Exploring the relativistic bound state structure in Minkowski space: applications to hadrons

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Motivation

space-time = Minkowski space

Develop methods in continuous nonperturbative QCD within a given dynamical simple framework

Solve the Bethe-Salpeter bound state equation

Observables: spectrum, SL/TL momentum region

Relation BSA to LF Fock-space expansion of the hadron wf

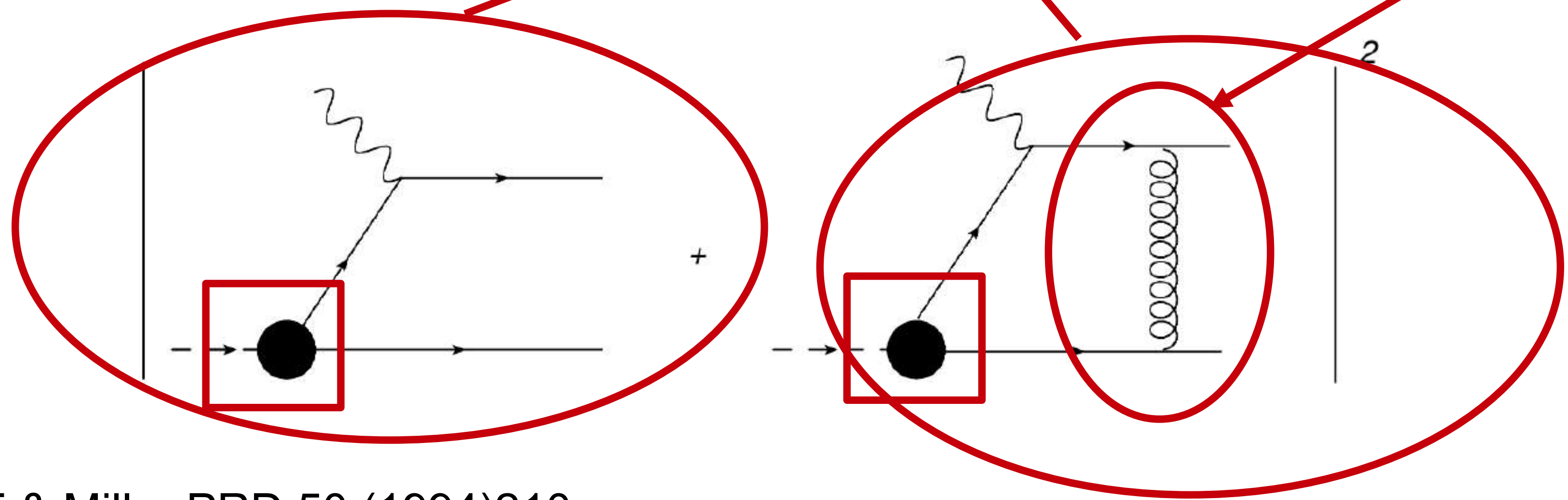
Problems to be addressed

Observables associated with the hadron structure in Minkowski space obtainable from BSA

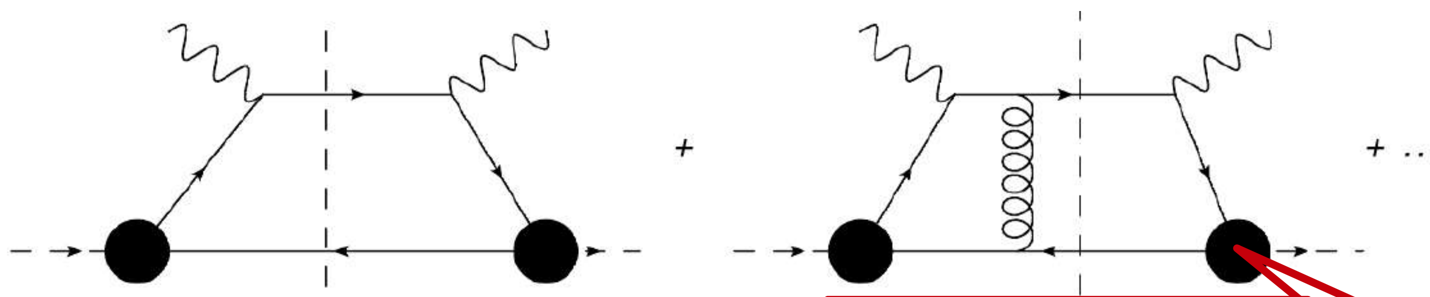
- parton distributions (pdfs)
- generalized parton distributions
- transverse momentum distributions (TMDs)
- Fragmentation functions
- TL form factors ….
- Inversion Problem: Euclidean→Minkowski

TMDs & PDFs

FSI gluon exchange: T-odd



TF & Miller PRD 50 (1994)210



$$q^2 = q^+q^- - q^2{}_T$$

$$q^+ = q^0+q^3 \quad q^- = q^0-q^3$$

$q^- \longrightarrow$ *infty*
DIS

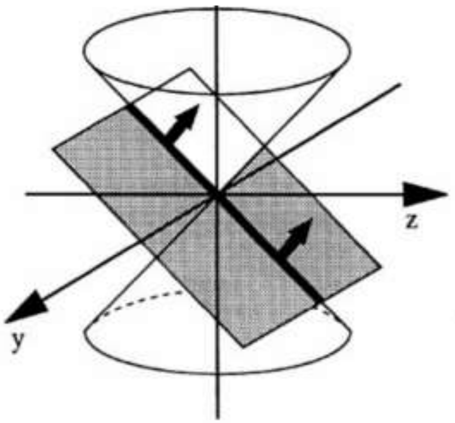
Bethe-Salpeter Amplitude @ x^+=0

Bethe-Salpeter Amplitude →Light-Front WF (LFWF)

- **basic ingredient in PDFs, GPDs and TMDs**

$$\tilde{\Phi}(x, p) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\Phi(k, p)$$

$$p^\mu = p_1^\mu + p_2^\mu \qquad k^\mu = \frac{p_1^\mu - p_2^\mu}{2}$$



$$\begin{aligned}\tilde{\Phi}(x, p) &= \langle 0|T\{\varphi_H(x^\mu/2)\varphi_H(-x^\mu/2)\}|p\rangle \\ &= \theta(x^+)\langle 0|\varphi(\tilde{x}/2)e^{-iP^- x^+/2}\varphi(-\tilde{x}/2)|p\rangle e^{ip^- x^+/4} + \cdots \\ &= \theta(x^+)\sum_{n,n'} e^{ip^- x^+/4}\langle 0|\varphi(\tilde{x}/2)|n'\rangle\langle n'|e^{-iP^- x^+/2}|n\rangle\langle n|\varphi(-\tilde{x}/2)|p\rangle + \cdots\end{aligned}$$

$x^+ = 0$ only valence state remains! How to rebuilt the full BS amplitude?

Iterated Resolvents: Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998)

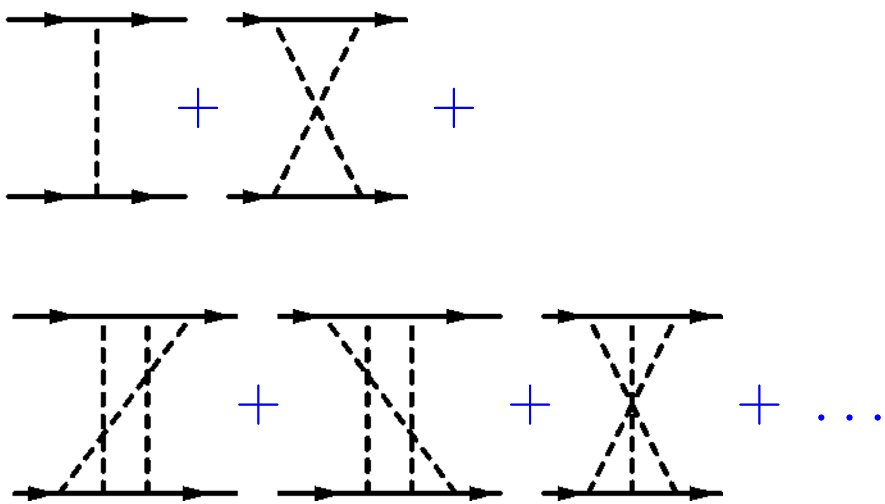
Reminder…

Bethe-Salpeter Bound-State Equation (2 bosons)

$$\Phi(k,p) = G_0^{(12)}(k,p) \int \frac{d^4k'}{(2\pi)^4} \mathrm{i}K(k,k';p)\Phi(k',p)$$

$$G_0^{(12)}(k,p) = \frac{\mathrm{i}}{\left[(p/2+k)^2 - m^2 + \mathrm{i}\epsilon\right]} \frac{\mathrm{i}}{\left[(p/2-k)^2 - m^2 + \mathrm{i}\epsilon\right]}$$

Kernel: sum 2PI diagrams



- **Valence LF wave function → BSA ?**
- **Valence→ full Fock Space w-f ?**

Sales, et al. PRC61, 044003 (2000)

BS amplitude from the valence LF wave function: sketch

- Quasi-Potential approach for the LF projection (3D equations);
- Derivation of an effective Mass-squared operator acting on the valence wave function;
- The effective interaction is expanded perturbatively in correspondence with the Fock-content of the intermediate states;
- $\Pi(p)$ reverse LF-time operator: computed perturbatively

Reverse operation: valence wave function $\Rightarrow$ BS amplitude

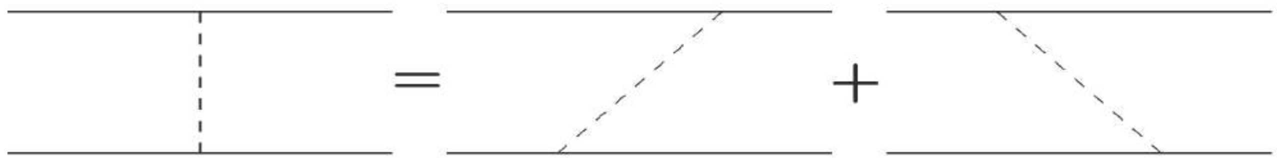
$$|\Psi\rangle = \Pi(p) \, |\phi_{LF}\rangle$$

Sales, et al. PRC61, 044003 (2000); PRC63, 064003 (2001); Frederico et al. NPA737, 260c (2004); Marinho et al., PRD 76, 096001 (2007); Marinho et al. PRD77, 116010 (2008); Frederico and Salmè, FBS49, 163 (2011).

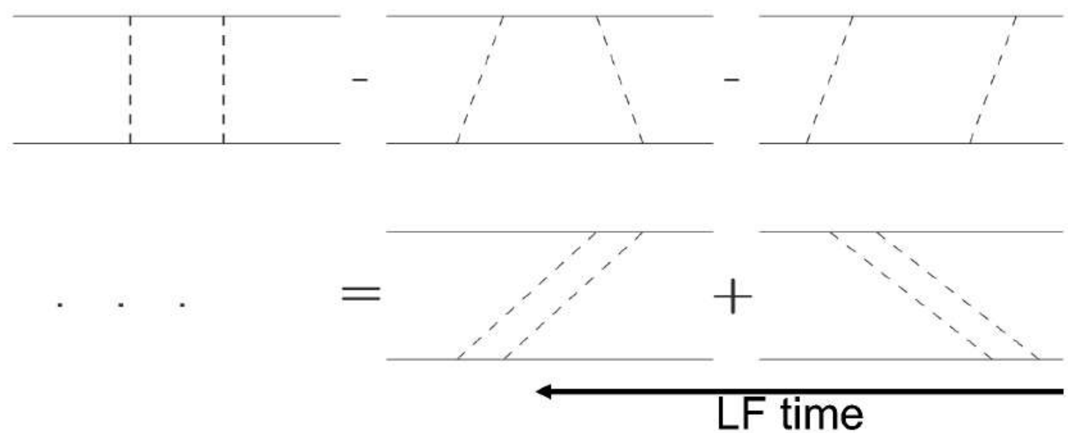
Example:Bosonic Yukawa model

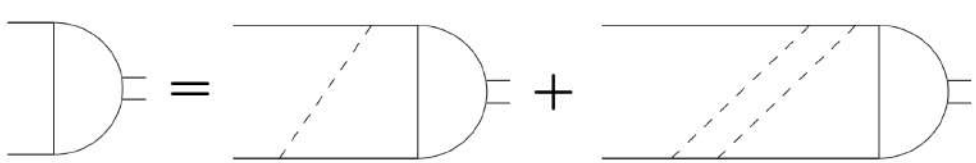
$$\mathcal{L}_I = g_S \phi_1^\dagger \phi_1 \sigma + g_S \phi_2^\dagger \phi_2 \sigma$$

$w^{(1)} =$



$w^{(2)} =$





$$\left[g_0^{-1} - w \right]$$

Mass^2 eigenvalue eq. & valence wf:

$$g(K_\lambda)^{-1} \left| \phi_\lambda \right\rangle = 0$$

Main Tool: Nakanishi Integral Representation (NIR)

"Parametric representation for any Feynman diagram for interacting bosons, with a denominator carrying the overall analytical behavior in Minkowski space" [Nakanishi PR130(1963)1230]

Bethe-Salpeter amplitude

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{g(\gamma',z')}{(\gamma' + \kappa^2 - k^2 - p.kz' - i\epsilon)^3}$$

$$\kappa^2 = m^2 - \frac{M^2}{4}$$

BSE in Minkowski space with NIR for bosons

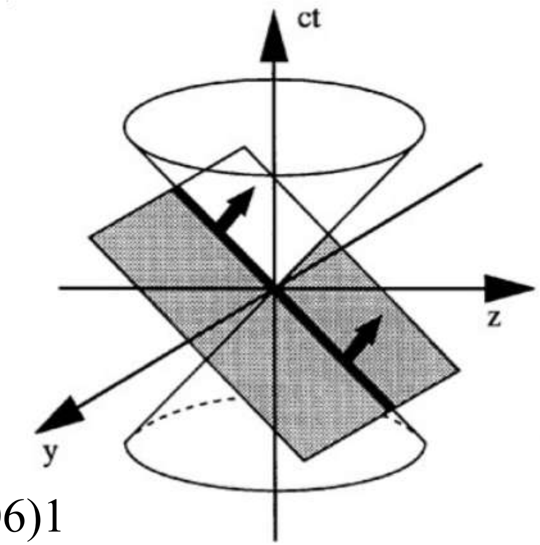
Kusaka and Williams, PRD 51 (1995) 7026;

Light-front projection: integration in k^-

Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11;

TF, Salme, Viviani PRD85(2012)036009;PRD89(2014) 016010,EPJC75(2015)398 (application to scattering)

LF wave function



& NAKANISHI INTEGRAL REPRESENTATION

Carbonell&Karmanov EPJA27(2006)1

$$\psi_{LF}(\gamma, z) = \frac{1}{4}(1 - z^2) \int_0^\infty \frac{g(\gamma', z) d\gamma'}{\left[\gamma' + \gamma + z^2 m^2 + \kappa^2(1 - z^2)\right]^2}$$

$$\gamma = k_\perp^2 \qquad z = 2x - 1$$

Solution Method of the Bethe-Salpeter eq.:

Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11

$$\Phi(k,p) = G_0(k,p) \int d^4k' \, \mathcal{K}_{BS}(k,k',p) \, \Phi(k',p)$$

$\Rightarrow$

$$\int_0^\infty d\gamma' \frac{g_b(\gamma', z; \kappa^2)}{[\gamma' + \gamma + z^2 m^2 + (1 - z^2)\kappa^2 - i\epsilon]^2} =$$

$$= \int_0^\infty d\gamma' \int_{-1}^{1} dz' \; V_b^{LF}(\gamma, z; \gamma', z') g_b(\gamma', z'; \kappa^2).$$

with $V_b^{LF}(\gamma, z; \gamma', z')$ determined by the irreducible kernel $\mathcal{I}(k, k', p)$!

UNIQUENESS OF THE NAKANISHI WEIGHT FUNCTION?

PERTURBATIVE PROOF BY NAKANISHI.

NON-PERTURBATIVE PROOF?

Generalized Stietjes transform and the LF valence wave function

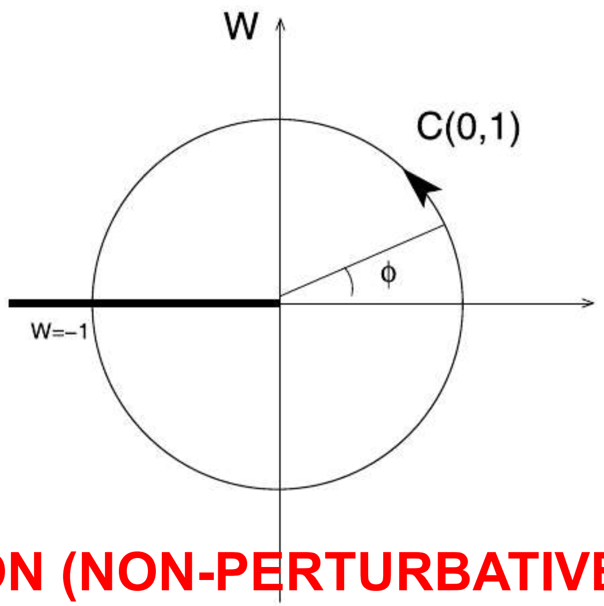
Jaume Carbonell, TF, Vladimir Karmanov PLB769 (2017) 418

$$\psi_{LF}(\gamma, z) = \frac{1-z^2}{4} \int_0^\infty \frac{g(\gamma', z)d\gamma'}{\left[\gamma' + \gamma + z^2 m^2 + (1-z^2)\kappa^2\right]^2}.$$

$$f(\gamma) \equiv \int_0^\infty d\gamma' \, L(\gamma, \gamma') g(\gamma') = \int_0^\infty d\gamma' \, \frac{g(\gamma')}{(\gamma' + \gamma + b)^2}$$

denoted symbolically as $f = \hat{L}\, g$.

$$g(\gamma) = \hat{L}^{-1} f = \frac{\gamma}{2\pi} \int_{-\pi}^{\pi} d\phi \, e^{i\phi} \, f(\gamma \, e^{i\phi} - b).$$



J.H. Schwarz, J. Math. Phys. 46 (2005) 014501,

- **UNIQUENESS OF THE NAKANISHI REPRESENTATION (NON-PERTURBATIVE)**
- **PHENOMENOLOGICAL APPLICATIONS from the valence wf → BSA!**

Two-Boson System: ground-state

Building a solvable model…

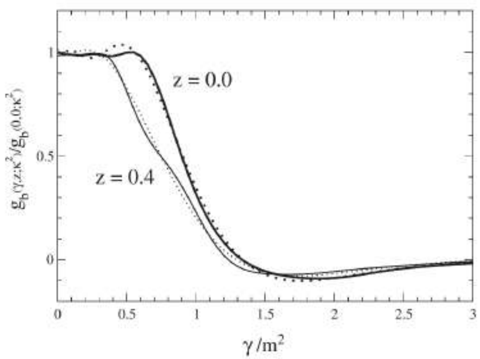
Nakanishi weight function

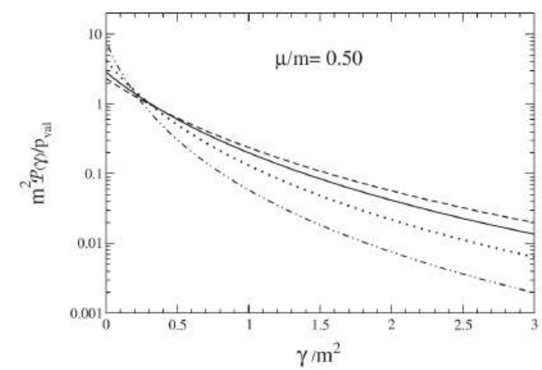
Valence wave function

3+1 n=1

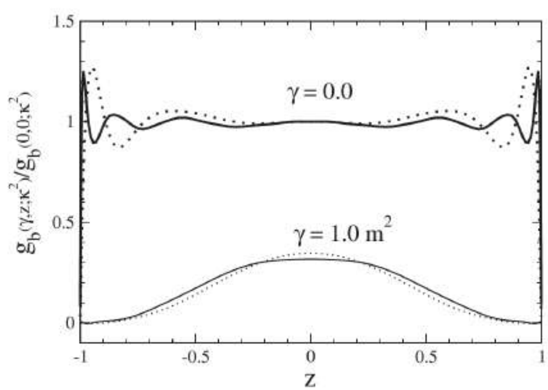
LADDER KERNEL

3+1 n=1





$\mu = 0.5 \ \ B/M = 1$



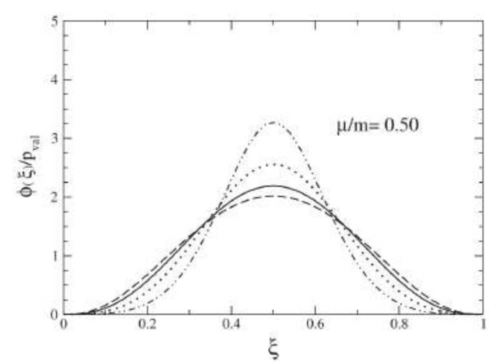


FIG. 3. The longitudinal LF distribution $\phi(\xi)$ for the valence component Eq. (34) vs the longitudinal-momentum fraction ξ for $\mu/m = 0.05$, 0.15, 0.50. Dash-double-dotted line: $B/m = 0.20$. Dotted line: $B/m = 0.50$. Solid line: $B/m = 1.0$. Dashed line: $B/m = 2.0$. Recall that $\int_0^1 d\xi \phi(\xi) = P_{val}$ (cf. Table III).

Karmanov, Carbonell, EPJA 27, 1 (2006)
Frederico, Salmè, Viviani PRD89, 016010 (2014)

(I) Valence LF wave function in impact parameter space

Miller ARNPS 60 (2010) 25

$$F(Q^2) = \int d^2\mathbf{b}\, \rho(\mathbf{b})\, e^{-i\mathbf{b}\cdot\mathbf{q}_\perp}$$

$$\rho(\mathbf{b}) = \rho_{\text{val}}(\mathbf{b}) + \text{ higher Fock states densities} \cdots$$

$$\rho_{\text{val}}(\mathbf{b}) = \frac{1}{4\pi} \int_0^1 \frac{d\xi}{\xi(1-\xi)^3} \left|\phi(\xi, \mathbf{b}/(1-\xi))\right|^2$$

» Burkardt IJMPA 18 (2003) 173

$$\phi(\xi, \mathbf{b}) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \psi(\xi, \mathbf{k}_\perp) e^{i\mathbf{k}_\perp \cdot \mathbf{b}}$$

$$\phi(\xi, b) = \frac{\xi(1-\xi)}{4\pi\sqrt{2}} F(\xi, b)$$

$$F(\xi, b) = \int_0^\infty d\gamma\, J_0(b\sqrt{\gamma}) \int_0^\infty d\gamma' \frac{g(\gamma', 1-2\xi; \kappa^2)}{[\gamma + \gamma' + \kappa^2 + (1/2-\xi)^2 M^2]^2}$$

(II) Valence LF wave function in impact parameter space

$$F(\xi,b)|_{b\to\infty} \to e^{-b\sqrt{\kappa^2+(\xi-1/2)^2M^2}}\, f(\xi,b)$$

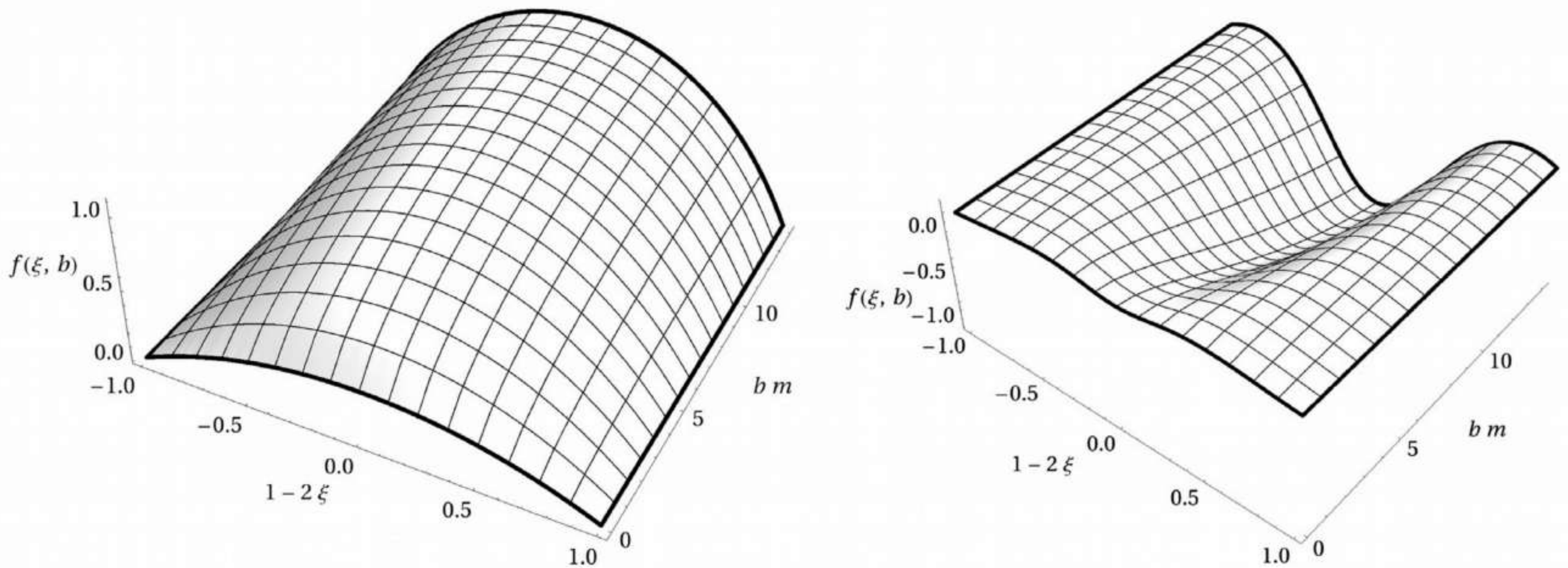


Fig. 7. The valence functions $f(\xi,b)$ in the impact parameter space. Left panel: the ground state, corresponding to $B(0)=1.9m$, $\mu=0.1m$ and $\alpha_{gr}=6.437$. Right panel: first-excited state, corresponding to $B(1)=0.22m$, $\mu=0.1m$ and $\alpha_{gr}=6.437$.

Gutierrez, Gigante, TF, Salmè,Viviani, Tomio PLB759 (2016) 131

Light-front valence wave function L+XL

Large momentum behavior

$$\psi_{LF}(\gamma,\xi) \rightarrow \alpha\, \gamma^{-2}\, C(\xi)$$

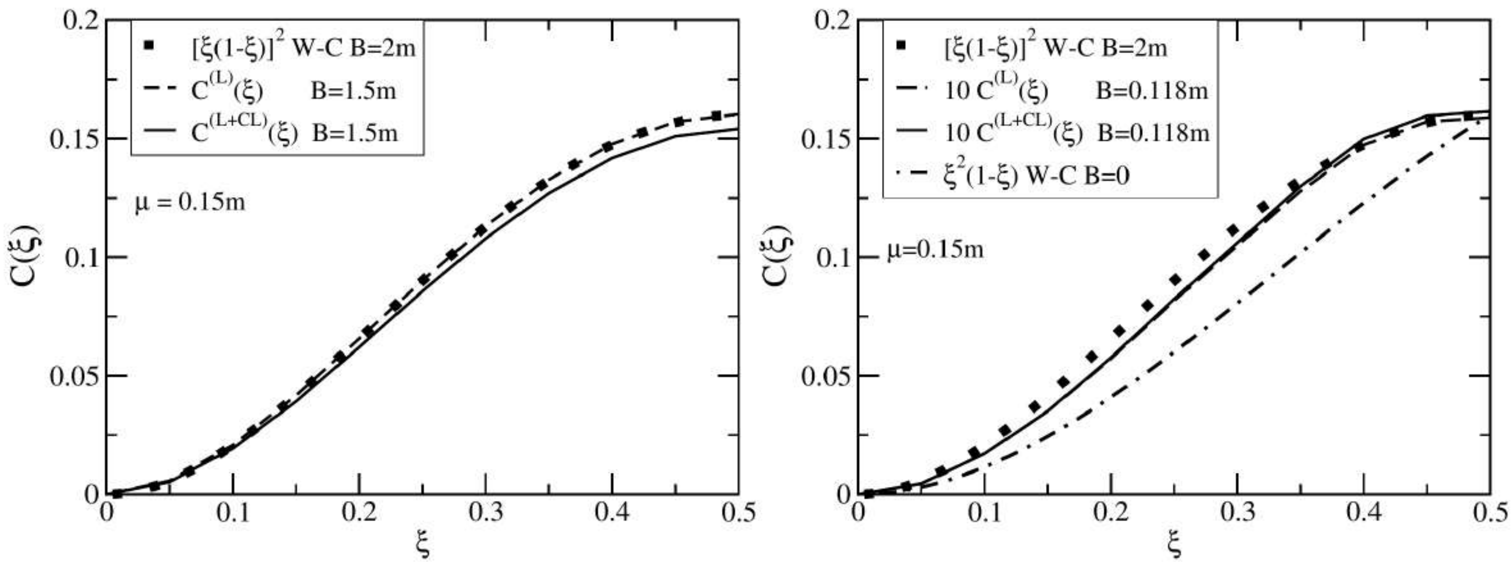


Fig. 2. Asymptotic function $C(\xi)$ defined from the LF wave function for $\gamma \to \infty$ (6) computed for the ladder kernel, $C^{(L)}(\xi)$ (dashed line), and ladder plus cross-ladder kernel, $C^{(L+CL)}(\xi)$ (solid line), with exchanged boson mass of $\mu = 0.15\,m$. Calculations are performed for $B = 1.5\,m$ (left frame) and $B = 0.118\,m$ (right frame). A comparison with the analytical forms of $C(\xi)$ valid for the Wick-Cutkosky model for $B = 2m$ (full box) and $B \to 0$ (dash-dotted line) both arbitrarily normalized.

Gigante, Nogueira, Ydrefors, Gutierrez, Karmanov, TF, PRD95(2017)056012.

Euclidean space: Nakanishi representation

Euclidean space (after the replacement $k_0 = ik_4$)

$$\Phi_E(k_v, k_4) = \int_{-1}^{1} dz' \int_0^{\infty} d\gamma' \frac{g(\gamma', z')}{(\gamma' + k_4^2 + k_v^2 + \kappa^2 - iMk_4 z')^3}$$

$k = (k_0, \vec{k})$ $\qquad \kappa^2 = m^2 - \frac{M^2}{4}$

- Note: Wick-rotation is the exact analytical continuation of the Minkowski space Nakanishi representation of the BS amplitude!

Transverse distribution: Euclidean and Minkowski

$$\phi_M^T(\mathbf{k}_\perp) \equiv \int dk^0 dk^3 \Phi(k,p) = \frac{1}{2}\int dk^+ dk^- \Phi(k,p) \text{ and}$$

$$\phi_E^T(\mathbf{k}_\perp) \equiv \mathrm{i}\int dk_E^0 dk^3 \Phi_E(k_E,p),$$

136 C. Gutierrez et al. / Physics Letters B 759 (2016) 131–137

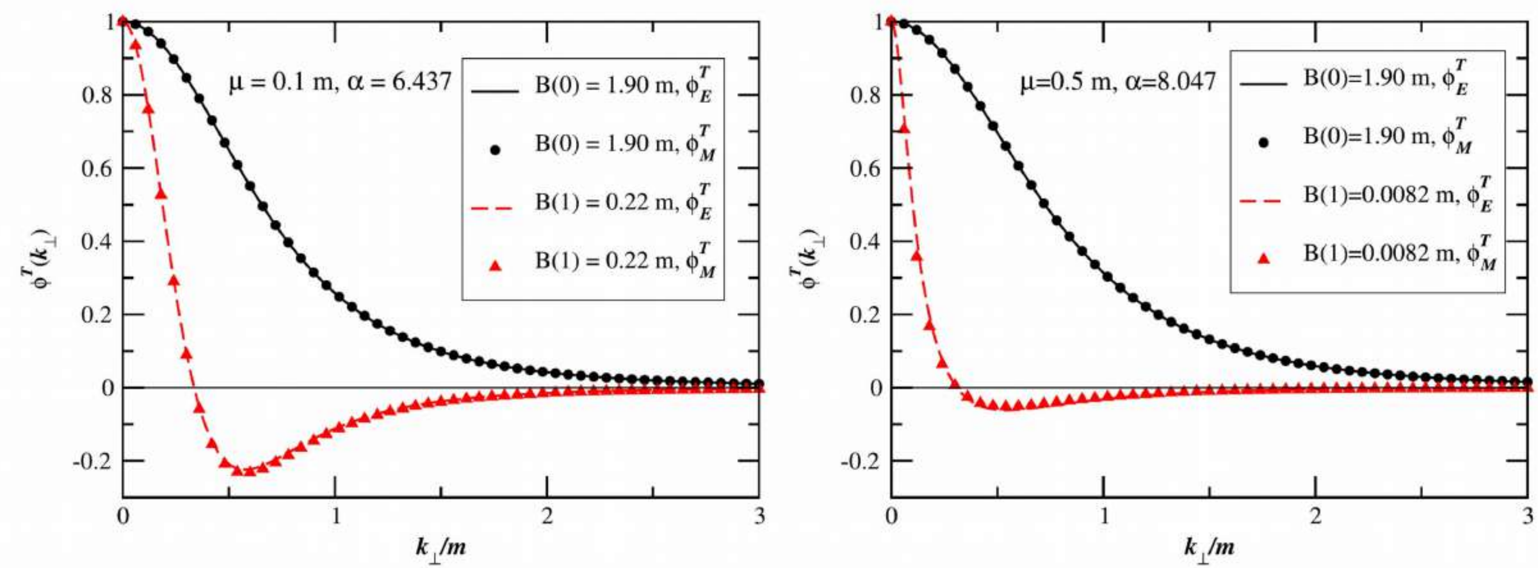


Fig. 6. Transverse momentum amplitudes *s*-wave states, in Euclidean and Minkowski spaces, vs $k_\perp$, for both ground- and first-excited states, and two values of μ/m and α_{gr} (as indicated in the insets). The amplitudes ϕ_E^T and ϕ_M^T, arbitrarily normalized to 1 at the origin, are not easily distinguishable.

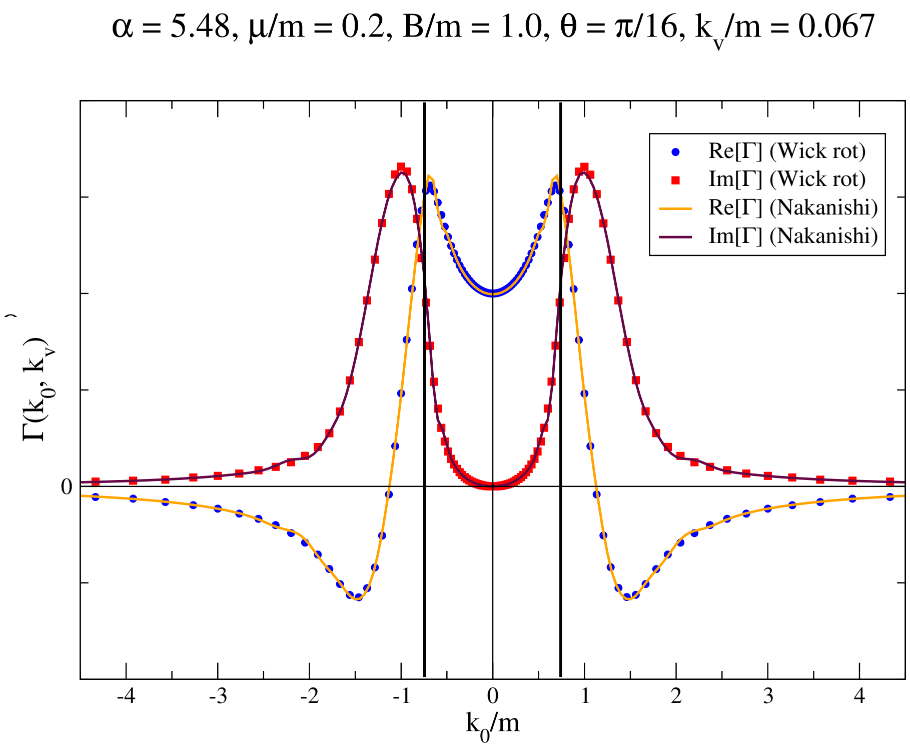
Gutierrez, Gigante, TF, Salmè,Viviani, Tomio PLB759 (2016) 131

Rotation in Complex Plane

Comparison between solution for the vertex function in the complex plane and NIR

$$\Gamma(k;P) = \imath g^2 \int \frac{d^4k'}{(2\pi)^4} \frac{\Gamma(k';P)}{((k-k')^2-\mu^2+i\epsilon)}$$
$$\times \frac{1}{((\frac{1}{2}P+k')^2-m^2+i\epsilon)((\frac{1}{2}P-k')^2-m^2+i\epsilon)}$$

$$k_0 \rightarrow k_0 \exp \imath\, \theta$$



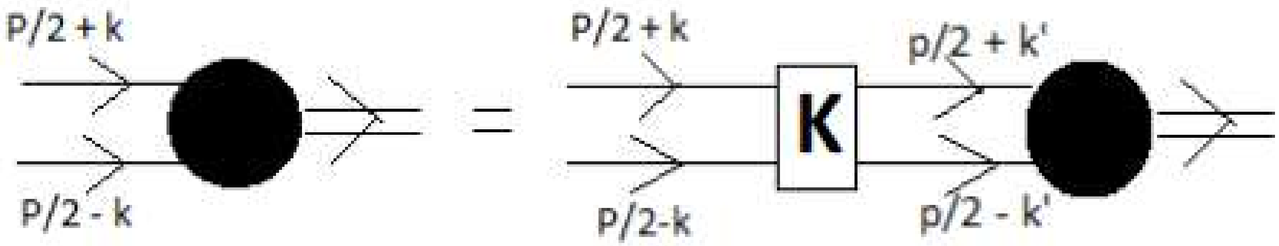
Peaks Branching points: $k_0^{\pm} = \pm\sqrt{(m+\mu)^2+(\vec{k})^2} \mp \frac{\sqrt{p^2}}{2}$

Castro, de Paula, TF, Maris, Nogueira, Ydrefors; in preparation

BSE for qqbar: pion

Carbonell and Karmanov EPJA 46 (2010) 387;

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;



$$\Phi(k,p) = S(k+p/2) \int \frac{d^4k'}{(2\pi)^4} F^2(k-k')\, i\mathcal{K}(k,k')\, \Gamma_1\, \Phi(k',p)\, \bar{\Gamma}_2\, S(k-p/2)$$

Ladder approximation (L): suppression of XL (non-planar diagram) for N_c=3

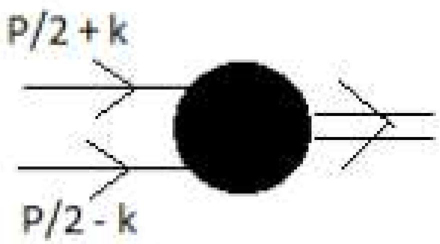
[A. Nogueira, CR Ji, Ydrefors, TF, PLB 777 (2018) 207]

Vector $$i\mathcal{K}_V^{(Ld)\mu\nu}(k,k') = -ig^2 \frac{g^{\mu\nu}}{(k-k')^2 - \mu^2 + i\epsilon}$$

Vertex Form-Factor $$F(q) = \frac{\mu^2 - \Lambda^2}{q^2 - \Lambda^2 + i\epsilon}$$

NIR for fermion-antifermion: 0- (pion)

BS amplitude



$$\Phi(k,p) = S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4$$

$$S_1 = \gamma_5 \quad S_2 = \frac{1}{M}\not{p}\gamma_5 \quad S_3 = \frac{k\cdot p}{M^3}\not{p}\gamma_5 - \frac{1}{M}\not{k}\gamma_5 \quad S_4 = \frac{i}{M^2}\sigma_{\mu\nu}p^\mu k^\nu\gamma_5$$

$$\phi_i(k,p) = \int_{-1}^{+1} dz' \int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{(k^2 + p\cdot k\ z' + M^2/4 - m^2 - \gamma' + i\epsilon)^3}$$

Light-front projection: integration over k- (LF singularities)

For the two-fermion BSE, singularities have generic form:

$$\mathcal{C}_j = \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} (k^-)^j \, \mathcal{S}(k^-, v, z, z', \gamma, \gamma') \qquad j = 1, 2, 3$$

with $\mathcal{S}(k^-, v, z, z', \gamma, \gamma')$ explicitly calculable

N.B., in the worst case

$$\mathcal{S}(k^-, v, z, z', \gamma, \gamma') \sim \frac{1}{[k^-]^2} \qquad \text{for} \quad k^- \to \infty$$

End-point singularities: T.M. Yan , Phys. Rev. **D 7**, 1780 (1973)

$$\mathcal{I}(\beta, y) = \int_{-\infty}^{\infty} \frac{dx}{\left[\beta x - y \mp i\epsilon\right]^2} = \pm \frac{2\pi i \; \delta(\beta)}{\left[-y \mp i\epsilon\right]}$$

→ Kernel with delta's and its derivatives!

End-point singularities– more intuitive: can be treated by the pole-dislocation method de Melo et al. NPA631 (1998) 574C, PLB708 (2012) 87

Numerical comparison: Scalar coupling

	$\mu/m = 0.15$			$\mu/m = 0.50$		
B/m	g^2_{dFSV}(full)	g^2_{CK}		g^2_{dFSV}(full)	g^2_{CK}	g^2_E
0.01	7.844	7.813		25.327	25.23	-
0.02	10.040	10.05		29.487	29.49	-
0.04	13.675	13.69		36.183	36.19	36.19
0.05	15.336	15.35		39.178	39.19	39.18
0.10	23.122	23.12		52.817	52.82	-
0.20	38.324	38.32		78.259	78.25	-
0.40	71.060	71.07		130.177	130.7	130.3
0.50	88.964	86.95		157.419	157.4	157.5
1.00	187.855	-		295.61	-	-
1.40	254.483	-		379.48	-	-
1.80	288.31	-		421.05	-	-

First column: binding energy.

Red digits: coupling constant g^2 for $\mu/m = 0.15$ and 0.50, with the analytical treatment of the fermionic singularities (present work). -

Black digits: results for $\mu/m = 0.15$ and 0.50, with a numerical treatment of the singularities (Carbonell & Karmanov EPJA **46**, (2010) 387).

Blue digits: results in Euclidean space from Dorkin et al FBS. **42** (2008) 1.

Scalar boson exchange

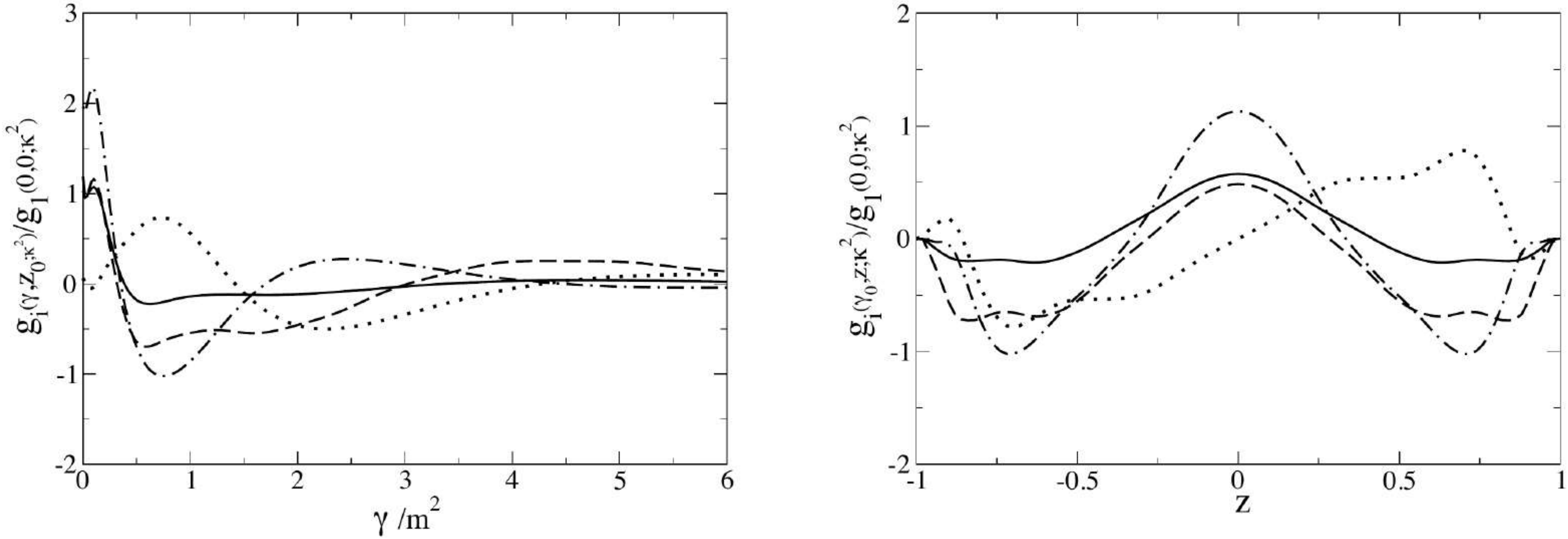
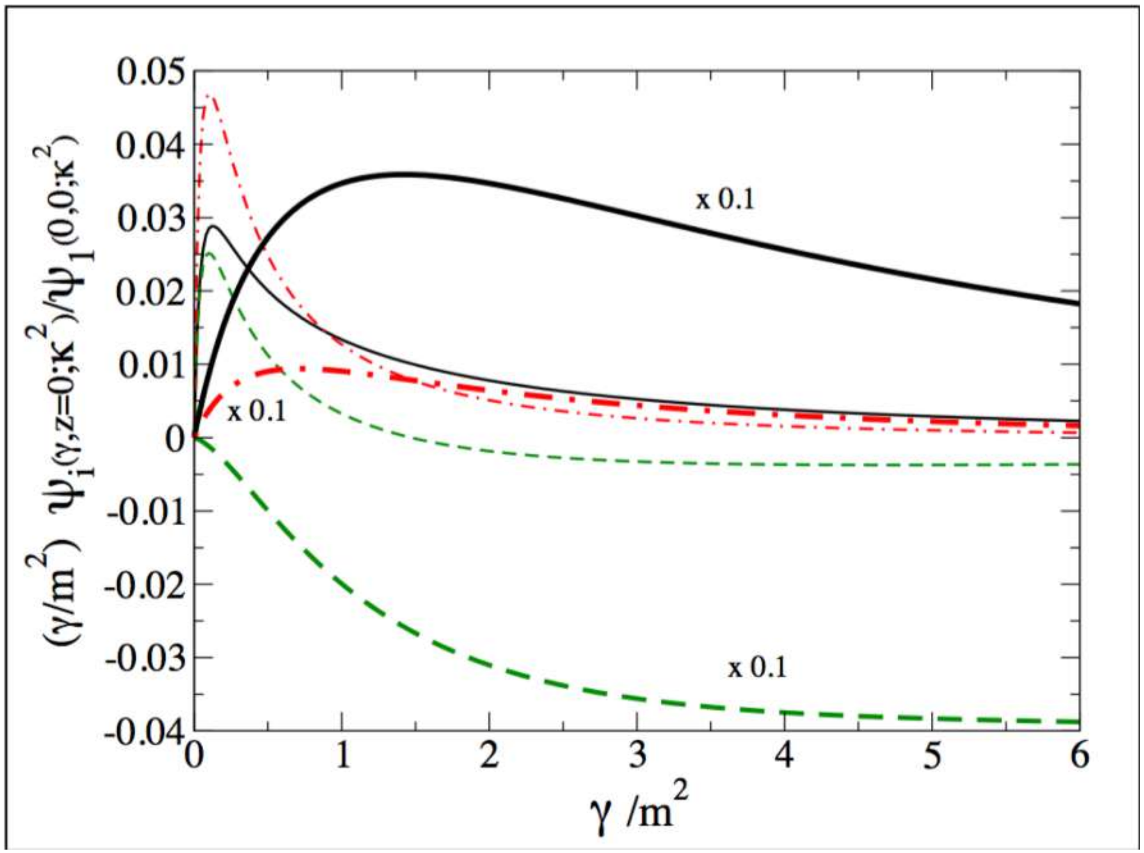


Figure 2. Nakanishi weight-functions $g_i(\gamma,\, z; \kappa^2)$, Eqs. 3.1 and 3.2 evaluated for the 0^+ two-fermion system with a scalar boson exchange such that $\mu/m = 0.5$ and $B/m = 0.1$ (the corresponding coupling is $g^2 = 52.817$ [17]). The vertex form-factor cutoff is $\Lambda/m = 2$. Left panel: $g_i(\gamma,\, z_0; \kappa^2)$ with $z_0 = 0.6$ and running γ/m^2. Right panel: $g_i(\gamma_0,\, z; \kappa^2)$ with $\gamma_0/m^2 = 0.54$ and running z, The Nakanishi weight-functions are normalized with respect to $g_1(0, 0; \kappa^2)$. Solid line: g_1. Dashed line: g_2. Dotted line: g_3. Dot-dashed line: g_4.

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;

Massless vector exchange: high-momentum tails

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;



LF amplitudes ψ_i times γ/m^2 at fixed $z=0$, for the vector coupling.
$B/m = 0.1$ (thin lines) and 1.0 (thick lines).

——— : $(\gamma/m^2)\ \psi_1$.

— —: $(\gamma/m^2)\ \psi_2$.

— • : $(\gamma/m^2)\ \psi_4$.

$\psi_3 = 0$ for $z = 0$

Power one is expected for the pion valence amplitude:

X Ji et al, PRL 90 (2003) 241601.

PION MODEL

W. de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

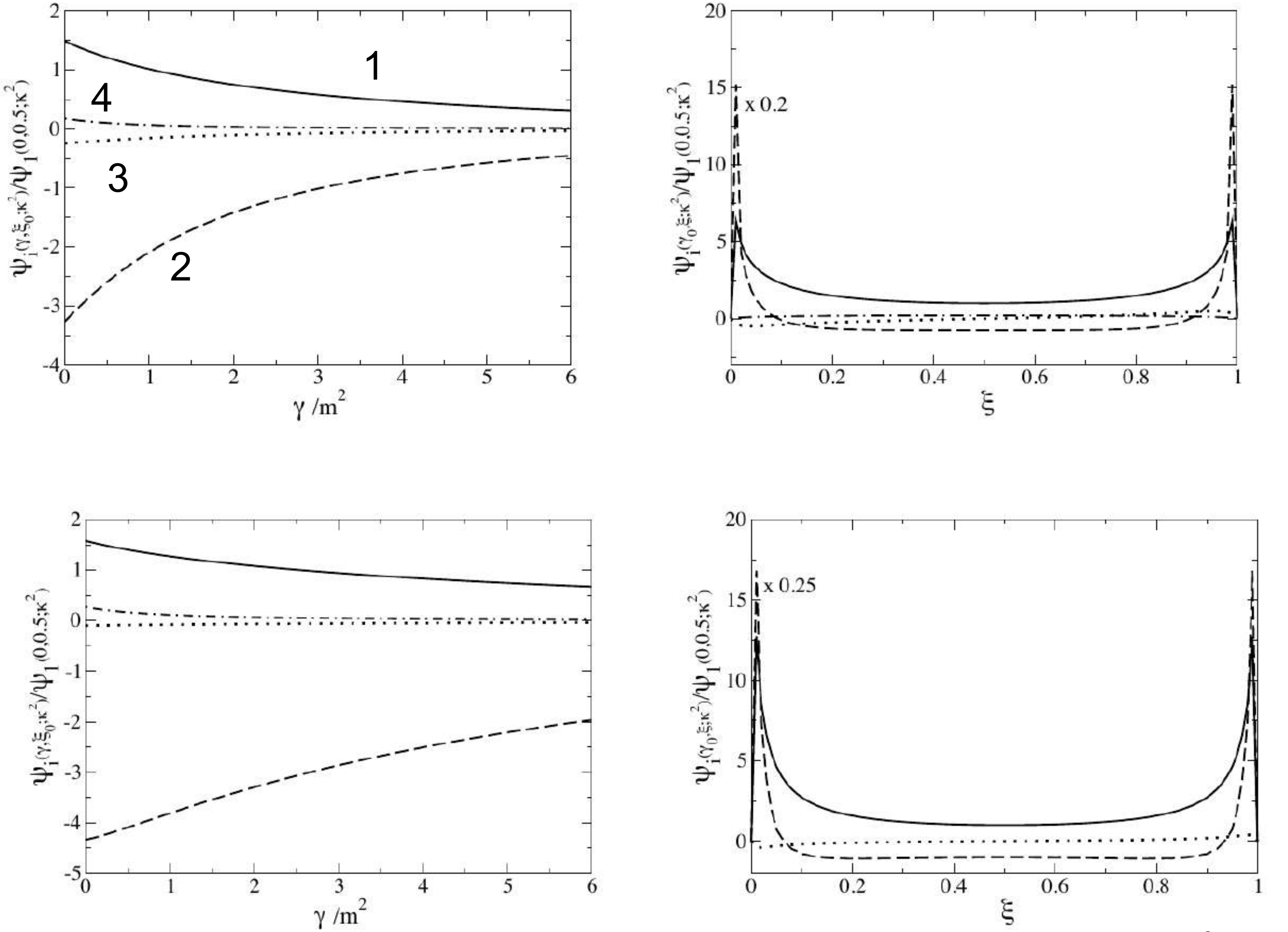
- **Gluon effective mass ~ 500 MeV – Landau Gauge LQCD**

[Oliveira, Bicudo, JPG 38 (2011) 045003;
Duarte, Oliveira, Silva, Phys. Rev. D 94 (2016) 01450240]

- **Mquark = 250 MeV**

[Parappilly, et al, PR D73 (2006) 054504]

- **Λ/m =1, 2, 3**



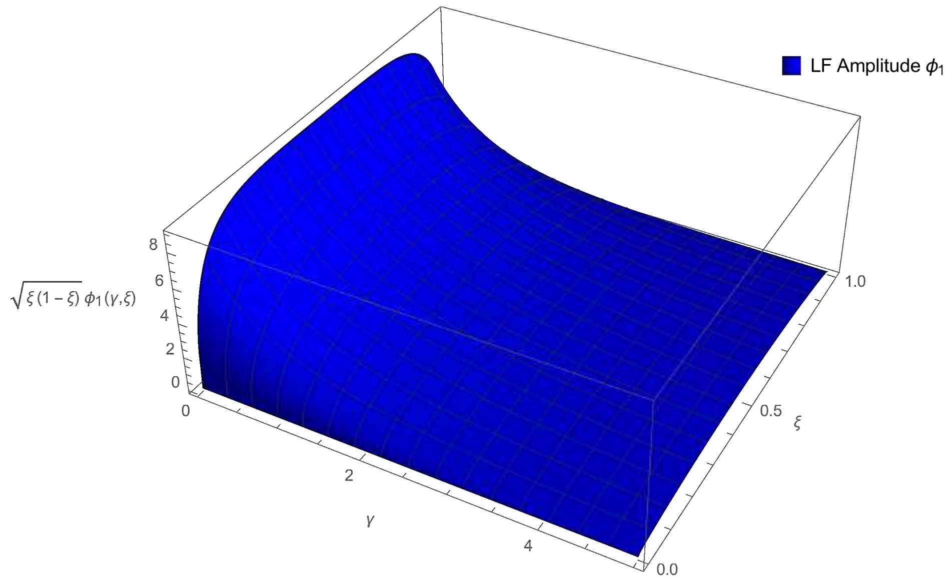
$f_\pi = 150\,\text{MeV}$

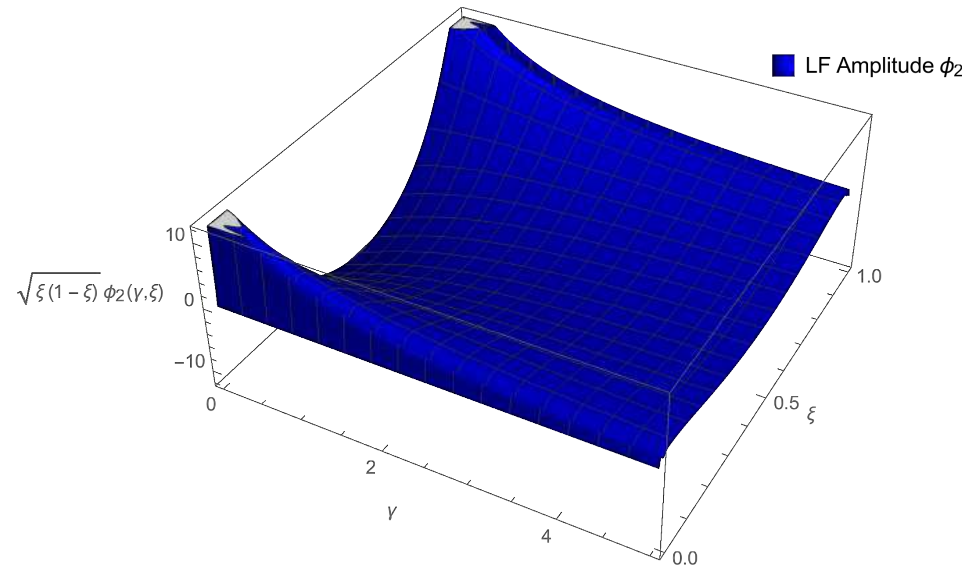
Figure 6. Light-front amplitudes $\psi_i(\gamma, \zeta)$, Eq. 3.11, for the pion-like system with a heavy-vector exchange ($\mu/m = 2$), binding energy of $B/m = 1.44$ and constituent mass $m = 250$ MeV. Upper panel: vertex form-factor cutoff $\Lambda/m = 3$ and $g^2 = 435.0$, corresponding to $\alpha_s = 10.68$ (see text for the definition of α_s). Lower panel: vertex form-factor cutoff $\Lambda/m = 8$ and $g^2 = 53.0$, corresponding to $\alpha_s = 3.71$. The value of the longitudinal variable is $\xi_0 = 0.2$ and $\gamma_0 = 0$. Solid line: ψ_1 . Dashed line: ψ_2. Dotted line: ψ_3. Dot-dashed line: ψ_4

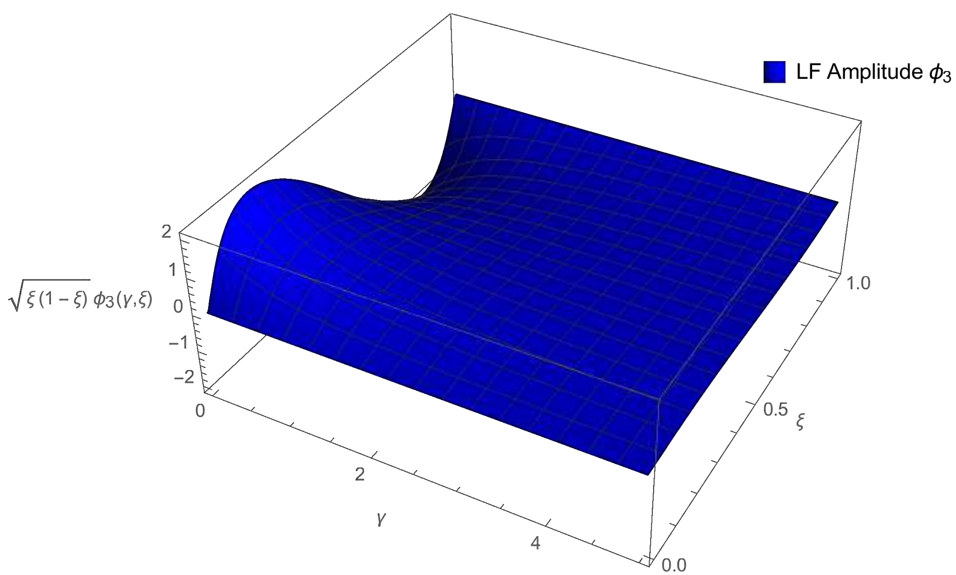
Light-front amplitudes

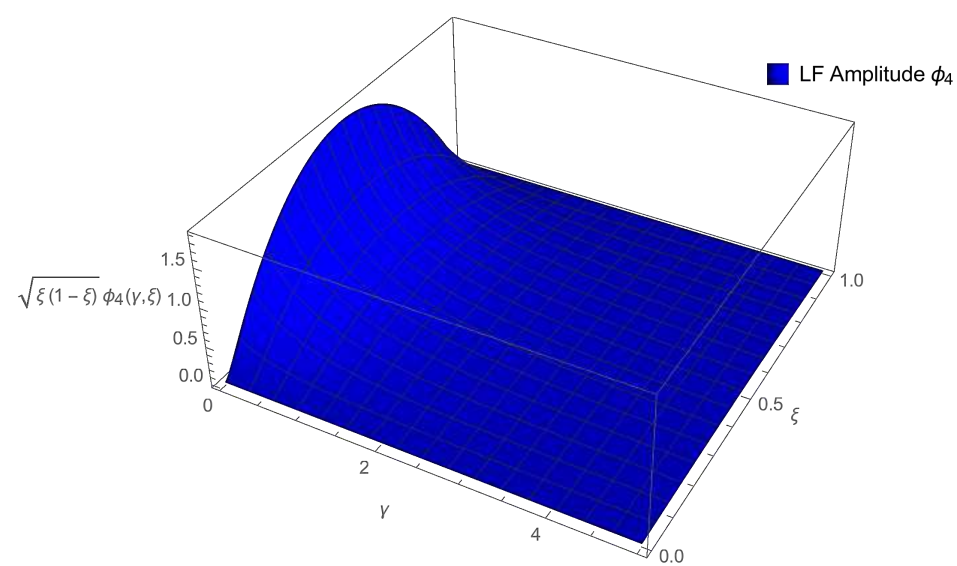
$(B/m = 1.35, \mu/m = 2.0, \Lambda/m = 1.0, \bar{m}_q$=215 MeV): $f_\pi = 96$ MeV

$P_{val} = 0.68$









Valence distribution functions

W. de Paula, et. al, in preparation

Valence probability:

$$N_2 = \frac{1}{32\,\pi^2}\int_{-1}^{1} dz \int_0^{\infty} d\gamma \left\{ \tilde{\psi}_{val}(\gamma,\xi)\ \tilde{\psi}_{val}(\gamma,\xi) + \frac{\gamma}{M^2}\ \psi_{val;4}(\gamma,\xi)\ \psi_{val;4}(\gamma,\xi) \right\}$$

$$\tilde{\psi}_{val}(\gamma,z) = -\frac{i}{M}\int_0^{\infty} d\gamma' \frac{g_2(\gamma',z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2-i\epsilon]^2}$$
$$-\frac{i}{M}\frac{z}{2}\int_0^{\infty} d\gamma' \frac{g_3(\gamma',z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2-i\epsilon]^2}$$
$$+\frac{i}{M^3}\int_0^{\infty} d\gamma' \ \frac{\partial g_3(\gamma',z)/\partial z}{[\gamma+\gamma'+z^2m^2+(1-z^2)\kappa^2-i\epsilon]}$$

$$\psi_{val;4}(\gamma,z) = -\frac{i}{M}\int_0^{\infty} d\gamma' \frac{g_4(\gamma',z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2-i\epsilon]^2}.$$

Valence probability

Table 1 Valence probability for a massive vector exchange, with $\mu/m = 0.15$ and a cut-off $\Lambda/m = 2$ for the vertex form-factor. The number of gaussian points is 72.

B/m	Prob.
0.01	0.96
0.1	0.78
1.0	0.68

Table 2 Valence probability for a massive vector exchange, with $\mu/m = 0.5$ and a cut-off $\Lambda/m = 2$ for the vertex form-factor. The number of gaussian points is 72.

B/m	Prob.
0.01	0.96
0.1	0.84
1.0	0.68

Lot of room for the higher LF Fock components of the wave function to manifest!

Valence distribution functions: longitudinal and transverse

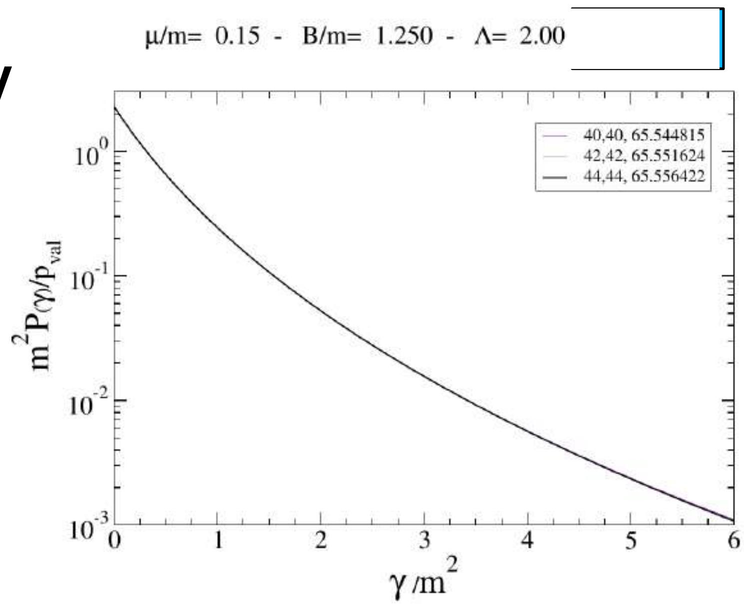
Mquark 187 MeV

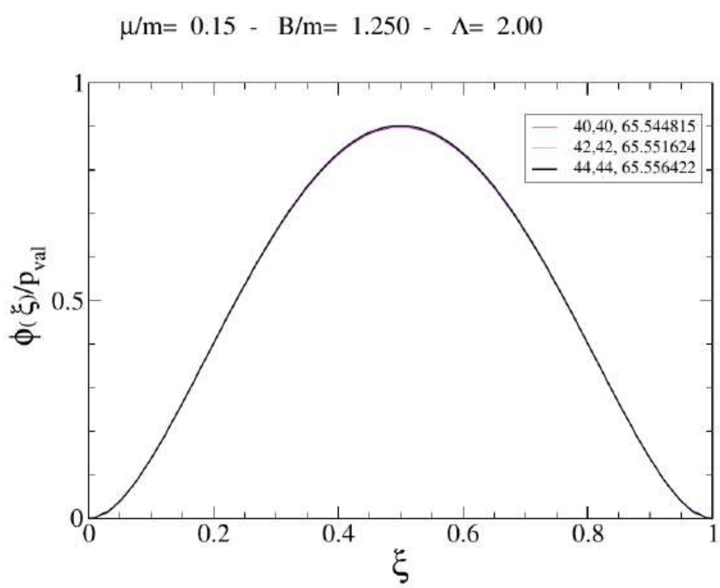
Mgluon 28 MeV

Λ/m =2

Pval=0.64

$f_\pi = 77\,\mathrm{MeV}$





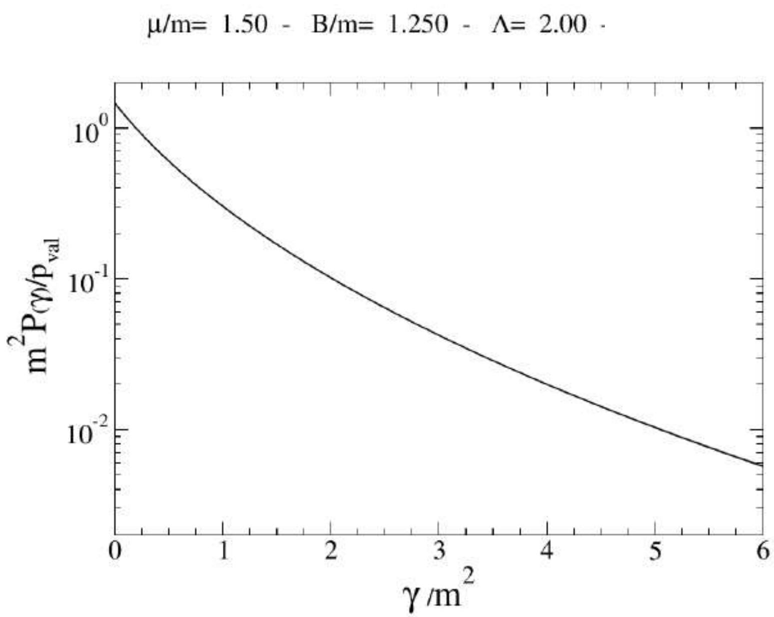
Mquark 187 MeV

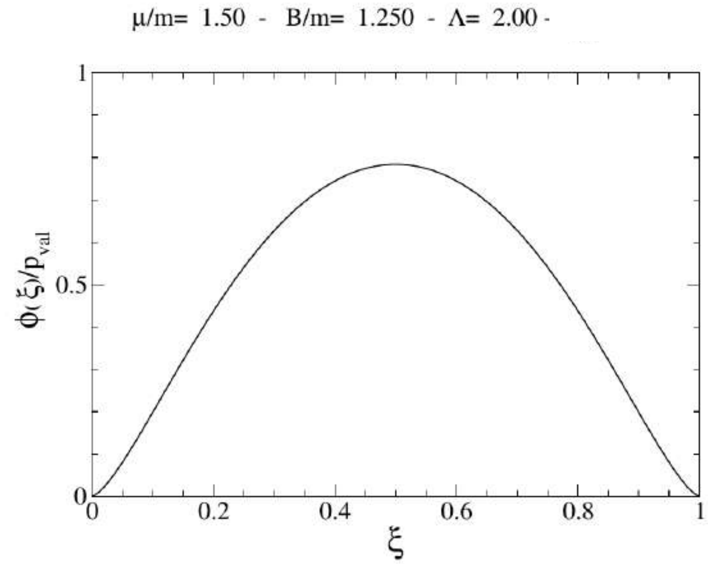
Mgluon 280 MeV

Λ/m =2

Pval=0.78

$f_\pi = 99\,\mathrm{MeV}$





Preliminary result for a fermion-scalar bound system

The covariant decomposition of the BS amplitude for a $(1/2)^+$ bound system, composed by a fermion and a scalar, reads

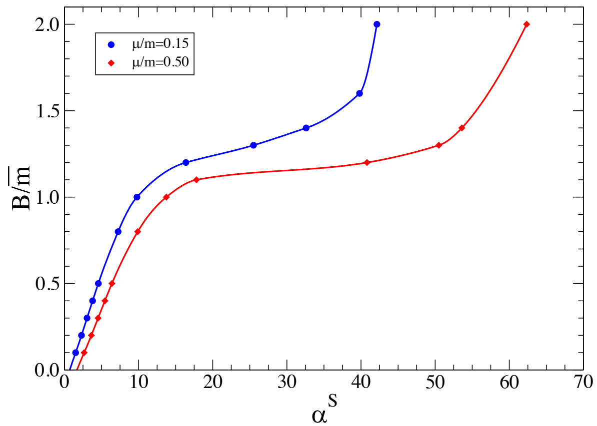
with A. Nogueira, Salmè and Pace

$$\Phi(k,p) = \Big[S_1 \ \phi_1(k,p) + S_2 \ \phi_2(k,p) \Big] \ U(p,s)$$

with $U(p,s)$ a Dirac spinor, $S_1(k) = 1$, $S_2(k) = \not{k}/M$,and $M^2 = p^2$

A first check: scalar coupling $\alpha^s = \lambda^s_F \lambda^s_S / (8\pi m_S)$, for $m_F = m_S$ and $\mu/\bar{m} = 0.15, \ 0.50$

$B/\bar{m}$	$\alpha^s_M(0.15)$	$\alpha^s_{WR}(0.15)$	$\alpha^s_M(0.50)$	$\alpha^s_{WR}(0.50)$
0.10	1.5057	1.5057	2.6558	2.6558
0.20	2.2969	2.2969	3.2644	3.6244
0.30	3.0467	3.0467	4.5354	4.5354
0.40	3.7963	3.7963	5.4505	5.4506
0.50	4.5680	4.5681	6.4042	6.4043
0.80	7.2385	7.2387	9.8789	9.8794
1.00	9.7779	9.7783	13.7379	13.7380

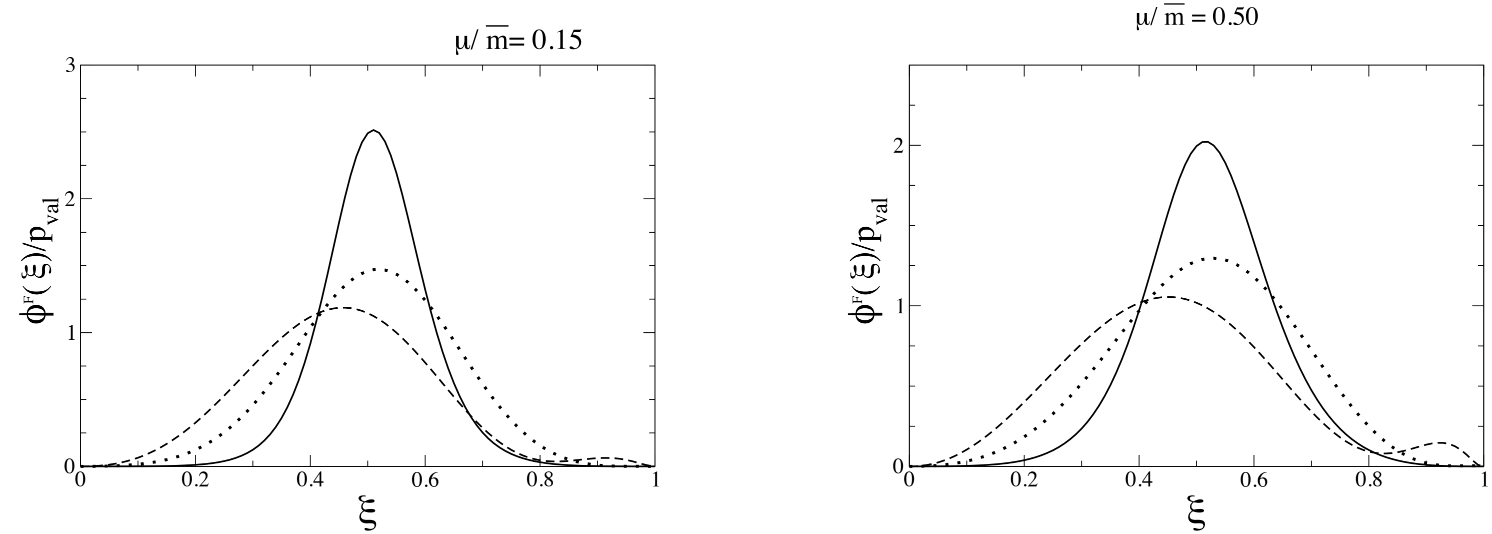


First column: the binding energy in unit of $\bar{m} = (m_S + m_F)/2$..

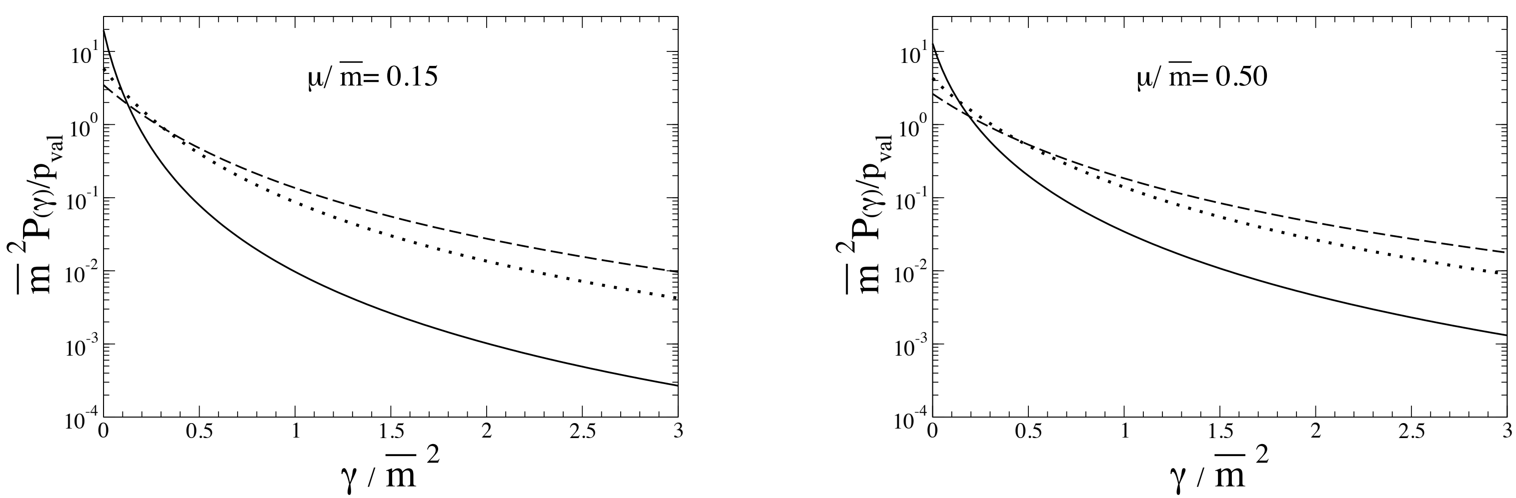
Second and fourth columns: coupling constant α_M, obtained by solving the BSE in Minkowski space, for given $B/\bar{m}$.

Third and fifth columns: Wick-rotated results, α_{WR}.

Fermion-scalar system interacting through a massive scalar exchange



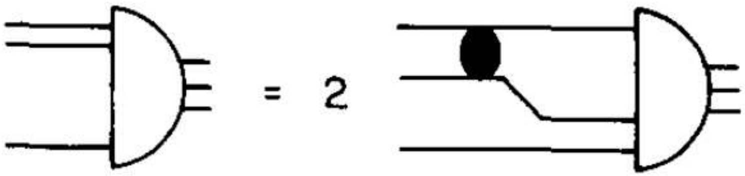
Longitudinal light-cone distribution for a fermion in the valence component. Solid line : $B/\bar{m} = 0.1$. Dotted line: $B/\bar{m} = 0.5$. Dotted line: $B/\bar{m} = 1.0$

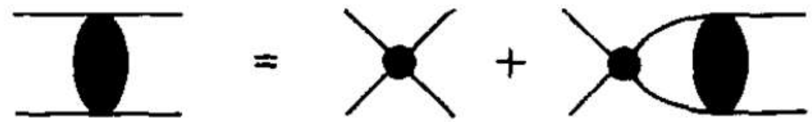


Transverse light-cone distribution for a fermion in the valence component.

Relativistic Three-body Bound states with contact interaction

Ydrefors, Alvarenga Nogueira, et al. PLB 770 (2017)131

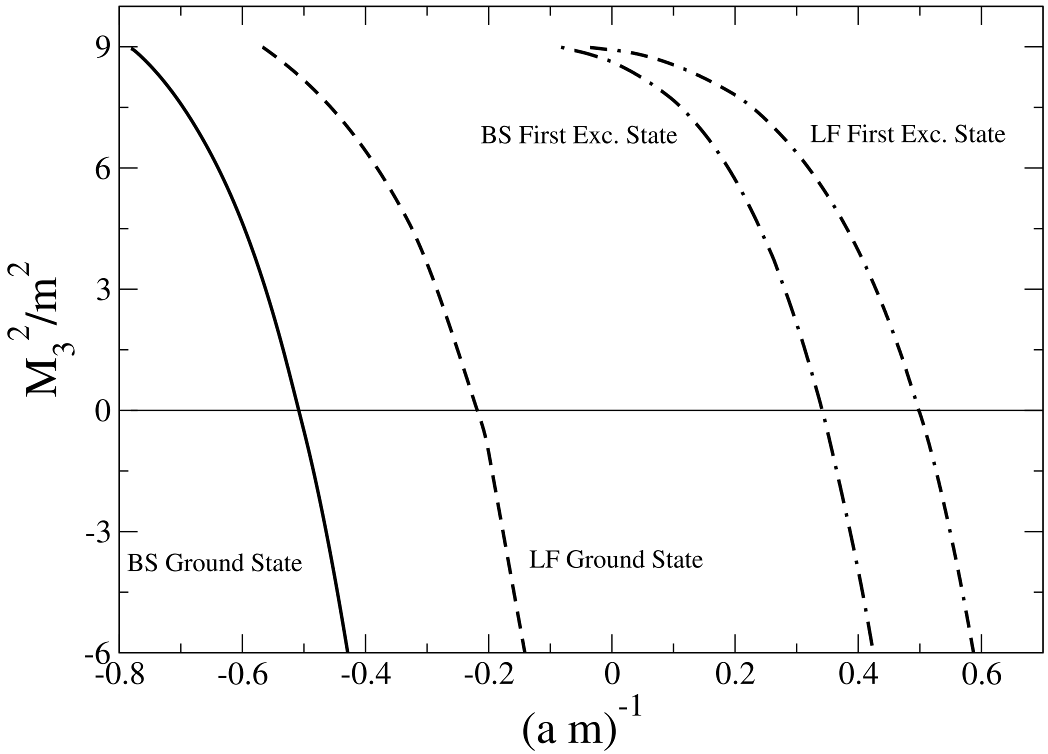




$$v(q,p) = 2iF(M_{12}) \int \frac{d^4k}{(2\pi)^4} \frac{i}{[k^2 - m^2 + i\epsilon]} \frac{i}{[(p-q-k)^2 - m^2 + i\epsilon]} v(k,p).$$

$$F(M_{12}) = \begin{cases} \dfrac{8\pi^2}{\dfrac{1}{2y'_{M_{12}}} \log \dfrac{1+y'_{M_{12}}}{1-y'_{M_{12}}} - \dfrac{\pi}{2am}}, & \text{if } M_{12}^2 < 0 \\[2ex] \dfrac{8\pi^2}{\dfrac{\arctan y_{M_{12}}}{y_{M_{12}}} - \dfrac{\pi}{2am}}, & \text{if } 0 \le M_{12}^2 < 4m^2 \end{cases}$$

Wick rotation after the transformation $k = k' + \frac{1}{3}p, \quad q = q' + \frac{1}{3}p.$



Faddeev-BSE in Eucl. space
vs.
Truncation in the LF valence sector

V.A. Karmanov, P. Maris, Few-Body Syst. 46 (2009) 95.

LF Missing induced three-body forces

E. Ydrefors et al. / Physics Letters B 770 (2017) 131–137

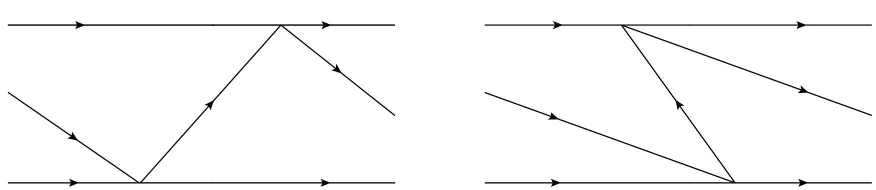


Fig. 2. The three-body LF graphs obtained by time-ordering of the Feynman graph shown in right panel of Fig. 1.

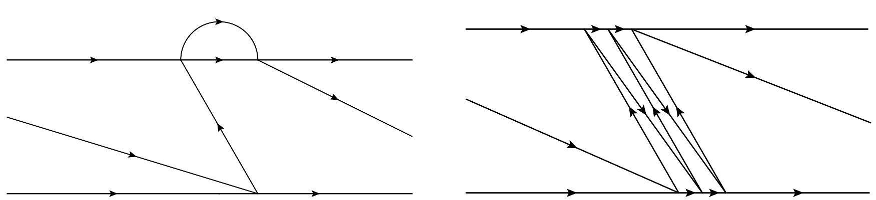


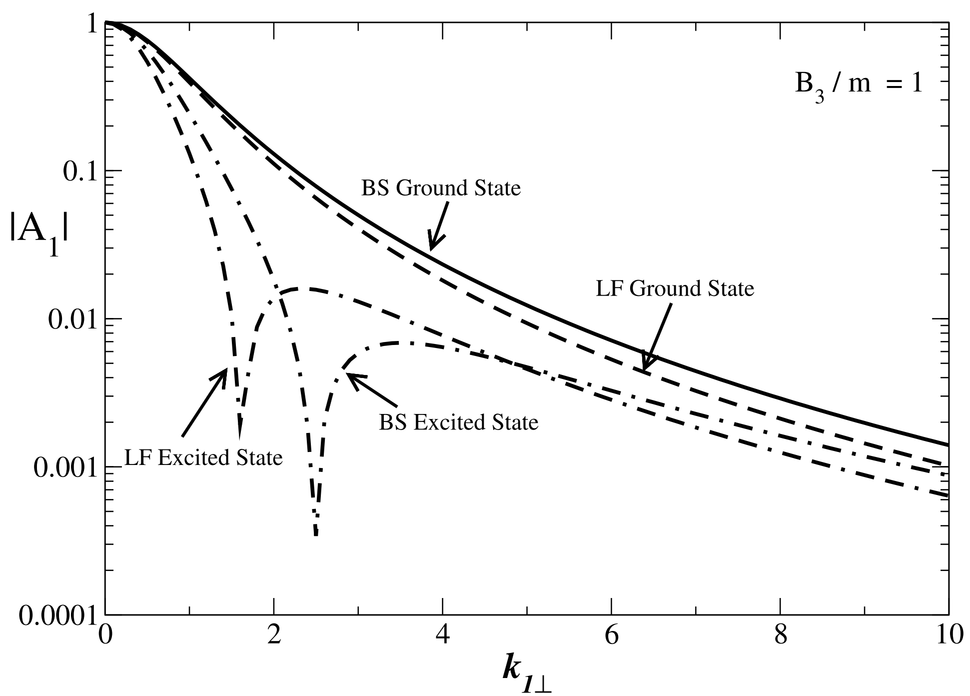
Fig. 3. Examples of many-body intermediate state contributions to the LF three-body forces.

Transverse amplitude

$$A_1^{BS} = \int \tilde{v}_E(k_{14}, k_{1v})\, \beta(k_{14}, k_{1z}; \vec{k}_{1\perp}, \vec{k}_{2\perp}) dk_{14} dk_{1z},$$

$$\beta(k_{14}, k_{1z}; \vec{k}_{1\perp}, \vec{k}_{2\perp}) = -\frac{\chi(k_{14}, k_{1z}; E_{2\perp}, E_{3\perp})}{\left[\left(k_{14} - \frac{i}{3} M_3\right)^2 + k_{1z}^2 + E_{1\perp}^2\right]},$$

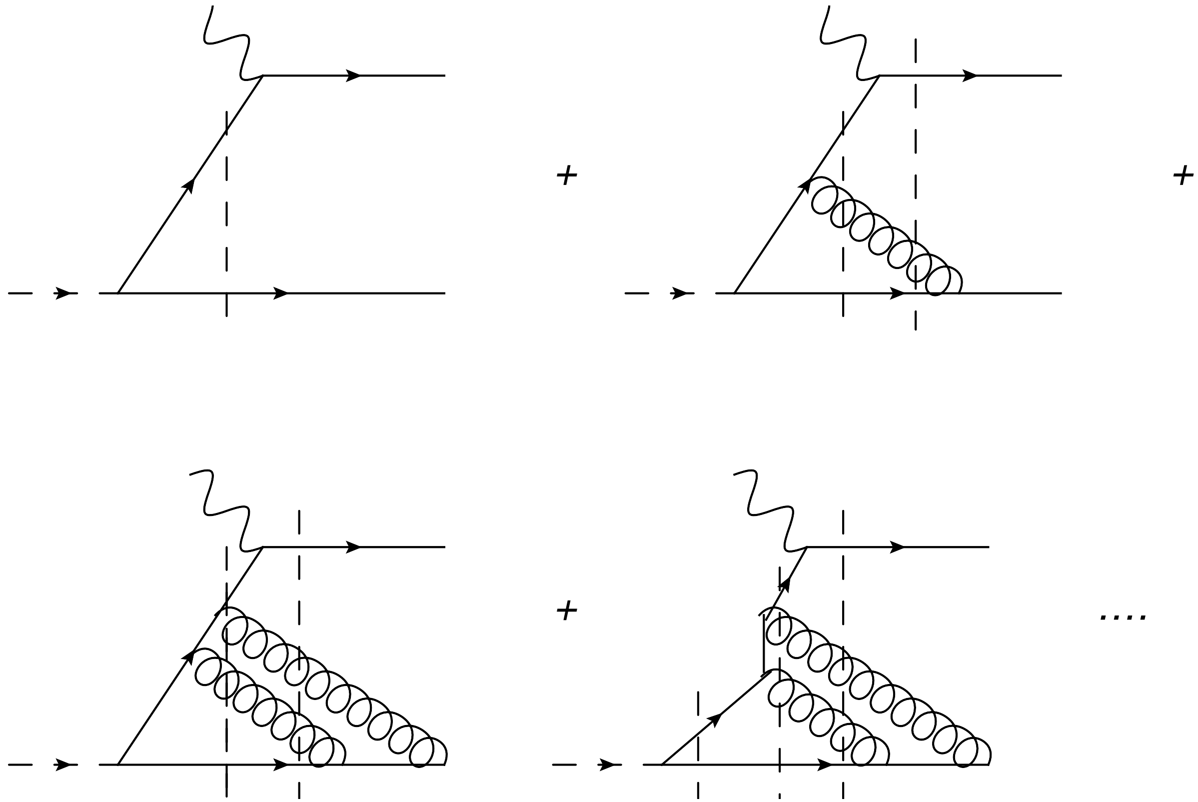
$$\chi(k_{14}, k_{1z}; \vec{k}_{1\perp}, \vec{k}_{2\perp}) = \int_0^1 \frac{\pi\, dy}{ay^2 + by + c},$$



Beyond the valence ….

Sales, TF, Carlson,Sauer, PRC 63, 064003 (2001)

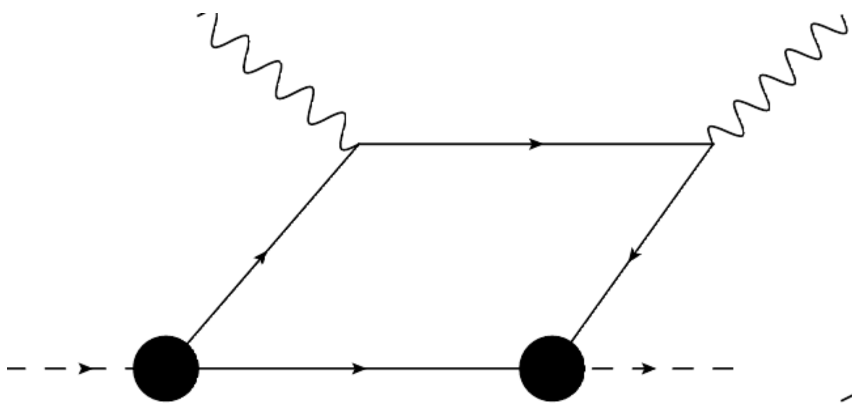
Marinho, TF, Pace,Salme,Sauer, PRD 77, 116010 (2008)

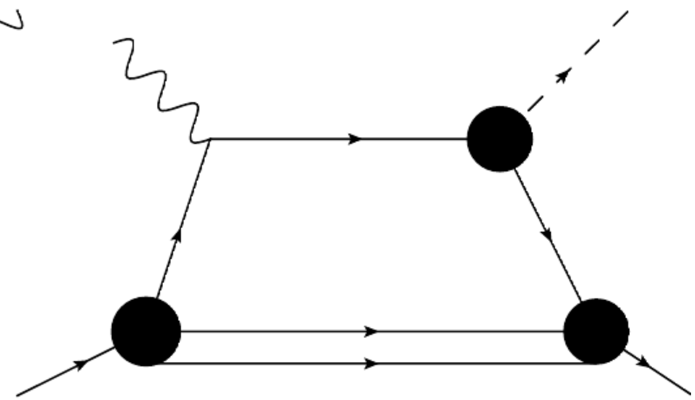


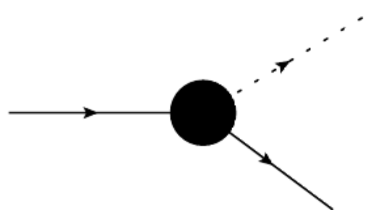
- **Population of lower x, due to the gluon radiation!**
- **Evolution?**

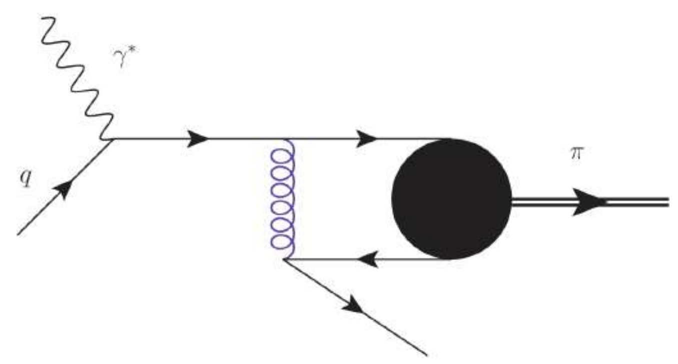
Beyond the valence ….

ERBL – DGLAP regions









Fragmentation function

Conclusions and Perspectives

- **A method for solving bosonic and fermionic BSE: NIR (LF singularities-fermions);**
- **Nakanishi Integral Representation and fermions and fermion-boson BSE's;**
- **Euclidean BSE for 3-bosons; [Minkowski space solution (under construction)]**
- **Self-energies, vertex corrections, Landau gauge, ingredients from LQCD….**
- **Confinement?**
- **Beyond the pion, kaon, D, B, rho…, and the nucleon**
- **Form-Factors, PDFs, TMDs, Fragmentation Functions...**

THANK YOU!

LIA/CNRS - SUBATOMIC PHYSICS: FROM THEORY TO APPLICATIONS

IPNO (Jaume Carbonell)…. + Brazilian Institutions …

Numerical method

$$g_b^{(Ld)}(\gamma, z; \kappa^2) = \sum_{\ell=0}^{N_z} \sum_{j=0}^{N_g} A_{\ell j} G_\ell(z) \mathcal{L}_j(\gamma)$$

$$G_\ell(z) = 4(1-z^2)\Gamma(5/2)\sqrt{\frac{(2\ell+5/2)(2\ell)!}{\pi\Gamma(2\ell+5)}} C_{2\ell}^{(5/2)}(z)$$

even Gegenbauer polynomials

$$\mathcal{L}_j(\gamma) = \sqrt{a}\, L_j(a\gamma) e^{-a\gamma/2}$$

Laguerre polynomials

Solution of the eigenvalue problem for g^2 for each given B

B=2m-M binding energy