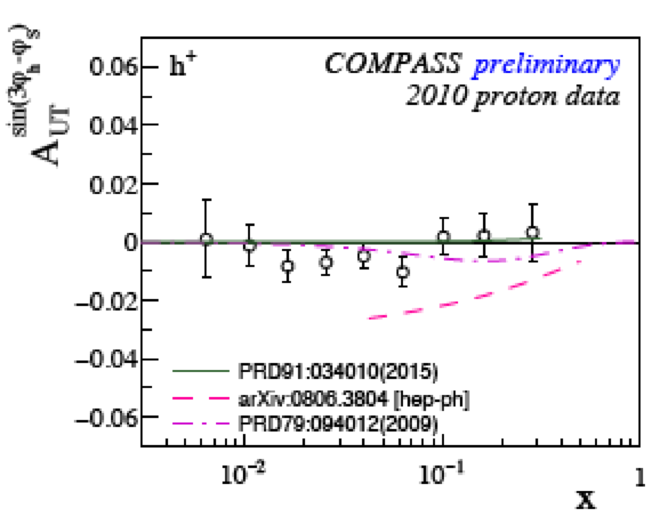
Leading-twist $A_{UT}^{\sin(3\phi_h-\phi_S)}$ asymmetry

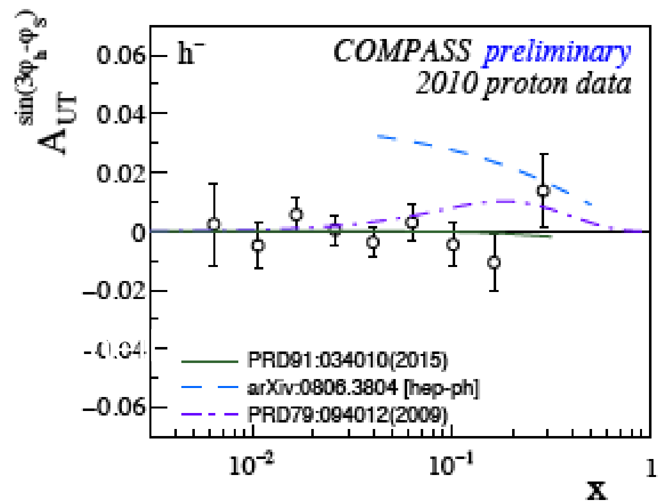
$$F_{UT}^{\sin(3\phi_h-\phi_S)}(x,z,\langle P_{hT}\rangle) = x\sum_q e_q^2\, h_{1T}^{\perp(2)q}(x)\, H_1^{\perp(1)q}(z)\, c^{(3)} \left(\frac{z}{\lambda^{1/2}}\right)^3 ; \quad c^{(3)} = 3/2\sqrt{\pi}\, M_N^2\, m_h$$

Boffi, Efremov, Pasquini, Schweitzer, PRD79 (2009) 094012, [0903.1271].

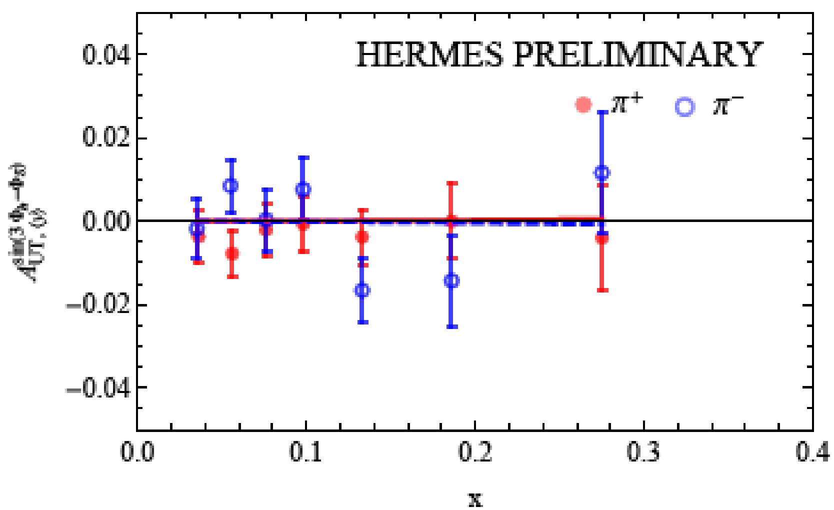
Lefky, Prokudin, PRD91 (2015)034010.

Avakian, Efremov, Schweitzer, Yuan, PRD 78(2008) 114024



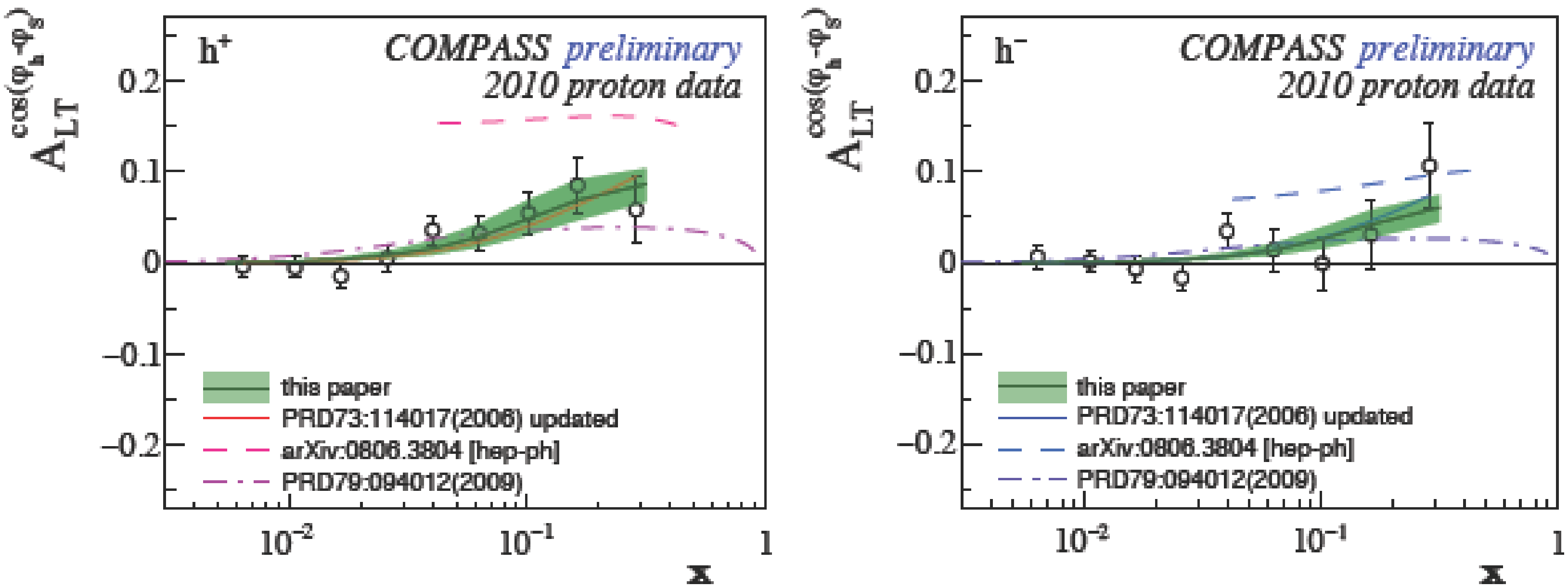


HERMES collaboration, G. Schnell, PoS DIS2010, 247



Leading-twist $A_{LT}^{\cos(\phi_h-\phi_S)}$ (Worm-gear T)

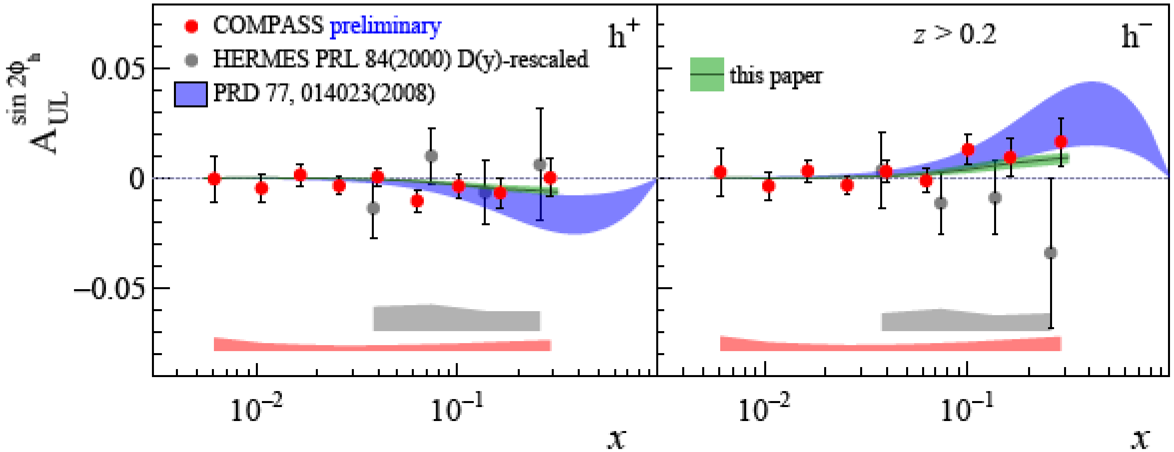
$$F_{LT}^{\cos(\phi_h-\phi_S)}(x,z,\langle P_{hT}\rangle) = x\sum_q e_q^2\, g_{1T}^{\perp(1)q}(x)\, D_1^q(z)\, c_B^{(1)}\left(\frac{z}{\lambda^{1/2}}\right) \qquad c_B^{(1)} = \sqrt{\pi}\, M_N$$

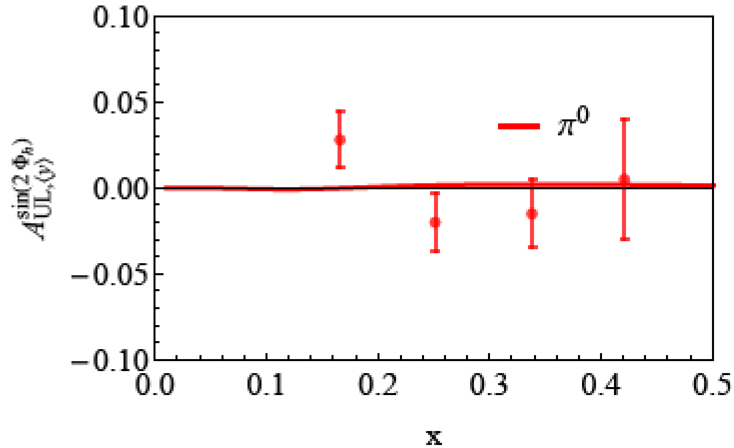


B. Parsamyan, PoS DIS2017 (2018) 259, [1801.01488].

Leading-twist $A_{UL}^{\sin 2\phi_h}$ Kotzinian–Mulders asymmetry

$$F_{UL}^{\sin(2\phi_h)}(x, z, \langle P_{hT} \rangle) = x \sum_q e_q^2 \, h_{1L}^{\perp(1)q}(x) \, H_1^{\perp(1)q/h}(z) \left(\frac{z}{\lambda^{1/2}} \right)^2 c_{AB}^{(2)} \qquad c_{AB}^{(2)} = 4 M_N m_h$$





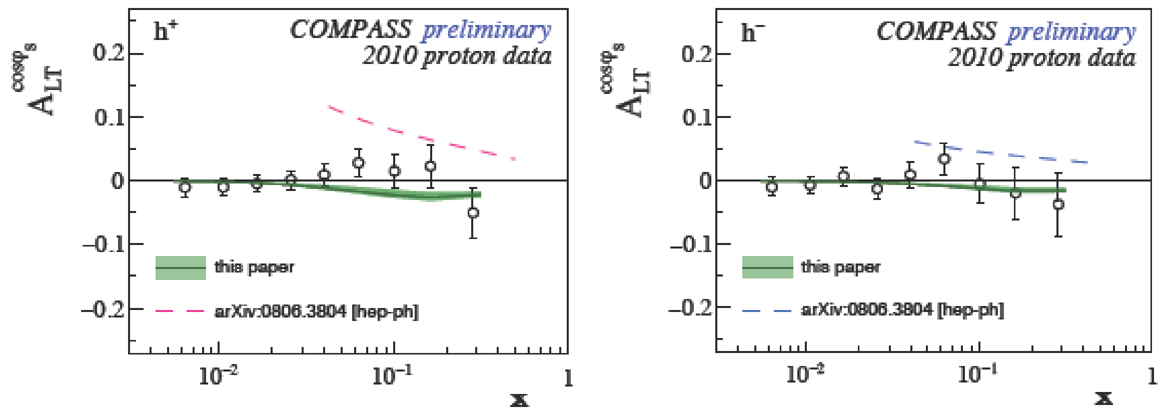
Kotzinian, Mulders, PRD54 (1996) 1229–1232, [hep-ph/9511420].

B. Parsamyan, Int. J. Mod. Phys. Conf. Ser. 40 (2016) 1660029, [1504.01599].

Subleading-twist asymmetries in WW-type approximation

Subleading-twist $A_{LT}^{\cos\phi_S}$

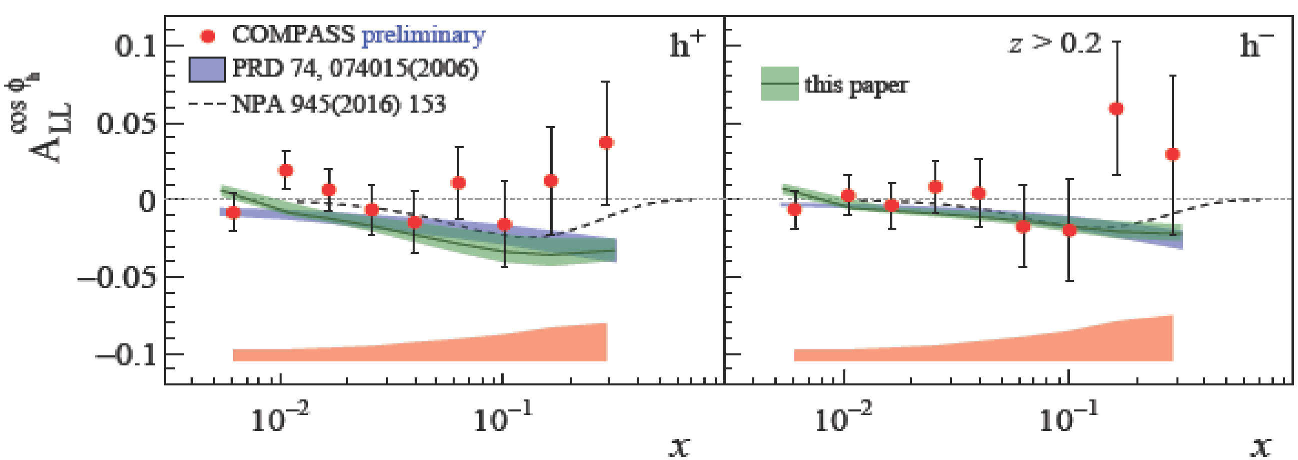
$$F_{LT}^{\cos\phi_S}(x,z) = -\frac{2M_N}{Q}\, x^2 \sum_q e_q^2\, g_T^q(x)\, D_1^q(z)$$



B. Parsamyan, PoS DIS2013 (2013) 231, [1307.0183].

Subleading-twist $A_{LL}^{\cos\phi_h}$

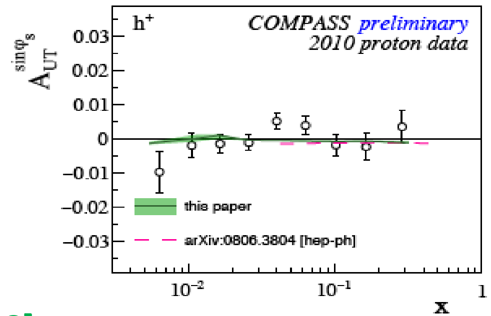
$$F_{LL}^{\cos\phi_h}(x,z,\langle P_{hT}\rangle) = -\frac{2M_N}{Q}\, x \sum_q e_q^2\, x\, g_L^{\perp(1)q}(x)\, D_1^q(z)\, c_B^{(1)}\left(\frac{z}{\lambda^{1/2}}\right) \qquad c_B^{(1)} = \sqrt{\pi}\, M_N$$

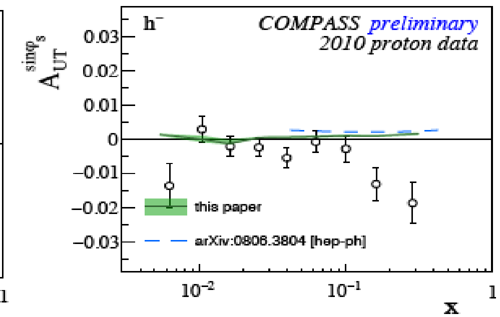


B. Parsamyan, PoS DIS2017 (2018) 259, [1801.01488].

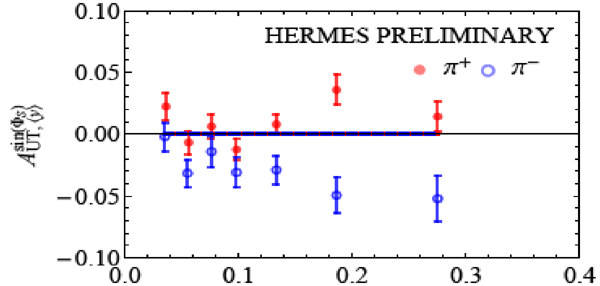
Subleading-twist $A_{UT}^{\sin\phi_S}$

$$F_{UT}^{\sin\phi_S}(x,z)=0\,.$$





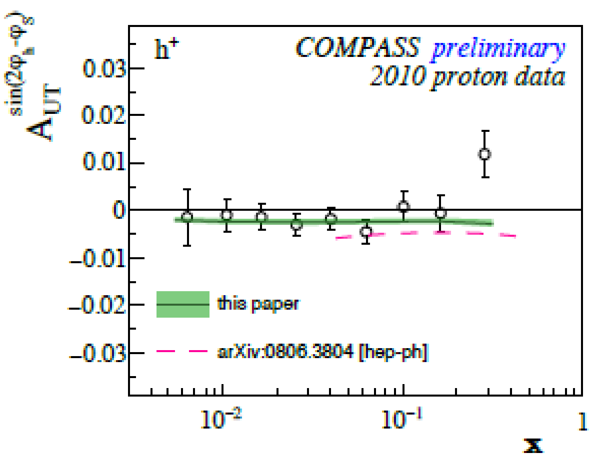
B. Parsamyan, PoS DIS2017 (2018) 259, [1801.01488].
HERMES coll., G. Schnell, PoS DIS2010 (2010) 247.

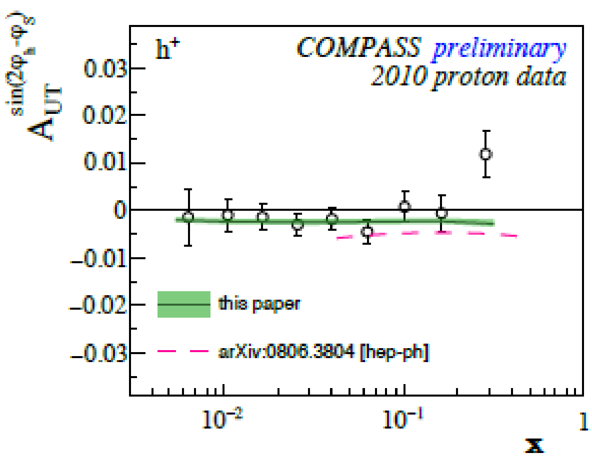


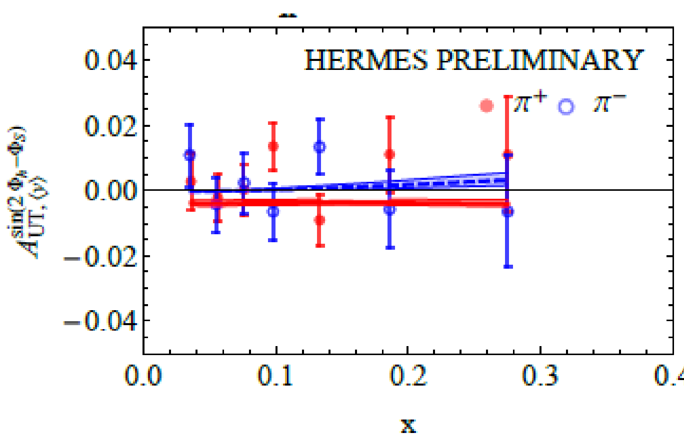
Subleading-twist $A_{UT}^{\sin(2\phi_h-\phi_S)}$

$$F_{UT}^{\sin(2\phi_h-\phi_S)}(x,z,\langle P_{hT}\rangle) \;=\; \frac{2M_N}{Q}\,x\sum_q e_q^2\left[x\,f_T^{\perp(2)q}(x)\;D_1(z)\;c_C^{(2)}\left(\frac{z}{\lambda^{1/2}}\right)^2+\frac{x}{2}\left(h_T^{(1)q}(x)+h_T^{\perp(1)q}(x)\right)H_1^{\perp(1)q}(z)\,c_{AB}^{(2)}\left(\frac{z}{\lambda^{1/2}}\right)^2\right]$$

$$\lambda = z^2\langle k_\perp^2\rangle_{h_T^\perp}+\langle P_\perp^2\rangle_{H_1^\perp}\quad c_{AB}^{(2)}=4M_N m_h$$

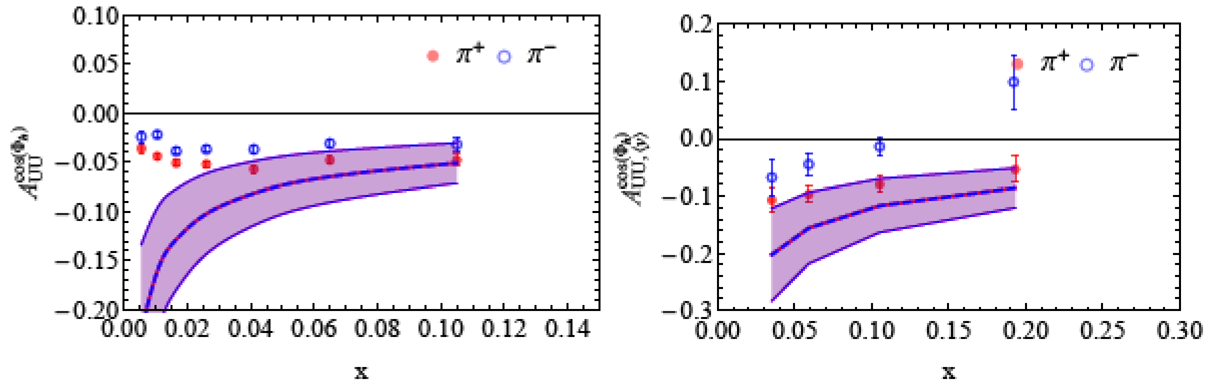






Subleading-twist $A_{UU}^{\cos\phi_h}$

$$F_{UU}^{\cos\phi_h}(x,z,\langle P_{hT}\rangle) = \frac{2M_N}{Q}\, x \sum e_q^2 \left[-x\, f^{\perp(1)q}(x)\, D_1^q(z)\, c_B^{(1)} \left(\frac{z}{\lambda^{1/2}}\right)\right] \qquad c_B^{(1)} = \sqrt{\pi}\, M_N$$

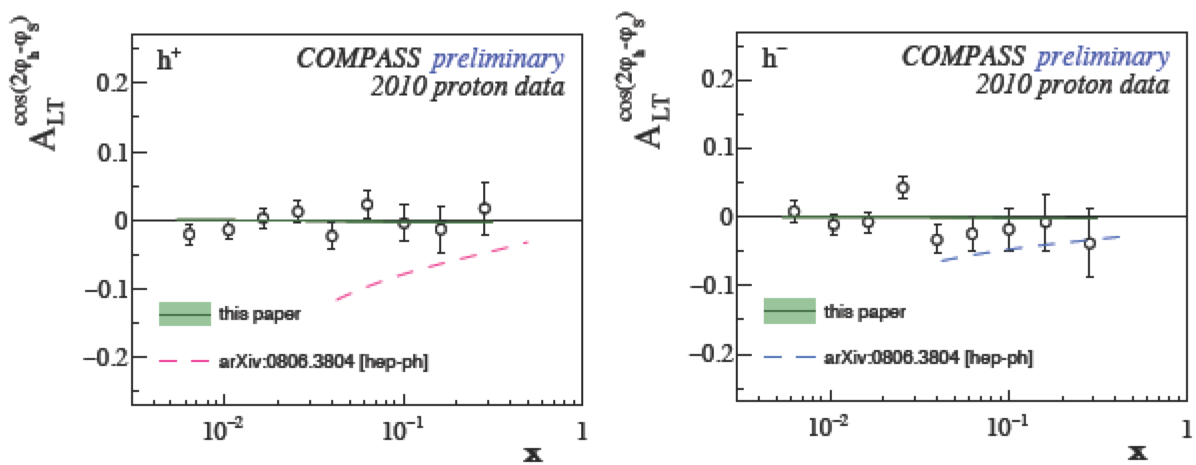


Left panel: asymmetry for positive and negative hadrons at COMPASS for a proton target.
Right panel: for from HERMES.

COMPASS coll., C. Adolph et al., Nucl. Phys. B886 (2014) 1046–1077.
HERMES coll., A. Airapetian et al., Phys. Rev. D87 (2013) 012010, [1204.4161].

Subleading-twist $A_{LT}^{\cos(2\phi_h-\phi_S)}$

$$F_{LT}^{\cos(2\phi_h-\phi_S)}(x,z,\langle P_{hT}\rangle) = -\frac{2M_N}{Q}\, x \sum_q e_q^2\, x\, g_T^{\perp(2)q}(x)\, D_1^q(z)\, c_C^{(2)} \left(\frac{z}{\lambda^{1/2}}\right)^2 \qquad c_C^{(2)} = M_N^2$$



Smallness of the asymmetry has two reasons: i) M_N/Q suppressed ii) proportional to P^2_{hT} with $P_{hT} << Q$ in TMD approach.

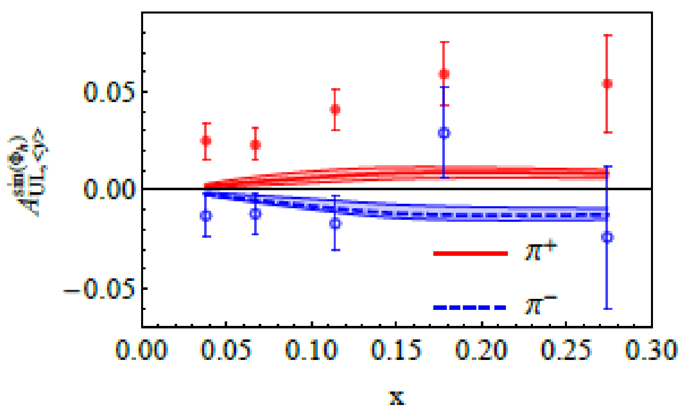
B. Parsamyan, PoS DIS2013 (2013) 231, [1307.0183].

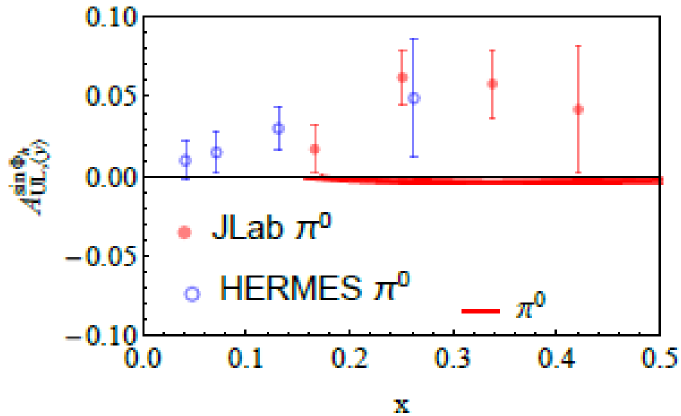
Subleading-twist $A_{LU}^{\sin\phi_h}$

$$A_{LU}^{\sin\phi_h} \propto \frac{\langle \bar{q} g q\rangle}{\langle \bar{q} q\rangle} \overset{\exp}{\sim} \mathcal{O}(2\,\%)\,.$$

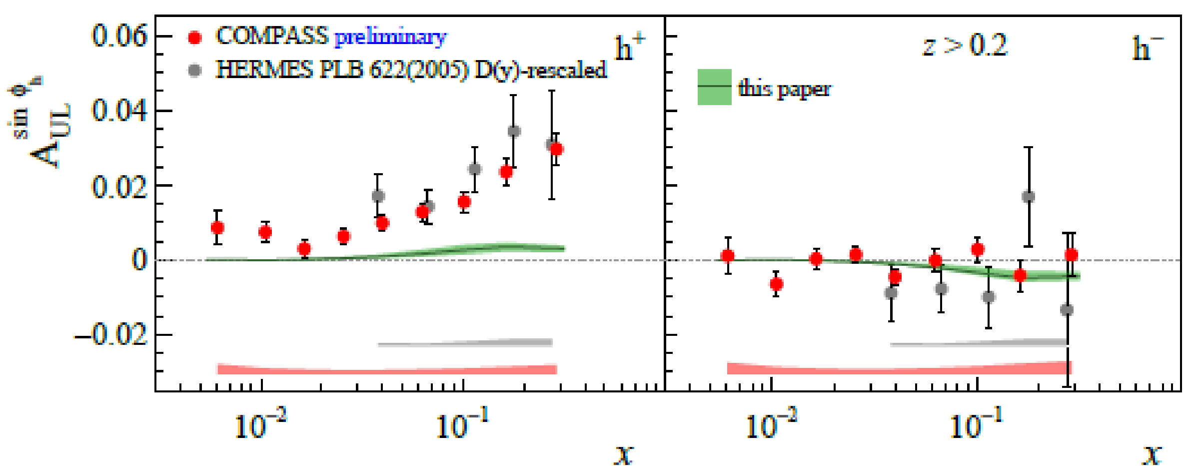
Subleading-twist $A_{UL}^{\sin\phi_h}$

$$F_{UL}^{\sin\phi_h}(x,z,\langle P_{hT}\rangle) = \frac{2M_N}{Q}\, x\sum e_q^2\, x\, h_L^q(x)\, H_1^{\perp(1)q}(z)\, c_A^{(1)}\left(\frac{z}{\lambda^{1/2}}\right) \qquad c_A^{(1)} = \sqrt{\pi}\, m_h$$





Upper panel left: from HERMES.
Upper panel right: p0 from HERMES (blue) [and JLab (red).
Lower panel: preliminary h^{+-} COMPASS data.



HERMES col., A. Airapetian et al.,PLB 622 (2005) 14–22, [hep-ex/0505042]. PRD 64 (2001) 097101, [hep-ex/0104005].

B. Parsamyan, PoS DIS2013 (2013) 231, [1307.0183].

Conclusions

1. A comprehensive and complete treatment of SIDIS azimuthal asymmetries in WW-approximations was presented.

2. For leading-twist SIDIS structure functions for the production of unpolarized hadrons factorization is proven, and each structure functions is expressed in terms of one of 6 basic twist-2 TMDs convoluted with one of 2 twist-2 FFs.

3. For subleading-twist the situation is far more complex for two reasons. First, factorization is not proven and must be assumed. Second, each of the functions receives several contributions from various TMDs and FFs one of which is twist-2 and the other twist-3.

4. Most importantly, we have conducted systematic tests of WW approximations with available published or preliminary data from HERMES, COMPASS, and JLab.

5. The results are useful for experiments prepared in the near term (JLab 12) or proposed in the long term (Electron Ion Collider), and provide helpful input for Monte Carlo event generators.

Remark: The generalized parton model approach of M. Anselmino et al. (PRD83 (2011) 114019) provides a description that is largely equivalent to ours.

THANK YOY FOR ATTENTION !