TSALLIS STATISTICS AND HEAVY QUARK TRANSPORT IN QUARK GLUON PLASMA

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Boltzmann-Gibbs (BG) distribution is not universal.

Many anomalous natural, and social systems exist for which *BG* statistical concepts appear to be inapplicable

Some of them can be handled using the techniques of Statistical Mechanics by introducing a more general entropy called the Tsallis (a.k.a Tsallis non-extensive) entropy.

Tsallis distribution is given by the following expression:

$$f = \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}}$$

$$q \to 1 \Rightarrow f \to f_{\mathrm{Boltzmann}} \equiv e^{-\frac{E-\mu}{T}}$$

I Bediaga, E M F Curado and J M de Miranda, Phys. A 286, 156 (2000); C Beck, Phys. A 286, 164 (2000);
J Cleymans, D Worku, Eur. Phys. J A 48, 160 (2012)

‘Effective’ Boltzmann factor

$$k_{eff} = \int f(\beta)\, e^{-\beta\, E}\, d\beta$$

Replace $f(\beta)$by a χ^2distribution

Boltzmann statistics: with a single value of Boltzmann temperature $T = 1/\beta$

Distribution of T

$$k_{\text{eff}} = \left(1 + \langle\beta\rangle(q-1)E\right)^{\frac{1}{1-q}} \equiv e_q^{-\langle\beta\rangle E}$$

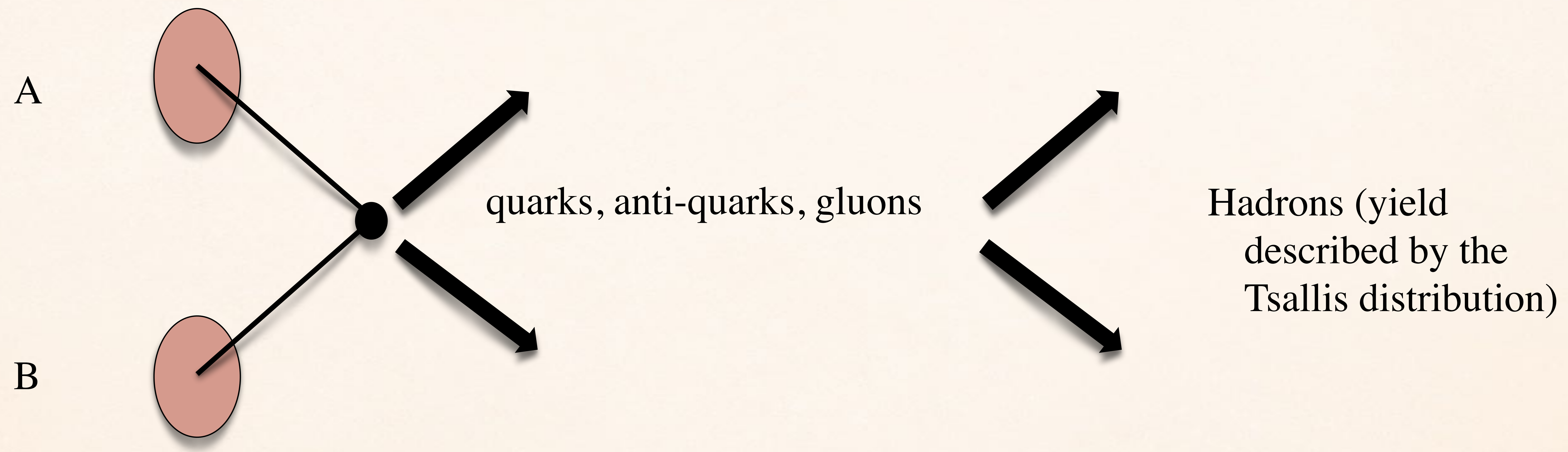
$$q - 1 = \frac{\left\langle \beta^2 \right\rangle - \langle\beta\rangle^2}{\langle\beta\rangle^2}$$

Tsallis (inverse) temperature, average of all Boltzmann (inverse) temperatures

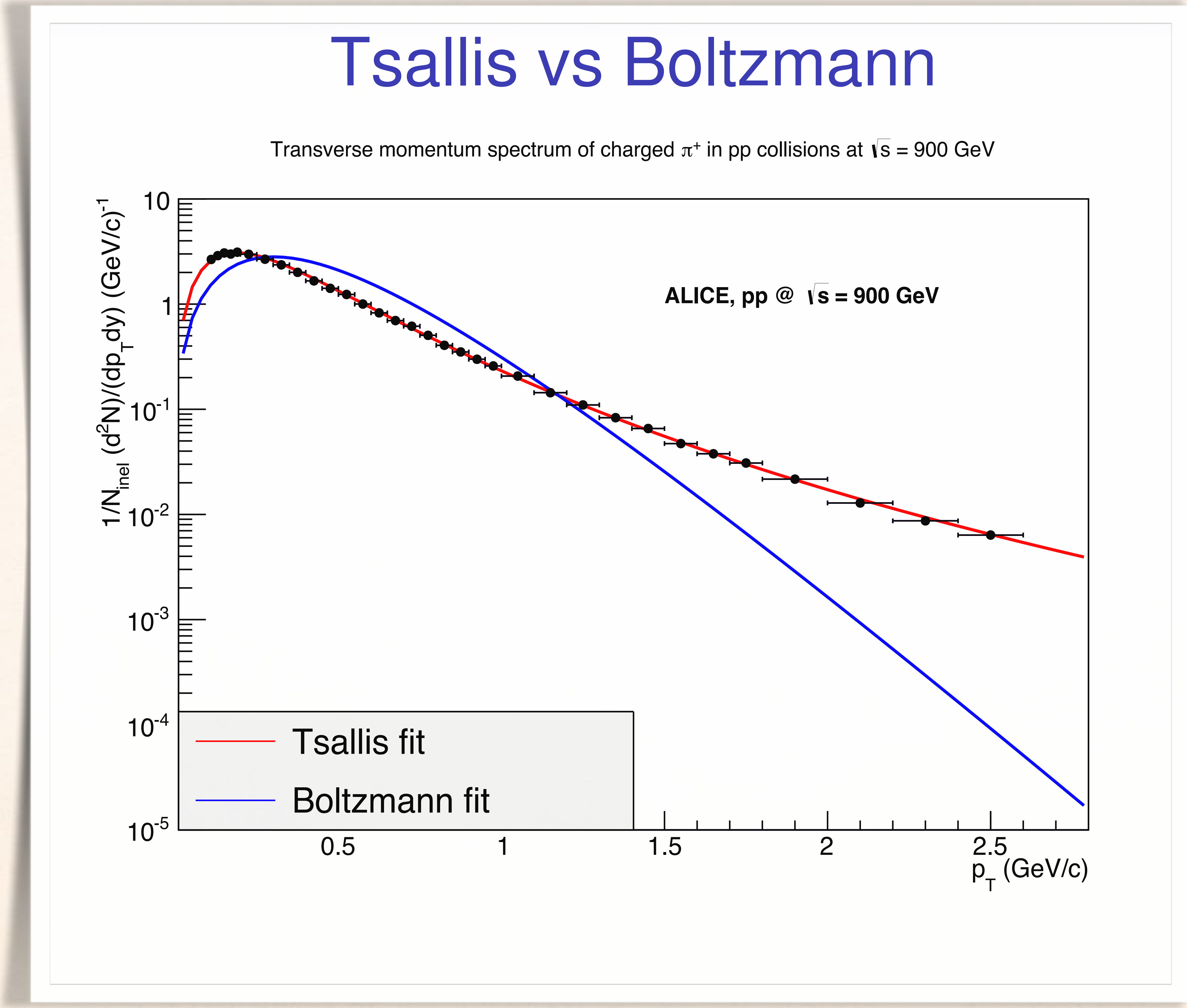
Tsallis parameter, relative variance in Boltzmann (inverse) temperature

G. Wilk and Z. Włodarczyk, Phys. Rev. Lett. 84, 2770(2000)

Tsallis distribution in high energy collision: Particle spectra

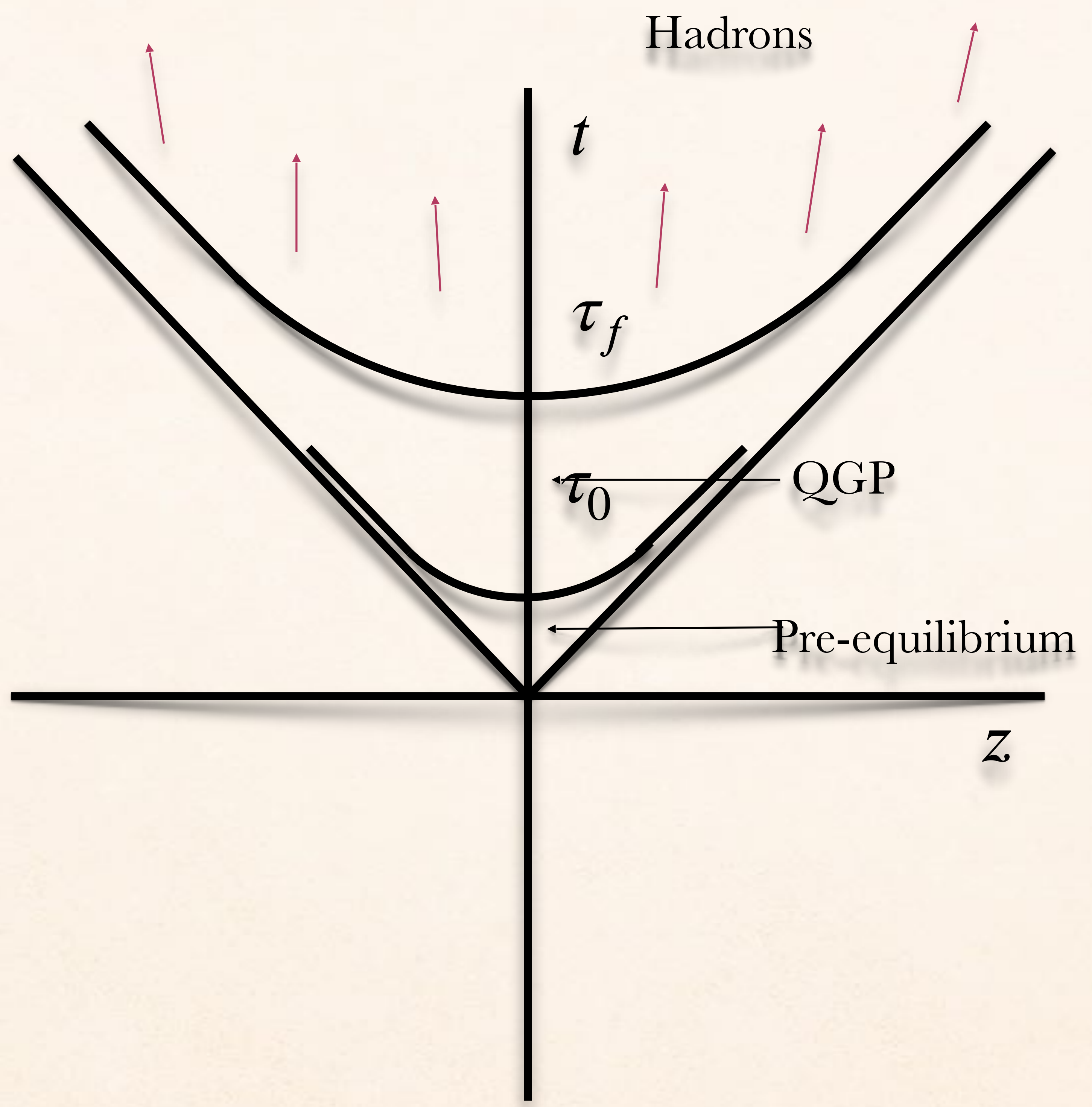


Spectrum for ALICE p-p 900 GeV: Tsallis vs Boltzmann



Tsallis distribution in high energy collision: Evolution equation

Evolution of what ? To answer this let us look at the space-time diagram



The low energy quarks and gluons created due to the collisions interact with each other and after a (proper) time τ_0 (thermalization time), they form evolving QGP. (Evolution 1)

High energy particles (like high energy charm and bottom quarks) barely thermalize with the medium and they act as the evolving 'probes' to evolving QGP.
(Evolution 2) $\longrightarrow$ Boltzmann Transport Equation (BTE)

An exercise: Boltzmann transport equation in the relaxation time approximation (RTA)

$$\frac{\partial f}{\partial t} = -\frac{f - f_{\mathrm{eq}}}{\tau}$$

$$f_{in} = \frac{gV}{(2\pi)^2} p_T m_T \left[1 + (q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}}$$

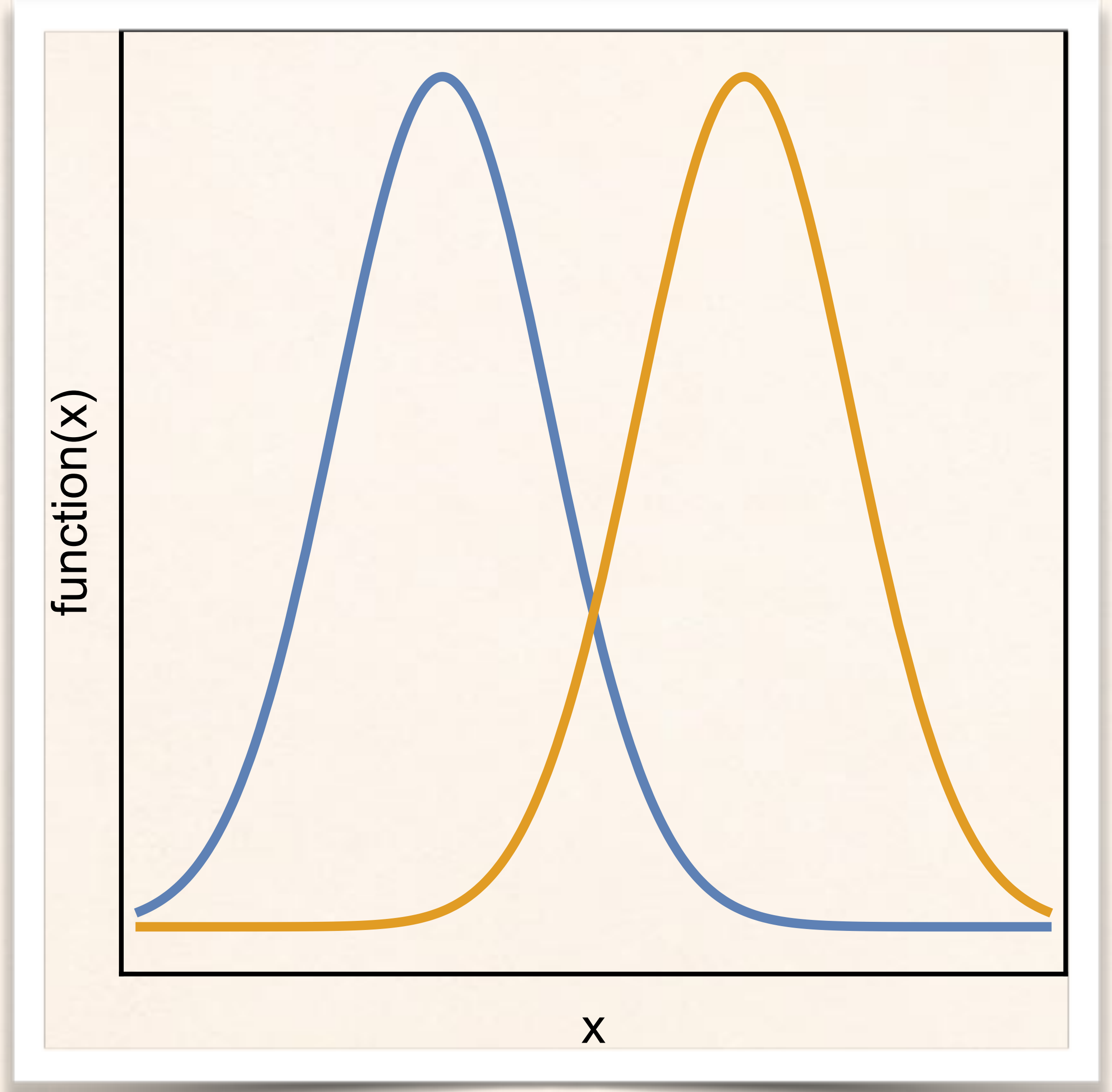
$$f_{eq} = \frac{gV}{(2\pi)^2} p_T m_T e^{-\frac{m_T}{T_{eq}}}$$

$$R_{AA} = \frac{e^{-\frac{m_T}{T_{eq}}}}{(1 + (q-1)\frac{m_T}{T})^{-\frac{q}{q-1}}} + \left[1 - \frac{e^{-\frac{m_T}{T_{eq}}}}{(1 + (q-1)\frac{m_T}{T})^{-\frac{q}{q-1}}}\right] e^{-\frac{t_f}{\tau}}$$

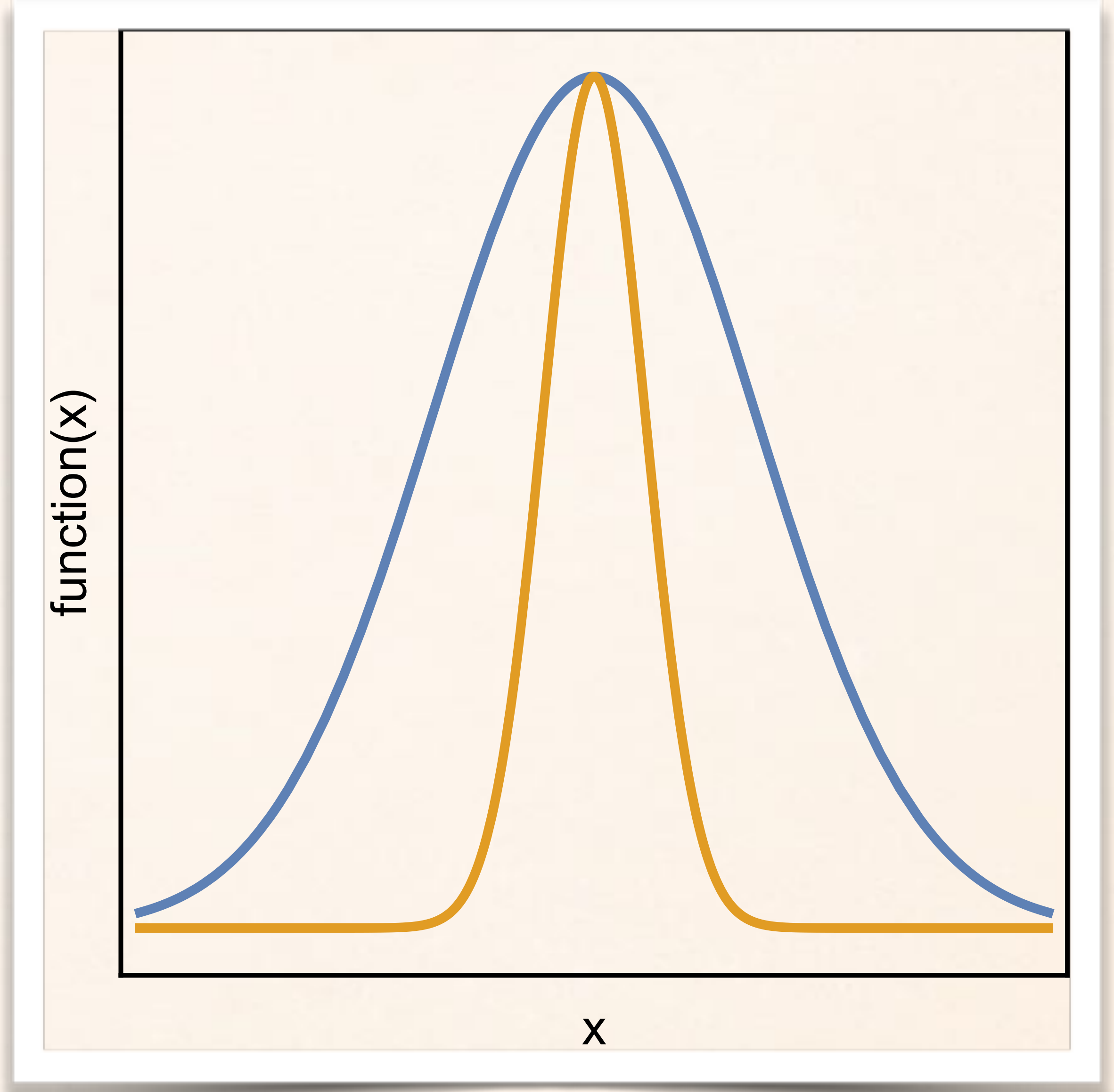
S Tripathy, T Bhattacharyya, P Garg, P Kumar, R Sahoo and J Cleymans Eur. Phys. J A 52, 289 (2016)

Too simplistic: Where is drag ? Where is diffusion ?

Drag



Diffusion



The quark-gluon plasma medium through which particles are moving is barely ideal.

Existence of the quasi-stationary states like the one given by the Tsallis distribution as opposed to the Boltzmann distribution

Indications that we need to have a modified kinetic equation (i.e. a modified Boltzmann transport equation) to deal with such situations.

Boltzmann Transport Equation: Recap

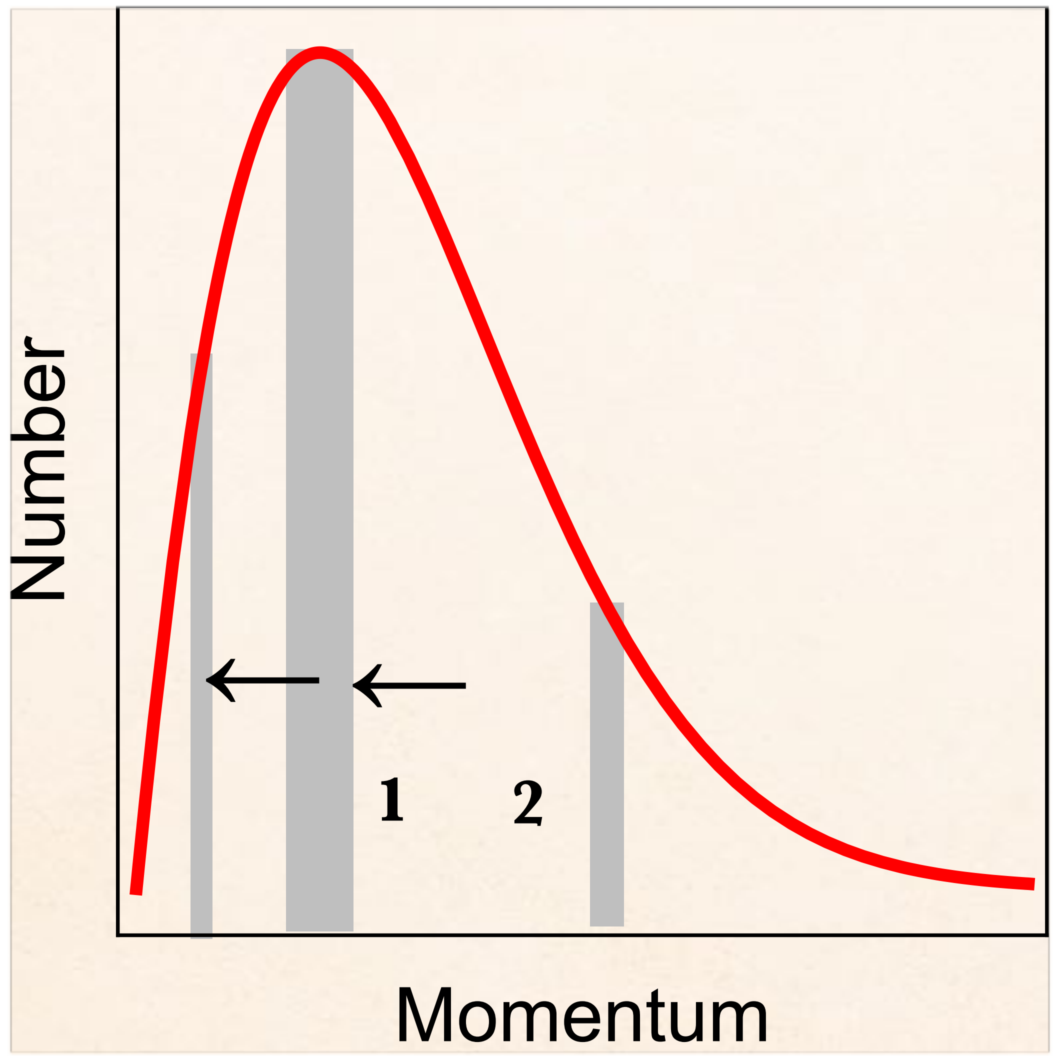
$$\frac{d}{dt} f(\vec{x}, \vec{p}; t) = C[f]$$

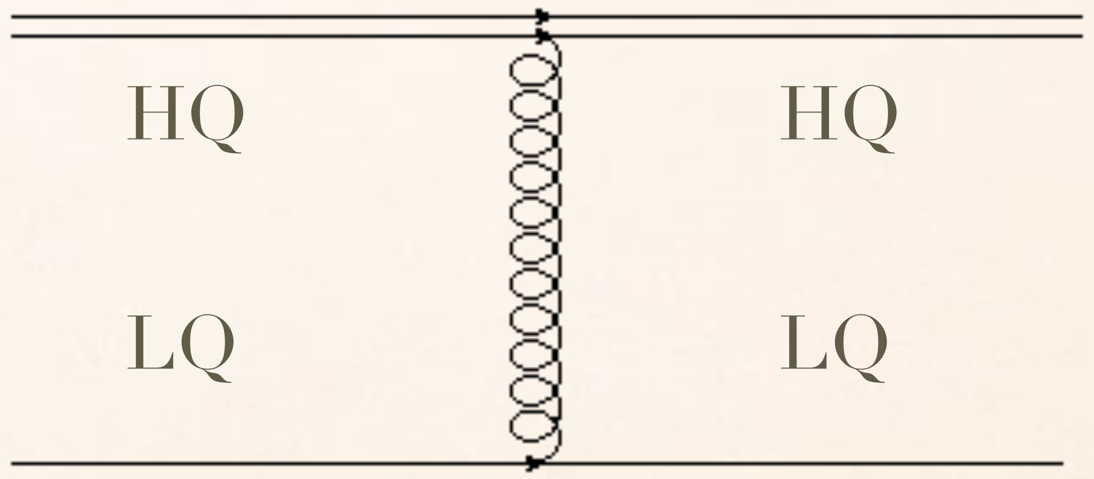
Molecular chaos

$$f_2(h, l) \equiv f_1(h) \times f_1(l)$$

$C[f]$

Kinematics $\otimes$ Kinetics





Generalized Boltzmann Transport Equation

Molecular chaos (MC)

$$f_1(h) \times f_1(l) = e^{\ln[f_1(h)]} \times e^{\ln[f_1(l)]}$$

Generalized Molecular chaos (GMC)

Ansatz

$$f_1(h) \otimes_q f_1(l) = e_q^{\ln_q[f_1(h)]} \times e_q^{\ln_q[f_1(l)]}$$

$$e_q(x) = [1-(q-1)x]_+^{1/(1-q)} \ ; \ \ln_q(x) = \frac{1-x^{1-q}}{q-1}$$

$$f_2(h,l) = f_1(h)\, f_1(l) + (q-1)\, f_1(h)\, f_1(l)\, \ln[f_1(h)]\, \ln[f_1(l)] + \mathcal{O}[(q-1)^2] + \ldots.$$

Generalized BTE: Low momentum transfer

Molecular chaos ⇒ BTE → Fokker-Planck

Generalized Molecular chaos
⇒ non-linear BTE → non-linear Fokker-Planck

$$\frac{\partial f}{\partial t} = -\frac{\partial[A_i^{\mathrm{NE}} f]}{\partial p_i} + \frac{\partial[B_{ij}^{\mathrm{NE}} f^{2-q}]}{\partial p_i \partial p_j}$$

G Wolschin, Phys. Lett. B 569, 67(2003), A. Lavagno Braz. Jour. of Phys. **35**, 516 (2005)

$$A_i = \frac{1}{2E_{\mathbf{p}}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{d^3\mathbf{q}'}{(2\pi)^3} \frac{d^3\mathbf{p}'}{(2\pi)^3} |\overline{M}|^2 (2\pi)^4 \times \delta^4(p + q - p^{'} - q^{'}) \;\; f(\mathbf{q}) \;\; (\mathbf{p} - \mathbf{p}')_i$$

$$B_{ij} = \frac{1}{2E_{\mathbf{p}}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{d^3\mathbf{q}'}{(2\pi)^3} \frac{d^3\mathbf{p}'}{(2\pi)^3} |\overline{M}|^2 (2\pi)^4 \times \delta^4(p + q - p^{'} - q^{'}) \;\; f(\mathbf{q}) \;\; \frac{1}{2}(\mathbf{p}' - \mathbf{p})_i (\mathbf{p}' - \mathbf{p})_j$$

Linear Fokker-Planck drag and diffusion coefficients of the heavy quarks

B. Svetitsky
Phys Rev D 37, 2484 (1988)

S Mazumder, TB, J Alam, S K Das
Phys Rev C 84, 044901 (2011)

S Mazumder, TB, J Alam
Phys. Rev. D 89, 014002 (2014)

D B Walton and J Rafelski
Phys. Rev. Lett. 84, 31 (2014)

Non-linear Fokker-Planck drag and diffusion coefficients of the heavy quarks

TB and J Cleymans arXiv: 1707.08425

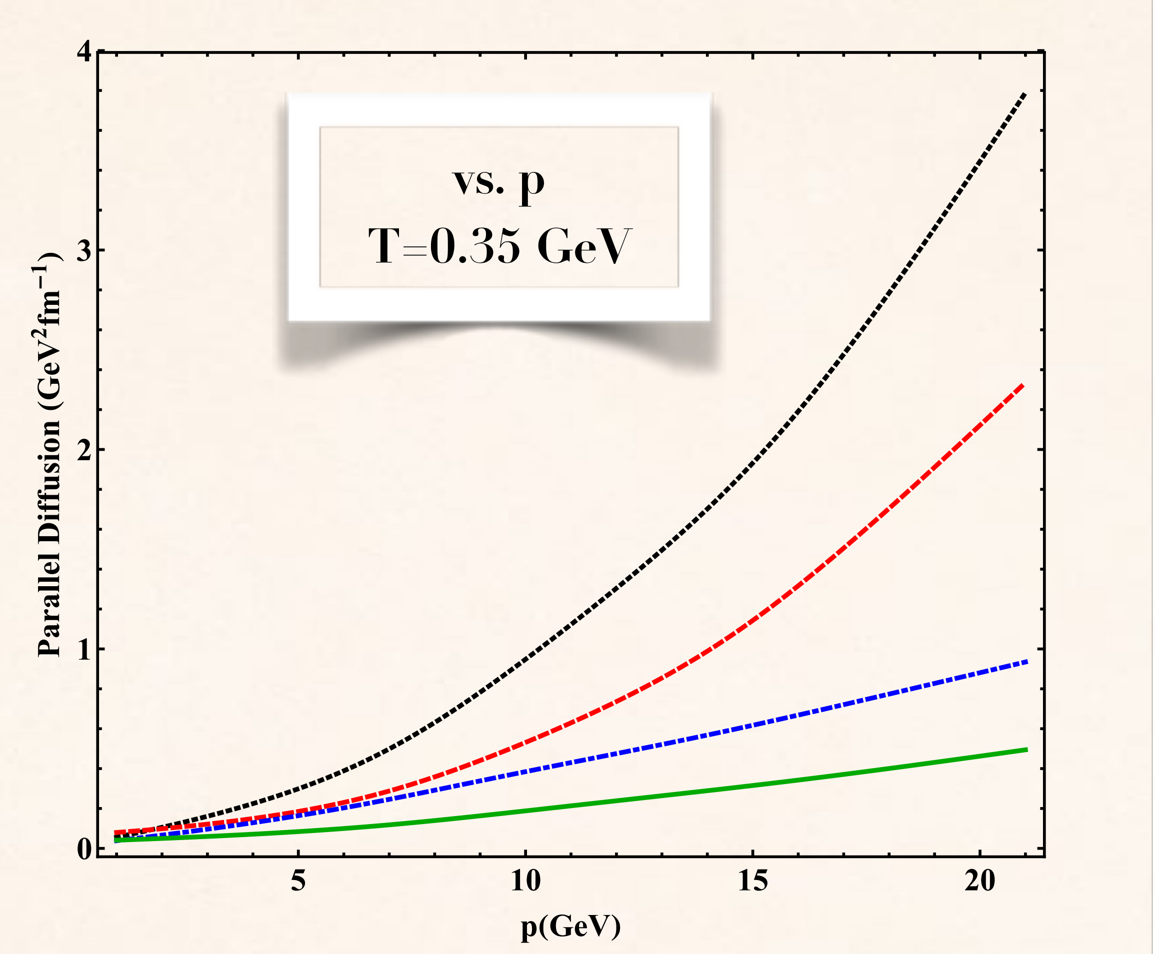
$$A_i^{\mathrm{NE}} = \frac{1}{2E_{\mathbf{p}}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{d^3\mathbf{q}'}{(2\pi)^3} \frac{d^3\mathbf{p}'}{(2\pi)^3} |\overline{M}|^2 (2\pi)^4 \times \delta^4(p + q - p^{'} - q^{'}) \times \mathcal{R}^1_{\mathbf{p},\mathbf{q}} \;\; (\mathbf{p} - \mathbf{p}')_i$$

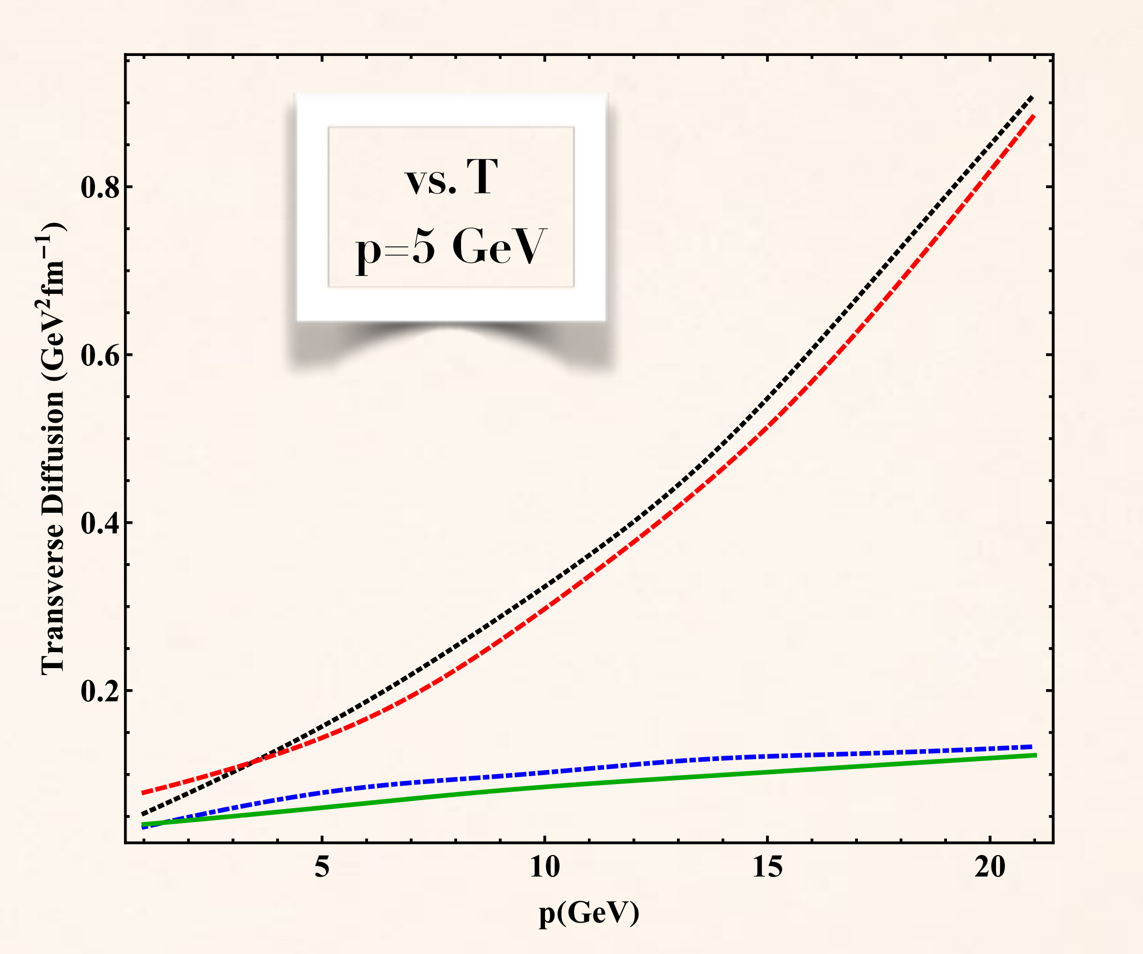
$$B_{ij}^{\mathrm{NE}} = \frac{1}{2E_{\mathbf{p}}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{d^3\mathbf{q}'}{(2\pi)^3} \frac{d^3\mathbf{p}'}{(2\pi)^3} |\overline{M}|^2 (2\pi)^4 \times \delta^4(p + q - p^{'} - q^{'}) \mathcal{R}^2_{\mathbf{p},\mathbf{q}} \;\; \frac{1}{2}(\mathbf{p}' - \mathbf{p})_i (\mathbf{p}' - \mathbf{p})_j$$

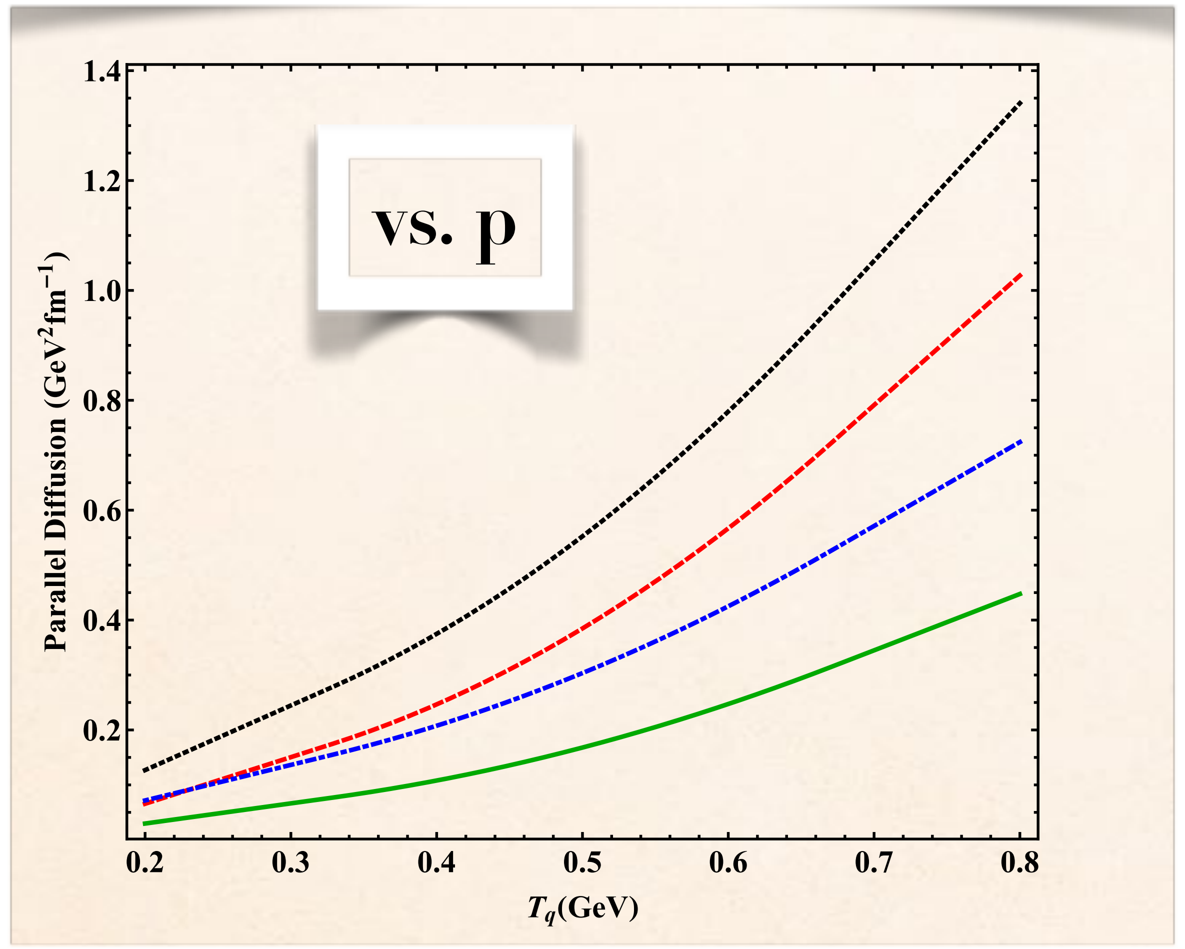
Parallel diffusion

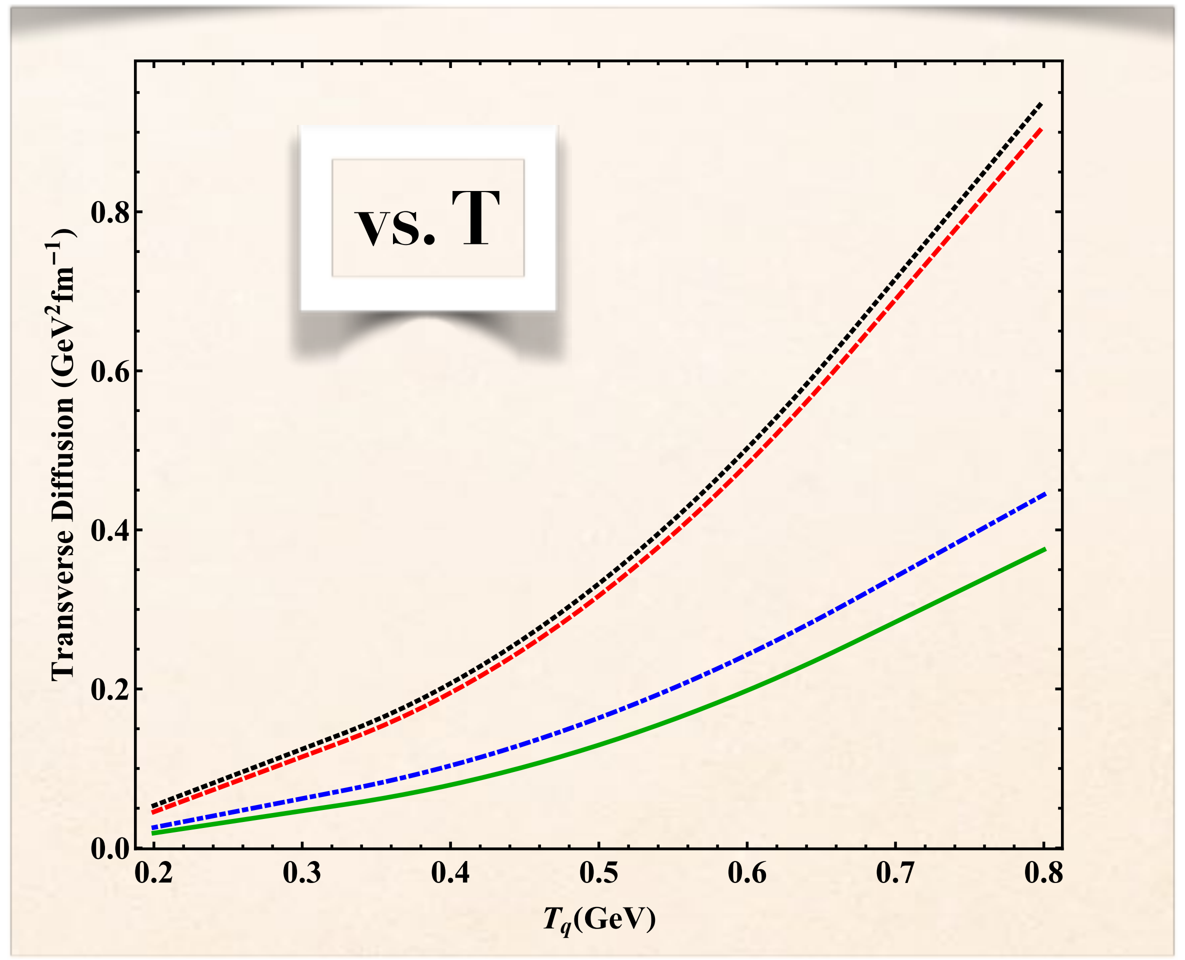
c: NL
b: NL
c: L
b:L

Transverse diffusion

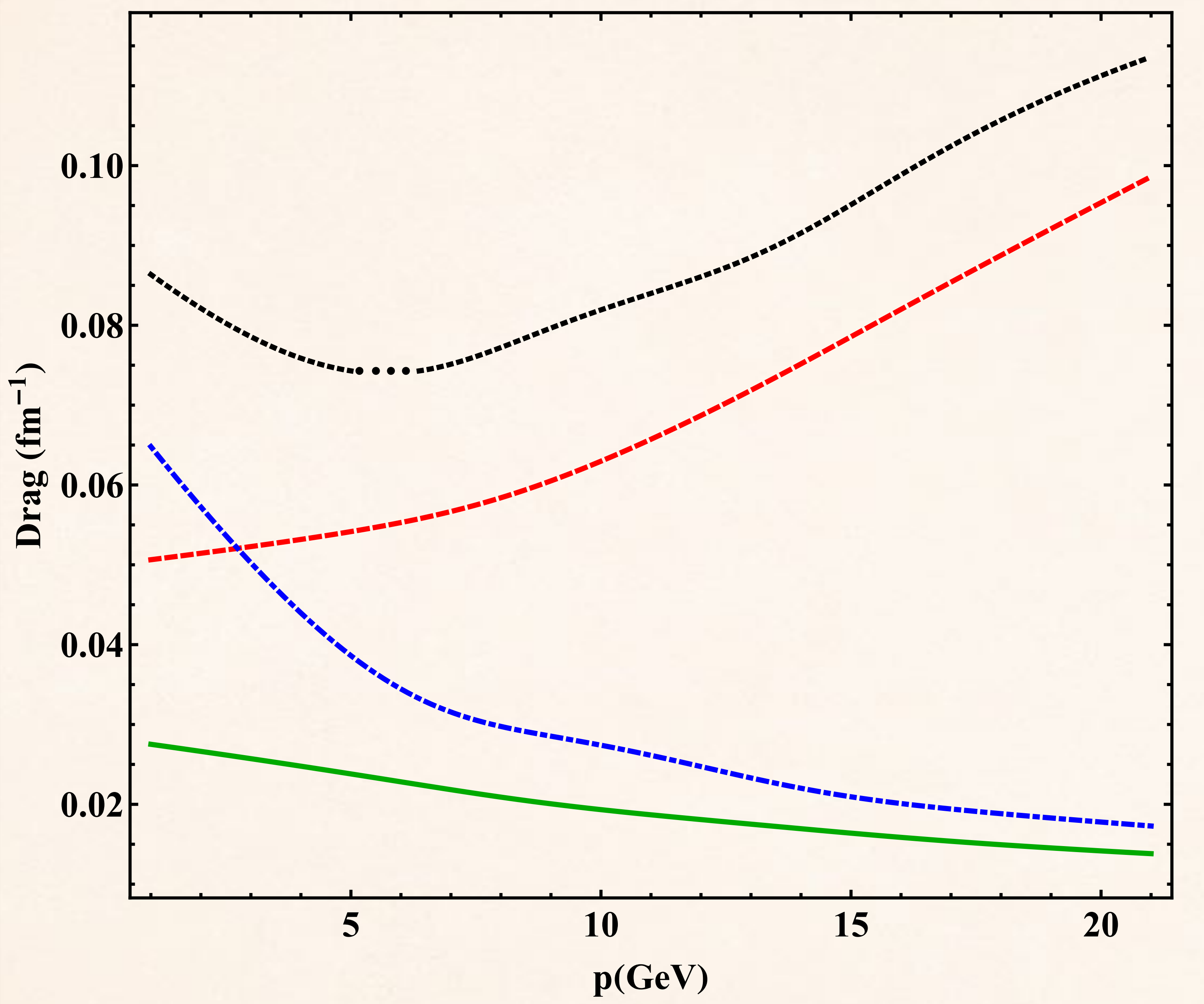


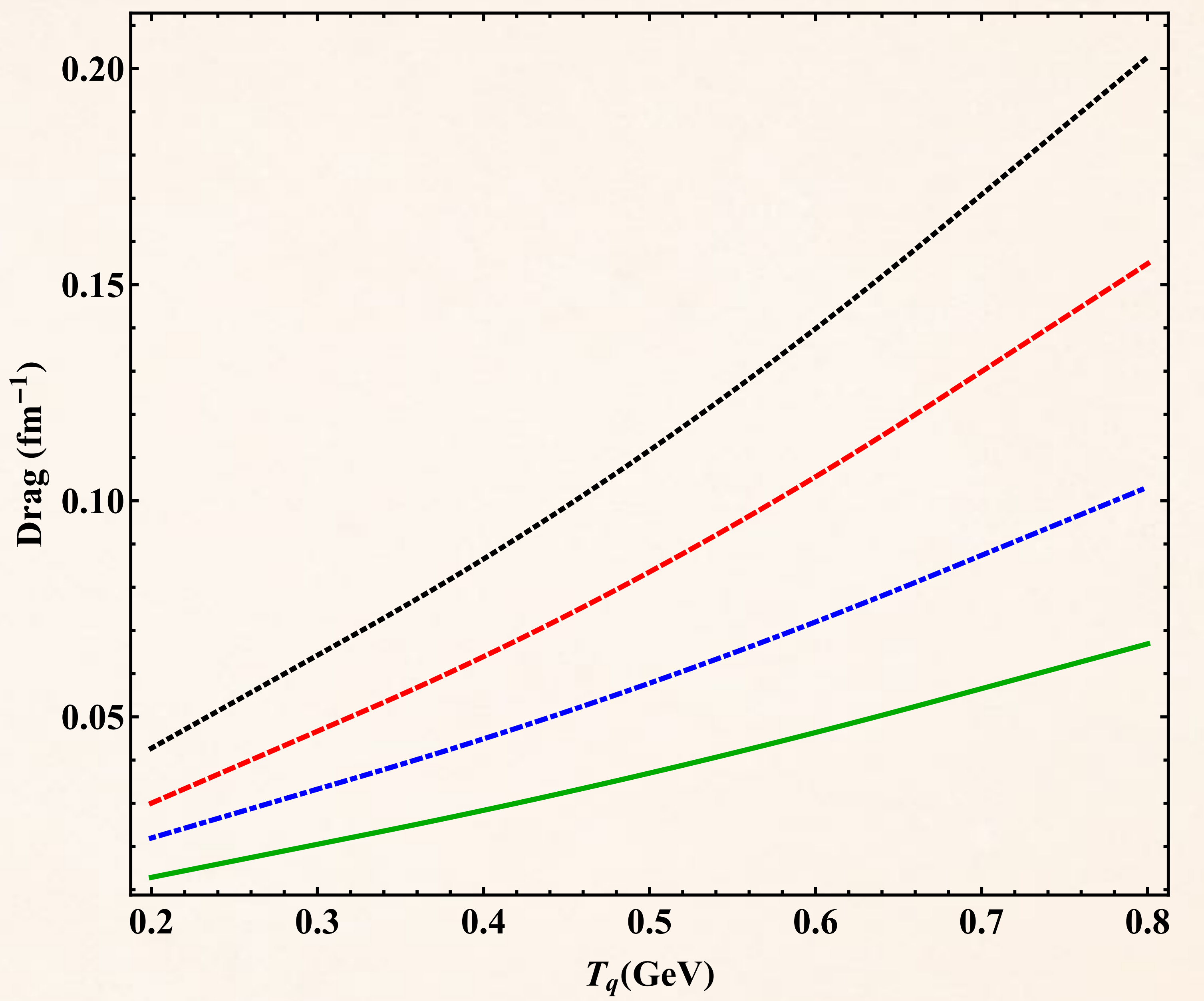






Drag





c: NL; b:NL; c:L; b:L

Summary, conclusion and outlook

Tsallis distribution is a generalisation of the Boltzmann distribution

Fluctuation, non-ideal plasma effects can be dealt with with the help of the Tsallis statistics

Inclusion of radiation

S Mazumder, TB, J Alam Phys. Rev. D 89, 014002 (2014)

Dokshitzer and Kharzeev, PLB 519, 199 (2001) JPG 17, 1481 (1991)

TB, Surasree Mazumder and Raktim Abir
Advances in High Energy Physics 2016 , 1298986 (2016)

Extension to dense systems

Connection with the experimental observables and finding out the values of Tsallis q-parameter from the nuclear suppression factor data.

Thank you !!