

Possible signals of two QCD phase transitions at NICA-FAIR energies

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Outline

1. Motivation

2. Novel and Old Irregularities at chemical freeze out

3. Shock adiabat model of A+A collisions

4. Newest results and possible evidence for two phase transitions

5. Conclusions

Experiments on A+A Collisions

AGS (BNL) up to 4.9 GeV
SPS (CERN) 6.1 - 17.1 GeV
RHIC (BNL) 62, 130, 200 GeV } Completed

Ongoing HIC experiments

LHC (CERN) > 1 TeV (high energy)

RHIC (BNL) low energy

SPS (CERN) low energy

Future HIC experiments

NICA (JINR, Dubna)

SIS300 = FAIR (GSI)

J-PARC

«collision»
of two droplets



Present Status of A+A Collisions

In 2000 CERN claimed indirect evidence for a creation of new matter

**In 2010 RHIC collaborations claimed to have created a quark-gluon
plasma/liquid**

However, up to now we do not know:

- 1. whether deconfinement and chiral symmetry restoration are the same phenomenon or not?**
- 2. are they phase transitions (PT) or cross-overs ?**
- 3. what are the collision energy thresholds of their onset?**

Recently 2 groups claimed to see the evidence of 2 QCD phase transitions

Recently Suggested Signals of QCD Phase Transitions 2014-2018

**During 2013-2017 our group developed
a very accurate tool to analyze data**

D. Oliinychenko, KAB, A. Sorin, Ukr. J. Phys. 58 (2013)

KAB, D. Oliinychenko, A. Sorin, G.Zinovjev, EPJ A 49 (2013)

KAB et al., Europhys. Lett. 104 (2013)

KAB et al., Nucl. Phys. A 970 (2018)

**Most successful
version of the
Hadron Resonance
Gas Model (HRGM)**

**The high quality description of data allowed us
to elucidate new irregularities at CFO from data and
to formulate new signals of two QCD phase transitions**

D. Oliinychenko et al., Ukr. J Phys. 59 (2014)

KAB et al., Phys. Part. Nucl. Lett. 12 (2015)

KAB et al., EPJ A 52 (2016) No 6

KAB et al., EPJ A 52 (2016) No 8

KAB et al., Phys. Part. Nucl. Lett. 15 (2018)

**First work on evidence of two
QCD phase transitions**

Recently Suggested Signals of QCD Phase Transitions 2016

Our results

1-st order PT of Chiral Symmetry Restoration in hadronic phase occurs at about $\sqrt{s} \sim 4.3-4.9$ GeV

and 2-nd order deconfinement PT exists at $\sqrt{s} \sim 9$ GeV

Giessen group results

W. Cassing et al., Phys. Rev. C 93, 014902 (2016);
Phys. Rev. C 94, 044912 (2016).

1-st order PT of ChSR in hadronic phase occurs at about $\sqrt{s} \sim 4$ GeV

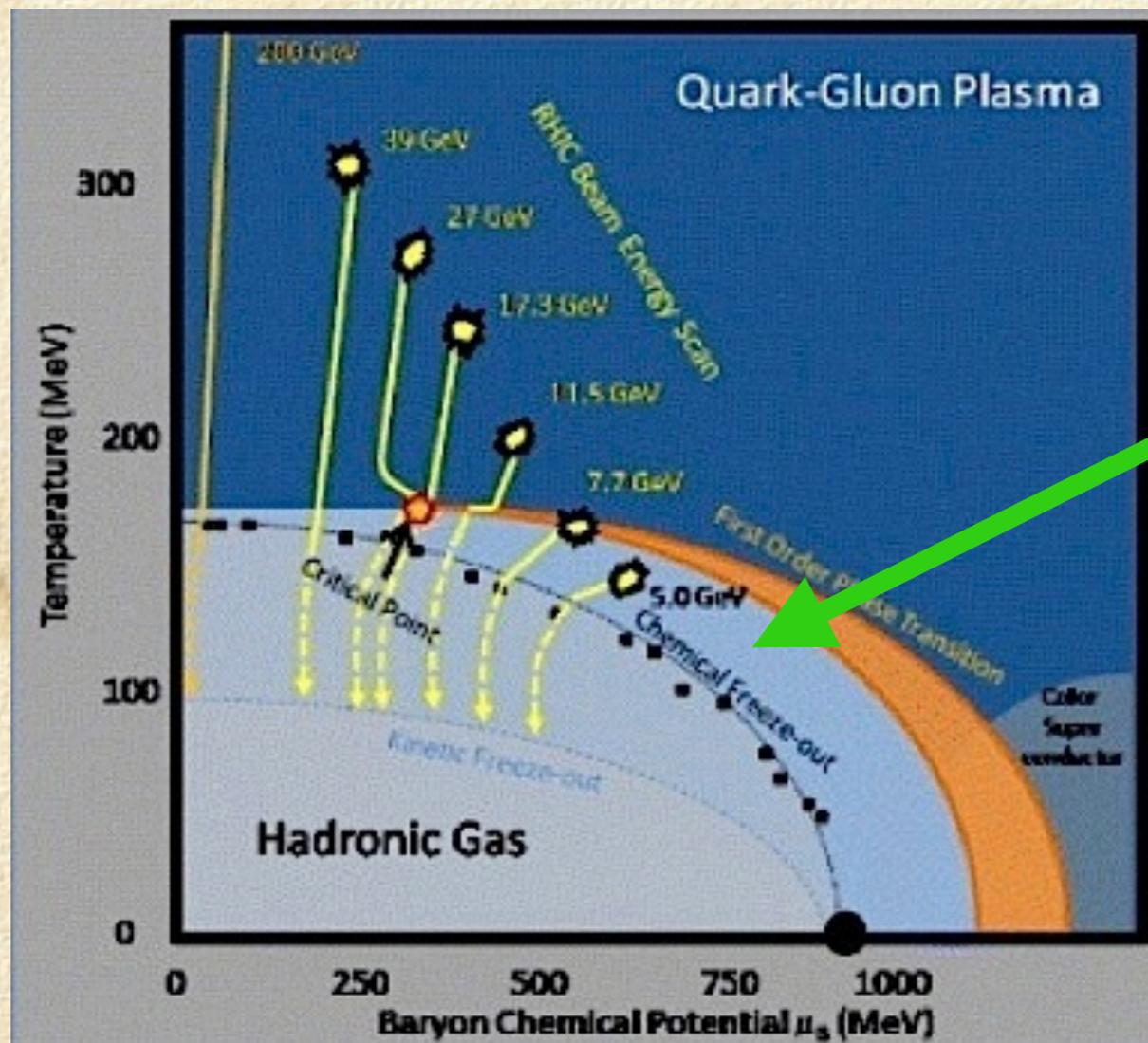
and 2-nd order deconfinement PT exists at $\sqrt{s} \sim 10$ GeV

Hard to locate them due to cross-over in Parton-Hadron-String-Dynamics model!

HRG: a Multi-component Model

HRG model is a truncated **Statistical Bootstrap Model** with the excluded volume correction a la VdWaals for all hadrons and resonances known from Particle Data Group.

For given temperature T , baryonic chem. potential, strange charge chem. potential, chem. potential of isospin 3-rd projection \Rightarrow thermodynamic quantities \Rightarrow all charge densities, to fit data.



Chemical freeze-out - moment after which hadronic composition is fixed and only strong decays are possible. I.e. there are no inelastic reactions.

Strangeness Enhancement as Deconfinement Signal

In 1982 J. Rafelski and B. Müller predicted that **enhancement of strangeness** production is a signal of deconfinement. **Phys. Rev. Lett. 48(1982)**

In 1991 J. Rafelski introduced strangeness fugacity **γ_S factor** **Phys. Lett. 62(1991)**

which quantifies strange charge chemical **oversaturation** (>1) or
strange charge chemical **undersaturation** (<1)

Idea: if s-(anti)quarks are created at QGP stage, then their number should not be changed during further evolution since s-(anti)quarks number is small and since density decreases \Rightarrow there is no chance for their annihilation!

Hence, we should observe chemical enhancement of strangeness with $\gamma_S > 1$

However, until 2013 the situation with strangeness was unclear:

P. Braun-Munzinger & Co found that γ_S factor is about 1

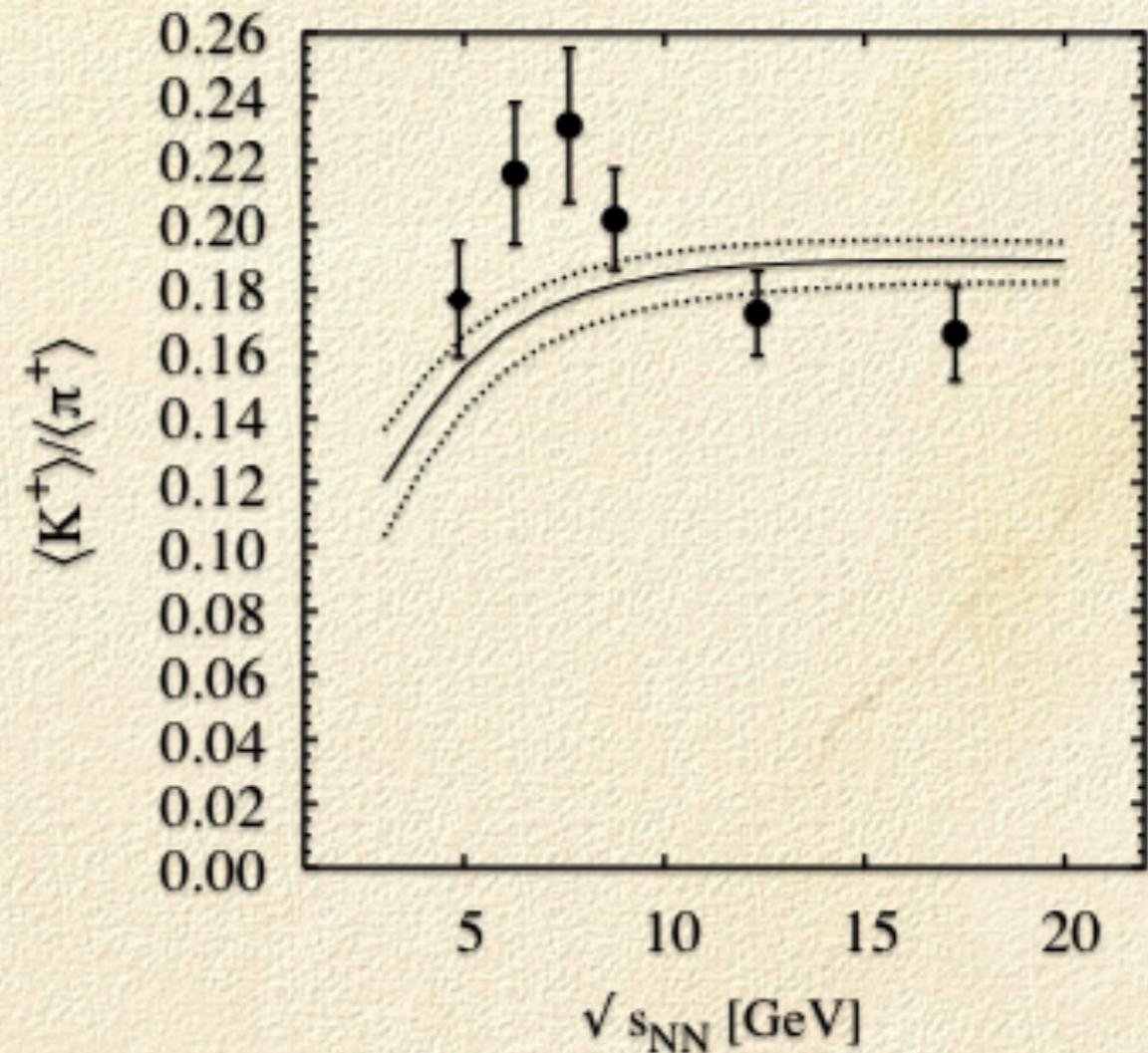
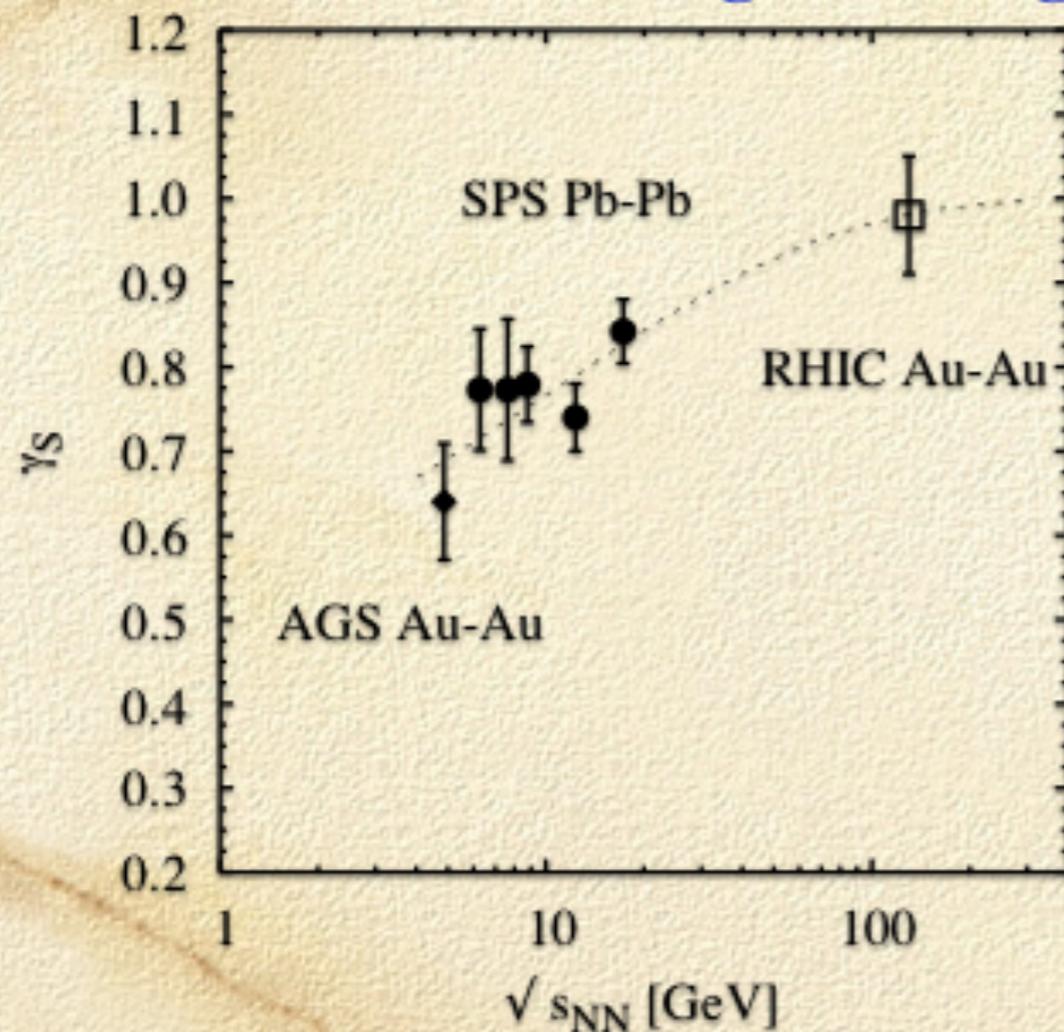
F. Becattini & Co found that γ_S factor is < 1

Systematics of Strangeness Suppression

Include γ_s factor $\phi_i(T) \rightarrow \phi_i(T)\gamma_s^{s_i}$, into thermal density

where s_i is number of strange valence quarks plus number of strange valence anti-quarks.

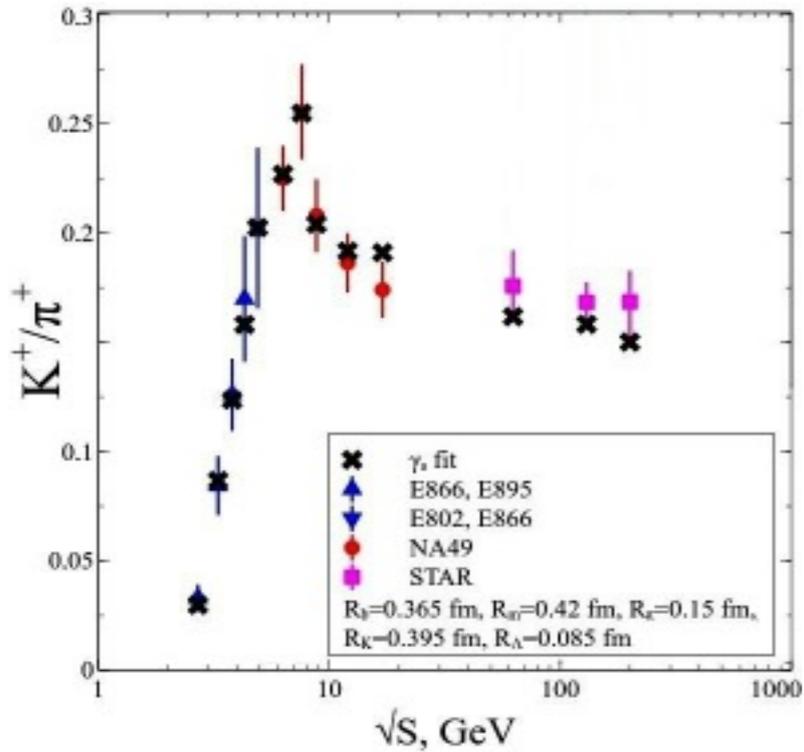
Thus, it is a strangeness fugacity



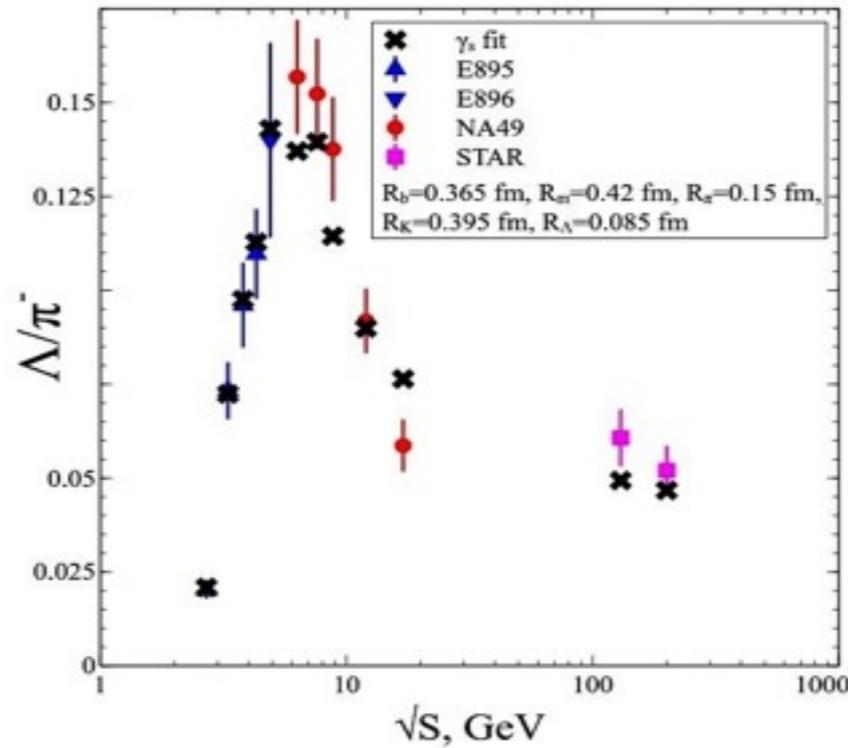
Single component model **F. Becattini, J. Manninen and M. Gazdzicki, PRC 73 (2006) 044905**

Typical values of $\chi^2/\text{dof} > 2$ at given energy!

Most Problematic ratios at AGS, SPS and RHIC energies



IST EOS: $\chi^2/dof \simeq 3.29/14$

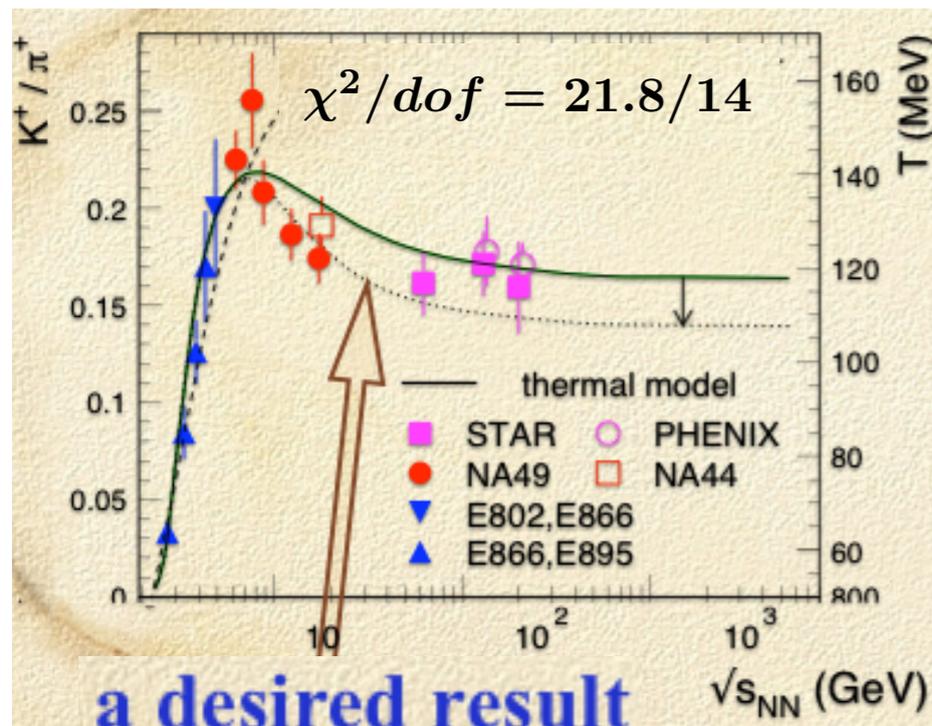


$\chi^2/dof \simeq 11.62/12$

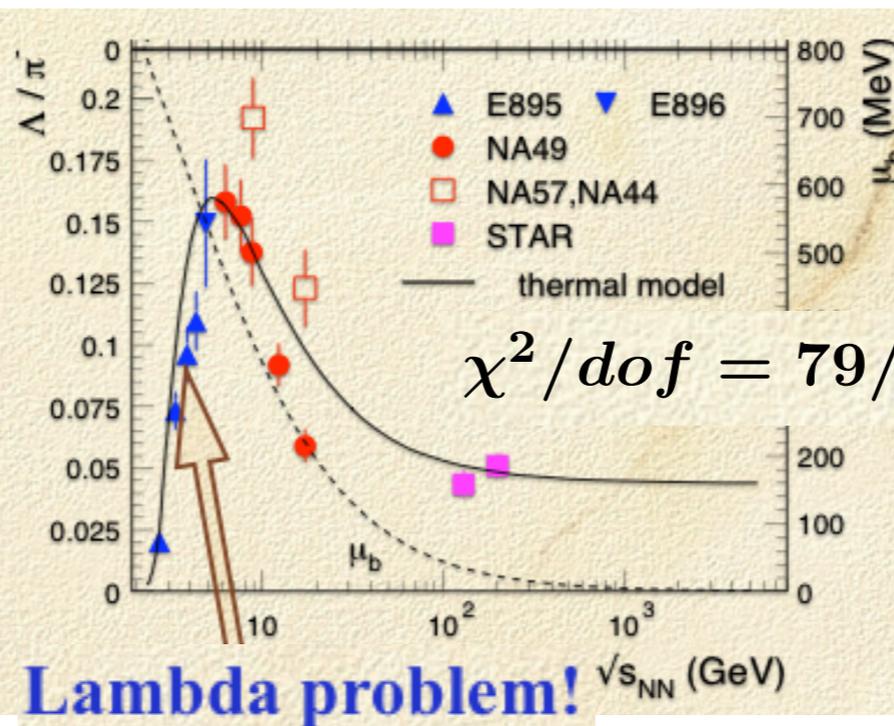
KAB et al., Nucl. Phys. A 970 (2018)

Note: RHIC BES I data have very large error bars and hence, are not analyzed!

Our IST EOS has 3 or 4 more fitting parameters compared to usual HRGM!



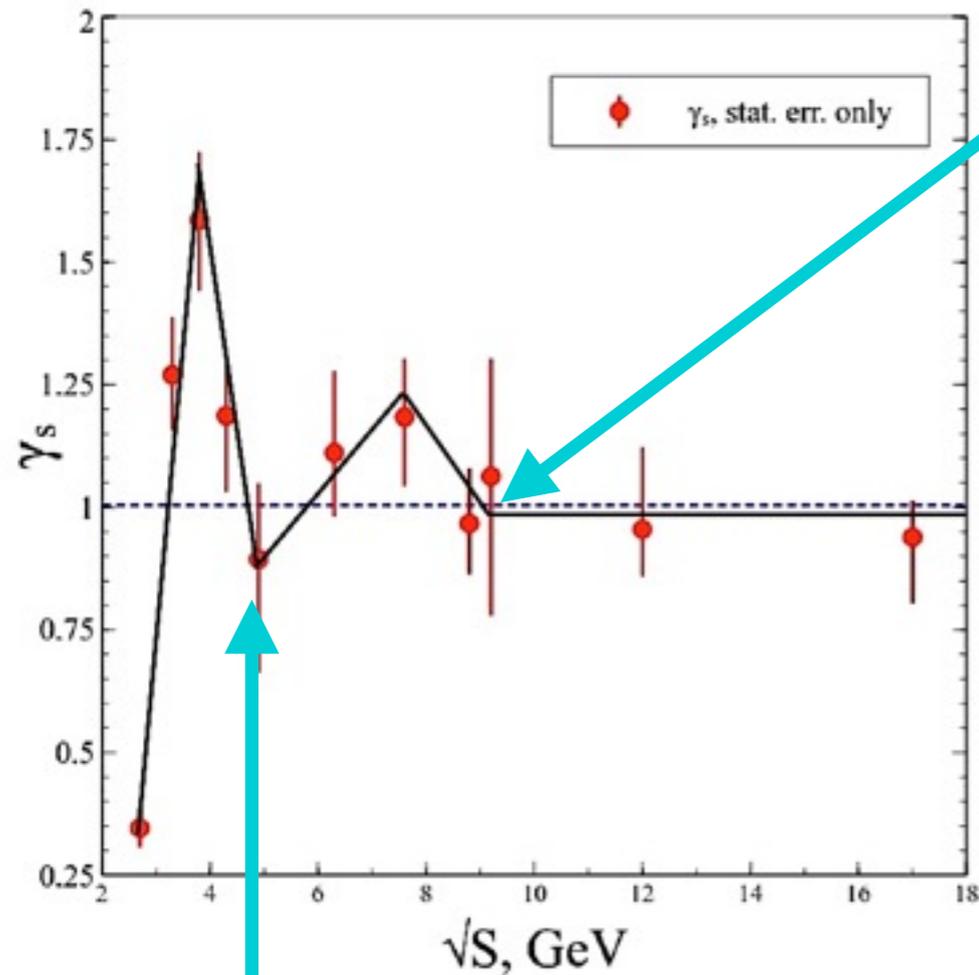
a desired result



Lambda problem!

Conventional one component HRGM by PBM and Co: A. Andronic, PBM, J. Stachel NPA (2006), PLB (2009)

Strangeness Irregularities



At c.m. energies above 8.8 GeV the strange hadrons are in chemical equilibrium! **Why?**

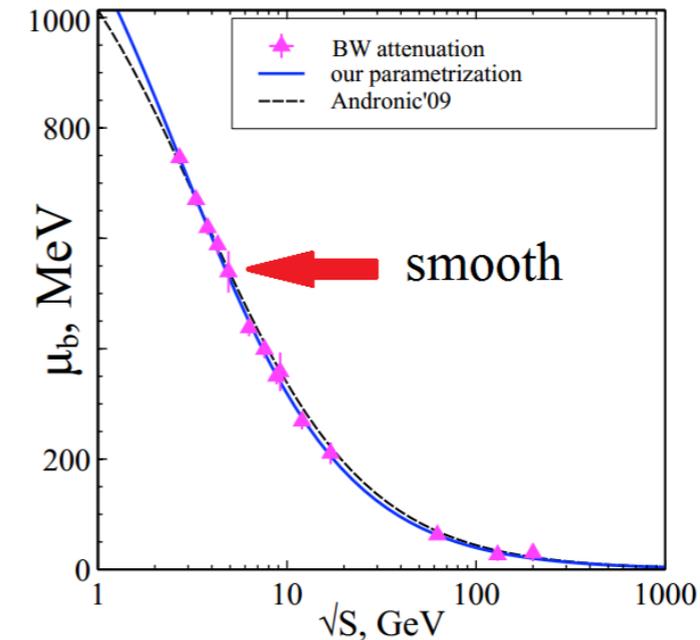
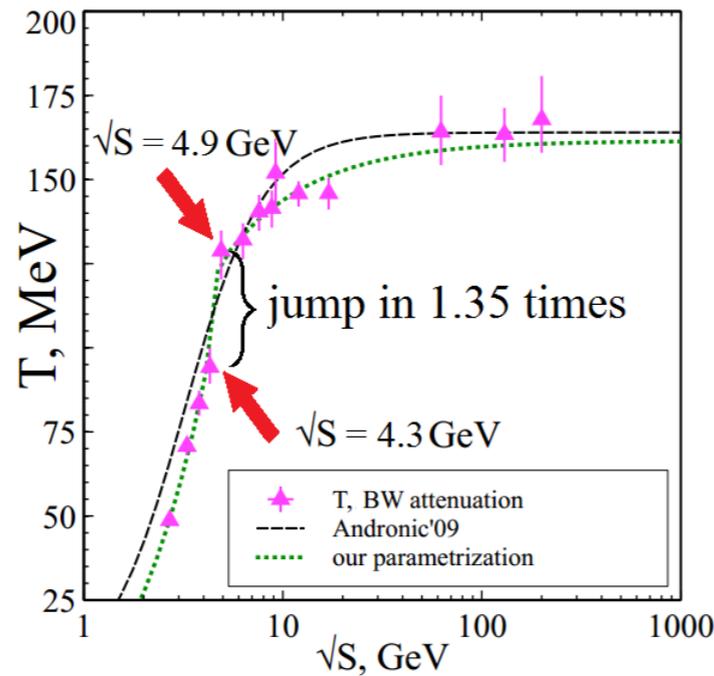
At c.m. energy below 4.9 GeV strange particles are also in chemical equilibrium, while at lower and higher energies of collision there is strangeness enhancement. **Why?**

Explanation of such peculiar behavior was found in 2017. See

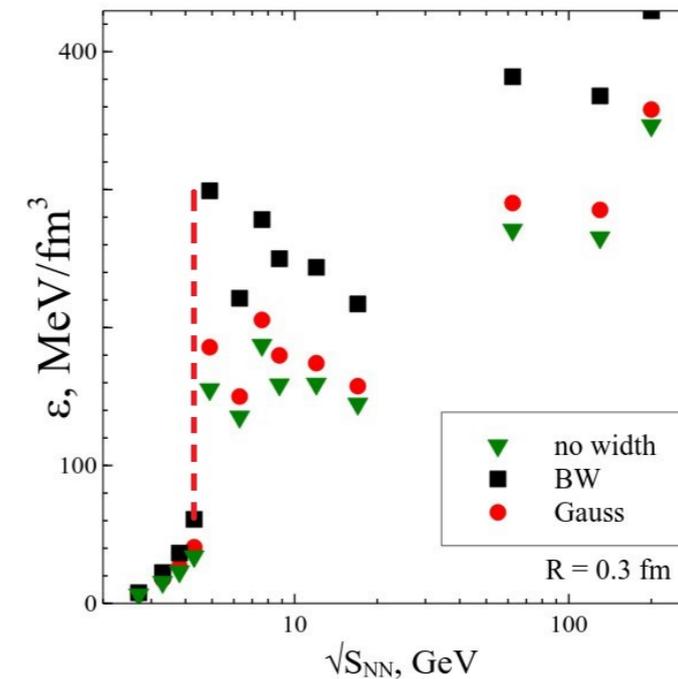
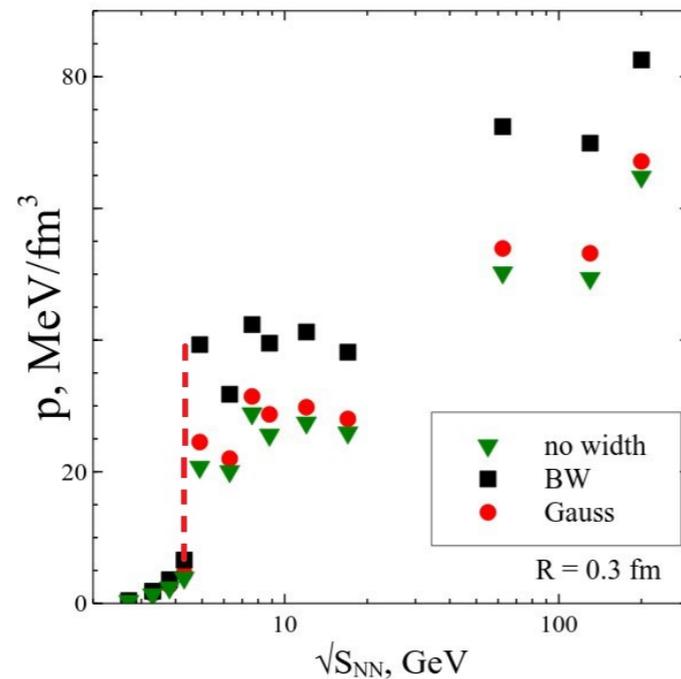
KAB et al., Phys. Part. Nucl. Lett. 15 (2018)

Jump of ChFO Pressure at AGS Energies

- Temperature T_{CFO} as a function of collision energy \sqrt{s} is rather non smooth



- Significant jump of pressure ($\simeq 6$ times) and energy density ($\simeq 5$ times)



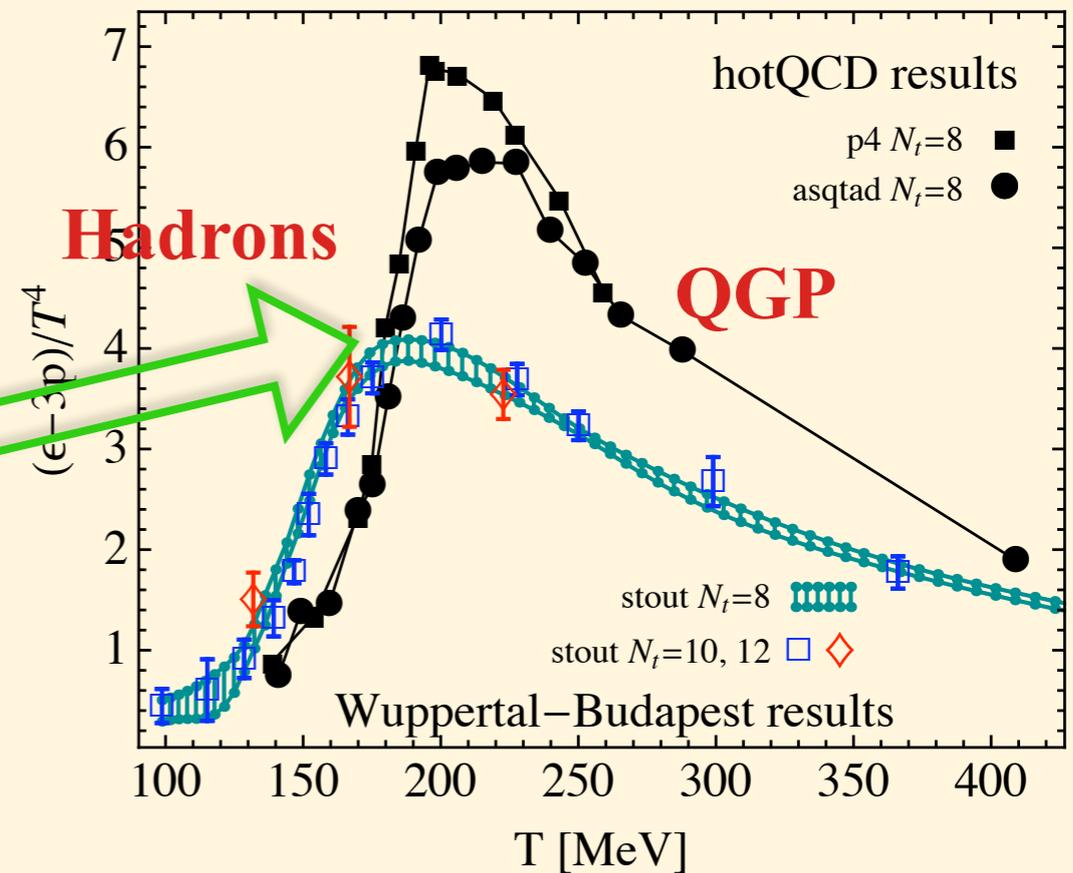
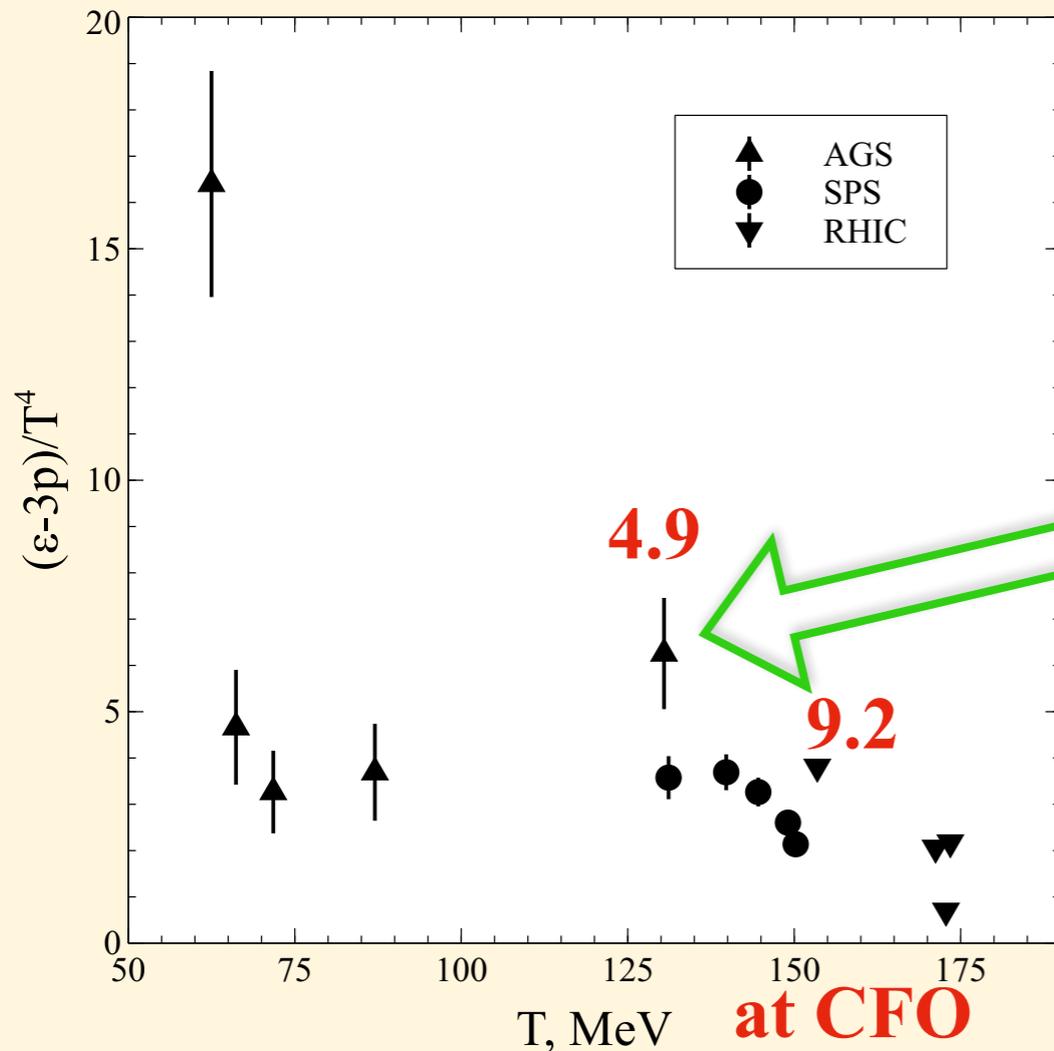
K.A. Bugaev et al., Phys. Part. Nucl. Lett. 12(2015) [arXiv:1405.3575];

Ukr. J. Phys. 60 (2015)

Trace Anomaly Peaks (Most Recent)

At chemical FO (large μ)

Lattice QCD (vanishing μ)



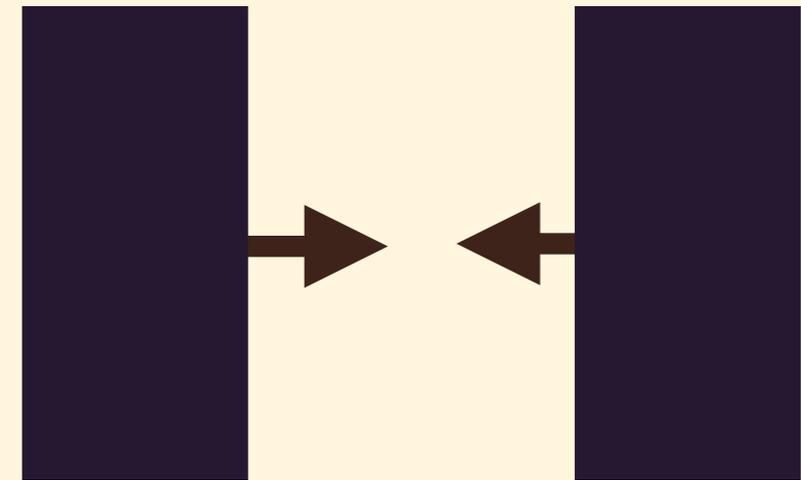
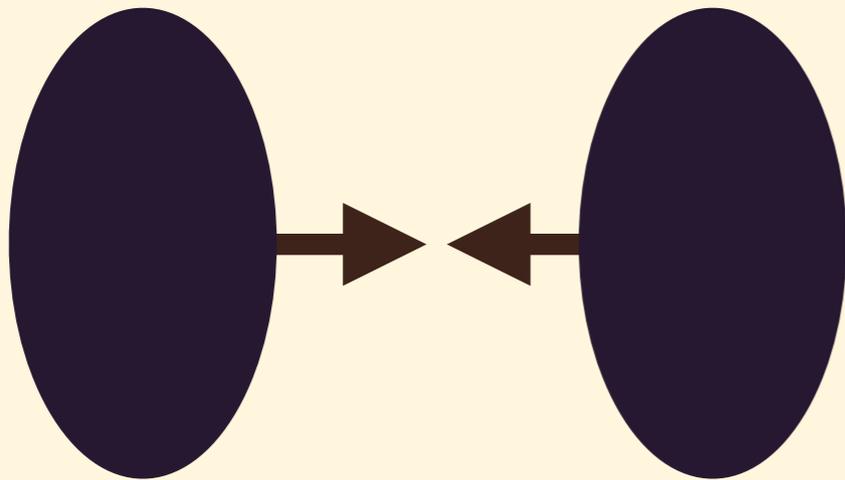
WupBud EOS arXiv: lat 1007.2580

Model from V.V. Sagun et al., Eur. Phys. J. A (2018) 54: 100,
arXiv:1703.00009 [hep-ph]

Are these trace anomaly peaks related to each other?

Shock Adiabatic Model for A+A Collisions

A+A central collision at $1 < E_{\text{lab}} < 30$ GeV Its hydrodynamic model



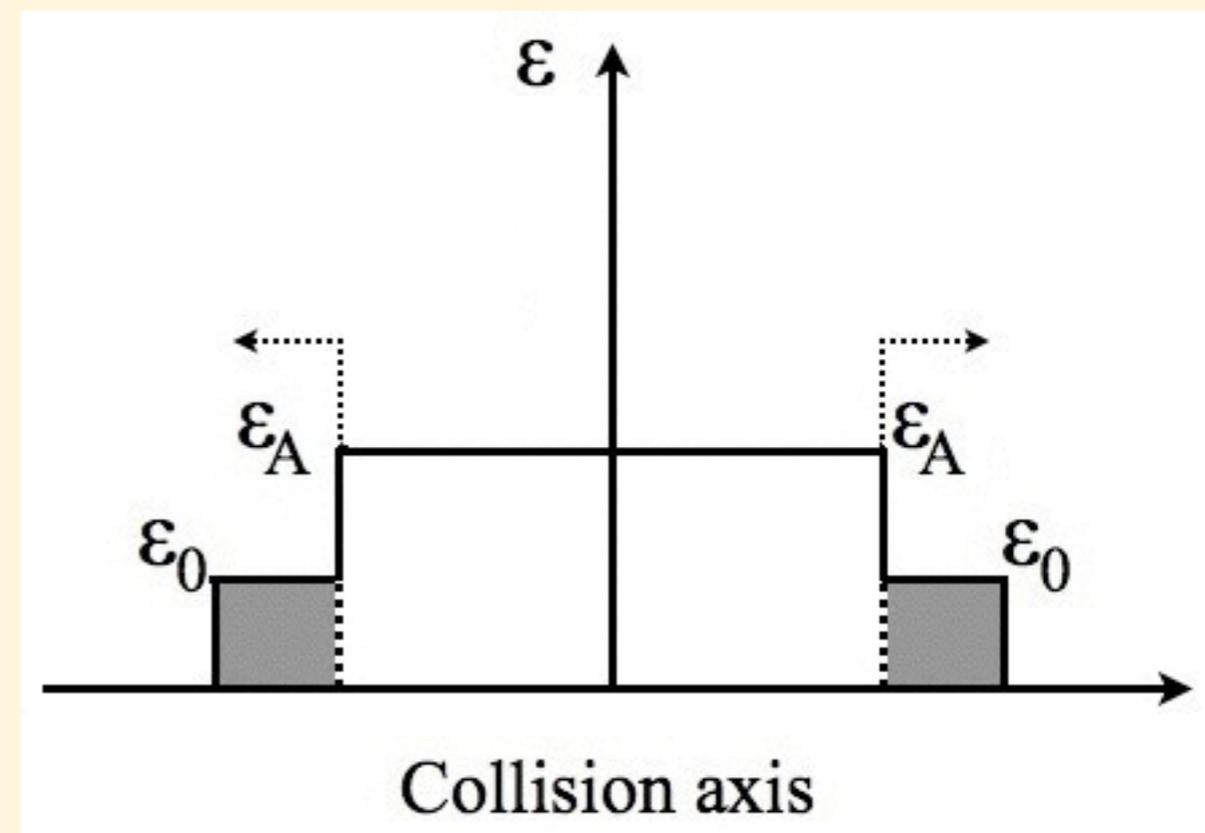
Works reasonably well at these energies.

H. Stoecker and W. Greiner, Phys. Rep. 137 (1986)

Yu.B. Ivanov, V.N. Russkikh, and V.D. Toneev,

Phys. Rev. C 73 (2006)

From hydrodynamic point of view
this is a problem of
arbitrary discontinuity decay:
in normal media there appeared
two shocks moving outwards



Medium with Normal and Anomalous Properties

Normal properties, if $\Sigma \equiv \left(\frac{\partial^2 p}{\partial X^2} \right)_{s/\rho_B}^{-1} > 0$ = convex down:

Usually pure phases (Hadron Gas, QGP) have normal properties

$X = \frac{\varepsilon + p}{\rho_B^2}$ – generalized specific volume

ε is energy density, p is pressure,

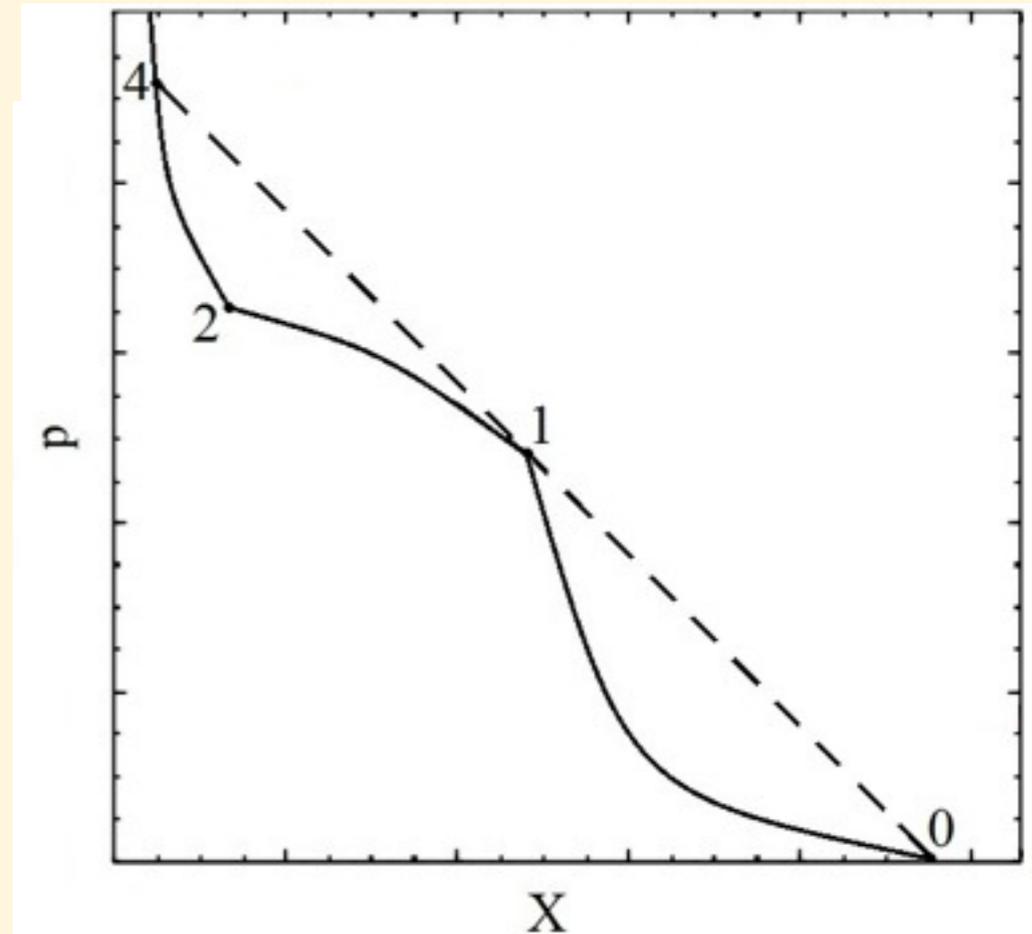
ρ_B is baryonic charge density

Anomalous properties otherwise.

Almost in all substances with liquid-gas phase transition the mixed phase has anomalous properties!

Then shock transitions to mixed phase are unstable and more complicated flows are possible.

Shock adiabat example



Region 1-2 is mixed phase with **anomalous properties.**

Highly Correlated Quasi-Plateaus

For realistic EoS at mixed phase entropy per baryon should have a plateau!

Since the main part of the system entropy is defined by thermal pions =>
thermal pions/baryon should have a plateau!

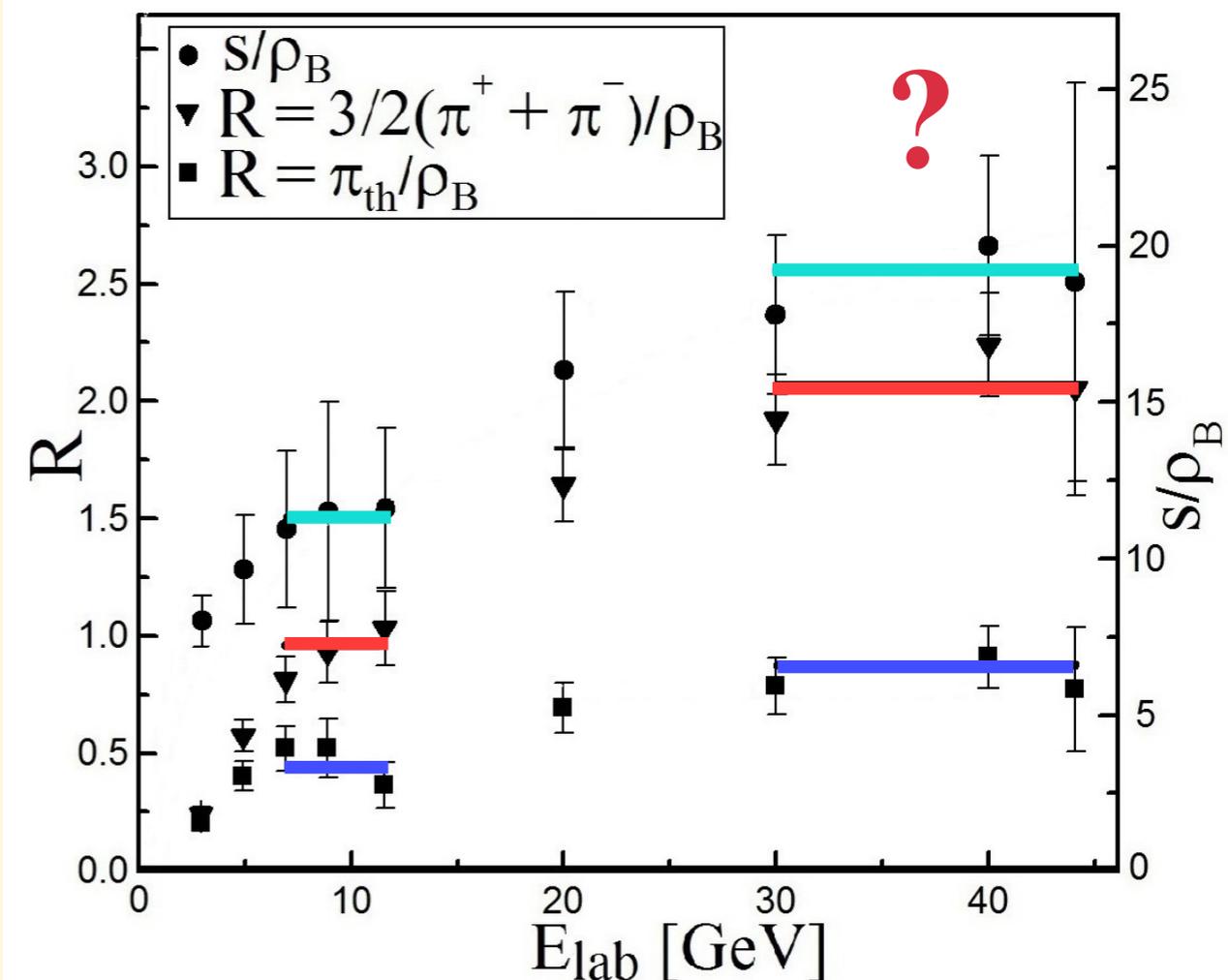
Also the total number of **pions per baryons should have a (quasi)plateau!**

K.A. Bugaev, M.I. Gorenstein, B. Kampher, V.I. Zhdanov, Phys. Rev. D 40, 9, (1989)
K.A. Bugaev, M.I. Gorenstein, D.H. Rischke, Phys. Lett. B 255, 1, 18 (1991)

Entropy per baryon has wide plateaus
due to large errors

Quasi-plateau in total number of
pions per baryon ?

Thermal pions demonstrate 2 plateaus

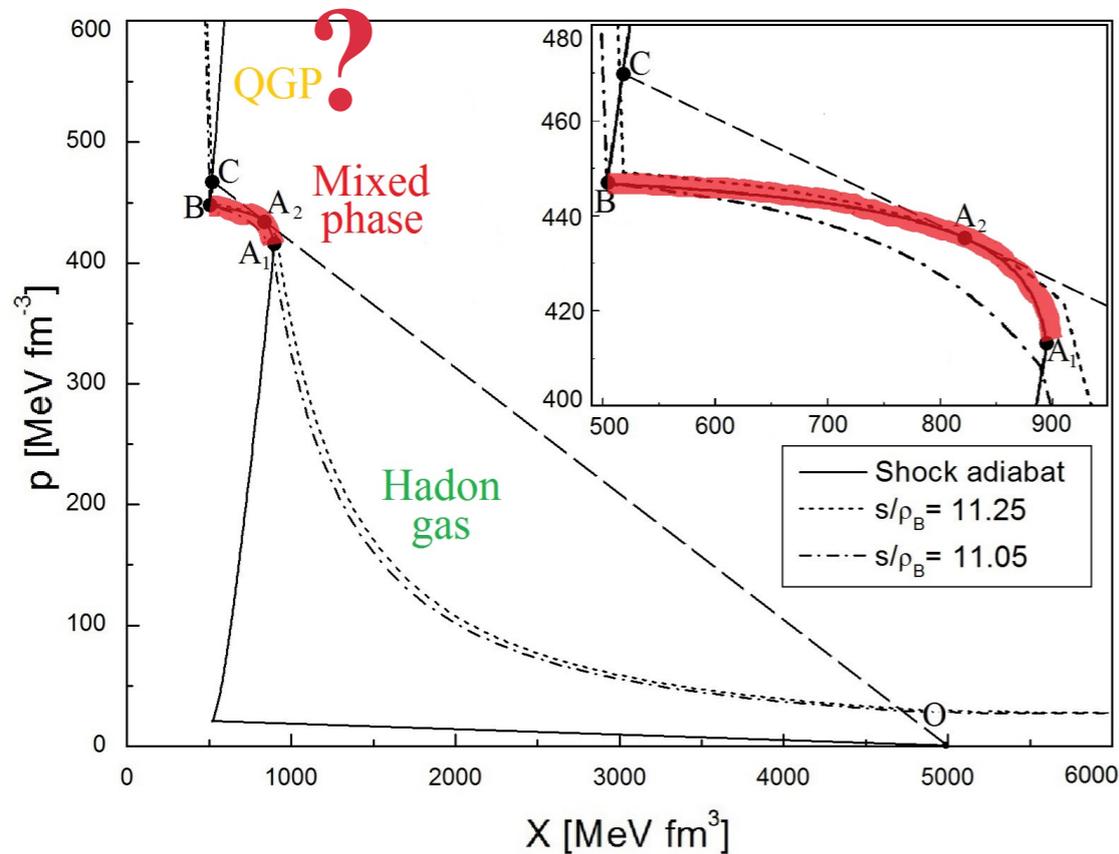


K.A. Bugaev et al., Phys. Part. Nucl. Lett. 12(2015)

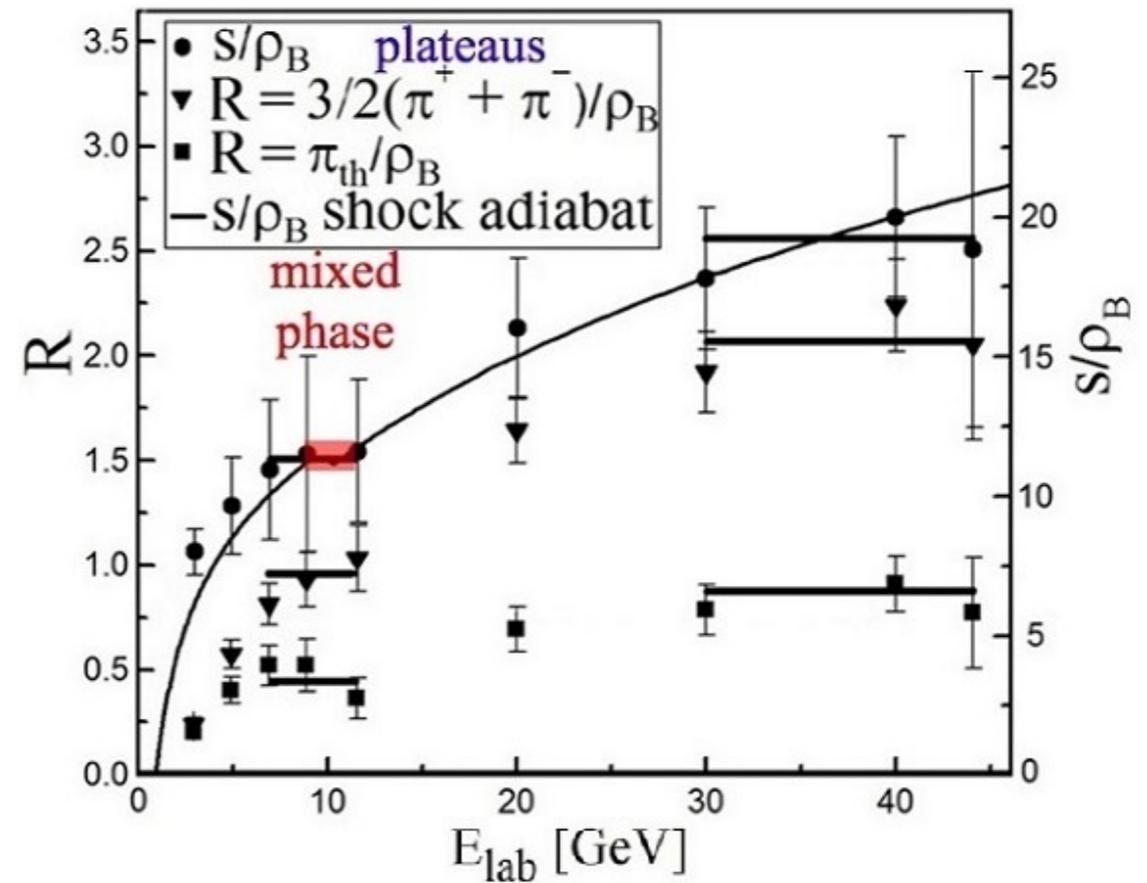
Unstable Transitions to Mixed Phase

$$X = \frac{\varepsilon + p}{\rho_B^2} - \text{generalized specific volume}$$

other PT?



K.A. Bugaev et al., arXiv:1405.3575[hep-ph]

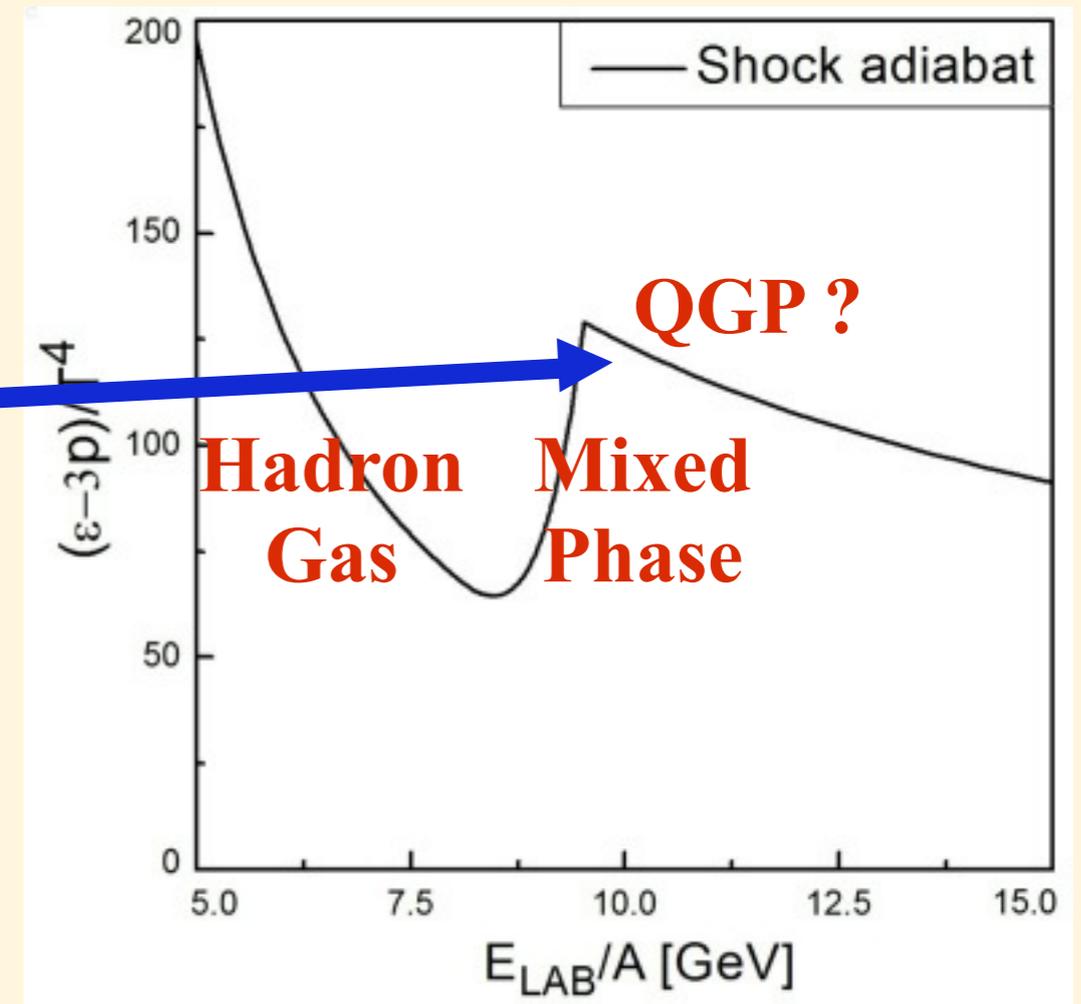
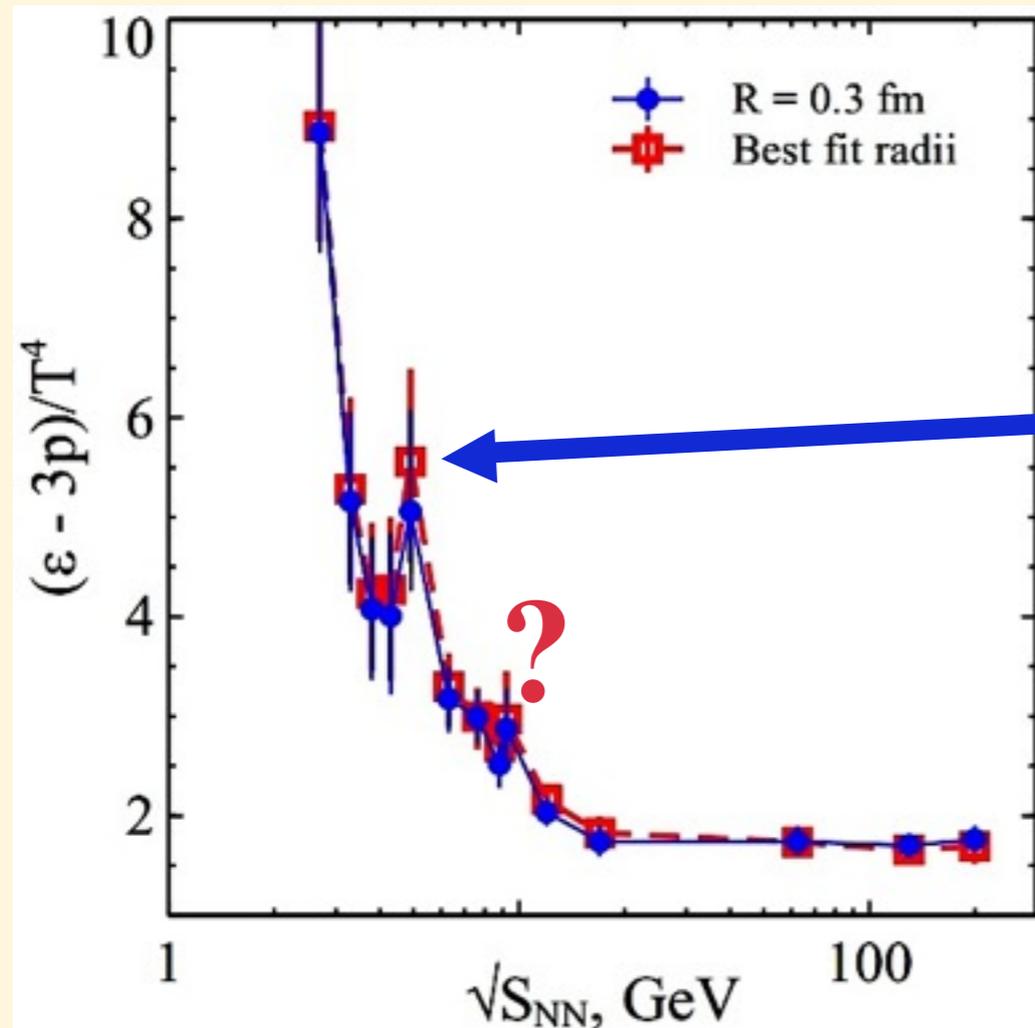


GSA Model explains irregularities at CFO as a signature of mixed phase

QGP EOS is MIT bag model with coefficients been fitted with condition $T_c = 150$ MeV at vanishing baryonic density!

HadronGas EOS is simplified HRGM discussed above.

Trace Anomaly Along Shock Adiabats 2016



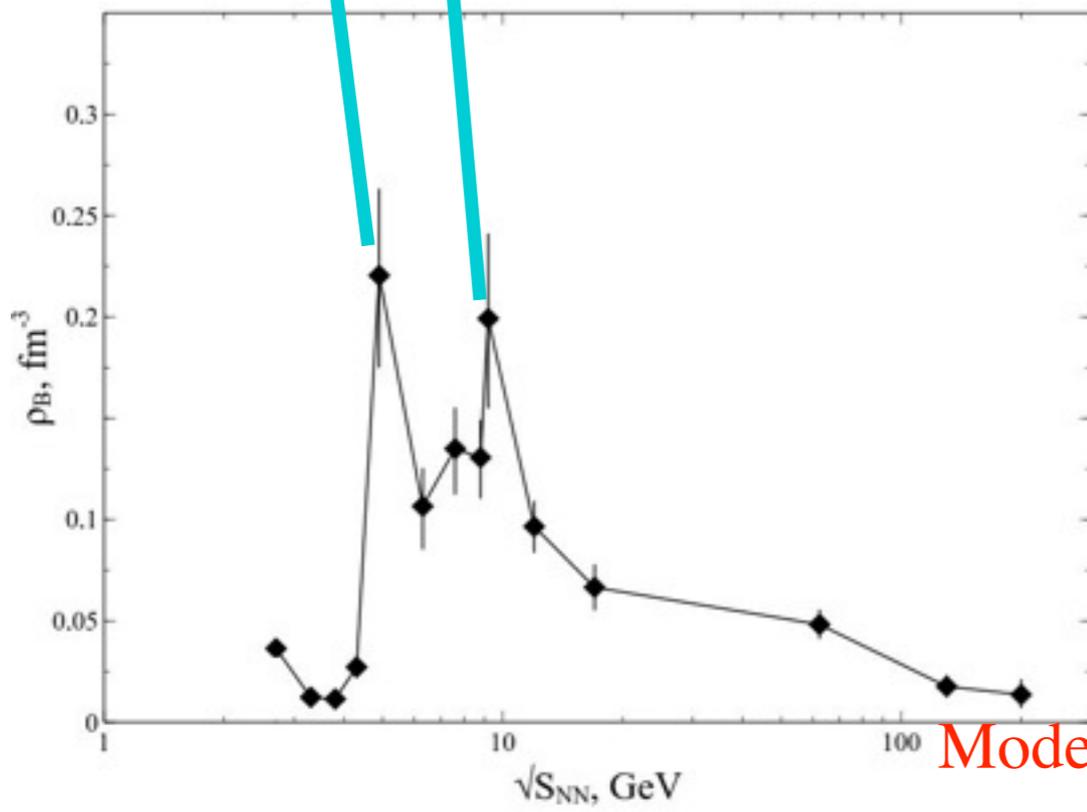
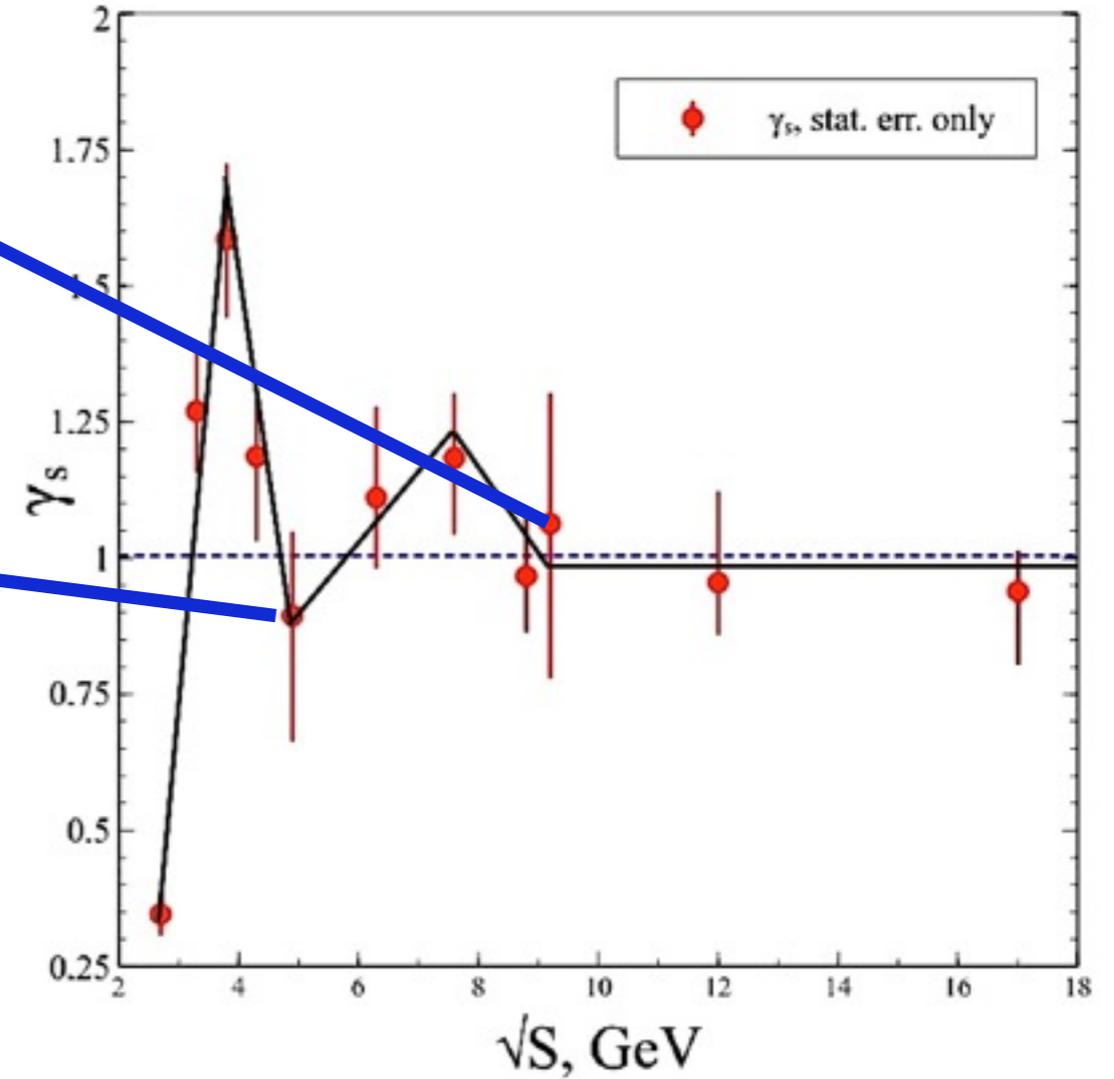
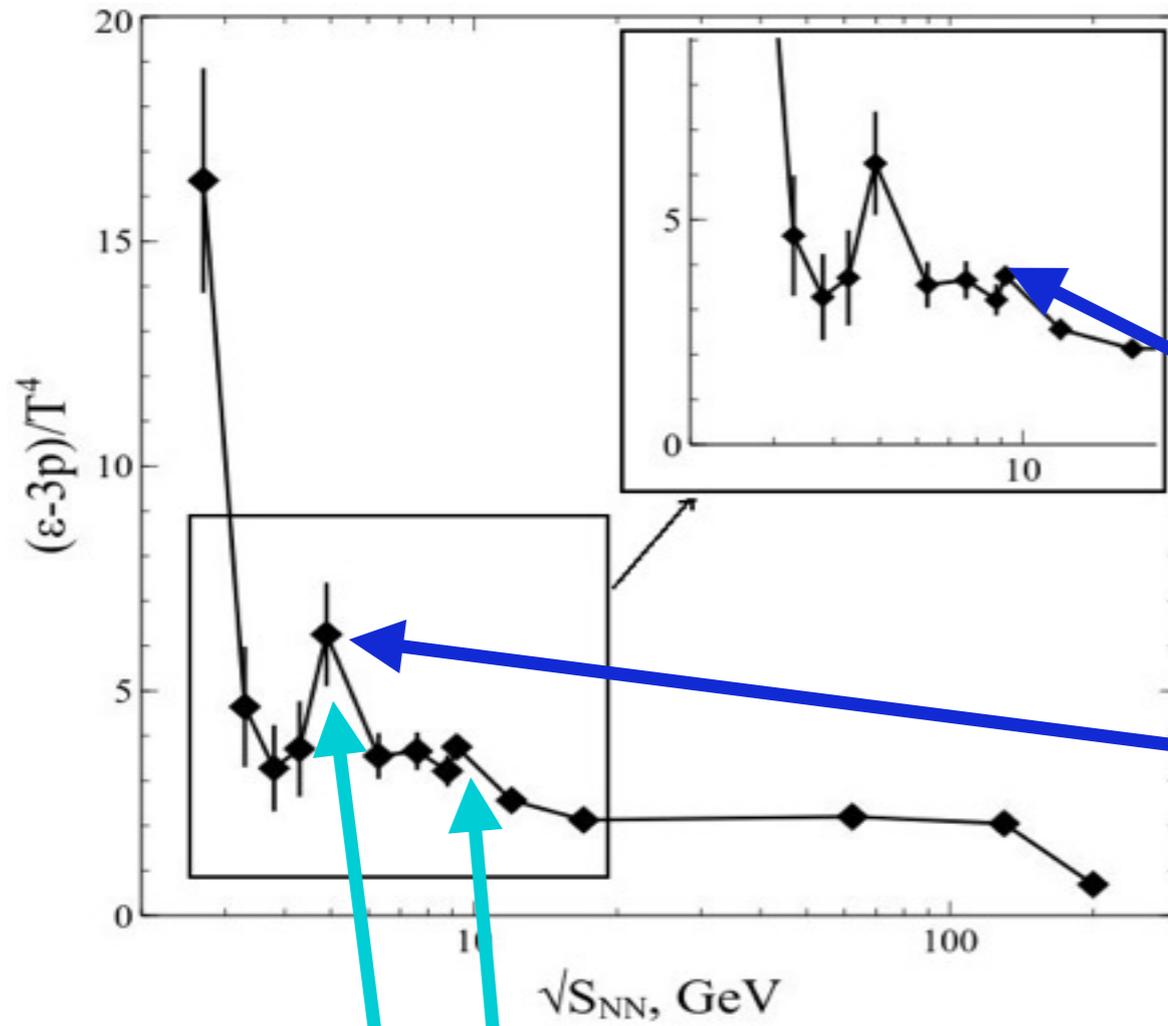
K.A. Bugaev et al., EPJ A (2016)

We found one-to-one correspondence between these two peaks.

Thus, sharp peak of trace anomaly at c.m. energy 4.9 GeV evidences for mixed phase formation. But what is it?

Is second peak at c.m. energy 9.2 GeV due to another PT?

Related Peaks (2017)

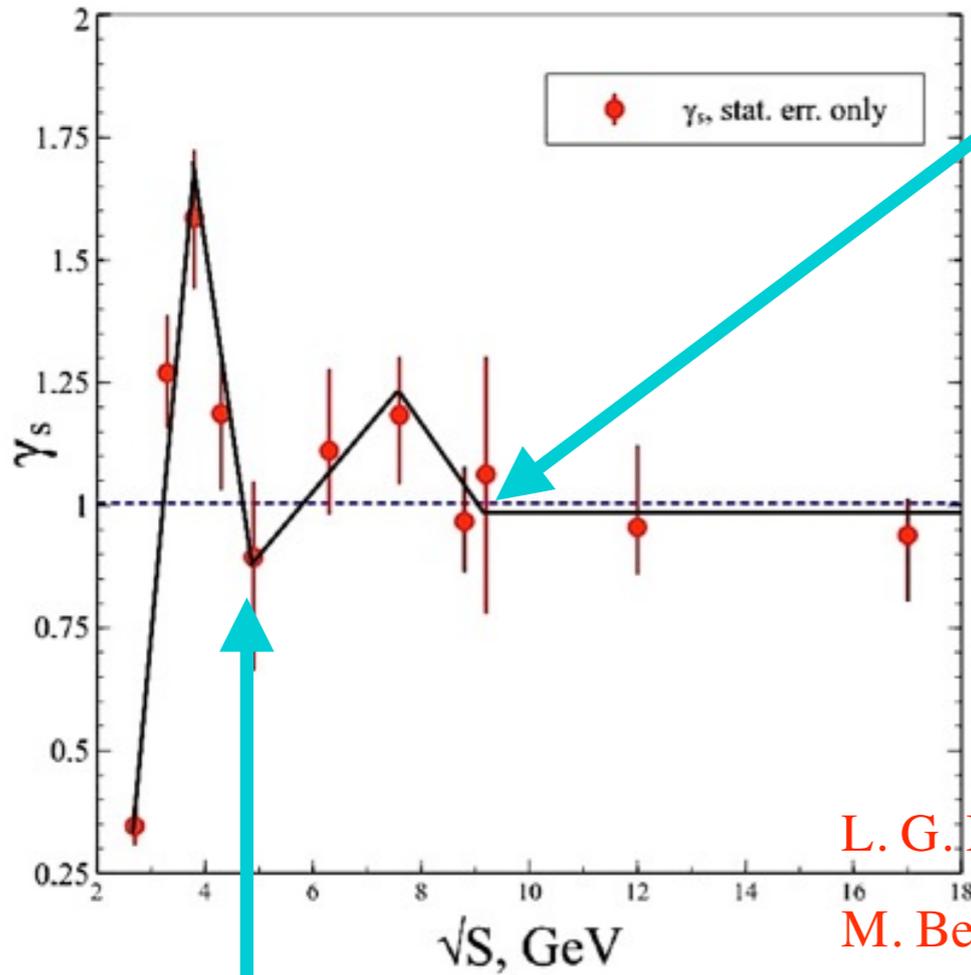


Trace anomaly peaks and baryonic density peaks are related to each other.

Can we relate them to γ_s irregularities?

Model from V.V. Sagun et al., Eur. Phys. J. A (2018) 54: 100

Strangeness Irregularities



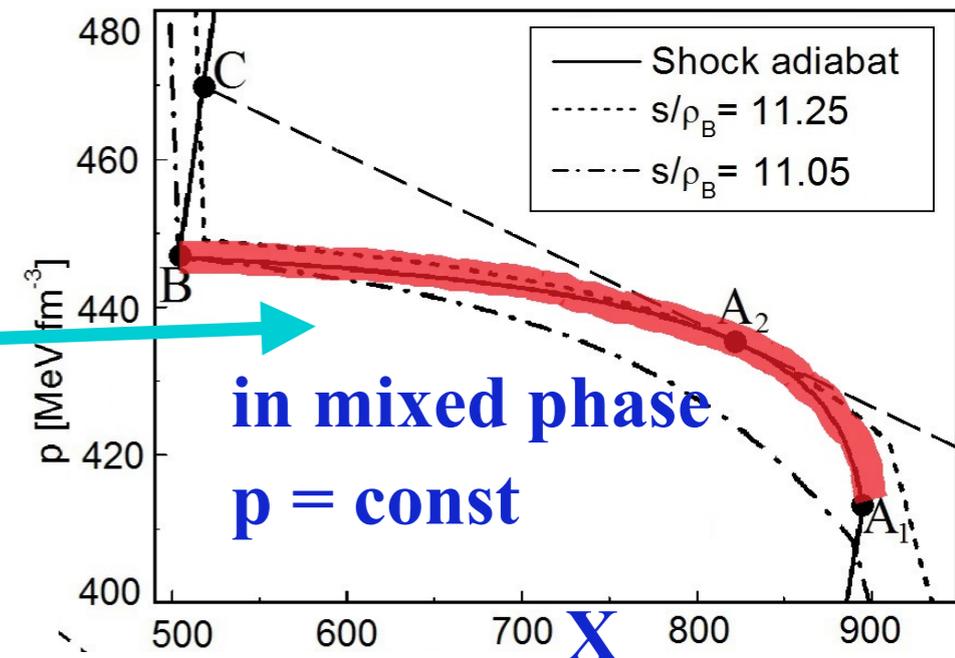
At c.m. energies above 8.8 GeV the strange hadrons are in chemical equilibrium due to formation of QG bags with Hagedorn mass spectrum!

Hagedorn mass spectrum is a **perfect thermostat** and a **perfect particle reservoir!** => Hadrons born from such bags will be in a full equilibrium!

L. G. Moretto, K. A. B., J. B. Elliott and L. Phair, Europhys. Lett. 76, 402 (2006)

M. Beitel, K. Gallmeister and C. Greiner, Phys. Rev. C 90, 045203 (2014)

At c.m. energy 4.9 GeV strange particles are in chemical equilibrium due to formation of mixed phase, since under **CONSTANT PRESSURE** condition the mixed phase of 1-st order PT is **explicit thermostat and explicit particle reservoir!**



Explicit Thermostats

1. At limiting temperature the Hagedorn mass spectrum is a perfect thermostat and a perfect particle reservoir since it is a kind of mixed phase!

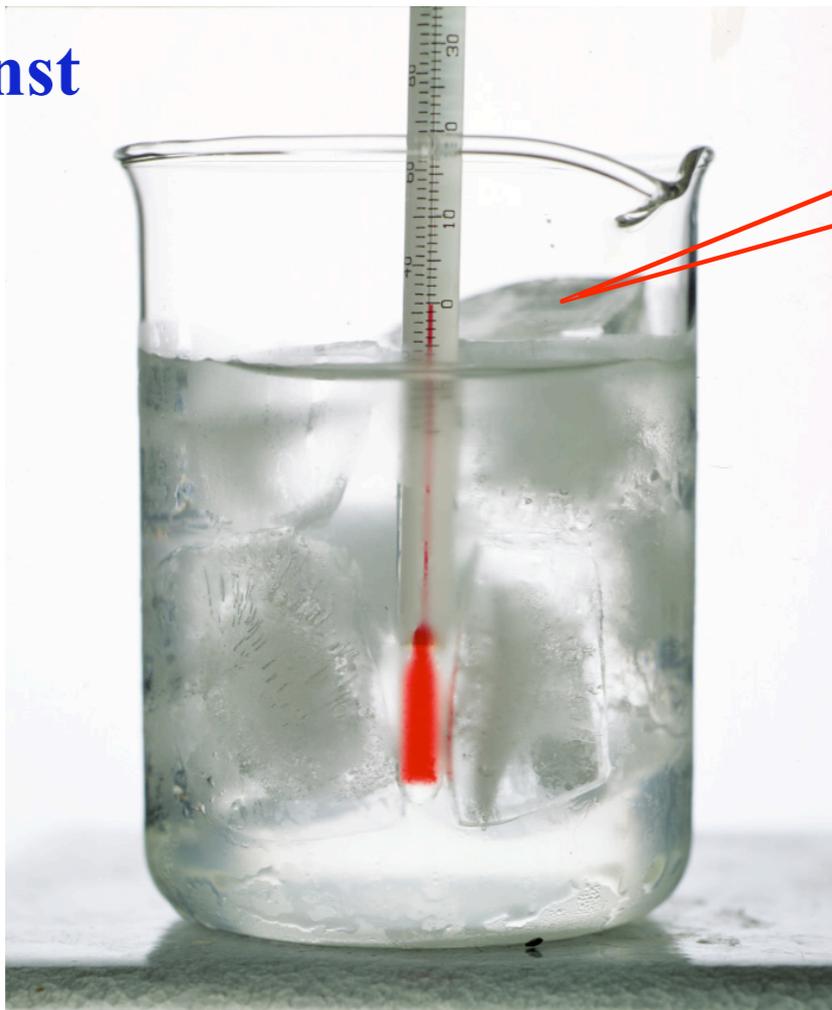
L. G. Moretto, K. A. B., J. B. Elliott, L. Phair, Europhys. Lett. 76, 402 (2006)

2. Under a constant external pressure ANY MIXED PHASE is a perfect thermostat and a perfect particle reservoir!

As long as two phases coexist

- Export/import of heat does not change T!

Pressure = const



$$T = T_c = 273\text{K} \quad ?$$

or

$$0 \leq T \leq 273\text{K} \quad \bullet$$

- ◆ First **take** heat $dQ=E$ from system with temperature T:
- ◆ Then give it to thermostat
 $\Rightarrow T = \text{const}, \mu = \text{const}$
- Export/import of **finite amount of phases** $\Rightarrow T = \text{const}, \mu = \text{const}$

Besides Quasi-plateaus There Exist Additional Hints for 2 Phase Transitions

Our: **K.A. Bugaev et al., Phys. Part. Nucl. Lett. 15 (2018)**

Each peak in trace anomaly δ corresponds to a huge peak
in baryonic charge density

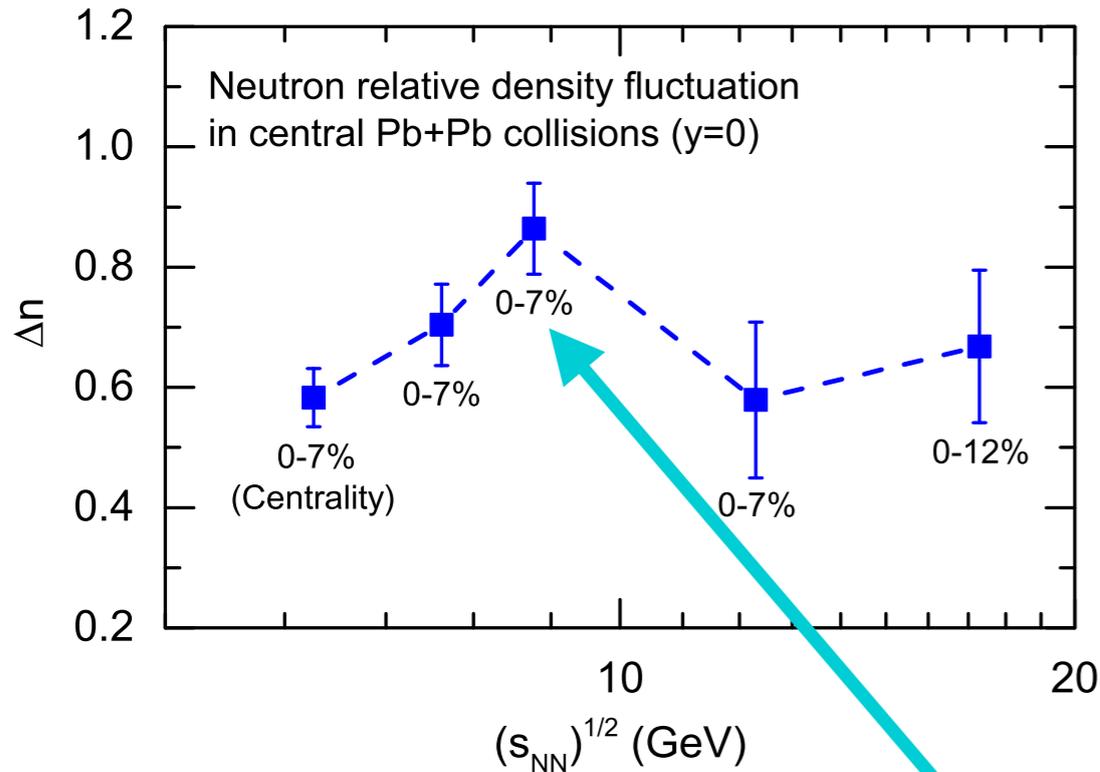
Thermostatic properties of Hagedorn mass spectrum of QGP bags
explain strangeness equilibration at $\sqrt{s} > 8.8$ GeV

Thermostatic properties of the 1-st order PT mixed phase explain
strangeness equilibration at $4.3 \text{ GeV} < \sqrt{s} < 4.9 \text{ GeV}$

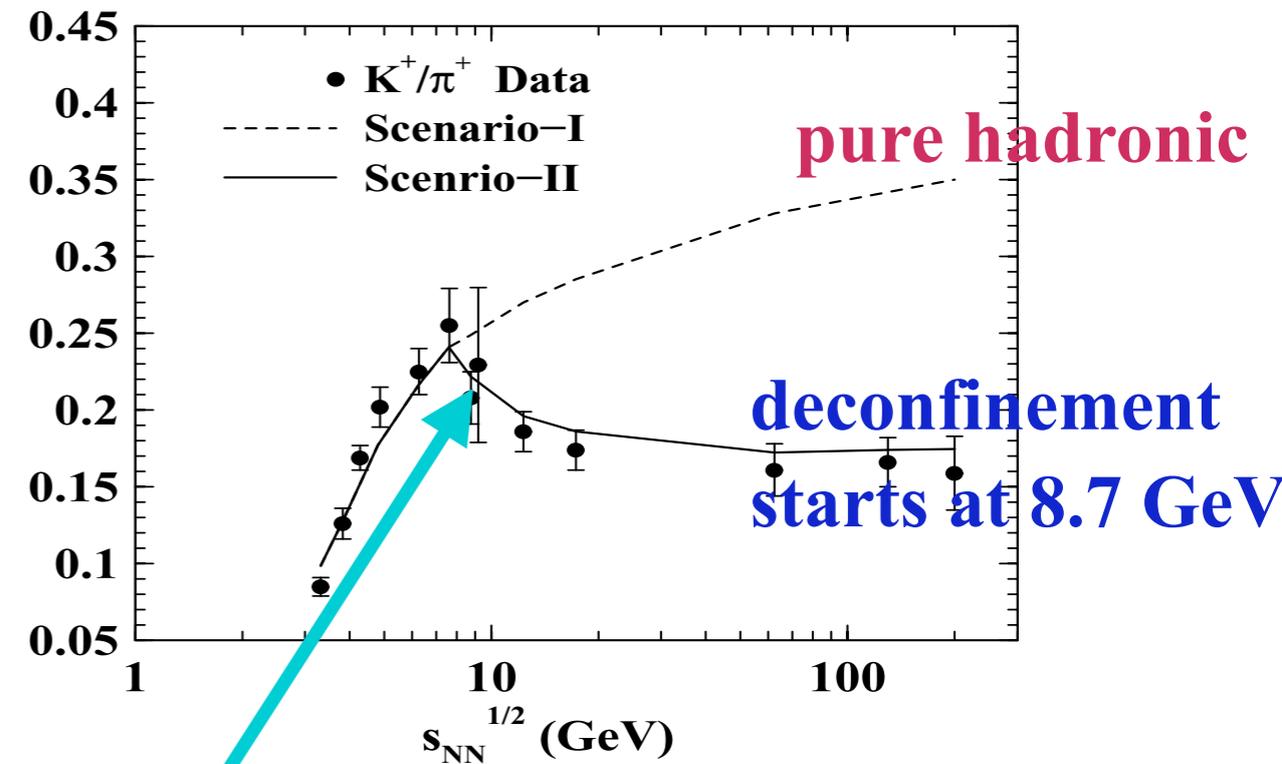
Other models predict deconfinement at $\sqrt{s} = 8.7- 9.2 \text{ GeV}$:

Onset of Deconfinement in Other Models

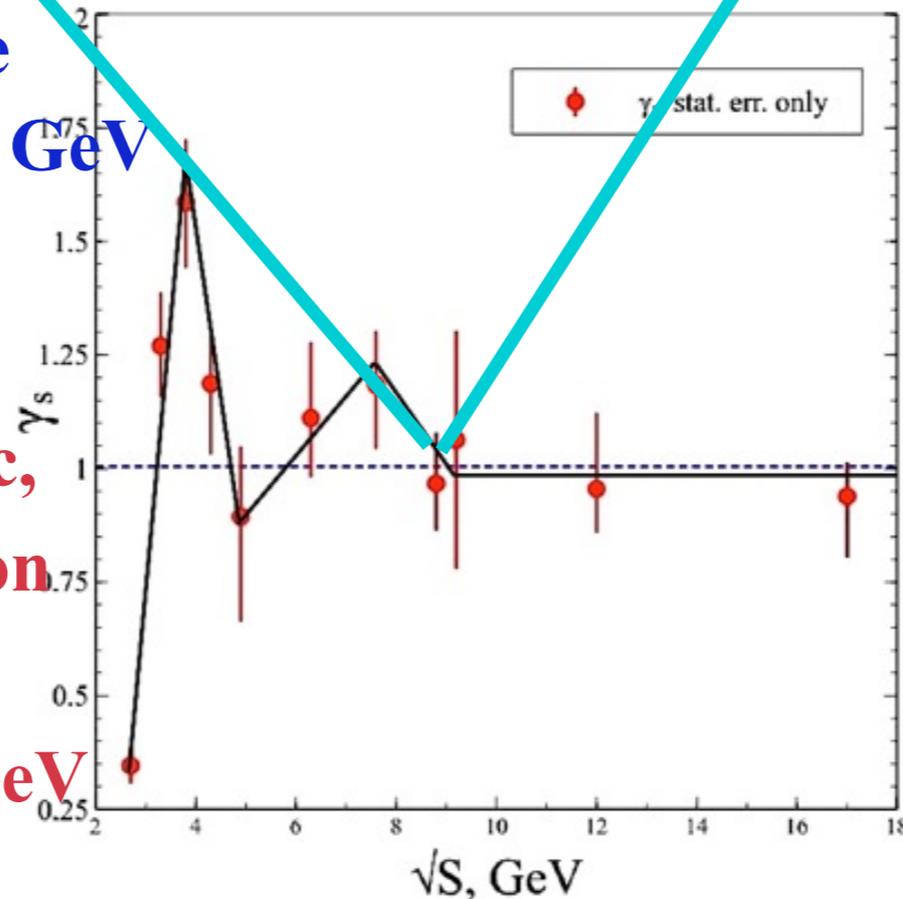
Che Ming Ko et al., arXiv 1702.07620 [nucl-th]



J. K. Nayak, S. Banik, Jan-e Alam, PRC **82**, 024914 (2010)



Light nuclei fluctuations are enhanced at c.m. energy 8.8 GeV \Rightarrow CEP is located nearby!



Strangeness Horn and other strange particles ratios can be explained, if the onset of deconfinement begins at c.m. energy 8.7 GeV!

Counting for thermodynamic, hydrodynamic and fluctuation signals we conclude that 3CEP may exist at 8.8-9.2 GeV

If There Are 2 Phase Transitions, then

1. What kind of phase exists at $\sqrt{s} = 4.9-9.2$ GeV?
2. Can we get any info about its properties?

Effective Number of Degrees of Freedom I

One look at this EoS:

$$p_{QGP} = \underbrace{A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B}_{\text{fitting}} = \underbrace{A_0^L T^4 + A_2^L T^2 \mu^2 + A_4^L \mu^4}_{LQCD} - B_{eff}$$

$$B_{eff}(T, \mu_B) = B - (A_0 - A_0^L) T^4 - (A_2 - A_2^L) T^2 \mu^2 - (A_4 - A_4^L) \mu^4$$

In our fit of entropy per baryon along the shock adiabat we used the QGP EoS

$$p_{QGP} = \underbrace{A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B}_{\text{fitting}}$$

$$A_0 \simeq 2.53 \cdot 10^{-5} \text{ MeV}^{-3} \text{ fm}^{-3}$$

$$A_2 \simeq 1.51 \cdot 10^{-6} \text{ MeV}^{-3} \text{ fm}^{-3}$$

$$A_4 \simeq 1.001 \cdot 10^{-9} \text{ MeV}^{-3} \text{ fm}^{-3}$$

$$B \simeq 9488 \text{ MeV fm}^{-3}$$

Effective Number of Degrees of Freedom II

One look at this EoS:

$$p_{QGP} = \underbrace{A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B}_{\text{fitting}} = \underbrace{A_0^L T^4 + A_2^L T^2 \mu^2 + A_4^L \mu^4}_{LQCD} - B_{eff}$$

$$B_{eff}(T, \mu_B) = B - (A_0 - A_0^L) T^4 - (A_2 - A_2^L) T^2 \mu^2 - (A_4 - A_4^L) \mu^4$$

Another look at this EoS:

$$p_{\text{New phase}} = \underbrace{A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B}_{\text{fitting}}$$

It corresponds to massless particles with strong interaction

Then one can find an effective #dof from A_0 !

For massless particles

$$A_0 = N_{dof} \frac{\pi^2}{90} \quad \text{with} \quad N_{dof} = N_{dof}^{Bosons} + \frac{7}{8} \times 2N_{dof}^{Fermions}$$

$$\Rightarrow N_{dof} = A_0 \hbar^3 \frac{90}{\pi^2} \simeq 1800$$

It's a huge number for QGP!

K.A. Bugaev et al., Phys. Part. Nucl. Lett. 15, 210 (2018), arXiv:1709.05419 [hep-ph]

Possible Interpretations

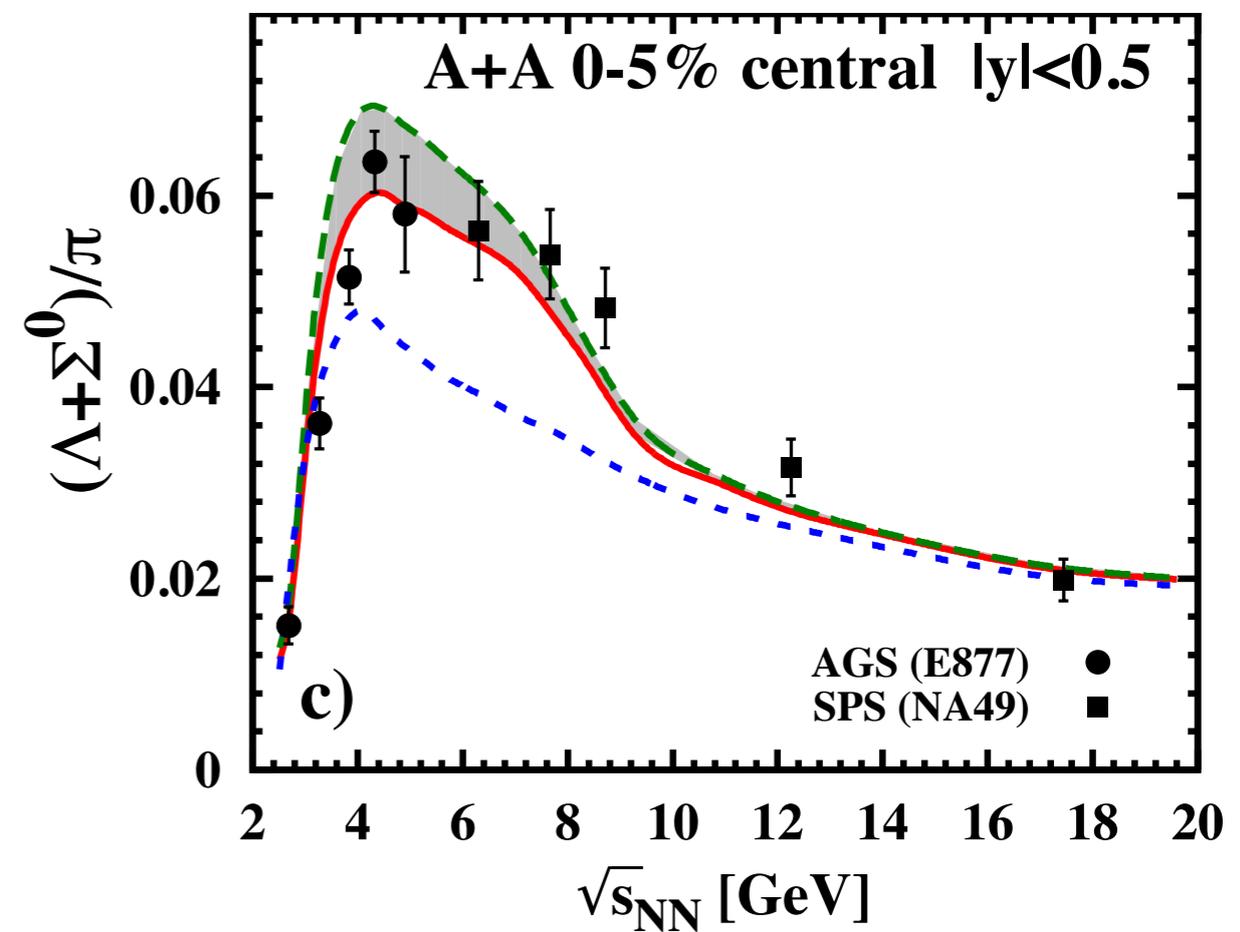
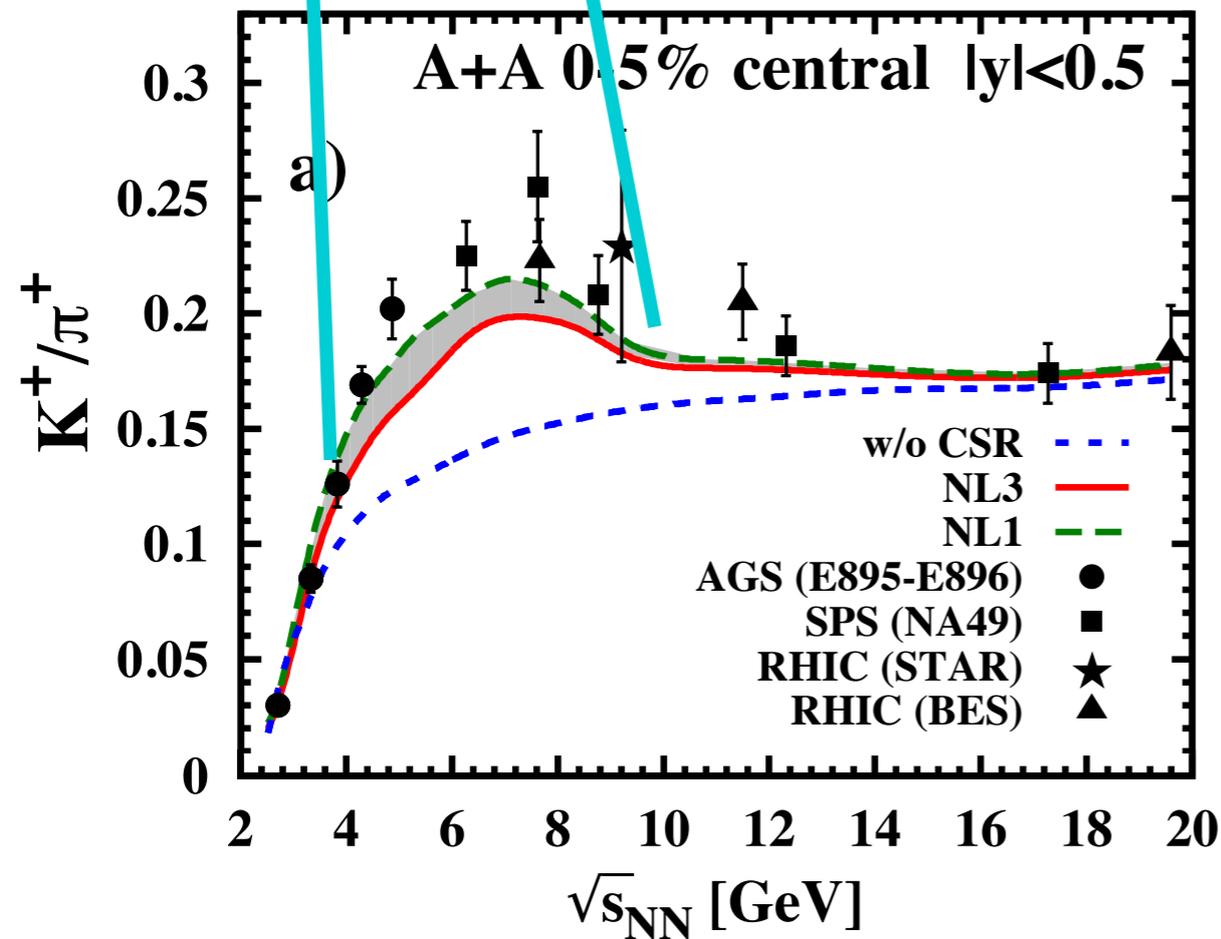
1. The phase emerging at $\sqrt{s} = 4.9-9.2$ GeV has **no Hagedorn mass spectrum**, since strange hadrons are not in chemical equilibrium.
2. 1800 of massless dof may evidence either about **chiral symmetry restoration in hadronic sector**.
3. Or 1800 of massless dof may evidence about tetra-quarks with massive strange quark!?
see Refs. in R.D. Pisarski, 1606.04111 [hep-ph]
4. Or 1800 of massless dof may evidence about quarkyonic phase!?
A. Andronic et. al, Nucl. Phys. A 837, 65 (2010)
5. 1800 of massless dof may evidence about something else...

Parton-Hadron-String-Dynamics Model

1-st order PT of Chiral Symmetry Restoration in
hadronic phase occurs at about $\sqrt{s} \sim 4$ GeV

and 2-nd order deconfinement PT exists at $\sqrt{s} \sim 9$ GeV
Hard to locate them due to cross-over in A+A!

W. Cassing et al., Phys. Rev. C 93, 014902 (2016);
Phys. Rev. C 94, 044912 (2016).



Conclusions

1. High quality description of the chemical FO data allowed us to find **few novel irregularities** at c.m. energies 4.3-4.9 GeV (pressure, entropy density jumps e.t.c.)
2. HRG model with multicomponent repulsion allowed us to find the **correlated (quasi)plateaus** at c.m. energies 3.8-4.9 GeV which were predicted about 27 years ago.
3. The second set of plateaus and irregularities may be a signal of another phase transition! Then the QCD diagram **3CEP may exist at the vicinity of c.m. energies 8.8-9.2 GeV.**
4. Generalized shock adiabat model allowed us to describe entropy per baryon at chemical FO and determine the parameters of the **EOS of new phase from** the data.
5. Hopefully, FAIR, NICA and J-PARC experiments will allow us to make more definite conclusions

Thank You for Your Attention!

For a summary of two QCD

PT signals see

K.A. Bugaev et al.,

arXiv:1801.08605 [nucl-th]

and references therein

Table 1. The summary of possible PT signals. The column II gives short description of the signal, while the columns III and IV indicate its location, status and references.

No and Type	Signal	C.-m. energy \sqrt{s} (GeV) Status	C.-m. energy \sqrt{s} (GeV) Status
1. Hydrodynamic	Highly correlated quasi-plateaus in entropy/baryon, thermal pion number/baryon and total pion number/baryon. Suggested in [11, 12].	Seen at 3.8-4.9 GeV [4, 5]. Explained by the shock adiabat model [4, 5].	Seen at 7.6-9.2 GeV [4, 5]. Require an explanation.
2. Thermodynamic	Minimum of the chemical freeze-out volume V_{CFO} .	In the one component HRGM it is seen at 4.3-4.9 GeV [13]. In the multicomponent HRGM it is seen at 4.9 GeV [14]. Explained by the shock adiabat model [4, 5].	Not seen.
3. Hydrodynamic	Minimum of the generalized specific volume $X = \frac{\epsilon+p}{\rho_b^2}$ at chemical freeze-out.	Seen at 4.9 GeV [4]. Explained by the shock adiabat model [4, 5].	Seen at 9.2 GeV [4]. Require an explanation
4. Thermodynamic	Peak of the trace anomaly $\delta = \frac{\epsilon-3p}{T^4}$.	Strong peak is seen at 4.9 GeV [5]. Is generated by the δ peak on the shock adiabat at high density end of the mixed phase [5].	Small peak is seen at 9.2 GeV [5]. Require an explanation
5. Thermodynamic	Peak of the baryonic density ρ_b .	Strong peak is seen at 4.9 GeV [10]. Is explained by $\min\{V_{CFO}\}$ [14].	Strong peak is seen at 9.2 GeV [10]. Require an explanation
6. Thermodynamic	Apparent chemical equilibrium of strange charge.	$\gamma_s = 1$ is seen at 4.9 GeV [10]. Explained by thermostatic properties of mixed phase at $p = const$ [10].	$\gamma_s = 1$ is seen at $\sqrt{s} \geq$ 8.8 GeV [10, 13]. Explained by thermostatic properties of QG bags with Hagedorn mass spectrum [10].
7. Fluctuational (statistical mechanics)	Enhancement of fluctuations	N/A	Seen at 8.8 GeV [9]. Can be explained by CEP [9] or 3CEP formation [10].
8. Microscopic	Strangeness Horn (K^+/π^+ ratio)	N/A	Seen at 7.6 GeV . Can be explained by the onset of deconfinement at [15]/above [8] 8.7 GeV .

Thank You for Your Attention!

Evidence for Chiral Symmetry Restoration?

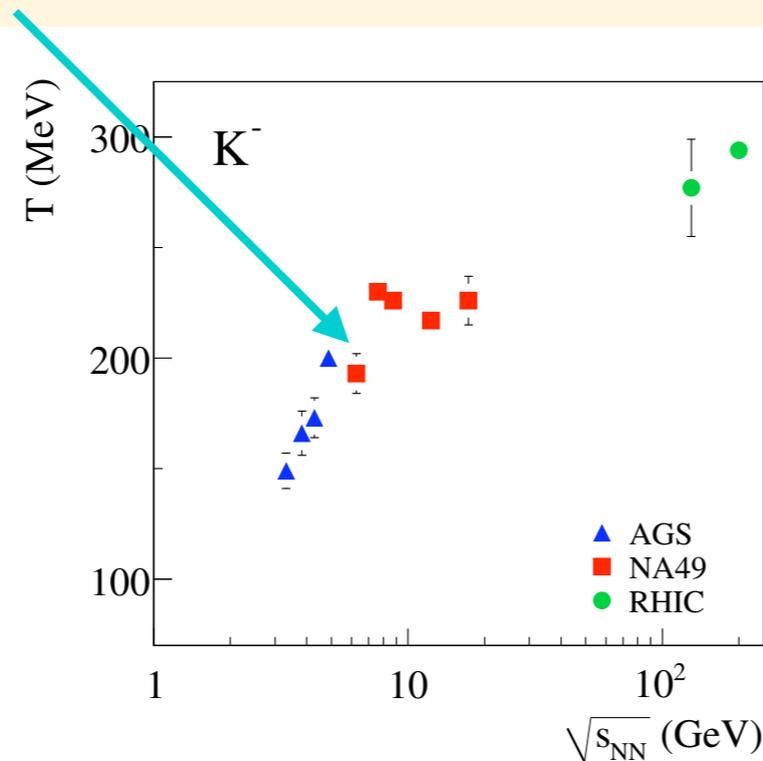
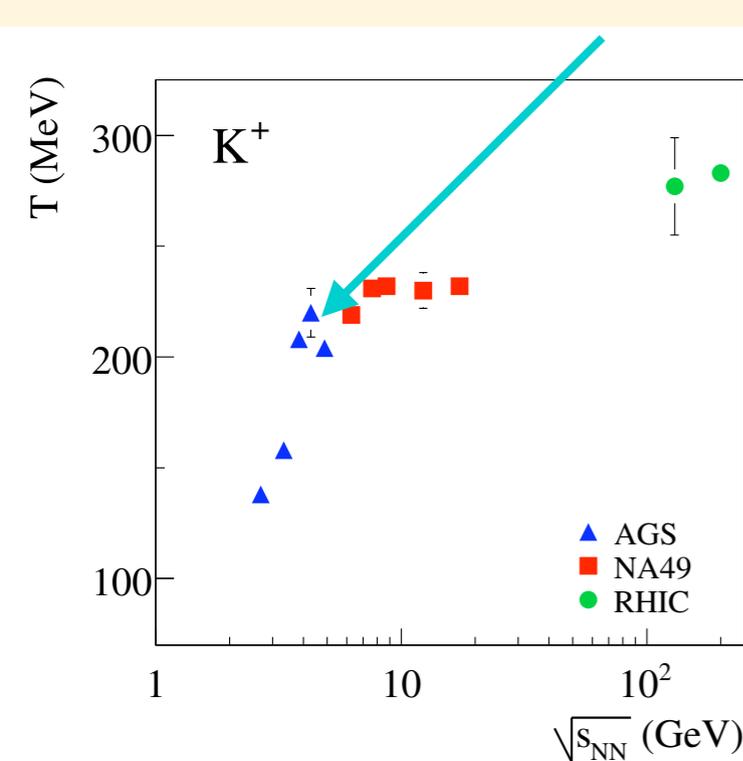
There are KINKs in apparent temperature of K^+ and K^- at 4.3-6.3 GeV

K.A. Bugaev et al., arXiv:1801.08605 [nucl-th]

$$T_k^*(p_T \rightarrow 0) = \frac{T_{fo}}{1 - \frac{1}{2} \bar{v}_T^2 (m_k/T_{fo} - 1)} \approx T_{fo} + \frac{1}{2} m_k \bar{v}_T^2$$

apparent temperature =
inverse slope of p_T spectra
at $p_T \rightarrow 0$:
depends on FO temperature
and mean transversal velocity

KINKs due to ChSR?



Simple (naive?) explanation:

1. FO temperature cannot decrease, if \sqrt{s} increases.
2. mean transversal velocity cannot decrease, if \sqrt{s} increases.

=> mass of Kaons gets lower due to ChSR restoration!?

M. Gazdzicki, M. I. Gorenstein and P. Seyboth, Acta Phys. Polon. B 42, 307 (2011)

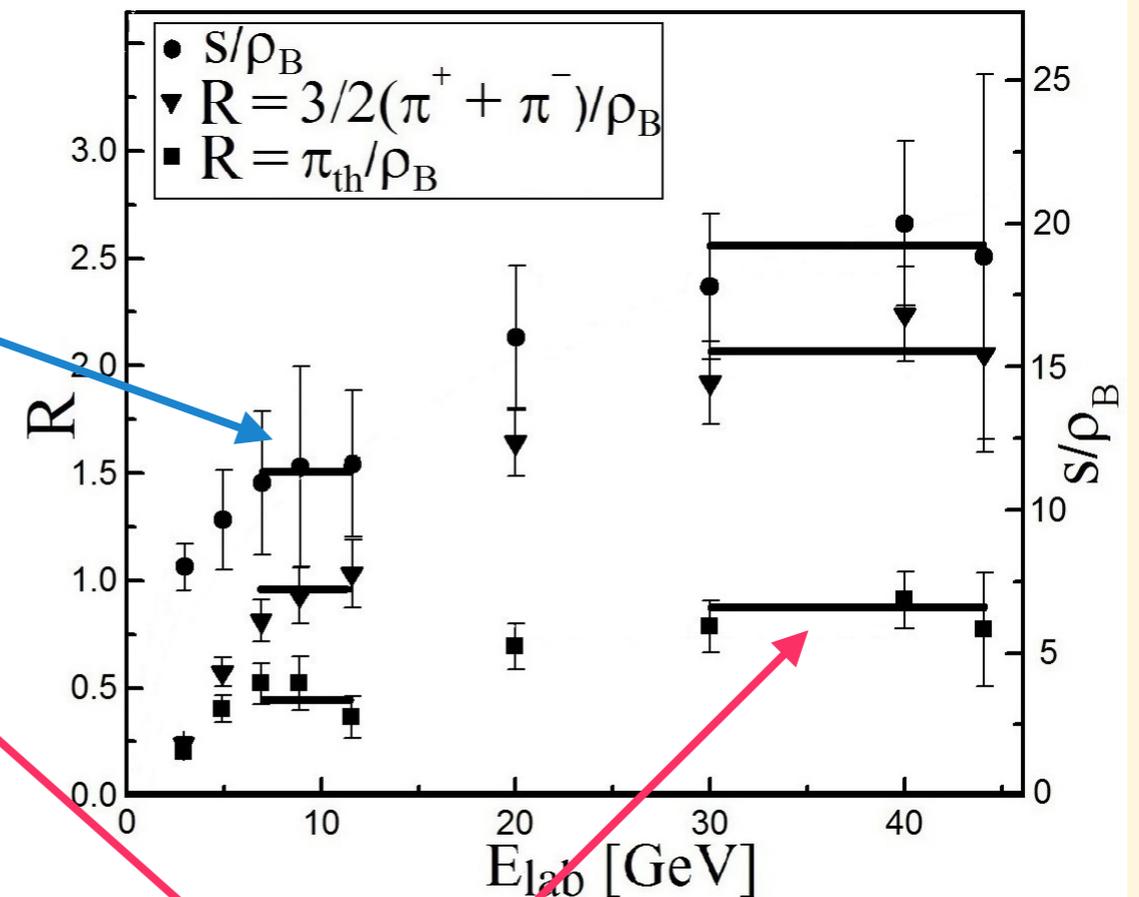
Suggestions for RHIC BESII, NICA and FAIR:
measure p_T spectra and apparent temperature of Kaons and
(anti) Λ hyperons at 4.3-6.3 GeV with high accuracy and
small collision energy steps!

Details on Highly Correlated Quasi-Plateaus

- Common width M – number of points belonging to each plateau
- Common beginning i_0 – first point of each plateau
- For every M , i_0 minimization of χ^2/dof yields $A \in \{s/\rho_B, \rho_\pi^{\text{th}}/\rho_B, \rho_\pi^{\text{tot}}/\rho_B\}$:

$$\chi^2/\text{dof} = \frac{1}{3M-3} \sum_A \sum_{i=i_0}^{i_0+M-1} \left(\frac{A - A_i}{\delta A_i} \right)^2 \Rightarrow A = \frac{\sum_{i=i_0}^{i_0+M-1} \frac{A_i}{(\delta A_i)^2}}{\sum_{i=i_0}^{i_0+M-1} \frac{1}{(\delta A_i)^2}}$$

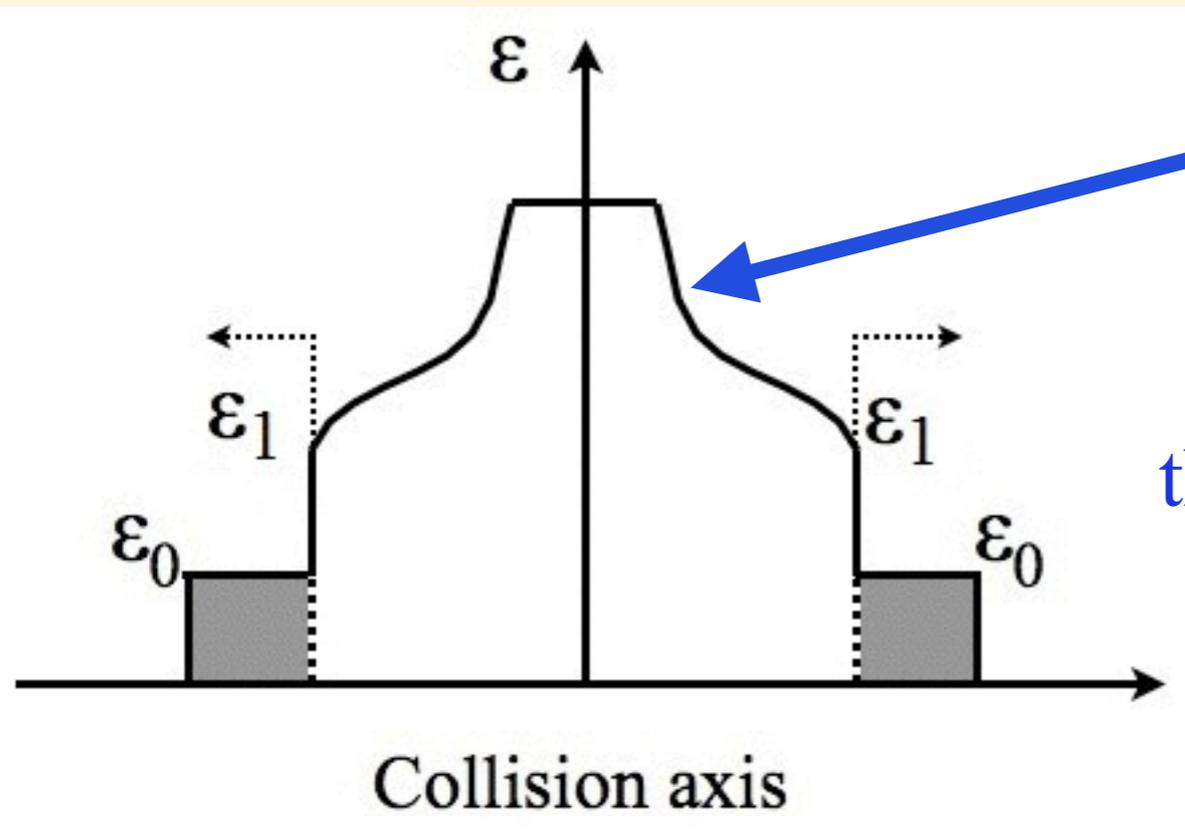
Low energy plateau					
M	i_0	s/ρ_B	$\rho_\pi^{\text{th}}/\rho_B$	$\rho_\pi^{\text{tot}}/\rho_B$	χ^2/dof
2	3	11.12	0.52	0.85	0.17
3	3	11.31	0.46	0.89	0.53
4	2	10.55	0.43	0.72	1.64
5	2	11.53	0.47	0.84	4.45
High energy plateau					
2	8	19.80	0.88	2.20	0.12
3	7	18.77	0.83	2.05	0.34
4	6	17.82	0.77	1.87	0.87
5	5	16.26	0.64	1.62	3.72



Generalized Shock Adiabats Model

In case of unstable shock transitions more complicated flows appear:

K.A. Bugaev, M.I. Gorenstein, B. Kampher, V.I. Zhdanov, Phys. Rev. D 40, 9, (1989)
 K.A. Bugaev, M.I. Gorenstein, D.H. Rischke, Phys. Lett. B 255, 1, 18 (1991)



shock 01 + compression simple wave

In each point of simple wave $\frac{s}{\rho_B} = \text{const}$

If during expansion entropy conserves, then unstable parts lead to entropy plateau!

Remarkably

Z model has stable RHT adiabat, which leads to quasi plateau!

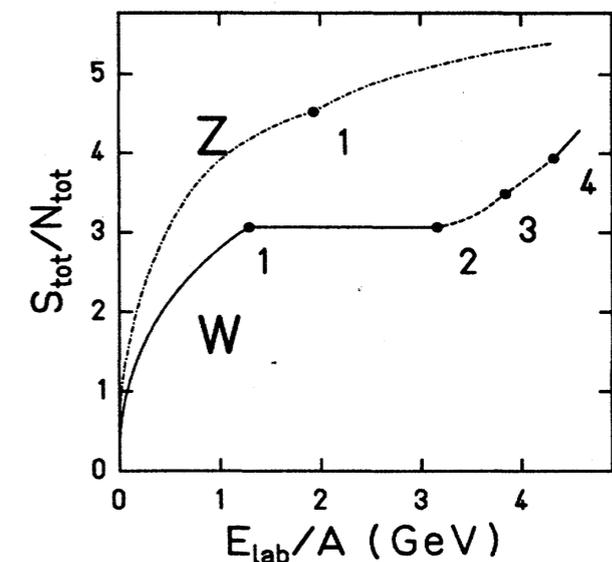


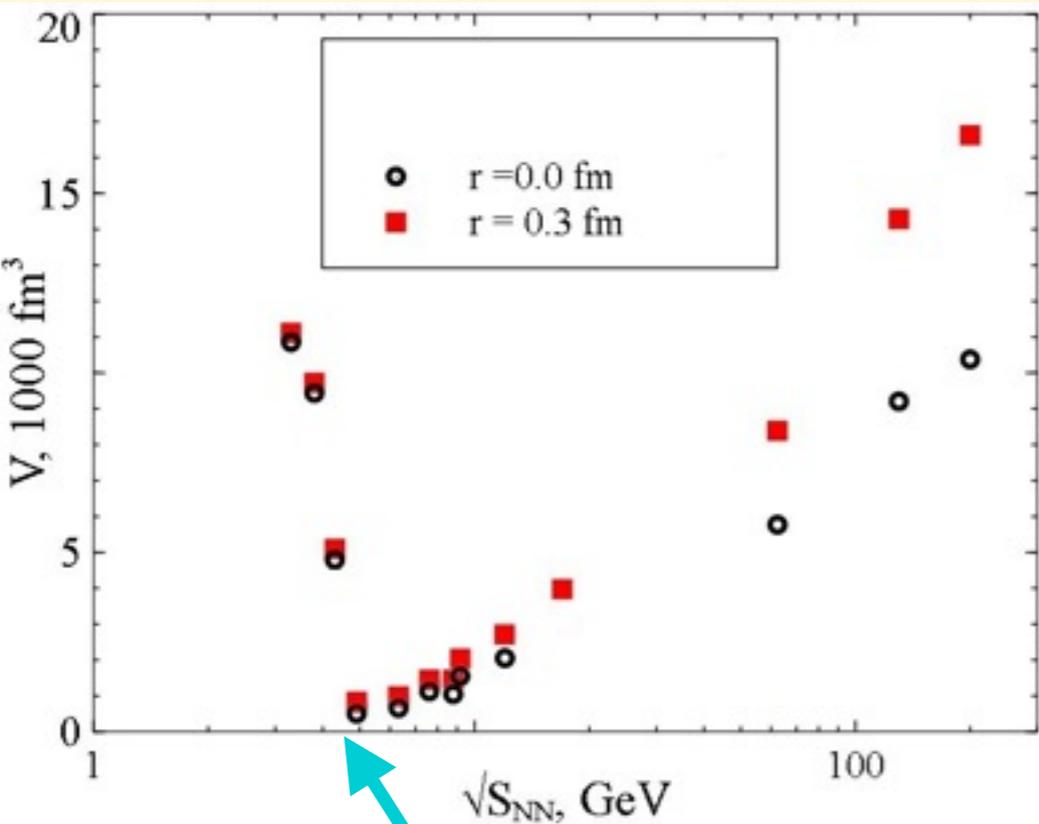
FIG. 9. The entropy per baryon as a function of the bombarding energy per nucleon of the colliding nuclei for models W and Z. The points 1, 2, 3, 4 on curve W correspond to those on the generalized adiabat as displayed in Fig. 7. The point 1 on curve Z marks the boundary to the mixed phase.

Other Minima at AGS Energies

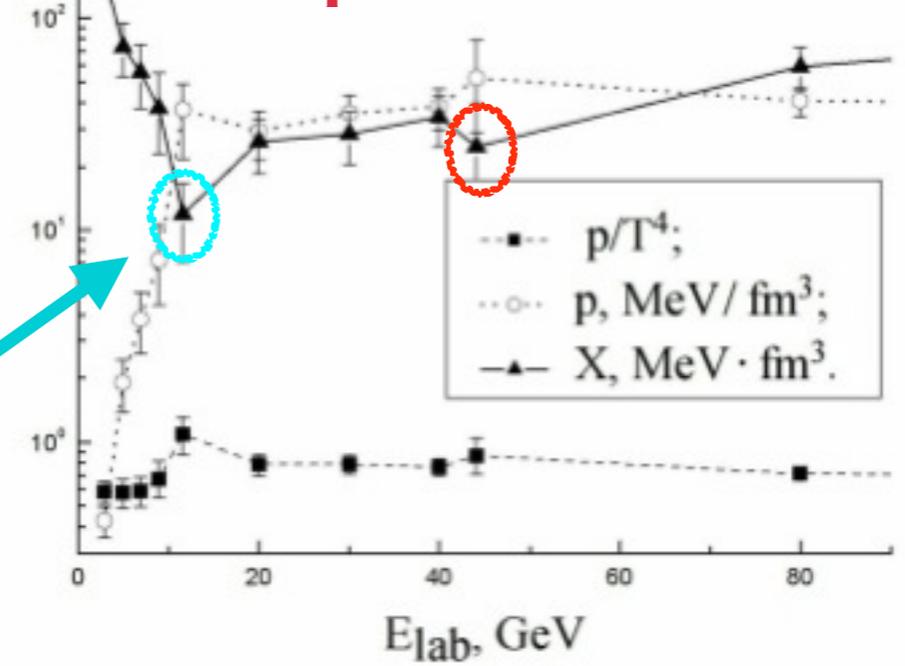
min V at ChFO

SAME energy!

min X at ChFO



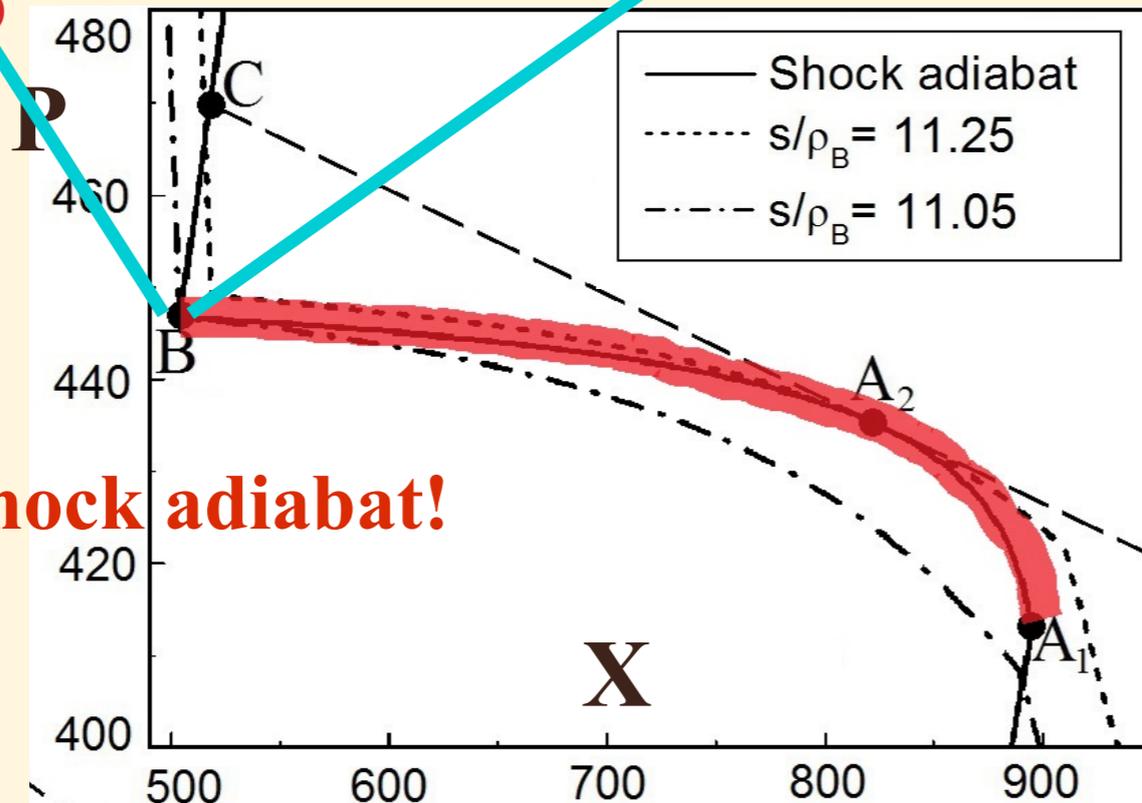
X is generalized specific volume
Is second X peak due to other PT?



D.R. Oliinychenko, K.A. Bugaev and A.S. Sorin,
Ukr. J. Phys. 58, (2013)

K.A. Bugaev et al., EPJ A (2016)

min X at shock adiabat!



In this work we gave
a proof that min X
at boundary between
QGP? and mixed phase
generates min X at ChFO
which leads to min V
of ChFO!

Induced Surface Tension EOS for HRGM

This EoS allows one to go beyond the Van der Waals approximation!

pressure $\frac{p}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right)$ **new term**

induced surface tension $\frac{\Sigma}{T} = \sum_i R_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right) \cdot \overbrace{\exp\left(\frac{(1 - \alpha)S_i \Sigma}{T}\right)}$

V_k and S_k are **eigenvolume and eigensurface of hadron of sort k**

α switches excluded and eigen volume regimes
high order virial coefficients

Advantages

- 1. Allows to go beyond the Van der Waals approximation**
- 2. Number of equations is 2 and it does not depend on the number different hard-core radii!**

see V.V. Sagun et al., arXiv:1703.00009 [hep-ph]

Consequent Problem and Its Possible Solution

If 1800 of massless dof exist then at high T and same μ_B the QGP cannot exist, since its pressure is too low to dominate!

⇒ Contradiction with Lattice QCD!

The only possibility to avoid the contradiction with LQCD is to assume hard-core repulsion for 1800 of massless dof!

Since they are almost massless ($m \ll T$), then the hard-core repulsion should be formulated for ultra-relativistic particles and include the effect of Lorentz contraction.

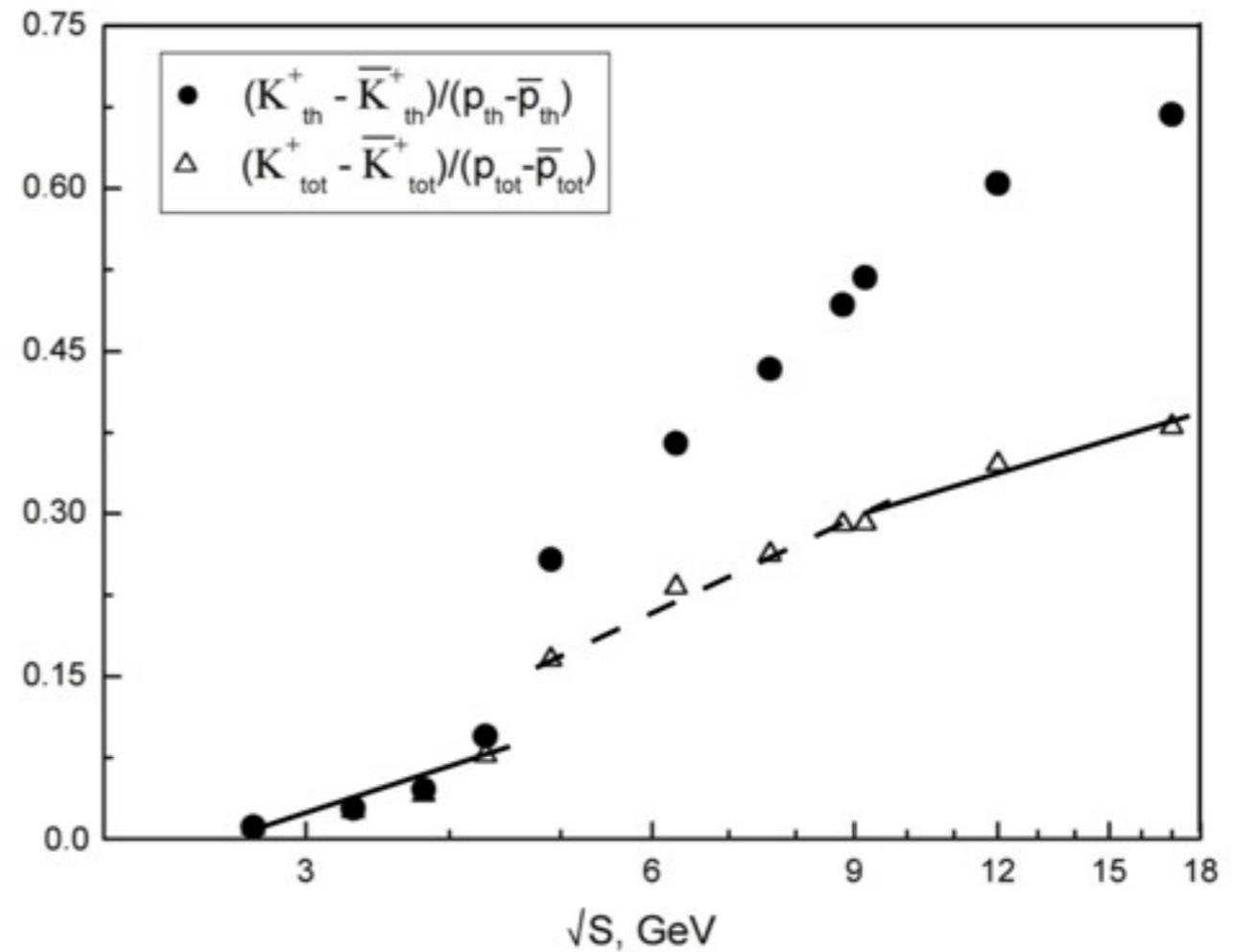
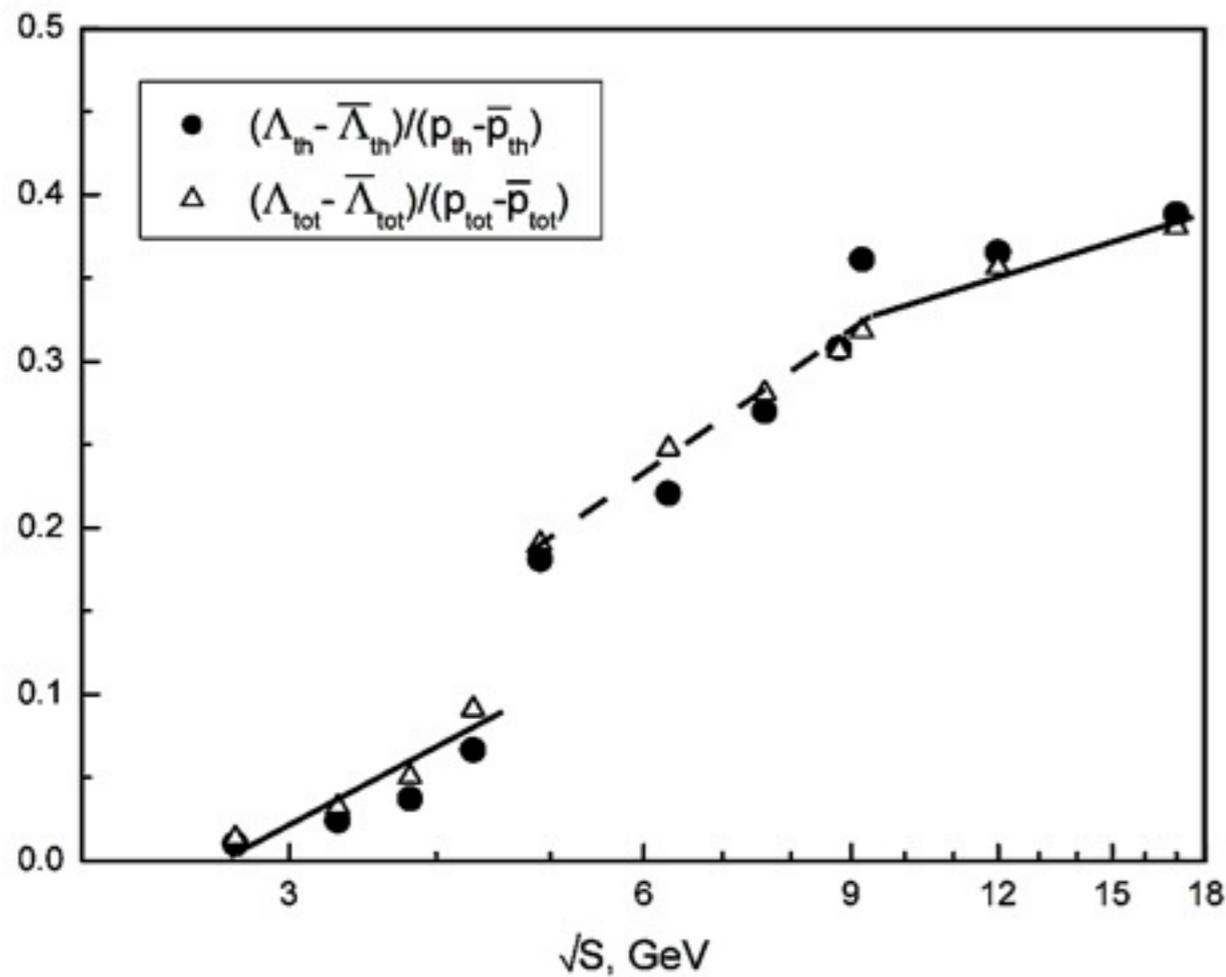
see K. A. Bugaev, Nucl. Phys. A 807, 251 (2008).

In the limit $\mu_B / T \ll 1$ and $\text{mass}/T \ll 1$ the pressure of such system is

$$p \simeq \frac{T^2}{V_0^{\frac{2}{3}}} N_{dof}^{\frac{1}{3}} C \quad \text{with} \quad C = \text{Const} \sim 1 \quad \text{here } V_0 \text{ is eigenvolume of hadron}$$

No mass dependence and very weak dependences on T and on #dof: $N_{dof}^{\frac{1}{3}} \simeq 12$

What To Measure at FAIR & NICA ?



We predicted **JUMPS** of these ratios at 4.3 GeV due to 1-st order PT and **CHANGE OF** their **SLOPES** at $\sim 8-10$ GeV due to 2-nd order PT (or weak 1-st order PT?)

To locate the energy of **SLOPE CHANGE** we need **MORE** data at 4-13 GeV