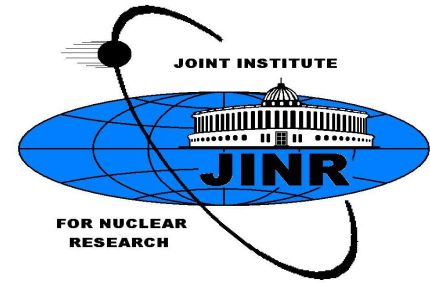




XXIV International Baldin Seminar  
on High Energy Physics Problems  
*Relativistic Nuclear Physics & Quantum Chromodynamics*

September 17 - 22, 2018, Dubna, Russia



# Energy dependence of the vector $A_y$ and tensor $A_{yy}$ and $A_{xx}$ analyzing powers in deuteron- proton elastic scattering at large scattering angles

**DSS**  
*pn*  
**deuteron** structure

*V.P. Ladygin on behalf of DSS collaboration*

# Outline

- **Motivation**
- **Review of dp-elastic scattering studies at intermediate energies**
- **Recent results on dp-elastic scattering obtained at Nuclotron JINR**
- **Conclusion**

**DSS collaboration: 12 Institutes and Universities from  
Bulgaria-JINR-Japan-Romania-Russia-Slovakia**

# Three body problem according H.Sakai



'How about the problem of the three bodies?' whispered Kritsov, smiling with great effort.

'The solution is difficult? '

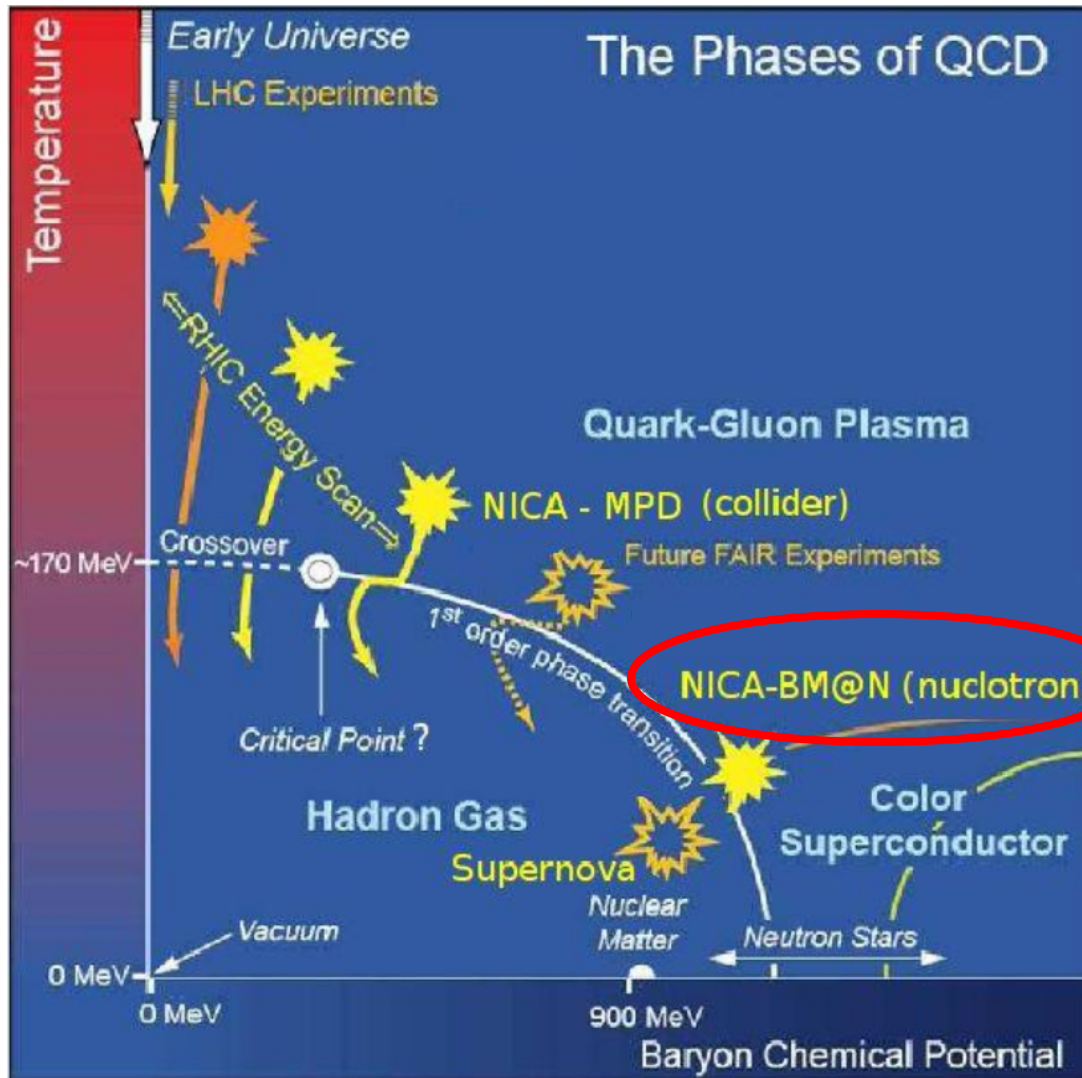
Resurrection, 1899, Chapter XX

by Leo Tolstoy

# Motivation of the **dp** interaction studies

- **Nucleon-nucleon interaction at short distances (including its mass off-shell behaviour)**
- **Relativistic effects**
- **Transition to the nonnucleonic degrees of freedom**
- **Contribution of three-nucleon forces (3NFs)**

# stalled from E.L.Bratkovskaya



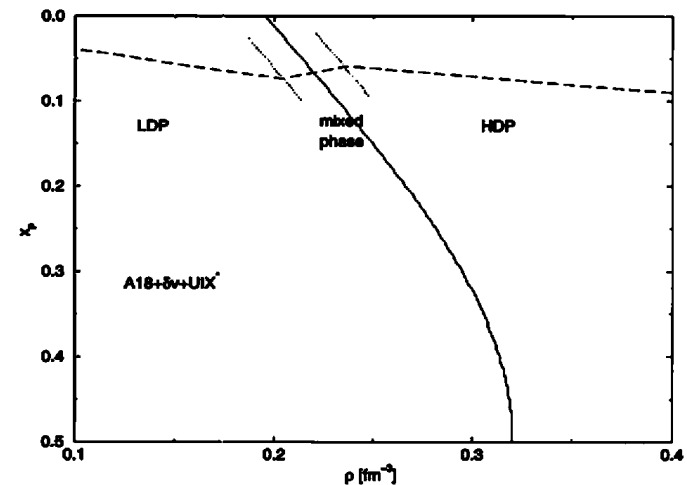
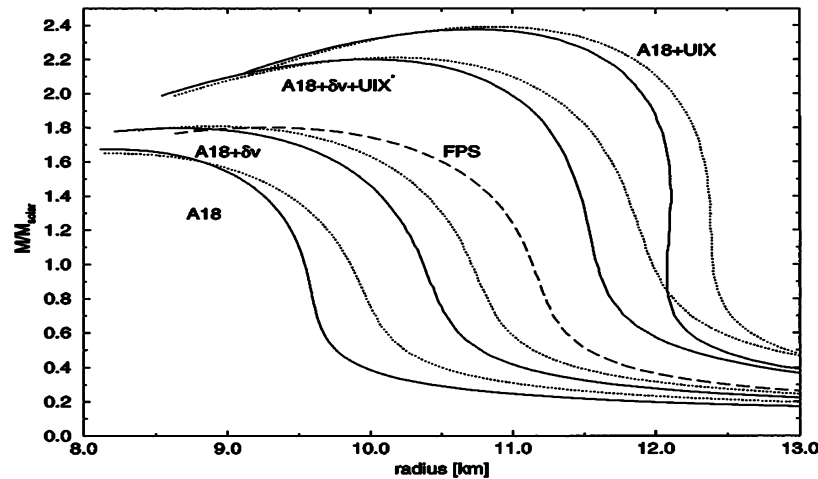
Study of the **phase transition** from hadronic to partonic matter – **Quark-Gluon-Plasma**

Many theoretical approaches: transport, thermodynamics etc. transferred to the event generators UrQMD, HSD, LAQGSM ...

**Lattice QCD calculations**

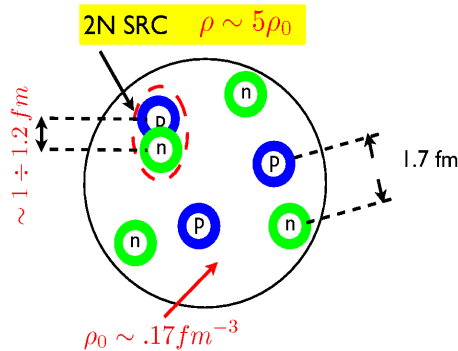
# Few nucleons systems as a tool for dense matter studies

Alternative way to obtain the information on the EOS at extreme densities (neutron stars) is the studies of the few nucleon systems.



Relativistic effects in 2NF and contribution of 3NF play very important role. (A.Akhmal et al, Phys.Rev. C58 (1998) 1804)

# Short range correlations (SRCs)



Summary of the theoretical analysis of the experimental findings  
*practically all of which were predicted well before the data were obtained*

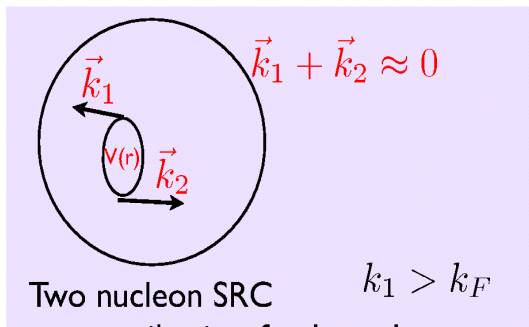
More than ~90% all nucleons with momenta  $k \geq 300 \text{ MeV/c}$  belong to two nucleon SRC correlations BNL + Jlab + SLAC

Probability for a given proton with momenta  $600 > k > 300 \text{ MeV/c}$  to belong to **pn** correlation is ~ 18 times larger than for **pp** correlation BNL + Jlab

Probability for a nucleon to have momentum  $> 300 \text{ MeV/c}$  in medium nuclei is ~25% BNL + Jlab 04 + SLAC 93

Probability of non-nucleonic components within SRC is small - < 20% - 2N SRC mostly build of two nucleons not  $6q, \Delta\Delta, \dots$  BNL + Jlab + SLAC

Three nucleon SRC are present in nuclei with a significant probability Jlab 05



**Poor data base on the spin parts of the 2N and 3N short-range correlations. This motivates the necessity to study light nuclei structure at short distances.**

**Experiments at Nuclotron allow to reach  $p_T \sim 1 \text{ GeV/c}$**

# Relativistic effects

- The principal feature of the relativistic quantum mechanics is the impossibility to separate the relative motion of the constituents and motion of the composite system as a whole. This leads to the dependence of the **relativistic** wave function not only on the relative momenta of the nucleons  $\vec{q}$  inside the composite system, but also on the total momentum  $\vec{p}$  of this system

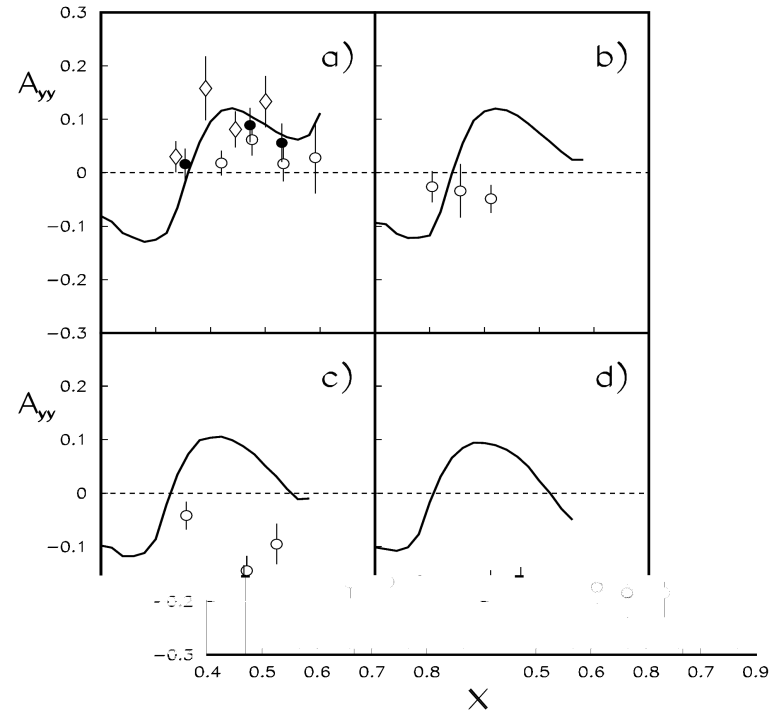
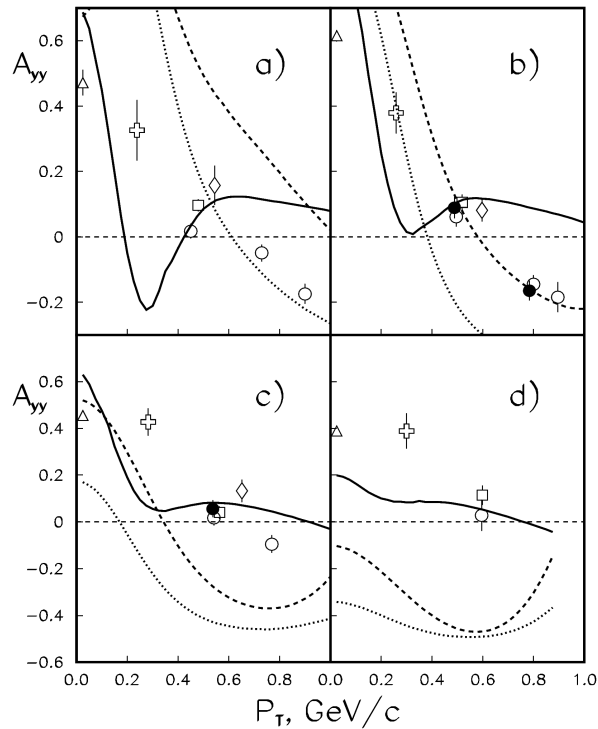
$$\Psi = \Psi(\vec{q}, \vec{p})$$

- Therefore, **relativistic** wave function is the function of the relative momentum  $\vec{q}$  in each new reference system.
- However, it is enough to know wave function in the infinite momentum frame,  $\vec{p} \rightarrow \text{inf}$ , where the structure of the wave function simplifies. Namely, the dependence on  $|\vec{p}|$  disappears, only the dependence on the direction of the vector  $\vec{n} = \vec{p}/|\vec{p}|$

$$\Psi = \Psi(\vec{q}, \vec{n})$$



# Relativistic effects in 2N SRCs (deuteron)

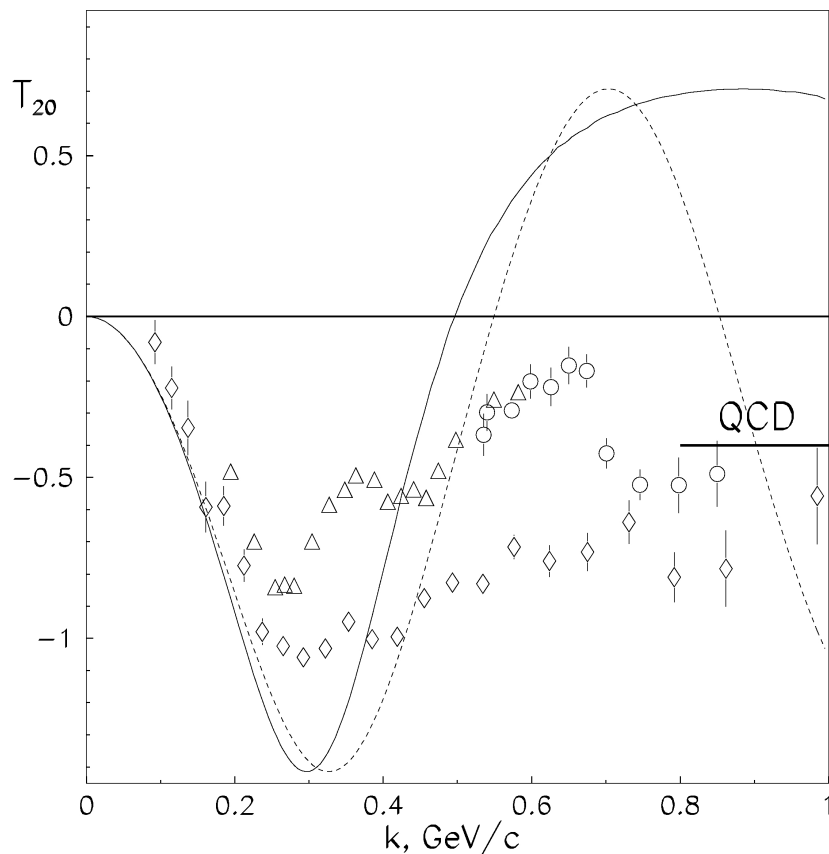


$A_{yy}$  in deuteron inclusive breakup demonstrates the dependence on 2 internal variables:  $p_T$  and  $x_F$ .

$A_{yy}$  changes the sign at  $p_T$  of about 600 MeV/c independently on  $x_F$ .

$A_{yy}$  demonstrates negative asymptotic at large  $p_T$ .

# Non-nucleonic degrees of freedom



When the distances between the nucleons are comparable with the size of the nucleon, the nucleon-nucleon interaction is a **non-local**.

The fundamental degrees of freedom, quark and gluons in the frame of QCD, begin also to play a role at the internucleonic distances comparable with the size of the nucleon.

They can manifest as  $\Delta\Delta$ ,  $NN^*$ ,  $N^*N^*$ ,  $6q$  etc. components.

## Data:

V.Punjabi et al., Phys.Lett.B350 (1995) 178

L.S.Azhgirey et al., Phys.Lett.B391 (1997) 22

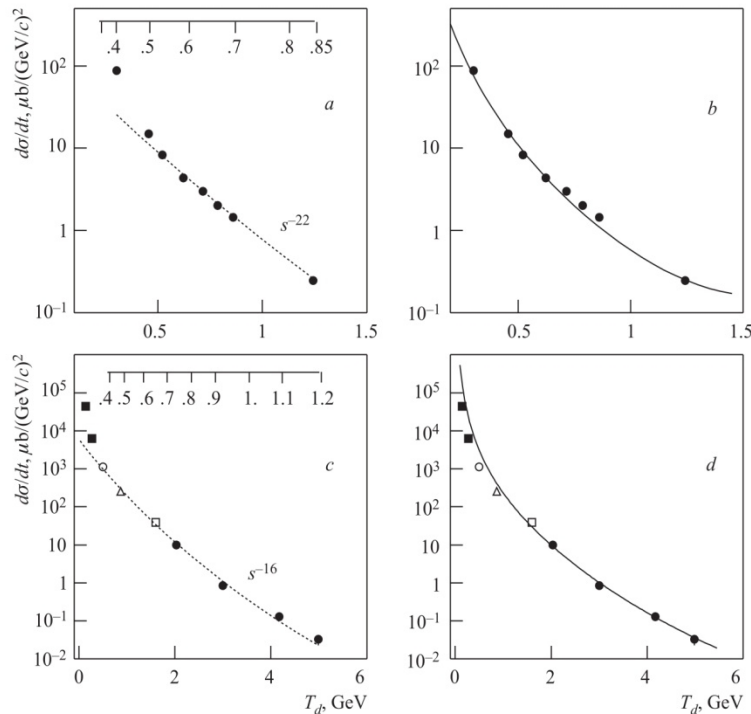
L.S.Azhgirey et al., Phys.Lett.B387 (1996) 37

# Fundamental (quark) degrees of freedom

At high energy  $s$  and large transverse momenta  $p_t$  the constituent counting rules (CCR) predict the following behavior of the differential cross section for the binary reactions:

$$\frac{d\sigma}{dt}(ab \rightarrow cd) = \frac{f(t/s)}{s^{n-2}} \quad ; \quad \mathbf{n} = \mathbf{N}_a + \mathbf{N}_b + \mathbf{N}_c + \mathbf{N}_d$$

(Matveev, Muradyan, Tavkhelidze, Brodsky, Farrar et al.)



**Yu. N. Uzikov**

**JETP Lett, 81 (2005) 303-306**

For the reaction  $dd \rightarrow {}^3\text{He}n$

$$N_A + N_B + N_C + N_D - 2 = 22$$

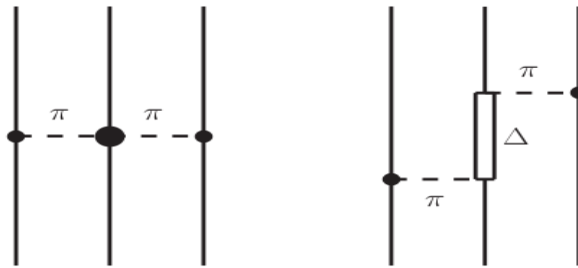
For the reaction  $dp \rightarrow dp$

$$N_A + N_B + N_C + N_D - 2 = 16$$

**The regime corresponding to CCR can occur already at  $T_d \sim 500$  MeV**

# Three Nucleon Forces

- Modern NN potentials (CD-Bonn, AV-18, Nijmegen etc.) accurately reproduce the NN data set up to about 350 MeV. However they fail in the description of the triton binding energy and data on unpolarized **dp**-elastic scattering and breakup.
- Incorporation of three nucleon forces (3NF), when interaction depends on the quantum numbers of the all three nucleon, allows to reproduce the binding energy of the three-nucleon bound systems and the data on unpolarized **dp**- interaction.



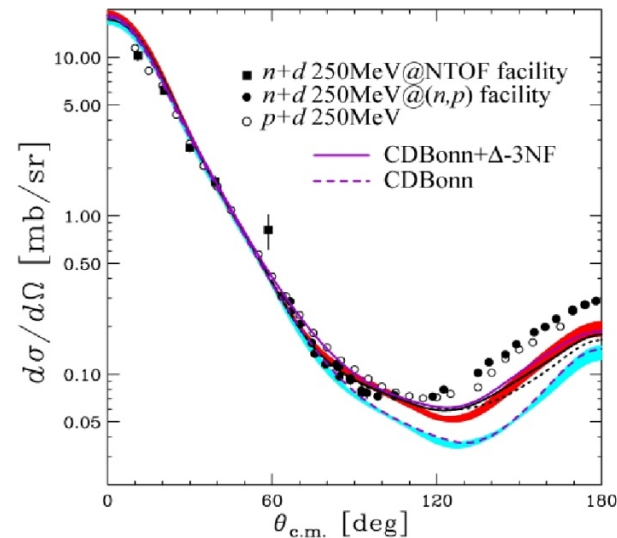
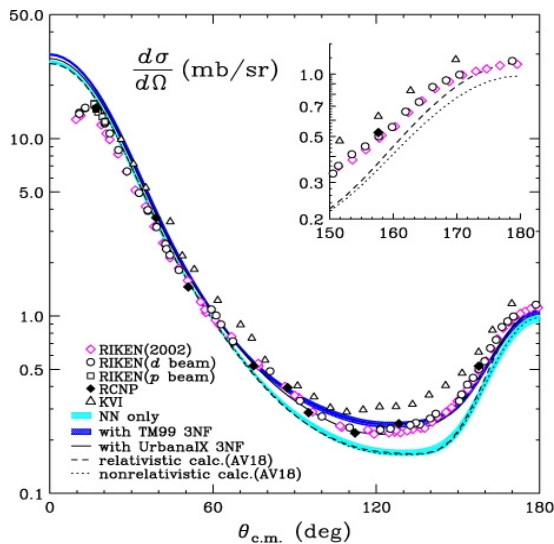
- Tucson-Melbourne
- Brazil
- Urbana-IX
- Fujita-Miyazawa ( $N\Delta$ )
- Chiral Effective Field Theory

Needs to be very careful: according to the theorem of **W.N.Polyzou and W.Gloeckle**, **Few Body Syst. 9 (1990) 97**, off-shell behaviour of 2NF can imitate 3NF effect.

Triton binding energy without 3NF:

**Y.Fujiwara et al., Phys.Rev.C66 (2002) 021001(R)**

# Cross section in **dp**- elastic scattering at intermediate energies



The differential cross section in elastic Nd scattering at the energy of 135 (left figure) and 250 (right figure) MeV/u.

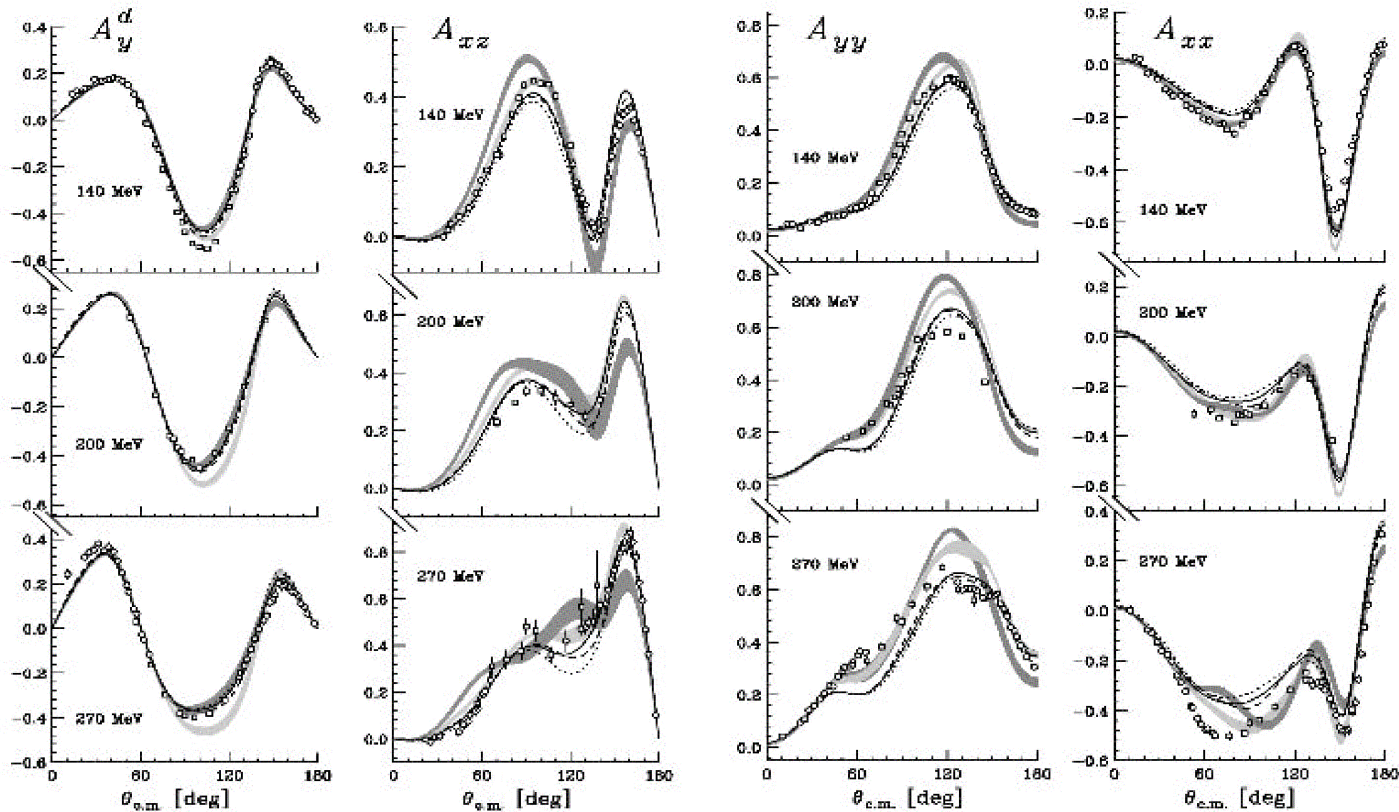
K. Sekiguchi et al., Phys. Rev. Lett. 95, 162301 (2005)

K. Hatanaka et al., Phys. Rev. C 66, 044002 (2002)

The cross section data for **dp**- elastic scattering are reproduced well up to 150 MeV taking into account 3NF. Manifestation of three-nucleon forces effect in the cross-section of **dp**-elastic scattering at this energy: up to **30%** in the vicinity of Sagara discrepancy.

But the problems in the description are at higher energies.

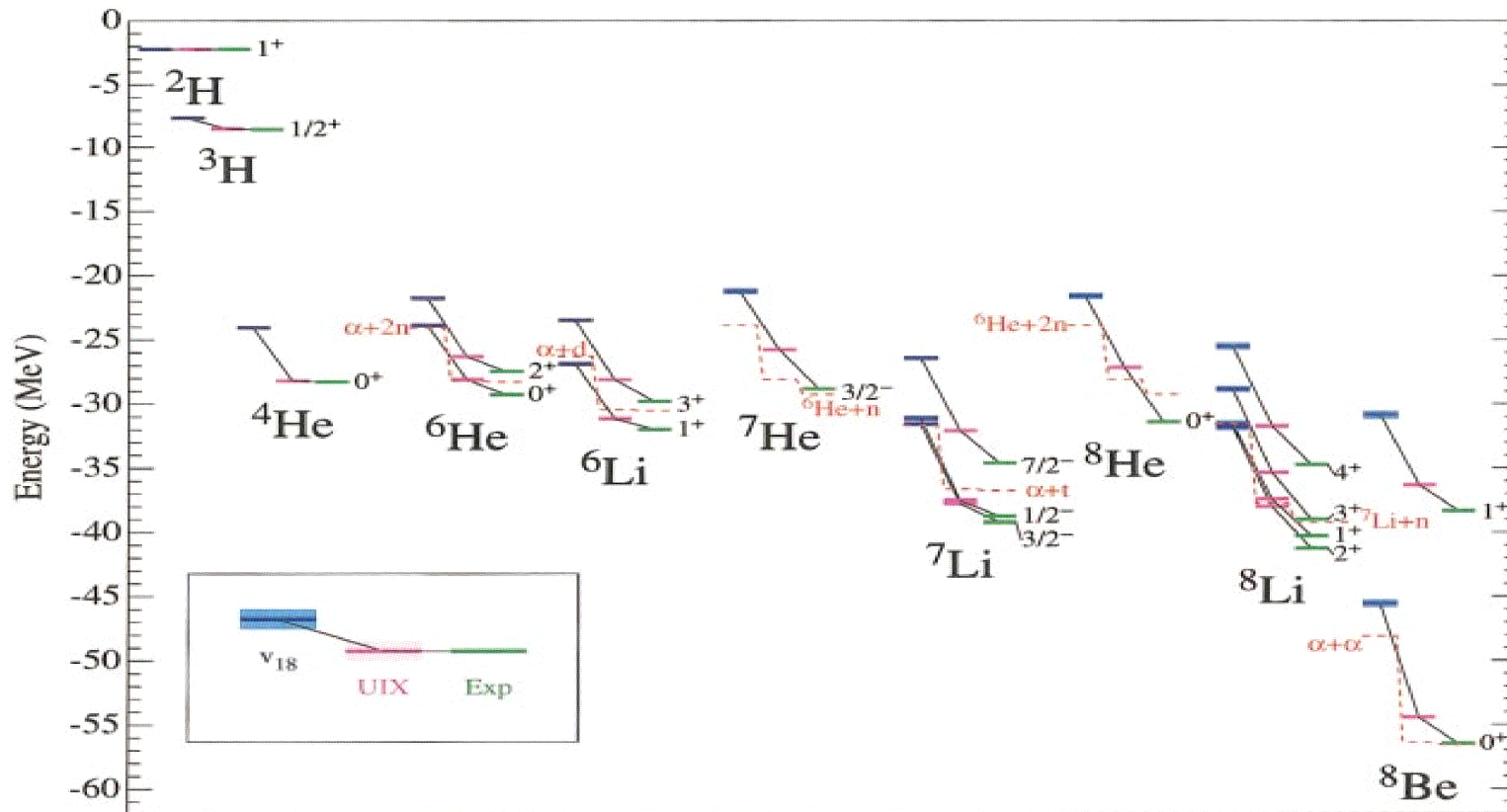
# Deuteron analyzing powers in **dp**- elastic scattering at intermediate energies (**140, 200, 270 MeV**)



Polarization data for **dp**- elastic scattering are not described even with the 3NFs inclusion (except for  $A_y$ ).

**The spin part of 3NFs is missed!**

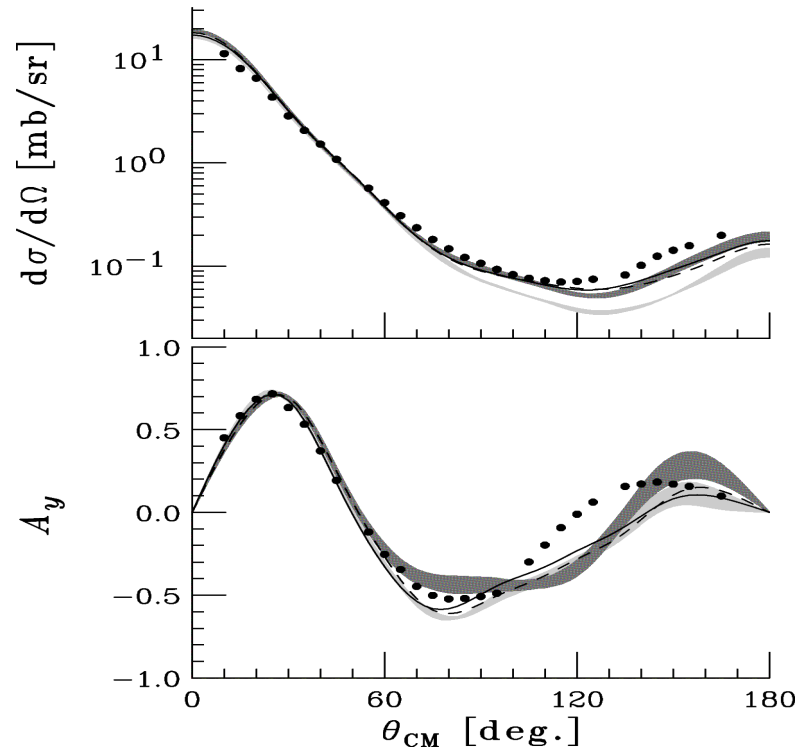
# Importance of the spin part of 3NF for the light nuclei binding energies



Spin parts of the 2N and 3N correlations are important to describe the light nuclei structure.

(S.C.Pieper et al., Phys.Rev.C64 (2001) 014001)

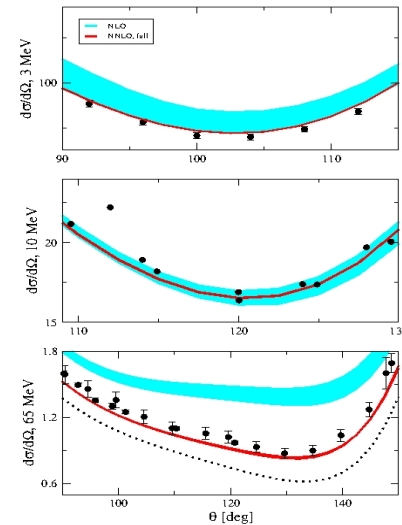
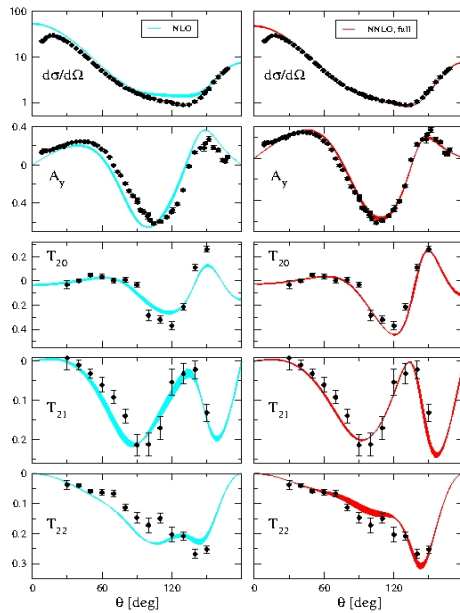
# Cross section and proton analyzing power $A_y$ in $pd$ - elastic scattering at 250 MeV



**Problems in description at backward angles.  
Relativistic effects become large ?  
Short range 3NFs manifestation ?**



# Chiral Effective Field Theory



**NNLO** allows to describe the data up to 65 MeV/n

**CFET** is out of game above the pion threshold production!  
However, new calculations exist at 200 MeV/nucleon.

# Status of **dp**- elastic scattering

Inclusion of modern 3NFs allows to describe cross section and deuteron vector analyzing power of **dp**- elastic scattering up to 135 MeV/nucleon, while the tensor observables are not described.

The data at higher energies (up to 300 MeV/nucleon) are not described even taking into account relativistic effects.

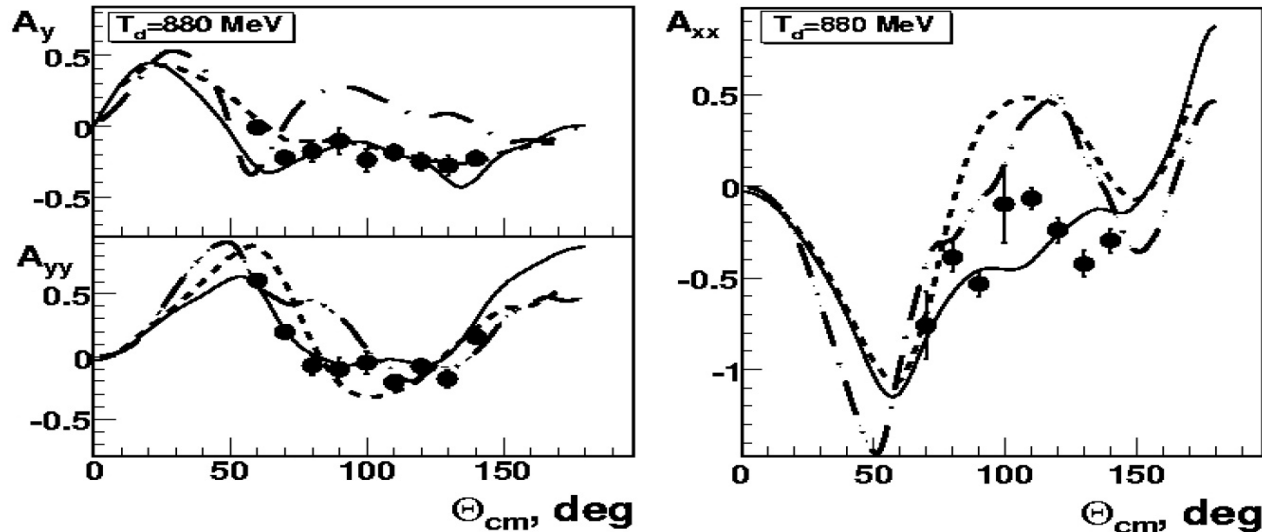
The reason of the discrepancy is nowadays called the importance of the **short range 3NFs** which are still not included.

1. The systematic study of hadronic reactions induced by deuterons at **Nuclotron** will allow to study the structure of **2N** and **3N forces**, including their short-range parts.
2. Development of the **relativistic** models for the description of these reactions is required.

The purpose of the **DSS** experimental program is to obtain the information about **2NF** and **3NF** (including their spin – dependent parts) from two processes:

- 1.dp-elastic scattering at the energies between **300 - 2000 MeV**;
- 2.dp-breakup with registration of two protons at deuteron energies of **300 - 500 MeV**. (talk of **M.Janek**)

# Analyzing powers in **dp**- elastic scattering at 880 MeV



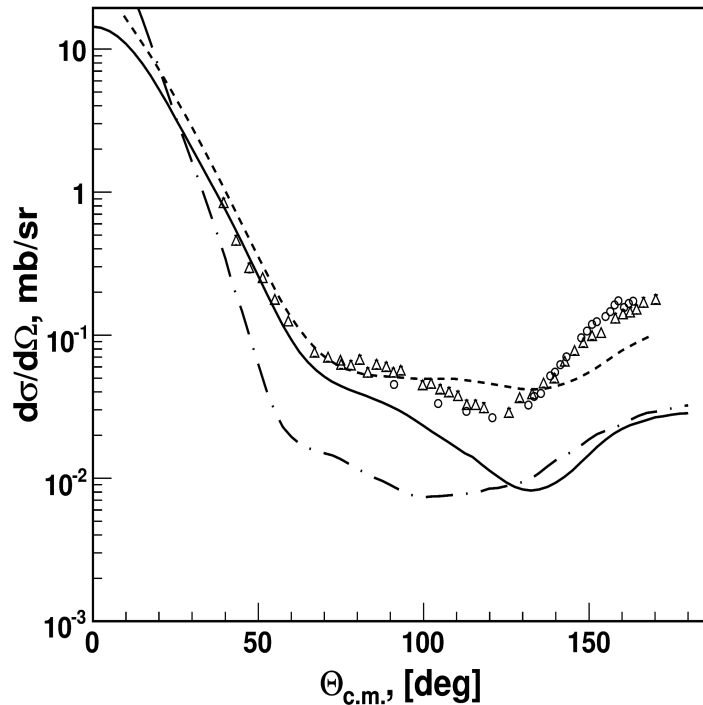
Dashed lines are the multiple scattering model calculations using **CD-Bonn DWF** (N.B.Ladygina, Phys.Atom.Nucl.71 (2008) 2039)

Solid lines are the Faddeev calculations using **CD-Bonn** potential (H.Witala, private communication)

Dott-dashed lines are the optical-potential calculations using **Dibaryon DWF** (M.Sikhalev, Phys.Atom.Nucl.72 (2009) 588 )

Published in **P.K.Kurilkin et al., Phys.Lett.B715 (2012) 61-65**

# Cross section in **dp**- elastic scattering at **880 MeV**



World data:

N.E.Booth et al., Phys.Rev.D4 (1971) 1261

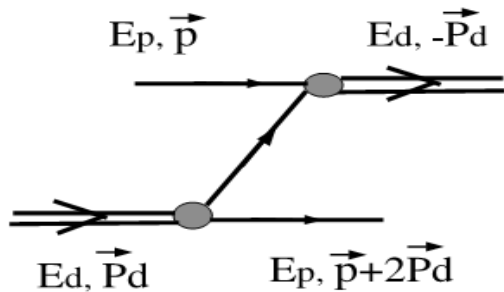
J.C.Alder et al., Phys.Rev.C6 (1972) 2010

- The results of the multiple scattering model are in agreement with the cross section data in the range **30 - 130°**.
- Double scattering dominates over single scattering at the angles larger than **70°**.
- Deviation of the data on the calculations at backward angles are related with the **s-type** of the **FM 3NF**.
- How to find the manifestation of 3N short range forces?

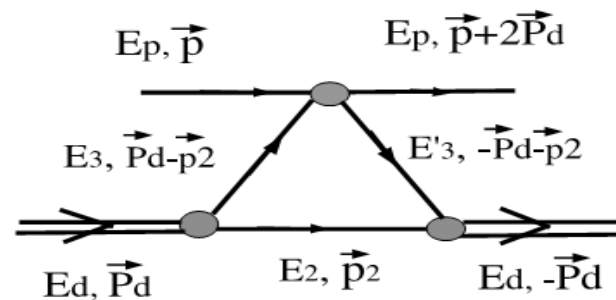
Relativistic multiple scattering model calculation:  
N.B.Ladygina, Eur.Phys.J, A42 (2009) 91

Picture is taken from  
P.K.Kurilkin et al., Phys.Lett.B715 (2012) 61-65

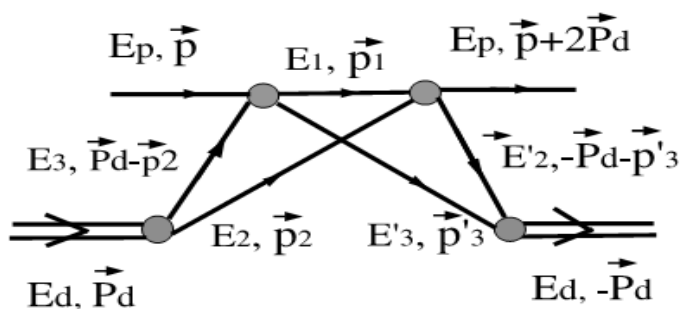
# Relativistic multiple scattering model for **dp**- elastic scattering at moderate energies



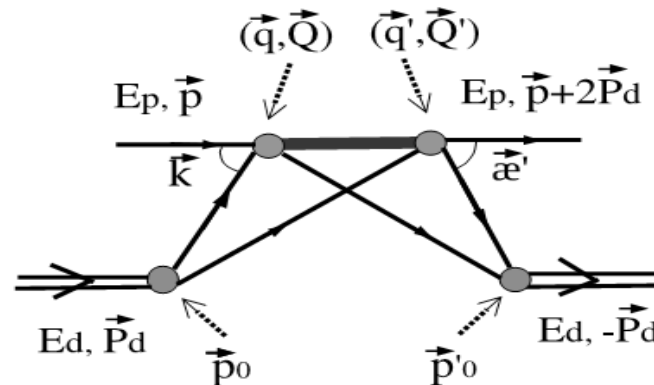
(a) **ONE**



(b) **SS**



(c) **DS**



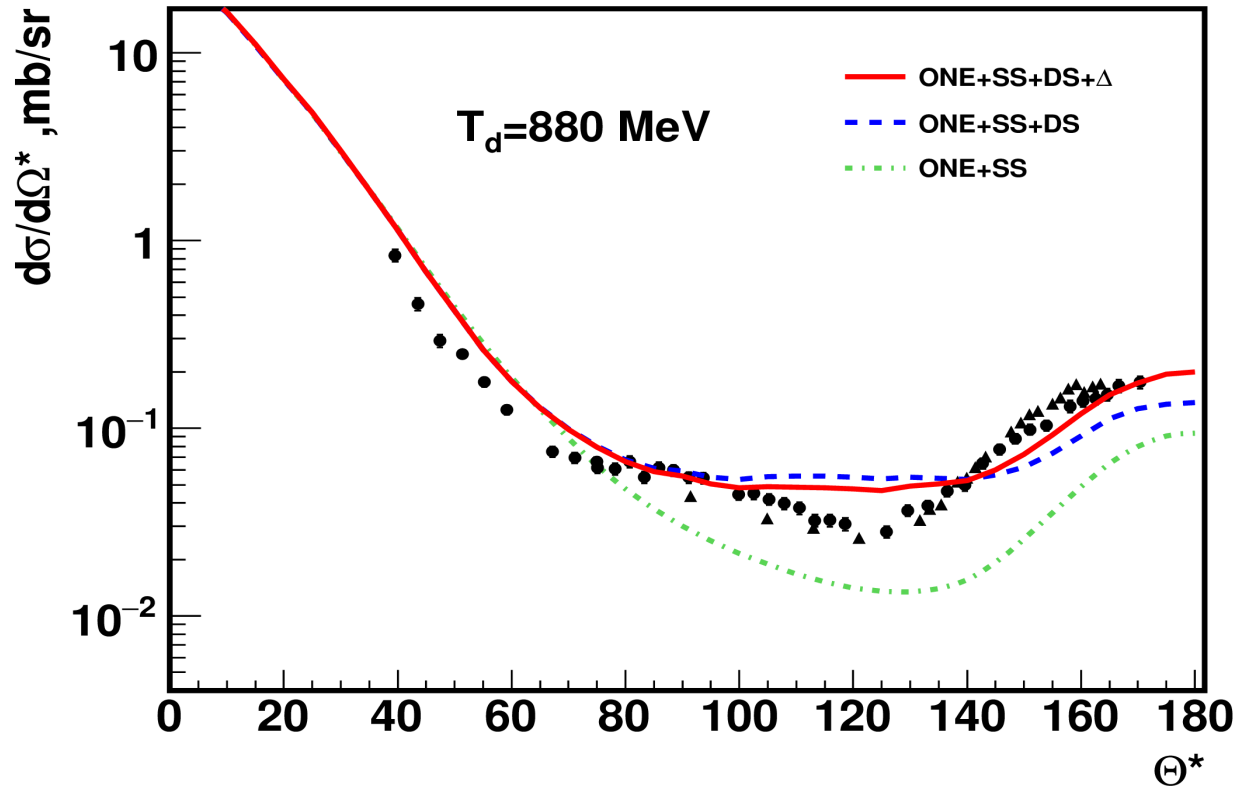
(d) **Δ**

ONE+SS+DS - **N.B.Ladygina, Phys.Atom.Nucl.71 (2008) 2039**

**N.B.Ladygina, Eur.Phys.J, A42 (2009) 91**

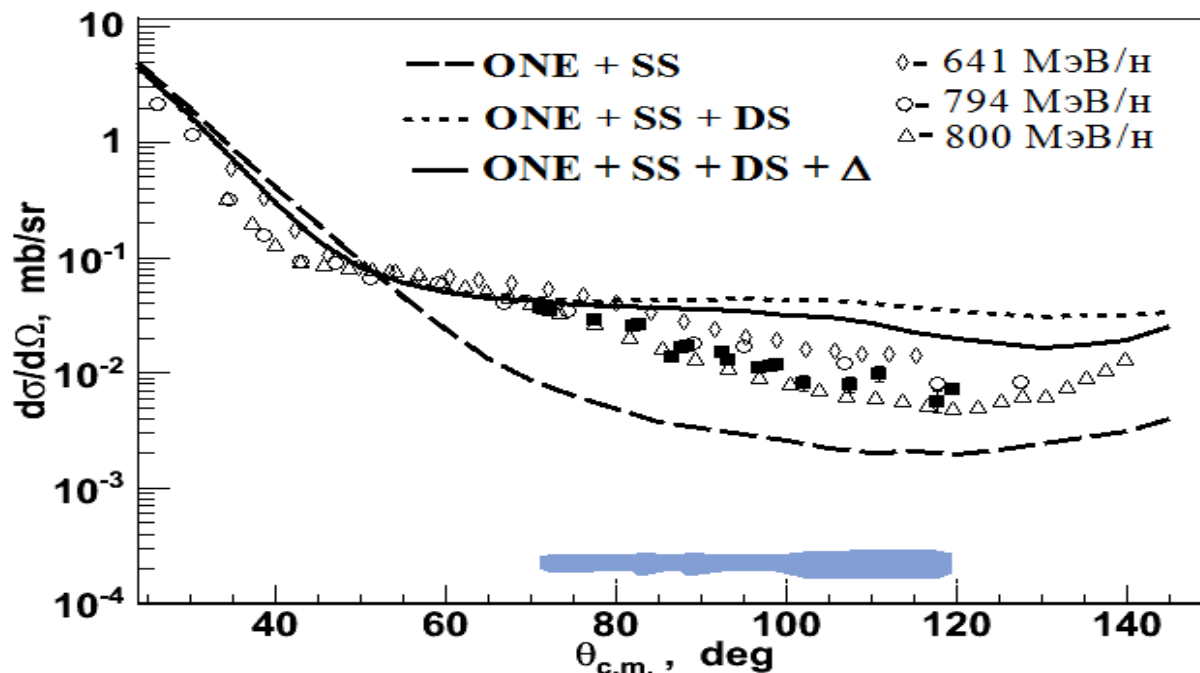
ONE+SS+DS +**Δ**- **N.B.Ladygina, Eur.Phys.J, A52 (2016) 199**

# dp- elastic scattering cross section at 880 MeV



Relativistic multiple scattering model calculation:  
N.B.Ladygina, Eur.Phys.J, A52 (2016) 199

# dp- elastic scattering cross section at 1400 MeV



A.A.Terekhin et al., Phys.Atom.Nucl. 80(2017) 1061.

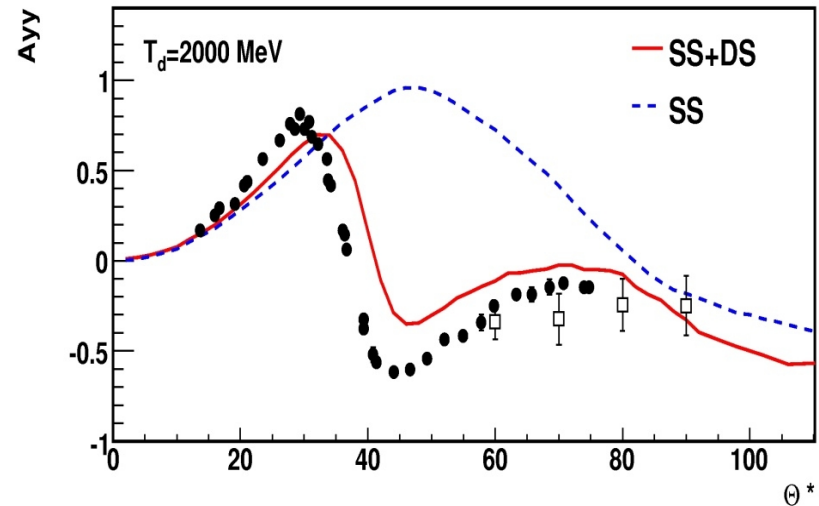
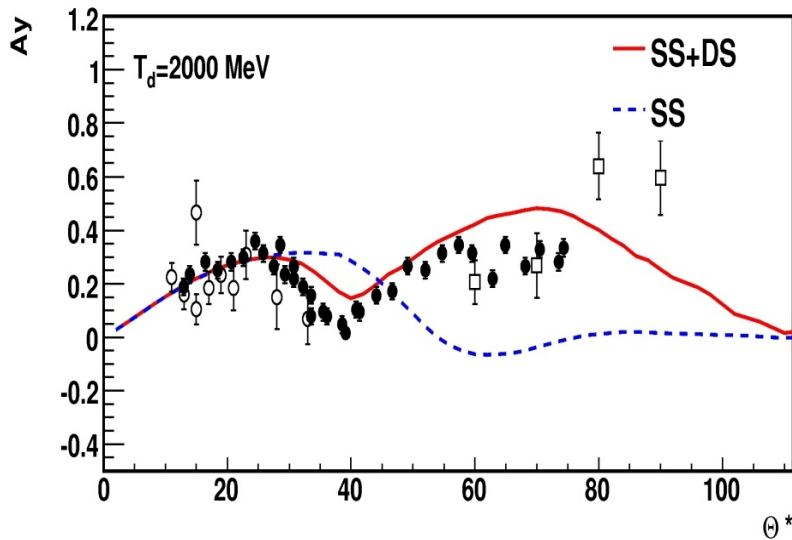
Relativistic multiple scattering model calculation:  
N.B.Ladygina, Eur.Phys.J, A52 (2016) 199

Final cross section data at 1000, 1300 and 1800 MeV

(talk of A.Terekhin)



# $A_y$ and $A_{yy}$ in **dp**- elastic scattering at 2000 MeV



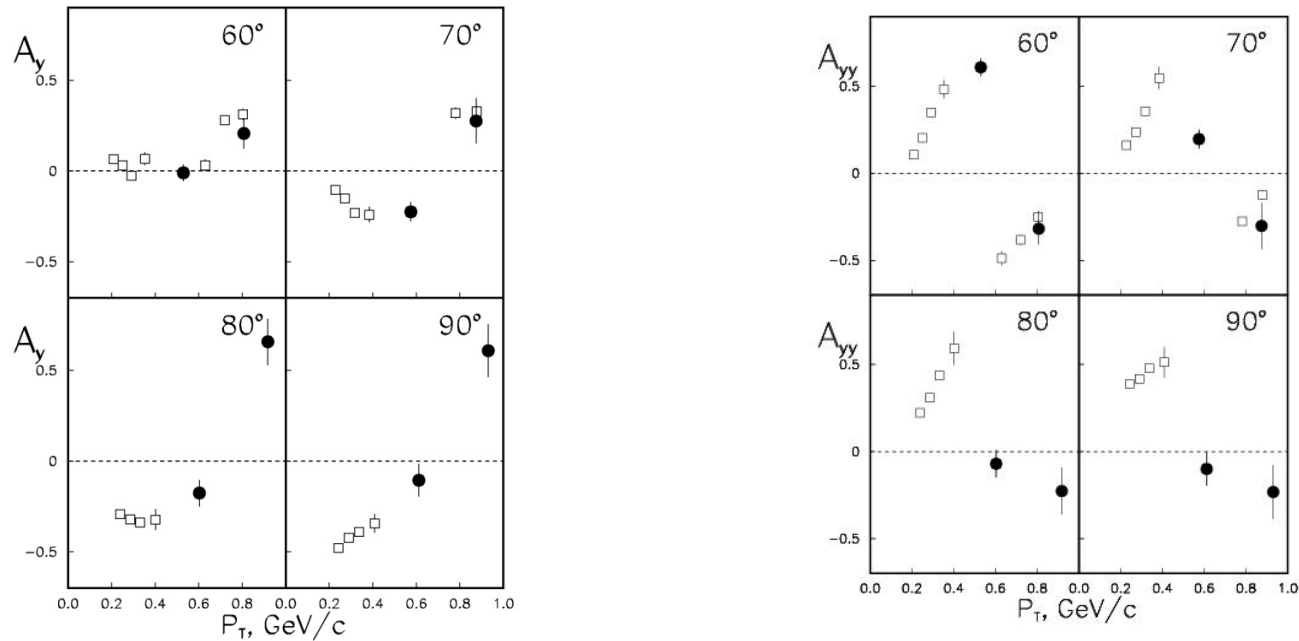
Open squares are the data obtained at Nuclotron **JINR**.

Open circles are the Synchrotron data (**V.V.Glagolev, Eur. Phys. J. A48 (2012) 182**)

Solid symbols are the data obtained by ANL group (**Haji-Saied et al., Phys.Rev.C.36 (1987) 2010**).

Dashed and solid lines are the relativistic multiple scattering model calculations using **CD- Bonn** DWF taking into account single scattering and single+double scattering, respectively.

# Energy dependence of the **dp**-elastic scattering analyzing powers at fixed scattering angles in the c.m.s.



- Full symbols are the data obtained at **JINR**
- Open symbols are the data obtained at RIKEN, Saclay and ANL
- The study of the energy dependence of the analyzing powers in **dp**- elastic scattering at large  $p_T$  is one of the tools to study spin effects in **cold dense matter**

# General View of SPI

Charge-Exchange Ionizer

Atomic Beam Source

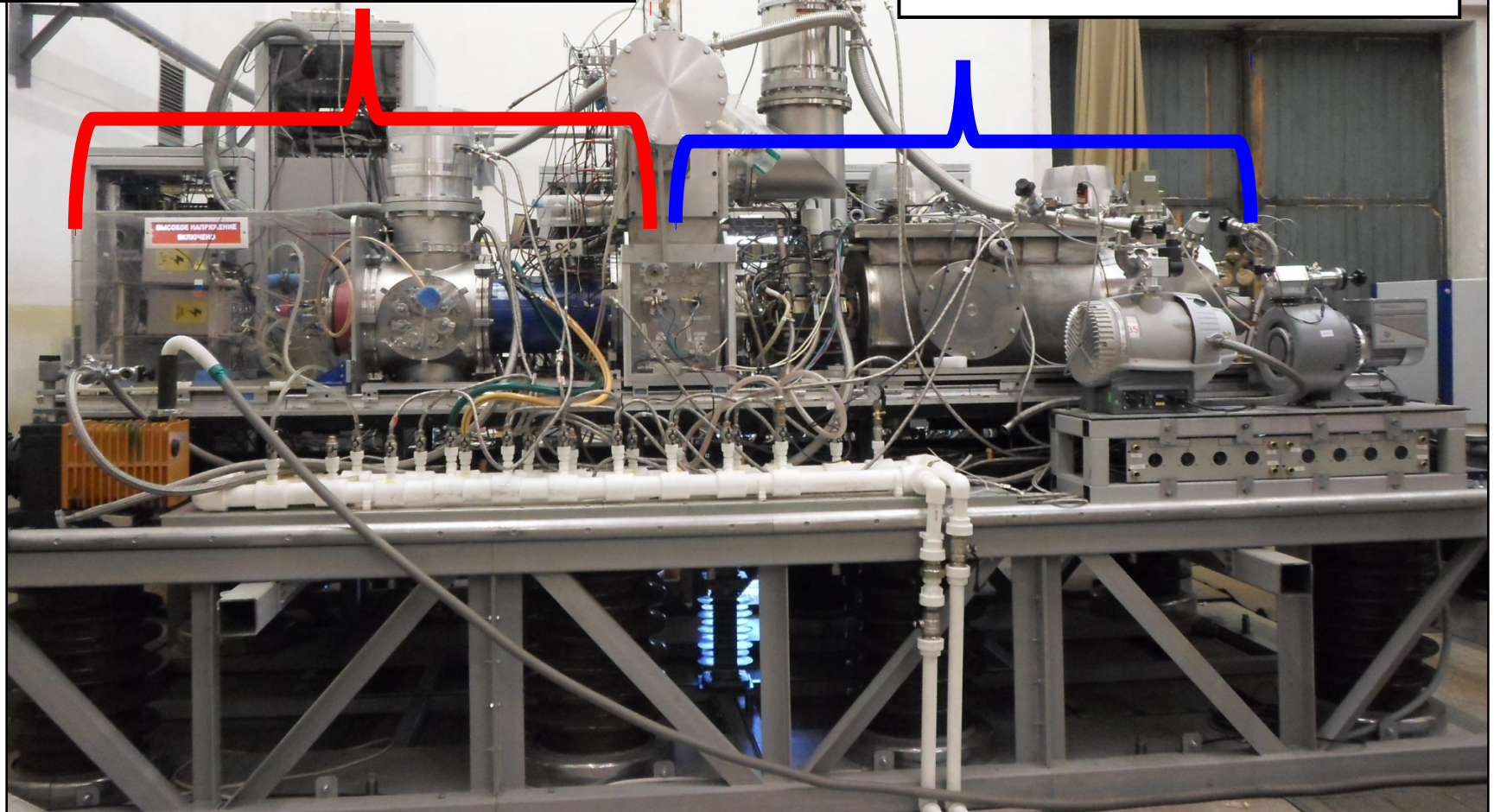
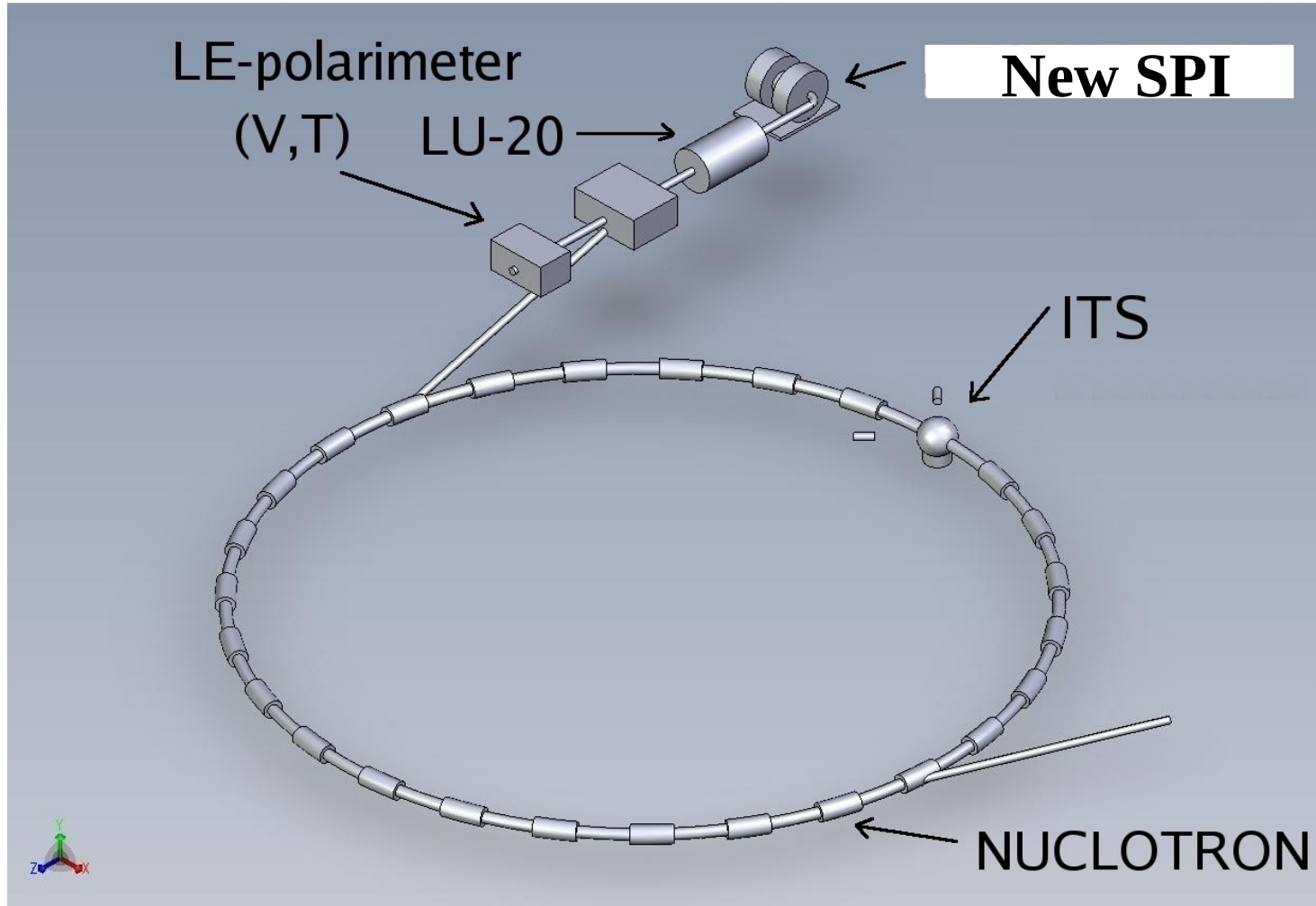


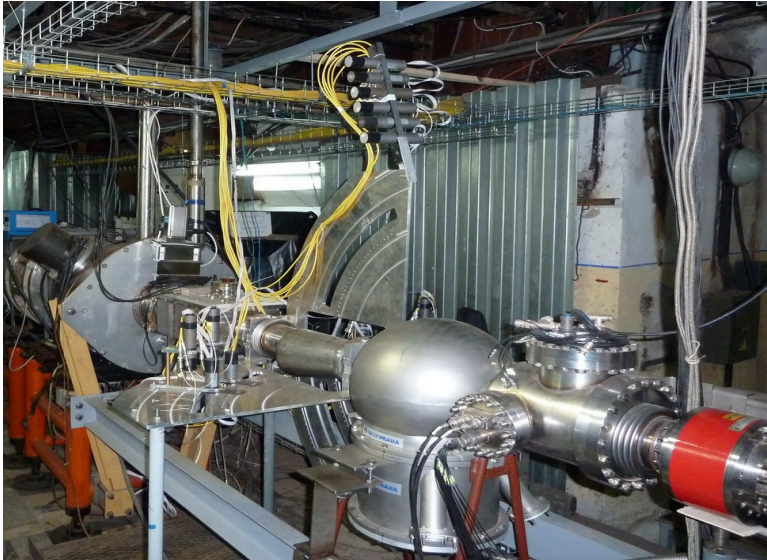
Figure of merit will be increased in future by a factor  $\sim 10^3$

# Nuclotron-M accelerator complex



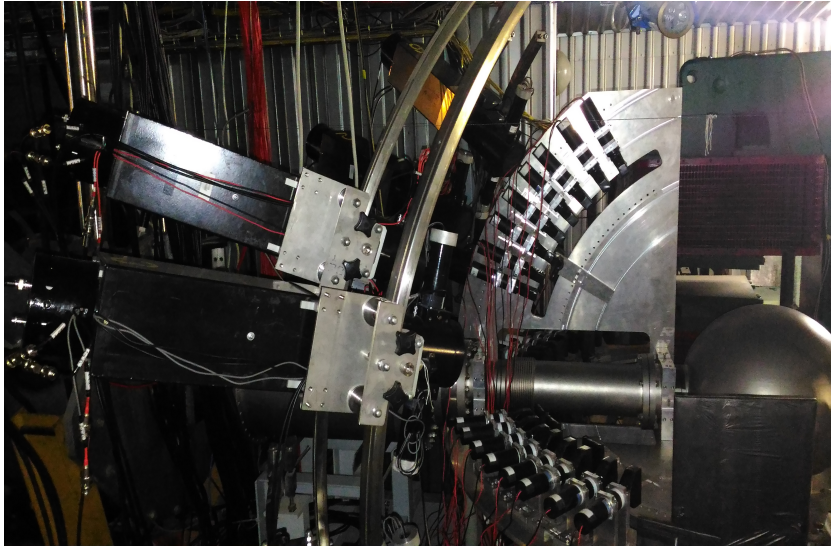
# Experiments at Internal Target Station at Nuclotron

## DSS-project



Internal Target Station is very well suited for the measurements of the **deuteron**- induced reactions observables at large scattering angles.

# Upgrade of the **Delta-LNS (DSS)** setup at ITS at Nuclotron



**New infrastructure, cabling**

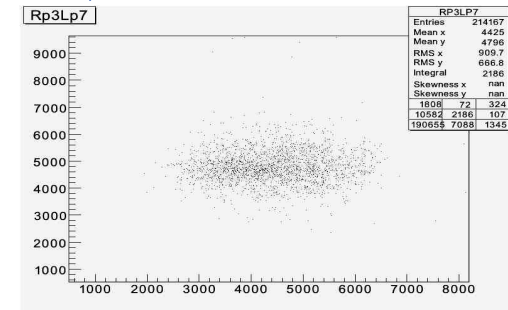
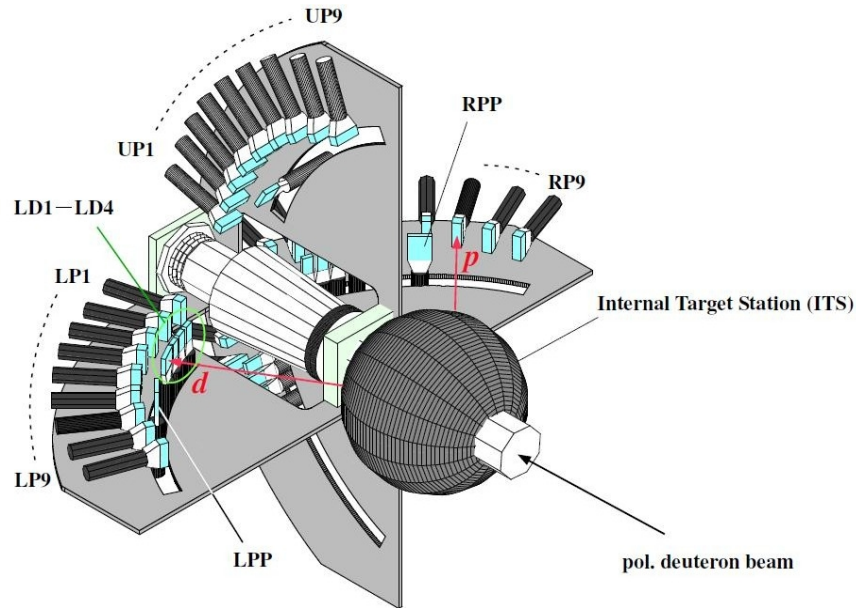
**New HV system (Mpod)**

**New VME DAQ**

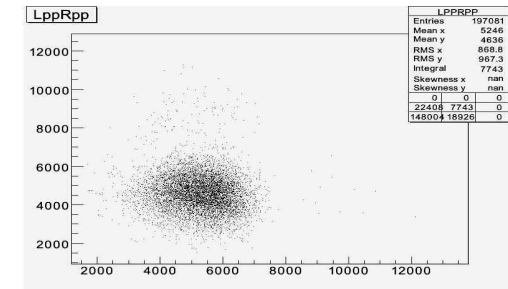
**40 counters for dp-elastic scattering studies**

**8 dE-E detectors for dp-breakup studies**

# Results from the commissioning run at Nuclotron at 270 MeV (June 2016)



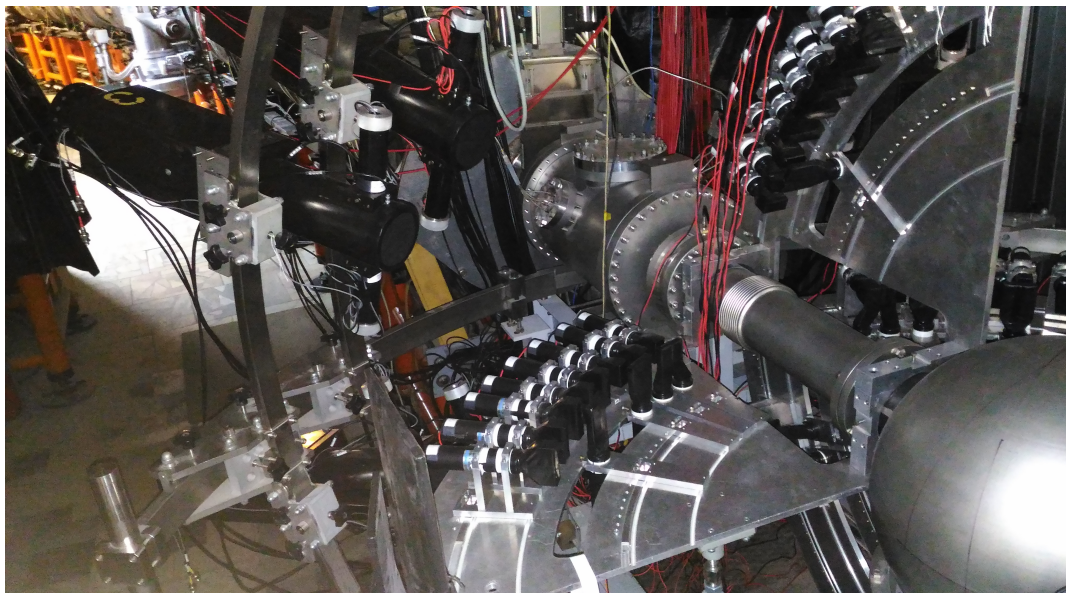
DP



PP

- Deuterons and protons in coincidences using scintillation counters
- Internal beam and thin  $\text{CH}_2$  target (C for background estimation)
- Measurements at 270 MeV
- The setup was ready to take the polarized data.

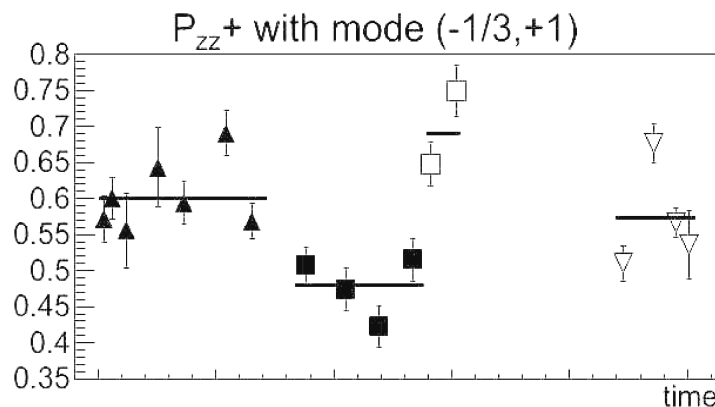
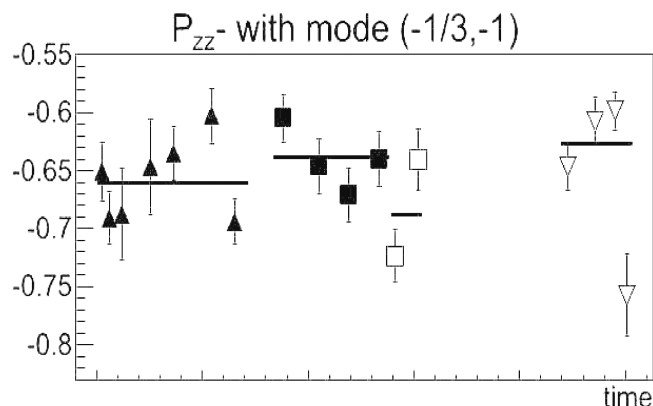
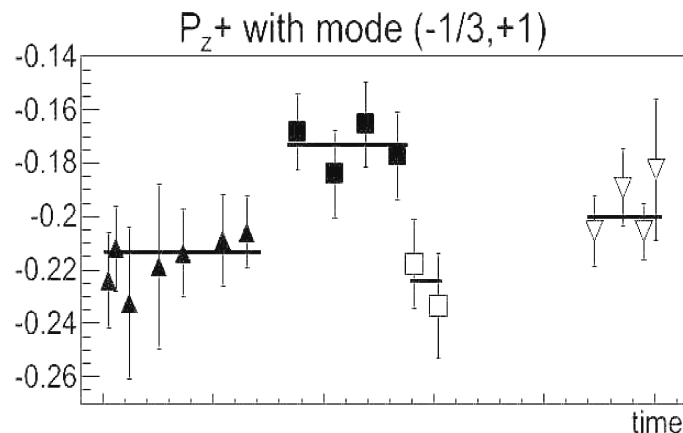
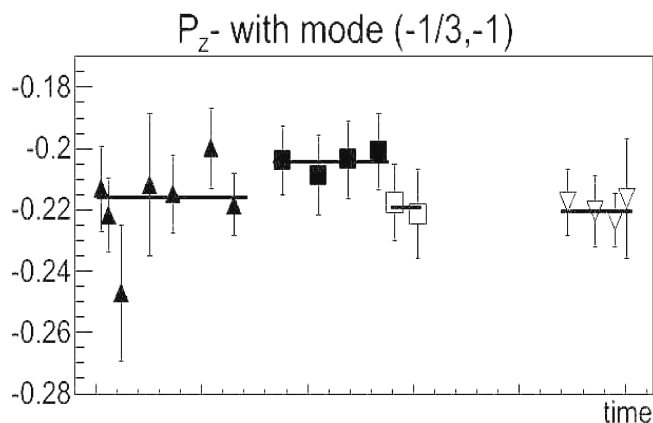
# Setup to study **dp**- elastic scattering at ITS at Nuclotron in 2016-2017.



- Deuterons and protons in coincidences using scintillation counters
  - Internal beam and thin **CH<sub>2</sub>** target (**C** for background estimation)
  - Permanent polarization measurement at **270 MeV** (between each energy).
  - Analyzing powers measurement at **400-1800 MeV**
  - The data were taken for three spin modes of SPI: unpolarized, “2-6” and “3-5” ( $p_z p_{zz}$ ) = (0,0), (-1/3,1) and (-1/3,-1).
  - Typical values of the polarization was 70-75% from the ideal values.
- (talks of **A.Yu.Isupov** and **Ya.Skhomenko**).

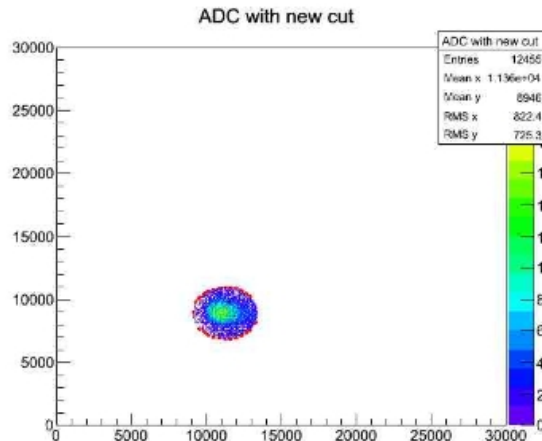
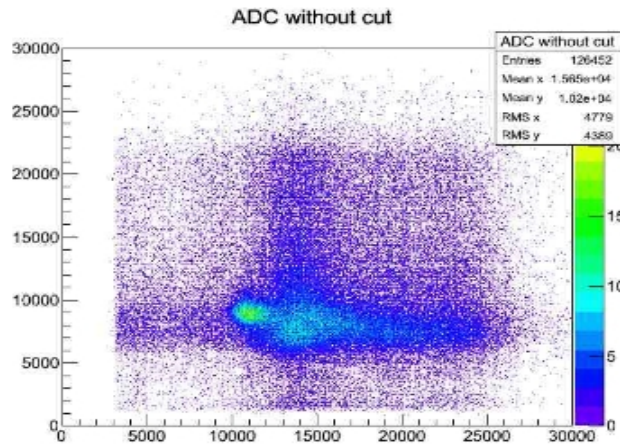
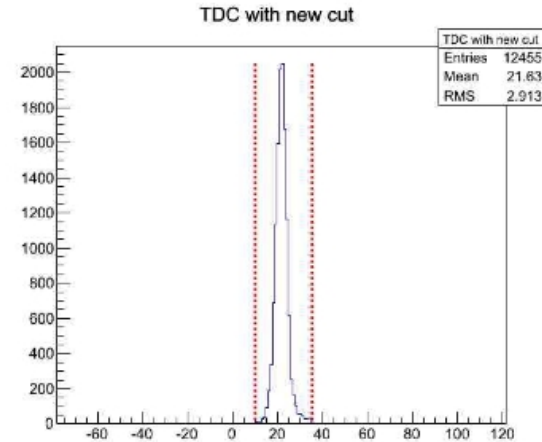
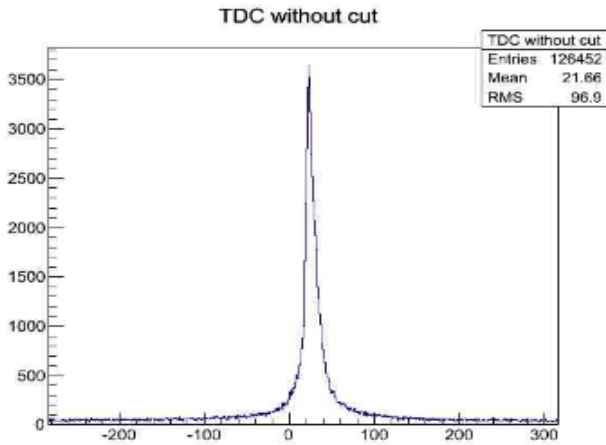


# Polarization measurements using **dp**- elastic scattering at **270 MeV**



SPI was tuned for 6 spin modes  $(p_z, p_{zz}) = (-1/3, 1), (-1/3, -1), (0, +1), (0, -2), (-2/3, 0), (+1, 0)$ .

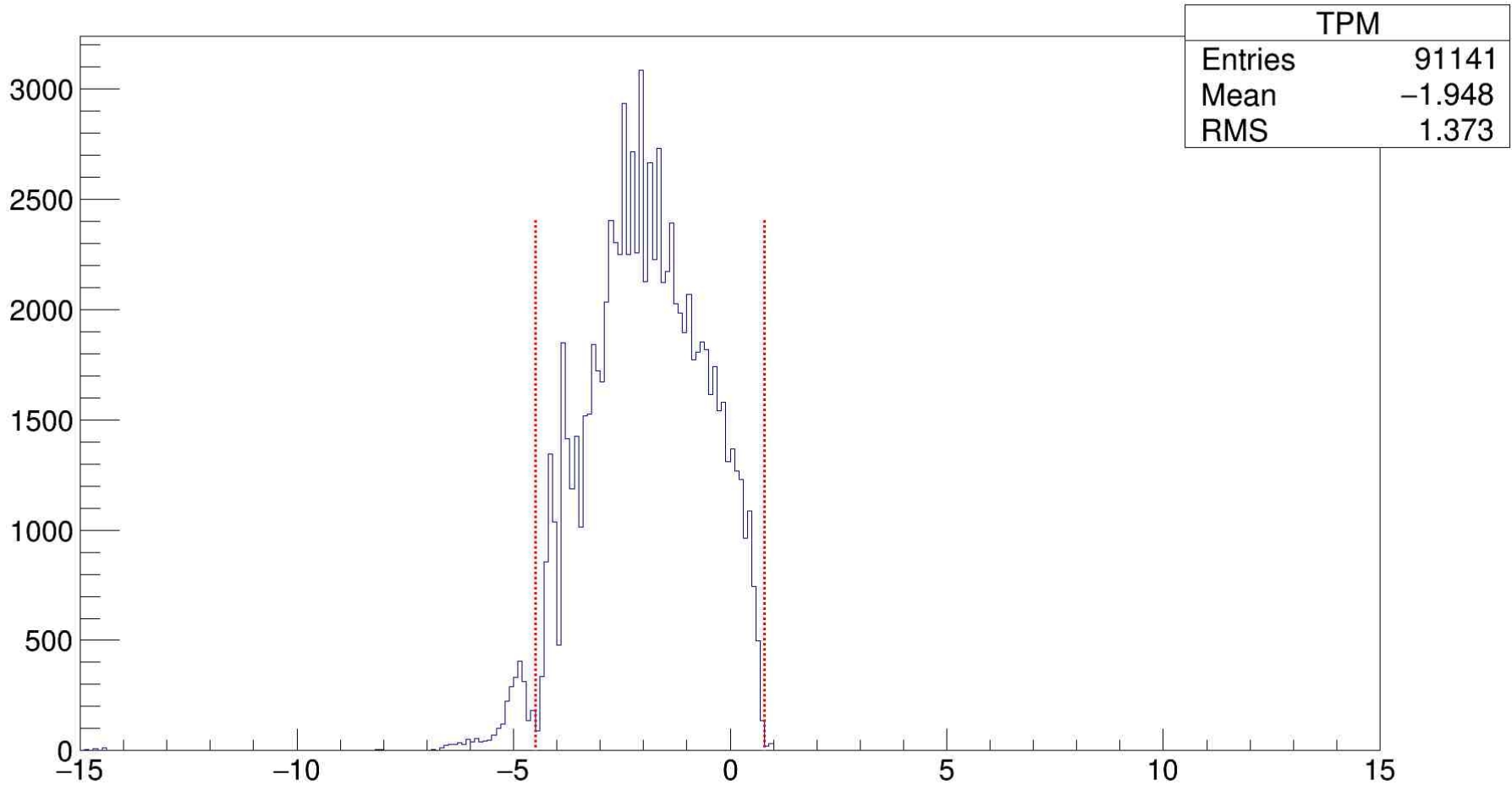
# The dp-elastic scattering events selection



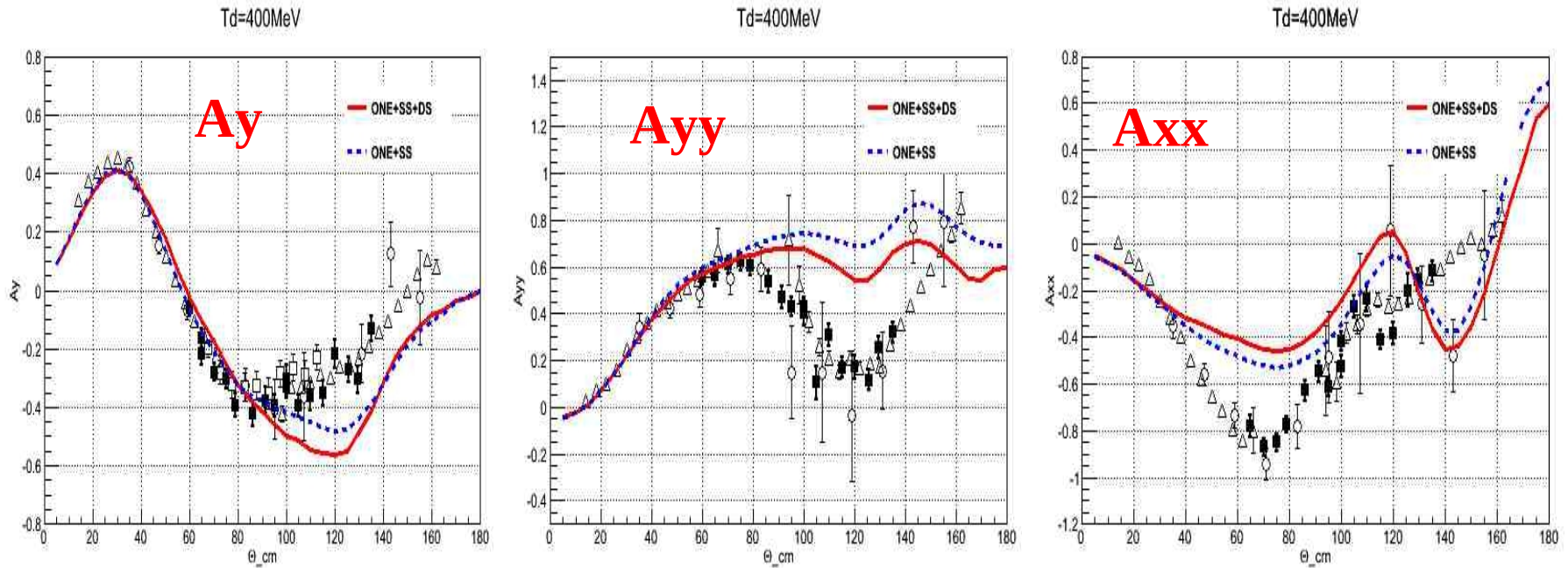
**Selection of the dp elastic events by the time difference between the signal appearance from deuteron and proton detectors with the criteria on the amplitude signal correlation.**

# Target Position Monitor cut

TPM



# Angular dependence of the vector and tensor analyzing powers in **dp**-elastic scattering at **400 MeV**



**Full squares are the data from Nuclotron (December 2016)**

**Curves are the relativistic multiple scattering model calculations**

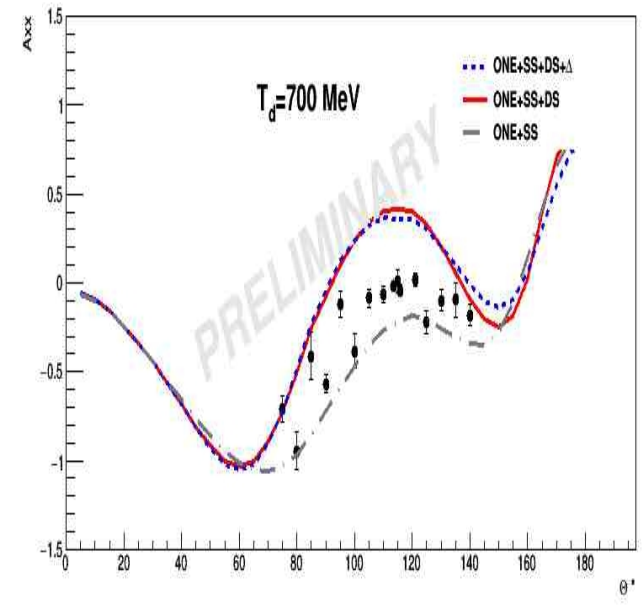
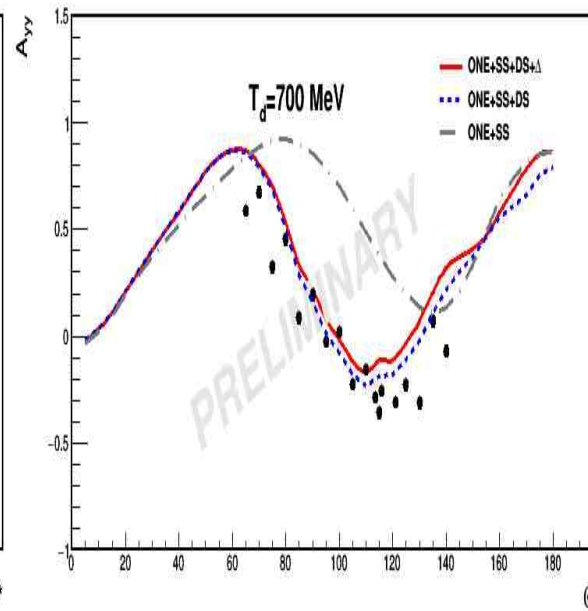
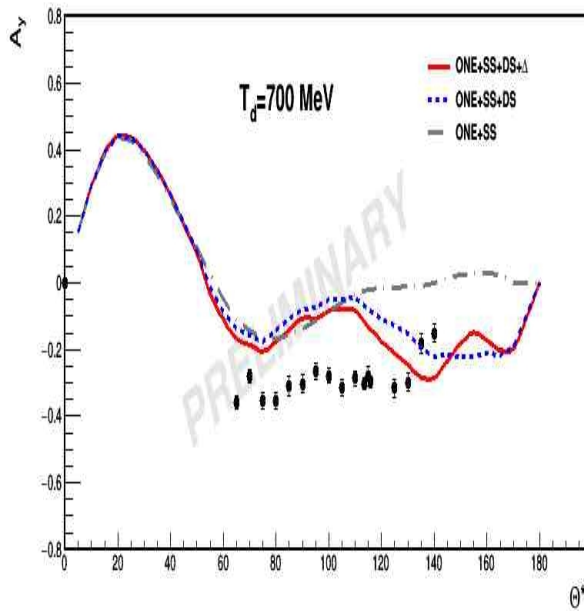
**N.B.Ladygina, Eur.Phys.J, A42 (2009) 91**

# Angular dependence of the vector and tensor analyzing powers in **dp**-elastic scattering at **700 MeV**

**$A_y$**

**$A_{yy}$**

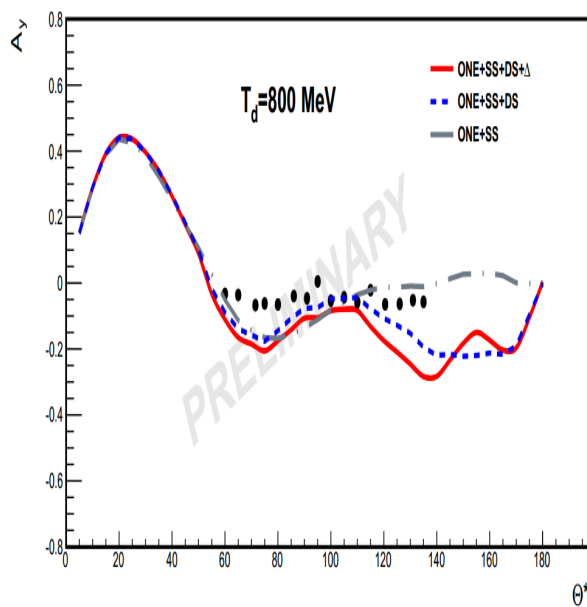
**$A_{xx}$**



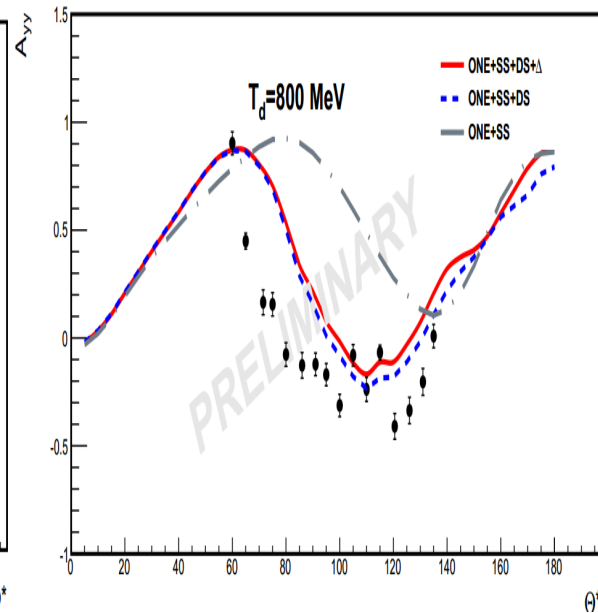
Curves are the relativistic multiple scattering model calculations  
**N.B.Ladygina, Eur.Phys.J, A52 (2016) 199**

# Angular dependence of the vector and tensor analyzing powers in **dp**-elastic scattering at **800 MeV**

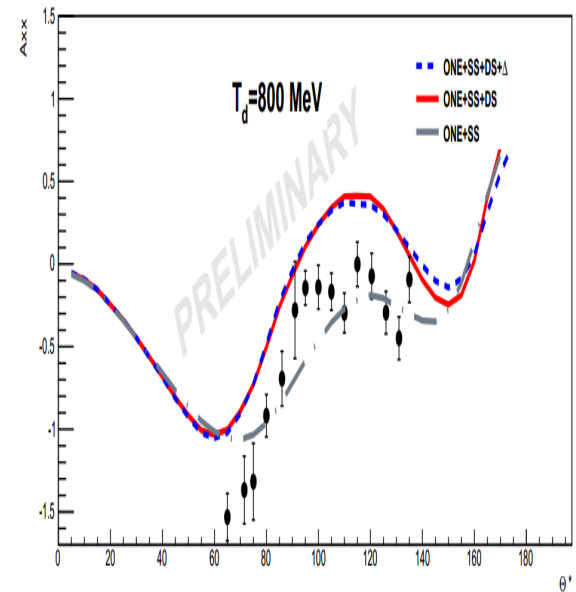
**$A_y$**



**$A_{yy}$**



**$A_{xx}$**

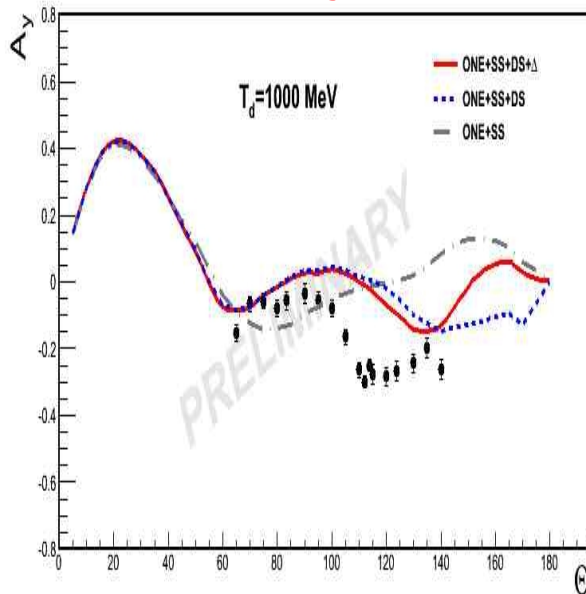


Curves are the relativistic multiple scattering model calculations  
N.B.Ladygina, Eur.Phys.J, A52 (2016) 199

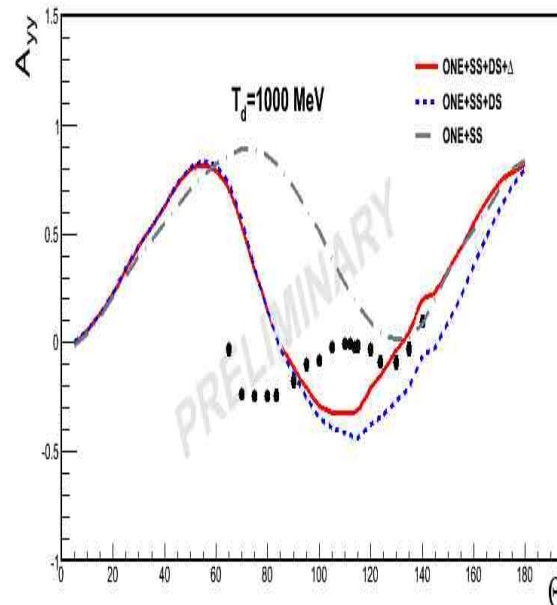
(talk of **O.Mezhenska**)

# Angular dependence of the vector and tensor analyzing powers in **dp**-elastic scattering at **1000 MeV**

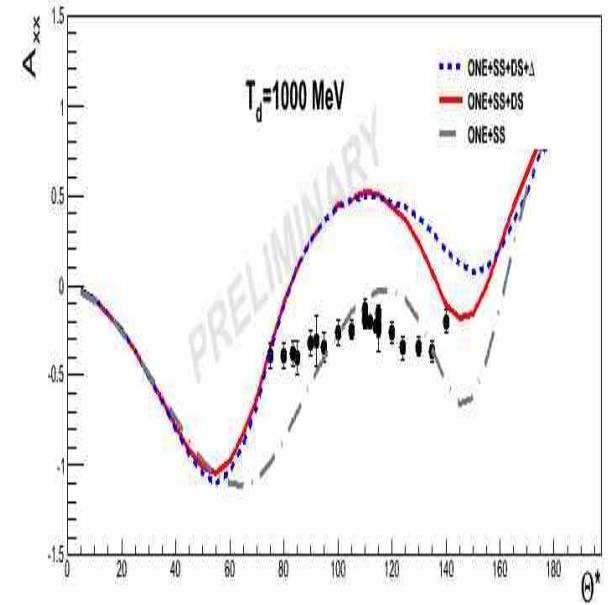
**$A_y$**



**$A_{yy}$**

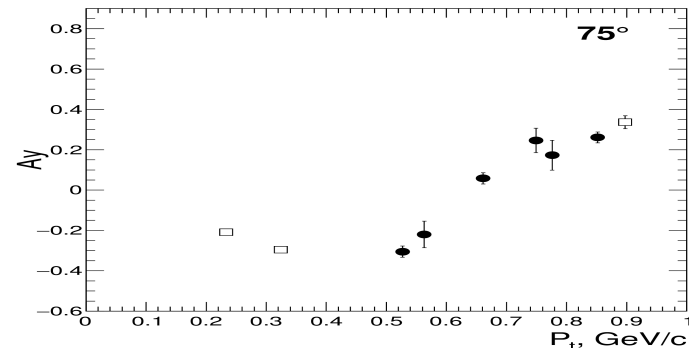
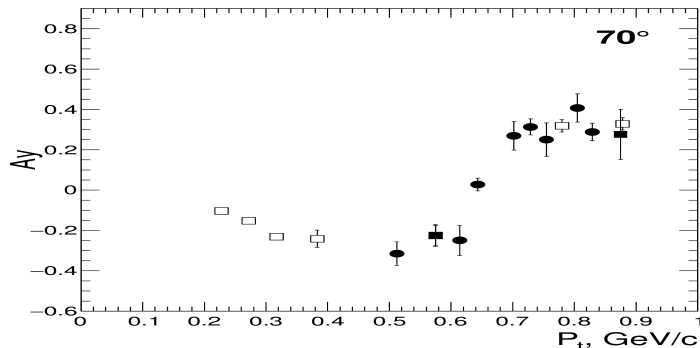
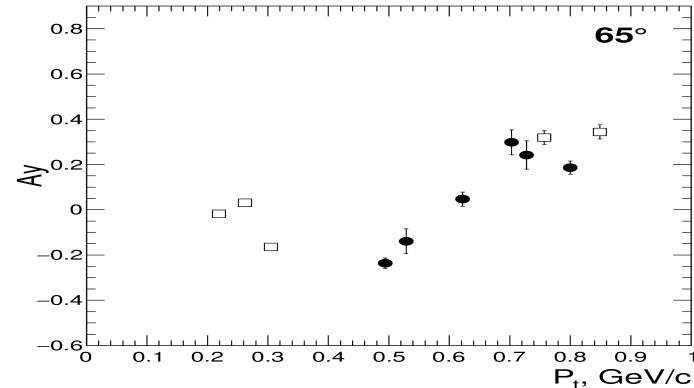
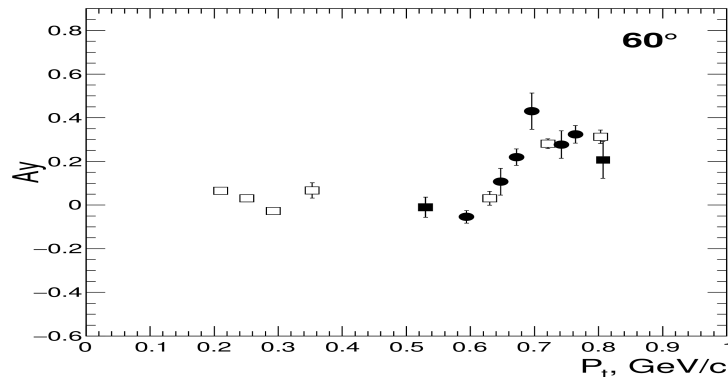


**$A_{xx}$**



Curves are the relativistic multiple scattering model calculations  
**N.B.Ladygina, Eur.Phys.J, A52 (2016) 199**

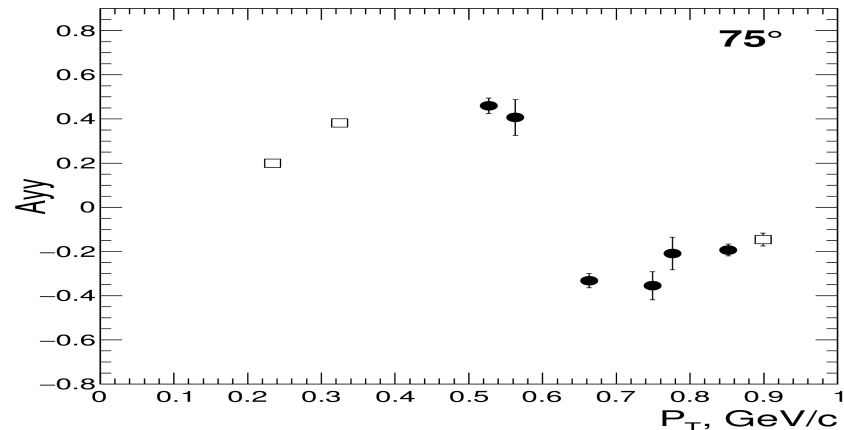
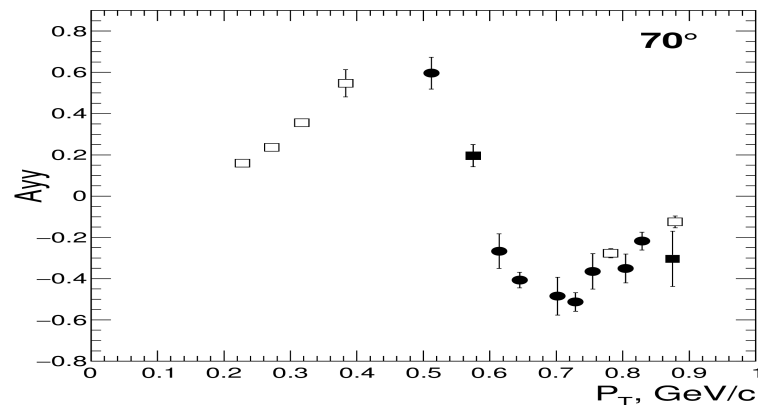
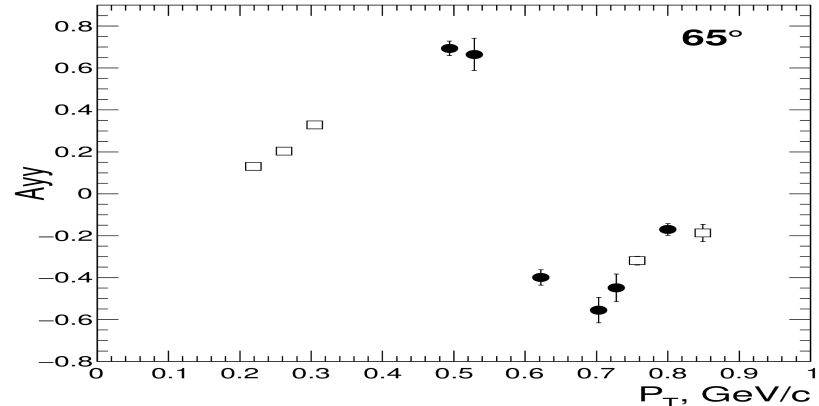
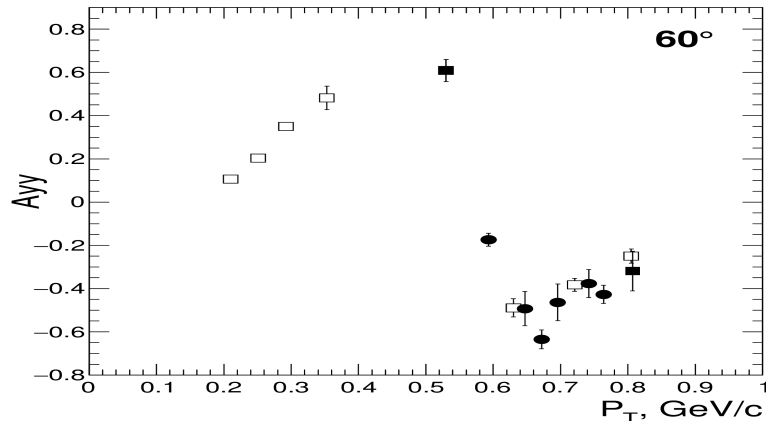
# Energy dependence of the vector analyzing power $A_y$ in dp-elastic scattering at 700-1800 MeV



Full circles are the new preliminary data from Nuclotron (2016-2017).  
Full squares are the data from Nuclotron (2005).  
Open symbols are the world data.



# Energy dependence of the tensor analyzing power $A_{yy}$ in $dp$ -elastic scattering at 700-1800 MeV



Full circles are the new preliminary data from Nuclotron (2016-2017).  
Full squares are the data from Nuclotron (2005).  
Open symbols are the world data.

# Conclusion

Upgraded Nuclotron with new **SPI** provides quite unique opportunity for the studies of the spin effects and polarization phenomena in few body systems.

The realization of the **DSS** program at **ITS** will allow to obtain the crucial data on the spin structure of 2-nucleon and 3- nucleon short range correlations.

The first natural step in these studies, namely, the energy scan of the deuteron analyzing powers in **dp**- elastic scattering has been performed in 2016-2017.

Next experiments using polarized deuterons and protons at **ITS** are in preparation.

The extention of the studies to the high energies is possible with the extracted polarized deuteron and **proton** beams.

# Polarized protons at ITS.

Injection of 5 MeV protons into Nuclotron ring.

Acceleration up to 500 MeV- no serious depolarization resonances.

Unpolarized protons:  $I \sim 1.5 \cdot 10^8$  ppp

Polarized protons:  $I \sim 2-3 \cdot 10^7$  ppp

IPol=1 P=-1 (WFT 1→3)

IPol=2 P=0 (unpolarized)

IPol=3 P=-1 (WFT 1→3) beam 2/3 of time.

Having the asymmetries for 8 angles ( $55^\circ$ - $125^\circ$  in the cms) we obtained the averaged value of the proton beam polarization

Unpolarized protons:  $P = 0.017 \pm 0.021$

Polarized protons:  $P = -0.354 \pm 0.022$

Need to produce new detection system for protons.

V.P.Ladygin et al., J.Phys.Conf.Ser. 938 (2017) 012008

## Further DSS plans

Final analysis of the systematic data on the cross section and analyzing powers  $A_y$ ,  $A_{yy}$  and  $A_{xx}$  in **dp**- elastic scattering between **270 MeV** and **2000 MeV** at ITS.

Preparation of the experiments on the systematic studies of the analyzing powers  $A_y$ ,  $A_{yy}$  and  $A_{xx}$  in **dp**- elastic scattering between **270 MeV** and **500 MeV** using new SPI at Nuclotron to study the manifestation of the short-range 3NFs.

Preparation for the taking of new polarized data for the **dp (pd)**- nonmesonic breakup at the energies between **300** and **500 MeV** for different kinematic configurations at ITS with polarized beams.

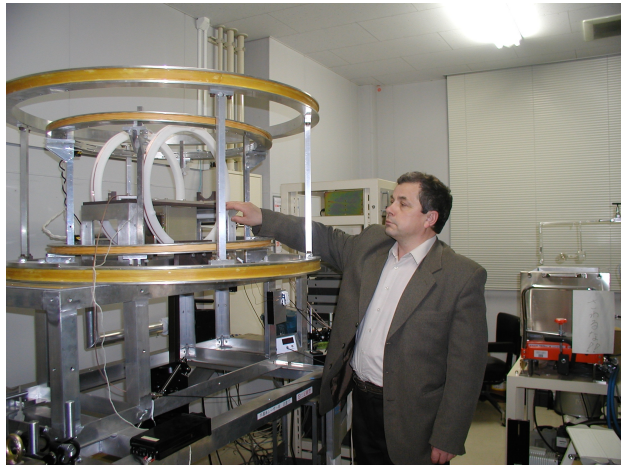
Preparation of the experiment on the energy scan of the nucleon analyzing power  $A_y^p$  in **pd**- elastic scattering between **135 MeV** and **1000 MeV** at ITS.

**Program is approved by JINR PAC for 2019-2021 yy.**

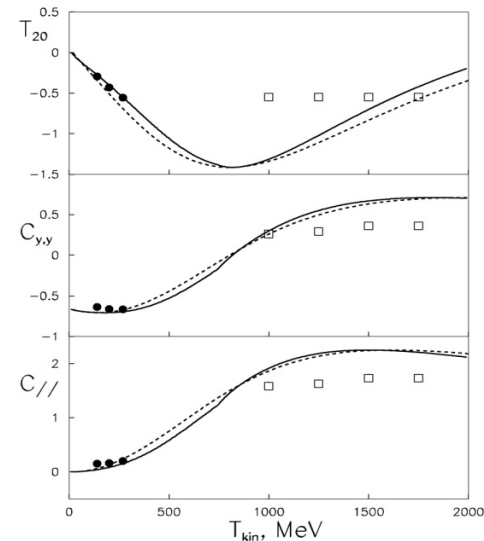
**Thank you for the attention!**

# Polarization observables for polarized deuteron induced reactions

Target position is in F5

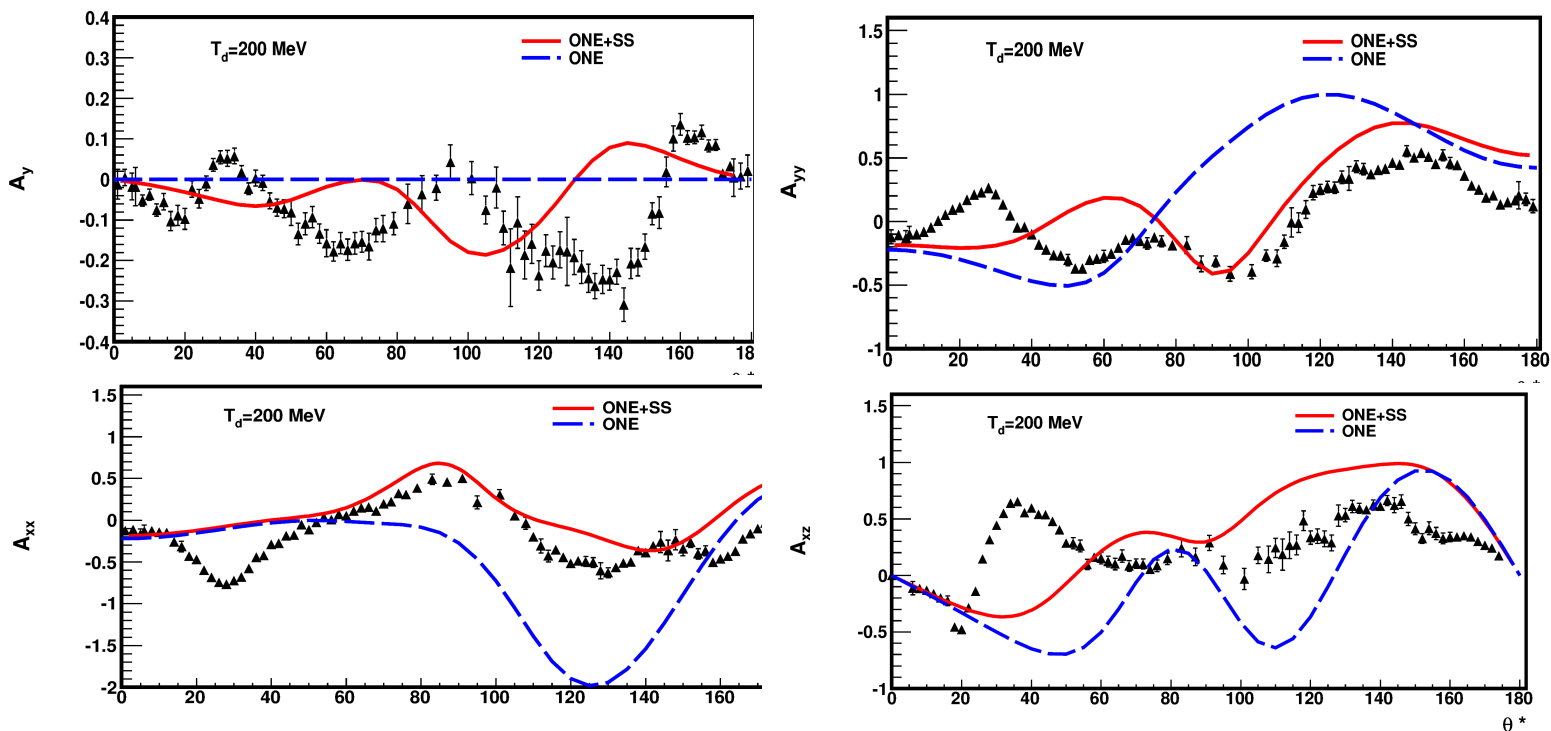


${}^3\text{He}(d,p){}^4\text{He}$



- The measurements of the tensor analyzing power  $T_{20}$  and spin correlation  $C_{y,y}$  in the  ${}^3\text{He}(d,p){}^4\text{He}$  reaction in the kinetic energy range between 1.0 and 1.75 GeV can be performed at the BM@N area.
- The polarization observables for the  $p(d,p)d$ ,  $d(d,p)t$  and  $d(A,p(0^\circ))X$  at intermediate and high energies also can be studied.
- Non-nucleonic degrees of freedom and baryonic resonances properties can be studied in the  $d(A,d(0^\circ))X$  and  $d(A,\pi^-(0^\circ))X$  reactions at different energies.
- The tensor analyzing power  $T_{20}$  can be studied for the meson production in the  $d(A,{}^3\text{He}(0^\circ))X$  reactions.

# Polarization effects in the $dd \rightarrow {}^3\text{He}({}^3\text{He})$ reactions at Nuclotron energies



The relativistic multiple scattering model was successfully used to describe the  $dd \rightarrow {}^3\text{He}({}^3\text{He})$  reactions in a GeV region at the Nuclotron.

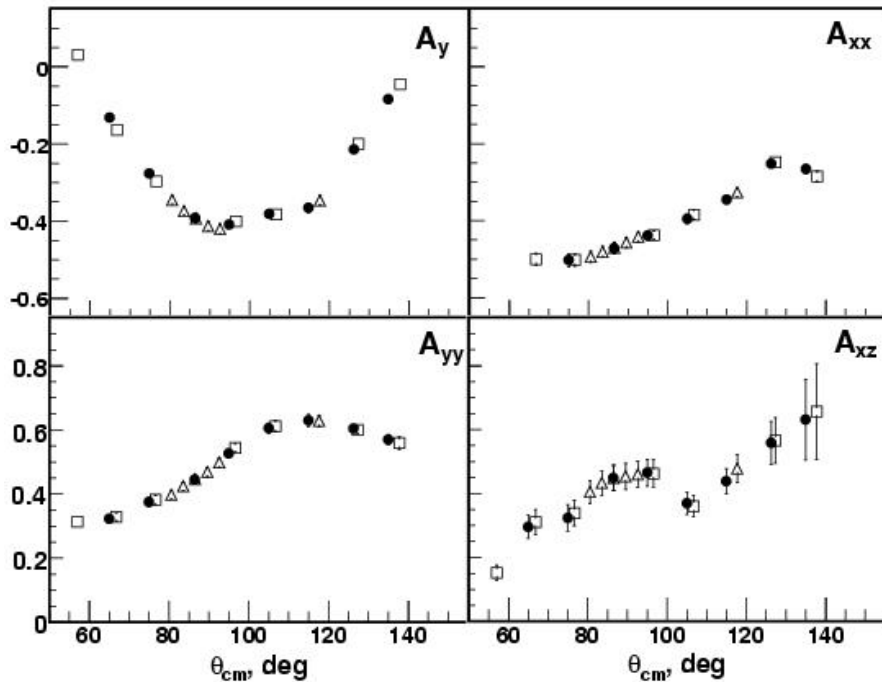
The calculations require a large amount of CPUs.

The results were published in FBS, PRC, PPN.

**N.Ladygina - theory**

**A.Kurilkin - experiment**

# Measurement of the deuteron beam polarization at ITS using DSS detection system at 270 MeV



Vector  $A_y$  and tensor analyzing powers  $A_{yy}$ ,  $A_{xx}$  and  $A_{yz}$  of dp- elastic scattering as a function of deuteron scattering angle in c.m.s. at deuteron beam energy of 270 MeV.  $\square$ ,  $\Delta$ - the world data. Extrapolated values of the analyzing powers are marked by  $\bullet$ .

Cubic spline interpolation:

$(x_i, y_i)$  на  $[A, B]$

$f(x) = ax^3 + bx^2 + cx + d$

$f''(A) = f''(B) = 0$

**K.Sekiguchi et al.,**

**Phys. Rev. C65 (2002) 034003**

**K.Sekiguchi et al.,**

**Phys. Rev.C70 (2004) 014001**

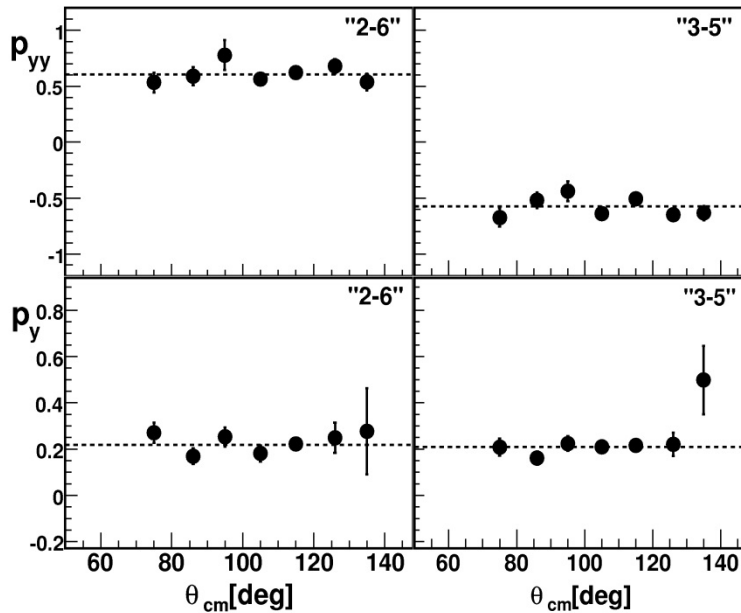
**K.Suda, et al.,**

**Nucl. Instr. Meth. in Phys.**

**Res. A572 (2007) 745**



# Measurement of the deuteron beam polarization at ITS using **DSS** detection system at **270 MeV**



Tensor  $p_{yy}$  and vector  $p_y$  polarization of the beam for "2-6" and "3-5" spin modes of PIS POLARIS as a function of the deuteron scattering angle in the cms.

$$\beta = -90.3^\circ \pm 1.2^\circ$$

$$F_i^2 = \int \epsilon A_i^2 d\Omega$$

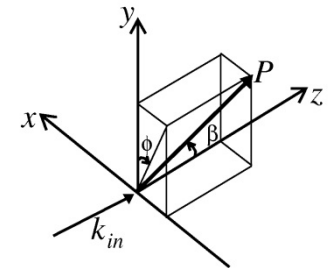
$$F_y \sim 1.0 \cdot 10^{-4}, F_{yy} \sim 1.8 \cdot 10^{-4}, F_{xx} \sim 0.8 \cdot 10^{-4}$$

$$P_{zz} = 0.605 \pm 0.025$$

$$P_z = 0.216 \pm 0.015$$

$$P_z = 0.208 \pm 0.012$$

$$P_{zz} = 0.69 \pm 0.03$$



- **Reference deuteron beam polarimeter at Nuclotron.**

**P.K.Kurilkin et al., Nucl. Instr. and Meth. A 642 (2011) 45**

# Relativization schemes

For the case of the deuteron vertex the internal momentum  $\mathbf{k}$ :

$$k = \sqrt{\frac{m_p^2 + \mathbf{k}_T^2}{4x(1-x)}} - m_p,$$
$$x = \frac{E_p + p_{pl}}{E_d + p_d},$$

where  $\mathbf{E}_d$  and  $\mathbf{p}_d$  are the energy and momentum of the initial deuteron, respectively,  $\mathbf{p}_{pl}$  is the longitudinal momentum of the proton,  $m_p$  and  $\mathbf{E}_p$  are the mass and energy of the proton, respectively.

- Minimal relativization scheme (Dirac, Weinberg, Frankfurt& Strikman)
- Bete-Salpeter equation solving (Tjon&Keisler, Bondarenko et al.)
- Quasi-potential wave functions (Gross, Braun&Tokarev, Kaptari et al.)
- Covariant theory on the light cone (Karmanov et al.)