# Polarization in HIC: comparison of methods

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# Chiral phenomena in relativistic hydrodynamics

## Investigation of Chiral Magnetic Effect (CME) and Chiral Vortical Effect (CVE)

- *A. Vilenkin, Phys. Rev. D 21 (1980) 2260; Phys. Rev. D 22 (1980) 3080.*
- *K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys.Rev. D 78, 074033 (2008).*
- *D. T. Son and P. Surowka, Phys. Rev. Lett. 103 (2009) 191601.*
- *A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, Phys. Rev. D 83 (2011) 105025.*
- *K. Landsteiner, E. Megias, F. Pena-Benitez. Phys. Rev. Lett. 107, 021601 (2011).*
- *M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018).*
- *O. Rogachevsky, A. Sorin and O. Teryaev, Phys. Rev. C 82 (2010) 054910.*
- *Karpenko, I. et al. Eur.Phys.J. C77 (2017) no.4, 213.*
- *F. Becattini and I. Karpenko, Phys. Rev. Lett. 120, no. 1, 012302 (2018).*

Some of the early works

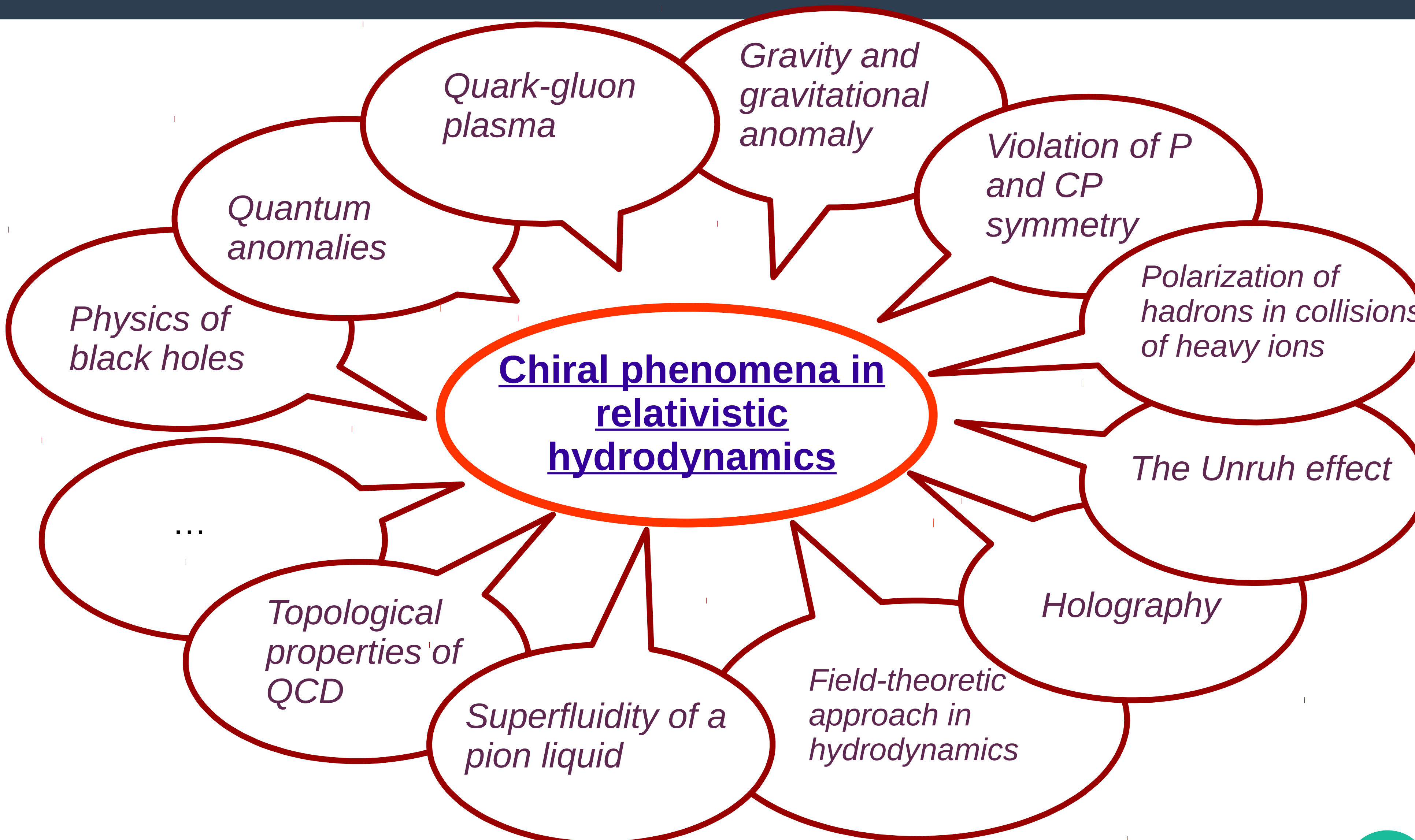
Connection with quantum anomalies

Polarisation in HIC

## Some applications of the theory of chiral liquids

- Hadron polarisation in heavy ion collisions.
- Chiral batteries.

# Areas of physics manifested in chiral phenomena



# Achievement of D. T. Son and P. Surowka

*D. T. Son and P. Surowka, Phys. Rev. Lett. 103 (2009) 191601. (formulas on the slide)*

**L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon, New York (1959).**

The law of conservation of energy/momentum in an external electromagnetic field

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

**+**

**Quantum anomaly in divergence axial current**

$$\partial_\mu j^\mu = C E^\mu B_\mu$$

Second law of thermodynamics $\partial_\mu s^\mu \geq 0$

Ideal fluid

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu},$$

$$j^\mu = n u^\mu + \nu^\mu,$$

Viscosity + electrical conductivity **+ axial anomaly**

Chiral vortical effect CVE

Chiral magnetic effect CME

$$\nu^\mu = -\sigma T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi \omega^\mu + \xi_B B^\mu,$$

$$s^\mu = s u^\mu - \frac{\mu}{T} \nu^\mu + D \omega^\mu + D_B B^\mu,$$

$$\xi = C \left(\mu^2 - \frac{2}{3} \frac{\mu^3 n}{\epsilon + P}\right)$$

# Chiral phenomena as manifestations of an axial anomaly in effective field theory

*A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, Phys. Rev. D 83 (2011) 105025. (formulas on the slide)*

**Chemical potential as an effective external electromagnetic field** $qA^\mu \;\rightarrow\; \mu u^\mu$

*was considered in many papers*

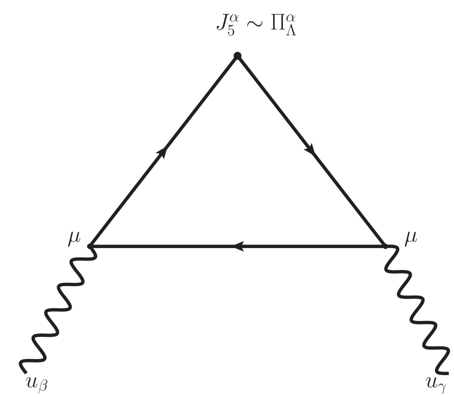
**Hydrodynamics as an effective field theory**

$$S_{eff} = \int dx\,\left(i\bar{\psi}\gamma^\rho D_\rho\psi + \mu u_\mu\bar{\psi}\gamma^\mu\psi + \mu_5 u_\mu\bar{\psi}\gamma^\mu\gamma_5\psi\right)$$

**Axial anomaly in hydrodynamics**

$$\partial_\mu j_5^\mu = -\frac{1}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}\left(\partial^\mu(A^\nu + \mu u^\nu)\partial^\alpha(A^\beta + \mu u^\beta) + \partial^\mu\mu_5 u^\nu\partial^\alpha\mu_5 u^\beta\right)$$

$$\partial_\mu j^\mu = -\frac{1}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\mu(A^\nu + \mu u^\nu)\partial^\alpha\mu_5 u^\beta$$




*A. Sorin and O. Teryaev, Phys. Rev. C95, no. 1, 011902 (2017)*

**Chiral effects in vector current**

$$j^\mu = nu^\mu + \frac{\mu\mu_5}{\pi^2}\omega^\mu + \frac{\mu_5}{2\pi^2}B^\mu$$

**Chiral effects in axial current**

$$j_5^\mu = n_5 u^\mu + \frac{\mu^2 + \mu_5^2}{2\pi^2}\omega^\mu + \frac{\mu}{2\pi^2}B^\mu$$

# Accounting for gravitational anomaly

- *K. Landsteiner, E. Megias, F. Pena-Benitez. Phys. Rev. Lett. 107, 021601 (2011).*
- *M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018).*
- *S. P. Robinson, F. Wilczek Phys.Rev.Lett. 95 (2005) 011303 MIT-CTP-3561 gr-qc/0502074.*

**Chiral effects in axial current**

$$j_5^\mu = n_5 u^\mu + \frac{\mu^2 + \mu_5^2}{2\pi^2}\omega^\mu + \frac{\mu}{2\pi^2}B^\mu$$

⇩

**Chiral vortical effect (CVE)**

$$j_5^\lambda = \left(\frac{T^2}{6} + \frac{\mu^2 + \mu_5^2}{2\pi^2}\right)\omega^\lambda$$

⇩

**While the gravitational chiral anomaly is negligible in the bulk, it turns to be crucial on the edge, or on the horizon**

# Literature


- F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].
- M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].
- G. Prokhorov and O. Teryaev, Phys. Rev. D97, no. 7, 076013 (2018) doi:10.1103/PhysRevD.97.076013 [arXiv:1707.02491 [hep-th]].
- G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to Phys. Rev. D.
- G. Y. Prokhorov, O. V. Teryaev and V. I. Zakharov, arXiv:1807.03584 [hep-th].

# Problems and methods

## Problems

- *Investigation of the effects of rotation and acceleration in the axial current.*
- **The manifestations of the Unruh effect in the axial current and boundary temperature.**
- **Polarization of $\Lambda$-hyperons in collisions of heavy ions.**
- **Effects of finite mass.**

## Methods

- **Covariant Wigner function for systems characterized by a thermal vorticity tensor.**
- **Summation of the complete series in the expansion of the mean value of the operator with respect to thermal vorticity - nonperturbative result.**

# Thermal vorticity tensor

$$\beta_\mu = \frac{u_\mu}{T}$$

4-vector of inverse temperature

**Thermal vorticity tensor**

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

**Global thermodynamic equilibrium with nonstationary movement**

$$b_\mu = \text{const} \quad \varpi_{\mu\nu} = \text{const} \quad \beta_\mu = b_\mu + \varpi_{\mu\nu} x_\nu$$

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

*(thermal) Acceleration*

$$\alpha_\mu = \varpi_{\mu\nu} u^\nu$$

$$\alpha_\mu = \frac{a_\mu}{T}$$

*(thermal)Vorticity*

$$w_\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} u^\nu \varpi^{\alpha\beta}$$

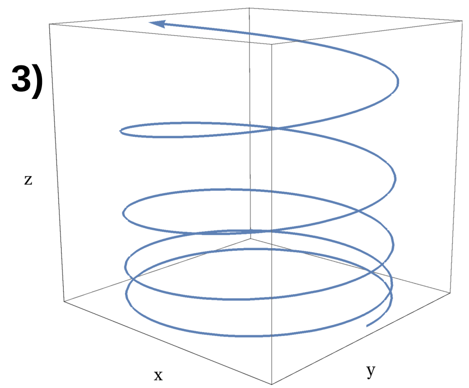
$$w_\mu = \frac{\omega_\mu}{T}$$

*Temperature gradient*

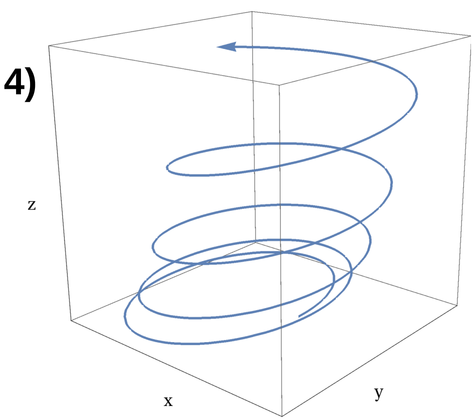
1) Pure uniformly accelerated motion

2) Rigid rotation

3)




4)




Trajectories in global equillibrium



*F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32.*

# Covariant Wigner function: a method of describing a system in quantum kinetic theory

*F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].*

**Wigner function for Dirac fields**

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y}\langle : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) :\rangle$$

⇒ Contains thermodynamic information about the system

⇓

The interaction is weak, W has inhomogeneities only on macroscopic scales

⇓

$$W(x,k) = \frac{1}{2}\int \frac{d^3p}{\varepsilon}\Big(\delta^4(k-p)U(p)f(x,p)\bar{U}(p) - \delta^4(k+p)V(p)\bar{f}^T(x,p)\bar{V}(p)\Big)$$

⇓

**Mean values of operators**

$$\langle : \overline{\Psi}(x)A\Psi(x) :\rangle = \int \mathrm{d}^4k\, \mathrm{tr}(AW(x,k))$$

⇓

$$f(x,p) = \frac{1}{\mathrm{e}^{\beta(x)\cdot p-\xi(x)}+1}$$

**Fermi-Dirac distribution under local thermodynamic equilibrium**

# Ansatz of the Wigner function, taking into account the thermal vorticity tensor

For weak interacting case Wigner function is expressed in terms of: $f(x,p)$

$$\frac{1}{2m}\bar{U}(p)X(x,p)U(p) = f(x,p)$$

*let's introduce X - function*

**Ansatz of the Wigner function**

$$X(x,p) = \left(\exp[\beta_\mu p^\mu - \zeta]\exp\left[-\frac{1}{2}\varpi_{\mu\nu}\Sigma^{\mu\nu}\right] + I\right)^{-1}$$

*1) F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].*
*2) W. Florkowski, A. Kumar and R. Ryblewski, arXiv:1806.02616 [hep-ph].*
*3) W. Florkowski, E. Speranza and F. Becattini, Acta Phys. Polon. B49(2018)1409.*

- Used to calculate corrections in $\varpi_{\mu\nu}$ in different physical quantities.
- Used to calculate polarisation of baryons.

- Explanation of decrease of polarisation of hyperons on STAR

*Generators of Lorentz transformations of spinors*

$$\Sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$$

- Limit of zero thermal vorticity tensor

$$f(x,p) = \frac{1}{e^{\beta(x)\cdot p - \xi(x)} + 1}$$

- Nonrelativistic limit

*F. Becattini and L. Tinti, Annals Phys. 325 (2010) 1566.*

# Axial current and CVE

**The axial current is expressed in terms of an integral with the Wigner function**

$$\langle : j^5_\mu : \rangle = -\frac{1}{16\pi^3}\epsilon_{\mu\alpha\nu\beta}\int \frac{d^3p}{\varepsilon}p^\alpha \Big\{ \mathrm{tr}\big(X\Sigma^{\nu\beta}\big) - \mathrm{tr}\big(\bar{X}\Sigma^{\nu\beta}\big)\Big\}$$

***Can be calculated analytically outside perturbation theory***

1) *M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].*

2) *F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].*

3) *G. Prokhorov and O. Teryaev, Phys. Rev. D97, no. 7, 076013 (2018) doi:10.1103/PhysRevD.97.076013 [arXiv:1707.02491 [hep-th]].*

**The existence of CVE was shown**

$$j^\mu_5(x) = \Big(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\Big)\omega^\mu$$

# Trace calculation, technical details

Particles $\mathrm{tr}\left(X\Sigma^{\nu\beta}\right)$ $\xi \to -\xi$ Common sign Antiparticles $\mathrm{tr}\left(\bar{X}\Sigma^{\nu\beta}\right)$

- **Expand in Taylor's series**

$$X = \sum_{n=0}^{\infty}(-1)^n \exp\left[t(n+l)(\beta\cdot p - \xi - \frac{1}{2}\varpi:\Sigma)\right] =$$

$$\sum_{n=0}^{\infty}(-1)^n \exp\left[t(n+l)(\beta\cdot p - \xi)\right]\sum_{m=0}^{\infty}\frac{1}{m!}\Big(t(n+l)(-\frac{1}{2}\varpi:\Sigma)\Big)^m$$

- Use the properties of gamma matrices

$$(\varpi:\Sigma)^{2k} = \eta^k\frac{1+\gamma^5}{2} + \theta^k\frac{1-\gamma^5}{2}, \quad k = 0,1,2\dots.$$

- One can find a trace in each term of the series

$$\mathrm{tr}\Big((\varpi:\Sigma)^{2k+1}\Sigma^{\nu\beta}\Big) = (\varpi^{\nu\beta} + i\tilde{\varpi}^{\nu\beta})\eta^k + (\varpi^{\nu\beta} - i\tilde{\varpi}^{\nu\beta})\theta^k$$

$$\mathrm{tr}\Big((\varpi:\Sigma)^{2k}\Sigma^{\nu\beta}\Big) = 0, \quad k = 0,1,2\dots.$$

- Sum up the series back



*G. Prokhorov and O. Teryaev, Phys. Rev. D97, no. 7, 076013 (2018) doi:10.1103/PhysRevD.97.076013 [arXiv:1707.02491 [hep-th]].*

# Axial current, mass is not zero

- General exact formula for axial current was obtained

*G. Prokhorov and O. Teryaev, Phys. Rev. D97, no. 7, 076013 (2018).*
*G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to PhysRevD.*

$$\langle j_\mu^5 \rangle = \frac{\omega_\mu + i\,\mathrm{sgn}(\omega a) a_\mu}{2(g_\omega - i g_a)} \int \frac{d^3 p}{(2\pi)^3} \Big\{ n_F(E_p - \mu - g_\omega/2 + i g_a/2) - n_F(E_p - \mu + g_\omega/2 - i g_a/2) + n_F(E_p + \mu - g_\omega/2 + i g_a/2) - n_F(E_p + \mu + g_\omega/2 - i g_a/2) \Big\} + c.c.\,,$$

Fermi-Dirac distribution

$$g_\omega = \frac{1}{\sqrt{2}} \left( \sqrt{(a^2 - \omega^2)^2 + 4(\omega a)^2} + a^2 - \omega^2 \right)^{1/2}$$

$$g_a = \frac{1}{\sqrt{2}} \left( \sqrt{(a^2 - \omega^2)^2 + 4(\omega a)^2} - a^2 + \omega^2 \right)^{1/2}$$

# Axial current, mass is not zero

- In particular case $\boldsymbol{\Omega} \,||\, \boldsymbol{a}$ in comoving frame it gives

$$\langle \boldsymbol{j}^5 \rangle = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\Big\{ n_F(E_p - \mu - \frac{\Omega}{2} + i\frac{|\boldsymbol{a}|}{2}) - n_F(E_p - \mu + \frac{\Omega}{2} + i\frac{|\boldsymbol{a}|}{2}) + n_F(E_p + \mu - \frac{\Omega}{2} + i\frac{|\boldsymbol{a}|}{2}) - n_F(E_p + \mu + \frac{\Omega}{2} + i\frac{|\boldsymbol{a}|}{2}) + c.c. \Big\}\, \boldsymbol{e}_\Omega\,,$$

Fermi-Dirac distribution

unit vector in the direction of the angular velocity

**Angular velocity and acceleration appear in combination with chemical potential**

$$\mu \pm (\Omega \pm i|\boldsymbol{a}|)/2$$



*G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to Phys. Rev. D.*

# Axial current, mass is not zero

**Combination appears**

$$\mu \pm (\Omega \pm i|\boldsymbol{a}|)/2$$

- Angular velocity $\dfrac{\Omega}{2}$ as a real chemical potential.
- Acceleration $i\dfrac{|\boldsymbol{a}|}{2}$ as an **imaginary** chemical potential.



*G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to Phys. Rev. D.*

# Angular velocity as a chemical potential

- For $T=0$, in region $|\mu|<m$, in stationary systems physical quantities equal zero

⇩

Let's consider

$\alpha_\mu = 0$

⇩

Modified chemical potential

$\mu \pm \Omega/2$

⇩

- Similar region with zero values of physical quantities for angular velocity. Integrals can be found analytically.

$$\langle \boldsymbol{j}^5 \rangle = \frac{1}{6\pi^2}\Big\{\theta(\mu+\frac{\Omega}{2}-m)\big[(\mu+\frac{\Omega}{2})^2-m^2\big]^{3/2} - \theta(\mu-\frac{\Omega}{2}-m)\big[(\mu-\frac{\Omega}{2})^2-m^2\big]^{3/2} + \theta(-\mu+\frac{\Omega}{2}-m)\big[(\mu-\frac{\Omega}{2})^2-m^2\big]^{3/2} - \theta(-\mu-\frac{\Omega}{2}-m)\big[(\mu+\frac{\Omega}{2})^2-m^2\big]^{3/2}\Big\}\boldsymbol{e}_\Omega\,,$$

Heaviside theta-function



G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to Phys. Rev. D.

# Angular velocity as a chemical potential

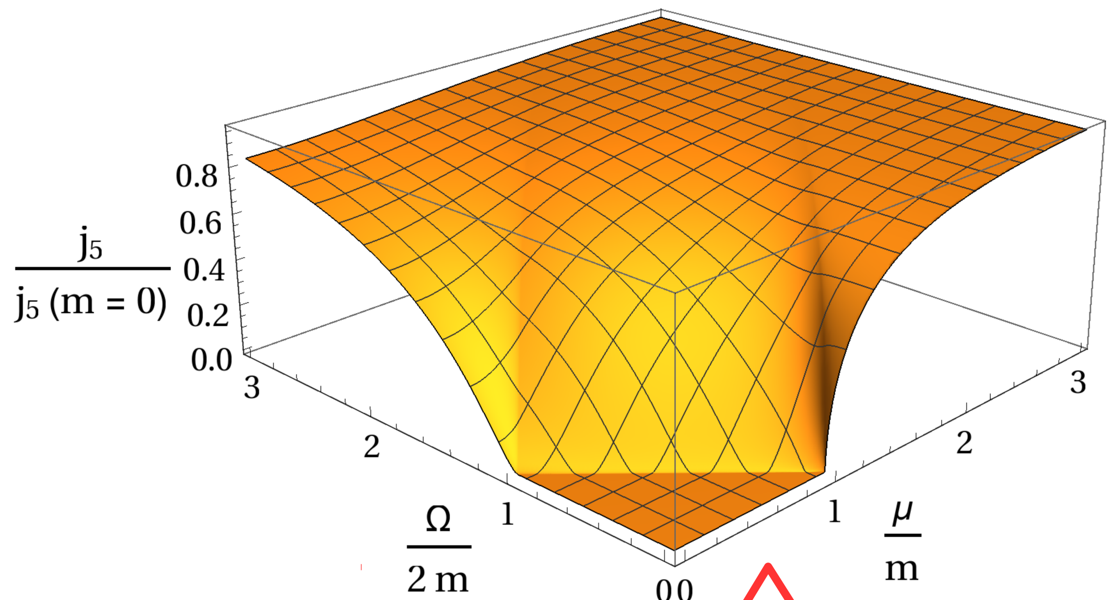
- For $T = 0$, in region $|\mu| < m$, in stationary systems physical quantities equal zero

Let's consider

$\alpha_\mu = 0$

Modified chemical potential

$$\mu \pm \Omega/2$$




**_Axial current equals zero at_** $\Omega < 2(m - |\mu|)$

- Similar region with zero values of physical quantities for angular velocity. Integrals can be found analytically.
- *Angular velocity as a chemical potential:*


*1) A. Vilenkin, Phys. Rev. D 20, 1807 (1979)*
*2) W. Florkowski, B. Friman, A. Jaiswal and E. Speranza, Phys. Rev. C 97, no. 4, 041901 (2018).*

# The limit of massless fermions

- **Integrals with a Fermi distribution can be found analytically in the massless limit** $m \to 0$

Remains for $T > \widetilde{T}_U$

$$\langle j^5_\mu \rangle = \left(\frac{1}{6}\left[T^2 + \frac{a^2-\omega^2}{4\pi^2}\right] + \frac{\mu^2}{2\pi^2}\right)\omega_\mu + \frac{1}{12\pi^2}(\omega a)\, a_\mu + \omega_\mu\Big[-\frac{4\pi T g_a}{g_a^2+g_\omega^2}\Big(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{g_a^2}{8\pi^2} - \frac{g_\omega^2}{8\pi^2}\Big)\left\lfloor \frac{g_a}{4\pi T}+\frac{1}{2}\right\rfloor - 2T^2 \left\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\right\rfloor^2 + \frac{8\pi T^3 g_a}{3(g_a^2+g_\omega^2)}\left\lfloor \frac{g_a}{4\pi T}+\frac{1}{2}\right\rfloor^3\Big] + a_\mu \mathrm{sgn}(\omega a)\Big[-\frac{4\pi T g_\omega}{g_a^2+g_\omega^2}\Big(\frac{T^2}{6}+\frac{\mu^2}{2\pi^2}+\frac{g_a^2}{8\pi^2}+\frac{g_\omega^2}{8\pi^2}\Big)\left\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\right\rfloor + \frac{8\pi T^3 g_\omega}{3(g_a^2+g_\omega^2)}\left\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\right\rfloor^3\Big]$$

Nonzero for $T < \widetilde{T}_U$



*G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to Phys. Rev. D.*

# The limit of massless fermions comparison with other approaches

- **In the area** $T > \widetilde{T}_U$ **one has:** $$\langle j^5_\mu \rangle = \Big(\frac{1}{6}\Big[T^2 + \frac{a^2-\omega^2}{4\pi^2}\Big] + \frac{\mu^2}{2\pi^2}\Big)\omega_\mu + \frac{1}{12\pi^2}(\omega a)\, a_\mu$$

**Standard CVE, a term along the vorticity of the first order**

**The third-order term in vorticity** $(-\frac{\omega^2}{24\pi^2})\omega_\mu$ **corresponds to**

**The term of order 3 along the 4-acceleration**

$$\langle : j^5_\mu : \rangle = \langle : j^5_\mu : \rangle_{\rm Tvort} + \langle : j^5_\mu : \rangle_{\rm vort} + \langle : j^5_\mu : \rangle_{\rm acc}\,,$$

$$\langle : j^5_\mu : \rangle_{\rm Tvort} = \Big(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\Big)\omega_\mu\,, \qquad \langle : j^5_\mu : \rangle_{\rm vort} = \frac{a^2-\omega^2}{24\pi^2}\omega_\mu\,,$$

$$\langle : j^5_\mu : \rangle_{\rm acc} = \frac{1}{12\pi^2}(\omega\cdot a)\, a_\mu\,.$$


*A. Vilenkin, Phys. Rev. D 20 (1979) 1807. doi:10.1103/PhysRevD.20.1807*

*A. Vilenkin, Phys. Rev. D 21 (1980) 2260. doi:10.1103/PhysRevD.21.2260*

*M. Stone and J. Kim, Phys.Rev. D98 (2018) no.2, 025012 DOI: 10.1103/PhysRevD.98.025012*





*G. Prokhorov and O. Teryaev, Phys. Rev. D97, no. 7, 076013 (2018) doi:10.1103/PhysRevD.97.076013 [arXiv:1707.02491 [hep-th]].*

# The limit of massless fermions

For $\boldsymbol{\Omega} \| \boldsymbol{a}$ , $m \to 0$ , $T > \widetilde{T}_U$

$$\langle \mathbf{j}^5 \rangle = \Big( \frac{T^2 \Omega}{6} + \frac{(\mu + \frac{\Omega}{2} + \frac{ia}{2})^3}{12\pi^2} - \frac{(\mu - \frac{\Omega}{2} - \frac{ia}{2})^3}{12\pi^2} + \frac{(\mu + \frac{\Omega}{2} - \frac{ia}{2})^3}{12\pi^2} - \frac{(\mu - \frac{\Omega}{2} + \frac{ia}{2})^3}{12\pi^2} \Big) \mathbf{e}_\Omega$$

**Combination appears**

$$\mu \pm (\Omega \pm i |\boldsymbol{a}|)/2$$



G. Y. Prokhorov, O. V. Teryaev and V. I. Zakharov, arXiv:1807.03584 [hep-th].

# The limit of massless fermions Diagonalization

$$\varphi_\mu = \frac{\omega_\mu}{2\pi} + i\frac{a_\mu}{2\pi}$$

Complex superposition of acceleration and vorticity vectors

$$\langle : j^5_\mu : \rangle = 2\pi \mathrm{Re}\Big[\Big(\frac{1}{6}(T^2 - \varphi^2) + \frac{\mu^2}{2\pi^2}\Big)\varphi_\mu\Big]$$

- The symmetry between vorticity and acceleration is an analog of the symmetry between the magnetic and electric fields in electrodynamics

# The limit of massless fermions, instabilities

$$\langle j^5_\mu \rangle = \Big(\frac{1}{6}\big[T^2 + \frac{a^2-\omega^2}{4\pi^2}\big] + \frac{\mu^2}{2\pi^2}\Big)\omega_\mu + \frac{1}{12\pi^2}(\omega a)\, a_\mu + \omega_\mu\Big[-\frac{4\pi T g_a}{g_a^2+g_\omega^2}\Big(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{g_a^2}{8\pi^2} - \frac{g_\omega^2}{8\pi^2}\Big)\Big\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\Big\rfloor - 2T^2\Big\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\Big\rfloor^2 + \frac{8\pi T^3 g_a}{3(g_a^2+g_\omega^2)}\Big\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\Big\rfloor^3\Big] +$$
$$a_\mu \mathrm{sgn}(\omega a)\Big[-\frac{4\pi T g_\omega}{g_a^2+g_\omega^2}\Big(\frac{T^2}{6}+\frac{\mu^2}{2\pi^2}+\frac{g_a^2}{8\pi^2}+\frac{g_\omega^2}{8\pi^2}\Big)\Big\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\Big\rfloor + \frac{8\pi T^3 g_\omega}{3(g_a^2+g_\omega^2)}\Big\lfloor\frac{g_a}{4\pi T}+\frac{1}{2}\Big\rfloor^3\Big]$$

Additional contribution for $T < \widetilde{T}_U$

Depends on scalars $g_\omega$ and $g_a$

Temperature $\widetilde{T}_U = \dfrac{g_a}{2\pi}$

$$g_\omega = \frac{1}{\sqrt{2}}\Big(\sqrt{(a^2-\omega^2)^2+4(\omega a)^2} + a^2 - \omega^2\Big)^{1/2}$$
$$g_a = \frac{1}{\sqrt{2}}\Big(\sqrt{(a^2-\omega^2)^2+4(\omega a)^2} - a^2 + \omega^2\Big)^{1/2}$$

*"Effective" acceleration and angular velocity*

$\boldsymbol{\Omega} \,||\, \boldsymbol{a}$ ⟹ $g_\omega = \Omega,\ g_a = |\boldsymbol{a}|$

**Also for zero acceleration or angular velocity**



*G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to Phys. Rev. D.*

# The limit of massless fermions, instabilities

$$\langle j^5_\mu \rangle = \Big(\frac{1}{6}\big[T^2 + \frac{a^2-\omega^2}{4\pi^2}\big] + \frac{\mu^2}{2\pi^2}\Big)\omega_\mu + \frac{1}{12\pi^2}(\omega a)\, a_\mu + \omega_\mu\Big[-\frac{4\pi T g_a}{g_a^2+g_\omega^2}\Big(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{g_a^2}{8\pi^2} - \frac{g_\omega^2}{8\pi^2}\Big)\Big\lfloor \frac{g_a}{4\pi T} + \frac{1}{2}\Big\rfloor - 2T^2\Big\lfloor \frac{g_a}{4\pi T} + \frac{1}{2}\Big\rfloor^2 + \frac{8\pi T^3 g_a}{3(g_a^2+g_\omega^2)}\Big\lfloor \frac{g_a}{4\pi T} + \frac{1}{2}\Big\rfloor^3\Big] +$$

$$a_\mu \mathrm{sgn}(\omega a)\Big[-\frac{4\pi T g_\omega}{g_a^2+g_\omega^2}\Big(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} + \frac{g_a^2}{8\pi^2} + \frac{g_\omega^2}{8\pi^2}\Big)\Big\lfloor \frac{g_a}{4\pi T} + \frac{1}{2}\Big\rfloor + \frac{8\pi T^3 g_\omega}{3(g_a^2+g_\omega^2)}\Big\lfloor \frac{g_a}{4\pi T} + \frac{1}{2}\Big\rfloor^3\Big]$$

**Instabilities due to the terms with integer part below** $\widetilde{T}_U = \frac{g_a}{2\pi}$

$$\Big\lfloor \frac{g_a}{4\pi T} + \frac{1}{2}\Big\rfloor$$

**In particular case** $\boldsymbol{\Omega} = 0$ **or** $\boldsymbol{\Omega} \,||\, \boldsymbol{a}$

$$\widetilde{T}_U \to T_U = \frac{|\boldsymbol{a}|}{2\pi}$$

**Unruh temperature**

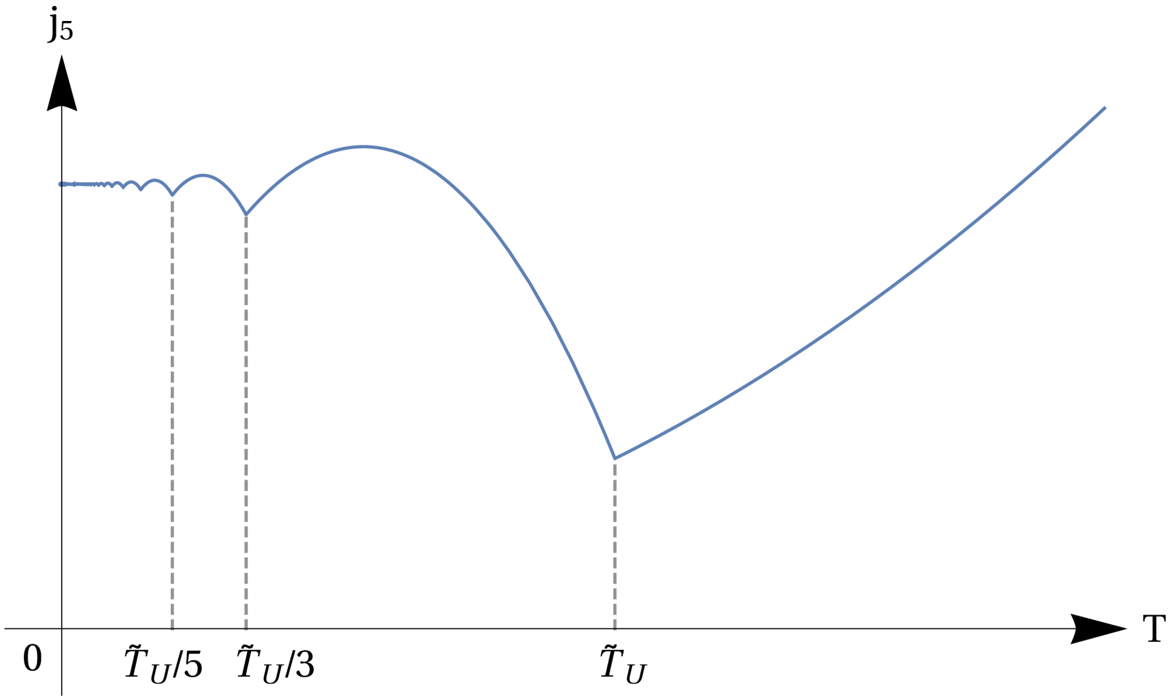
- **Instabilities below the temperature $\widetilde{T}_U$ are a manifestation of the Unruh-Hawking radiation.**
- **From this point of view, the temperature $\widetilde{T}_U(\Omega, |\boldsymbol{a}|, \theta)$ should be considered as a generalization of the Unruh-Hawking temperature to the case of systems having simultaneously non-zero acceleration and angular velocity.**



*G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to Phys. Rev. D.*

# The limit of massless fermions, instabilities

- **Instabilities below the temperature $\widetilde{T}_U$ are a manifestation of the Unruh-Hawking radiation.**




**Acceleration as imaginary chemical potential**

- **The appearance of instabilities below Unruh temperature is a direct consequence of appearance of acceleration as an imaginary chemical potential.**
- **Periodic instabilities due to the terms with $\lfloor \frac{g_a}{4\pi T} + \frac{1}{2} \rfloor$ at $\frac{g_a}{2} = (2n+1)\pi T, n = 0, 1, 2..$ correspond to the theories with imaginary chemical potential: Roberge-Weiss phase transitions.**

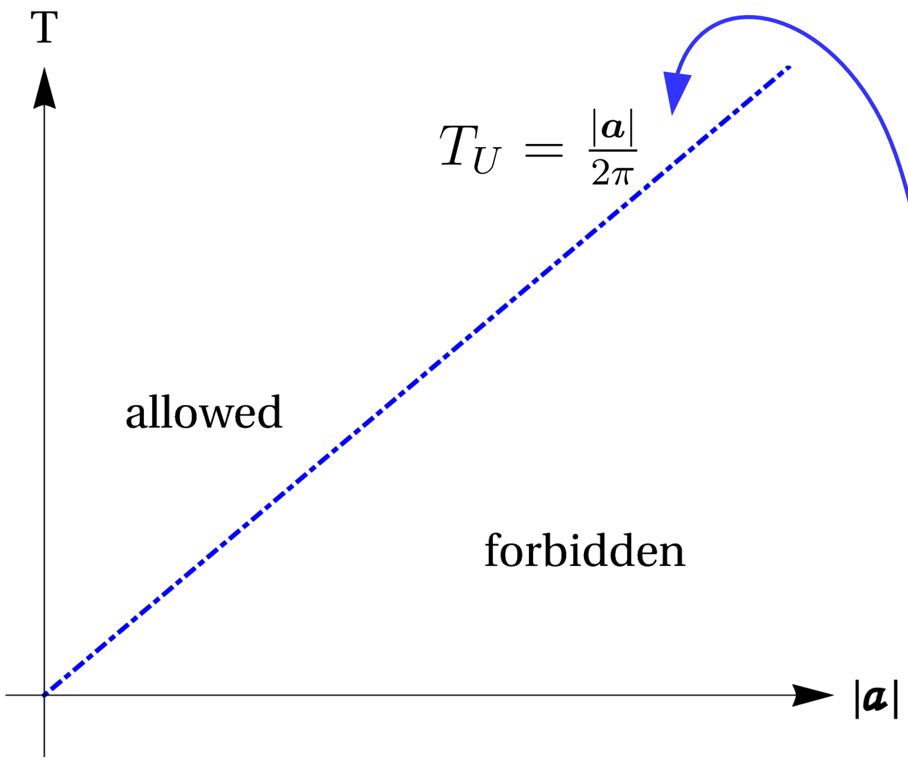
  *A. Roberge and N. Weiss, Nucl. Phys. B 275, 734 (1986); Y. Sakai, K. Kashiwa, H. Kouno and M. Yahiro, Phys. Rev. D 78, 036001 (2008).*

- **The periodicity of the axial current with an acceleration change $\frac{|a|}{2} \to \frac{|a|}{2} + 2\pi T n$ corresponds to the periodicity of the partition function in the theories with an imaginary chemical potential with respect to this potential.**



*G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to Phys. Rev. D.*

# Minimal temperature in the medium with acceleration and rotation

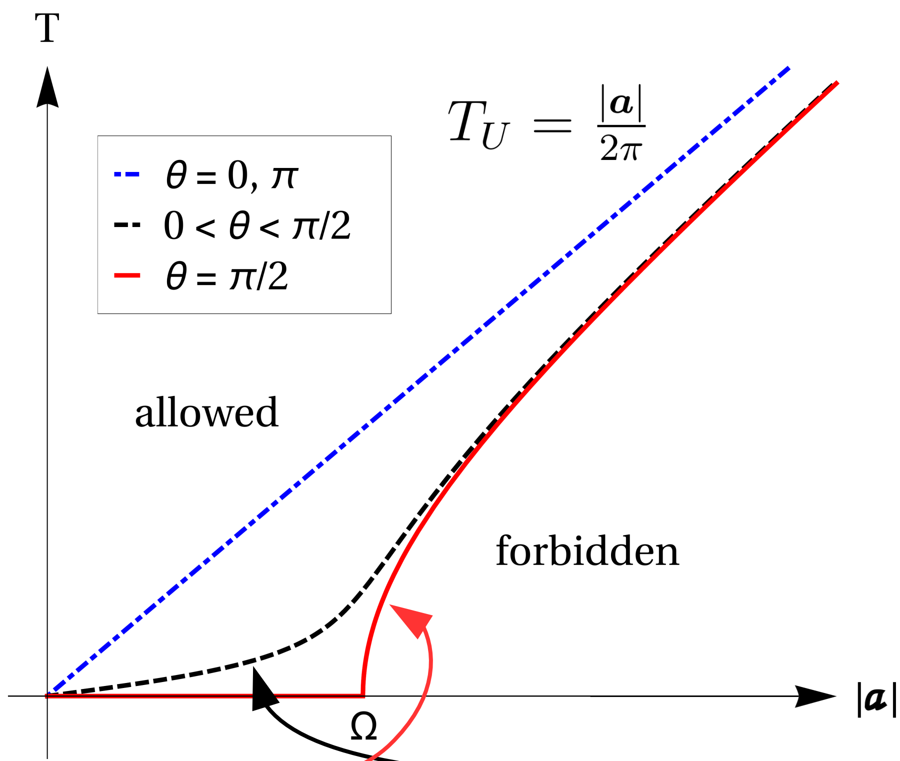



**Unruh temperature is minimal temperature for accelerated medium**

*F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018).*

- **From the analysis of quantum correlation functions for scalar fields.**
- **Equal zero after vacuum subtraction.**
- **In fact, rephrasing of Unruh effect.**

# Minimal temperature in the medium with acceleration and rotation




**For**

$\boldsymbol{\Omega} \perp \boldsymbol{a}$ **:**

$$\widetilde{T}_U = \frac{\sqrt{|\boldsymbol{a}|^2 - \Omega^2}}{2\pi}$$

**Minimal temperature for systems with acceleration and rotation:**

$$\widetilde{T}_U = \frac{g_a}{2\pi}$$

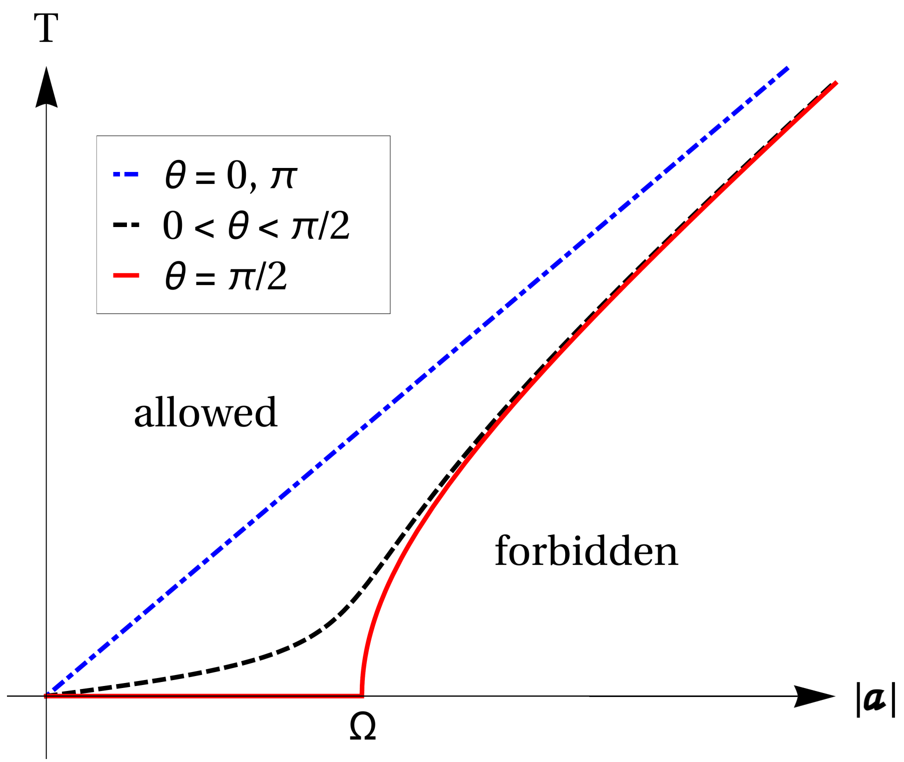
## What will be for fermions?

*Look: W. Florkowski, E. Speranza and F. Becattini, arXiv:1803.11098 [nucl-th], for fermions bound temperature is twice Unruh temperature*

## What will change, if we consider acceleration and rotation simultaneously?

**Let's use the absence of instabilities in axial current as a criterion to define minimal possible temperature**

# Minimal temperature in the medium with acceleration and rotation




- Minimal temperature shifts down for the medium with rotation and acceleration.
- No bound temperature for $\boldsymbol{\Omega} \perp \boldsymbol{a}$ if $|a| < \Omega$.

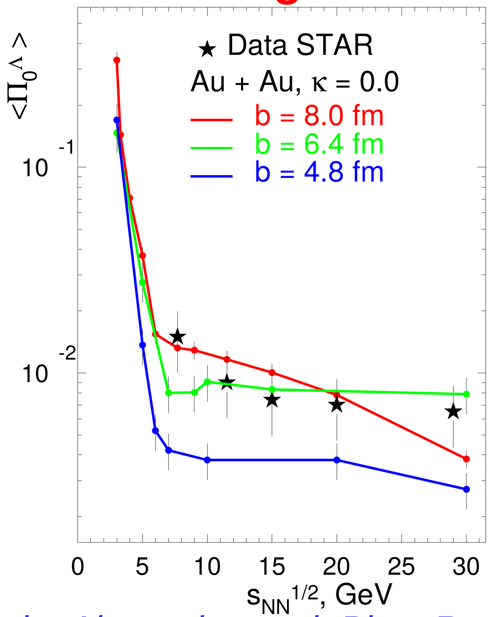


G. Prokhorov, O. Teryaev and V. Zakharov, arXiv:1805.12029 [hep-th]. Sent to Phys. Rev. D.

# 2 approaches to the calculation of the polarization of hyperons

## 1-st approach

**The fall of polarization of hyperons with increasing collision energy.**




- *Sorin, Alexander et al. Phys.Rev. C95 (2017) no.1, 011902 arXiv:1606.08398 [nucl-th]*
- *M. Baznat, K. Gudima, A. Sorin and O. Teryaev, EPJ Web Conf. 138 (2017) 01008.doi:10.1051/ epjconf/201713801008; arXiv:1701.00923*

The polarization is determined by the axial charge of strange quarks

from CVE

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6},$$

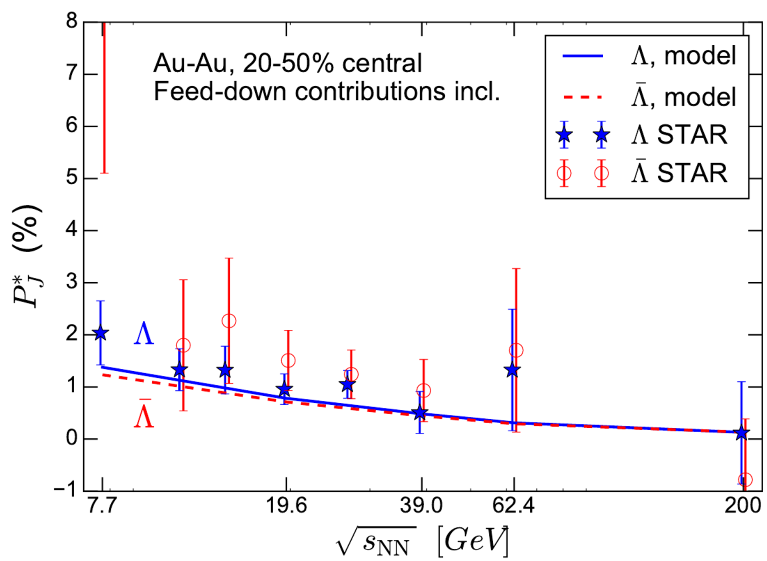
$$Q_5^s = N_c \int d^3x\, C(r)\, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

## 2-nd approach

- *Karpenko, Iu. et al. Nucl.Phys. A967 (2017) 764-767 arXiv:1704.02142 [nucl-th]*

**Based on the Wigner function**

$$S^\mu(x,p) = -\frac{1}{8m}(1 - f(x,p))\epsilon^{\mu\nu\rho\sigma} p_\sigma \varpi_{\nu\rho}$$




**The fall of polarization is caused by the decrease of the strange chemical potential**

# 2 approaches to the calculation of the polarization of hyperons

- **It was proved that the Wigner function used in the second approach leads to CVE.**
- **CVE underlies the first approach.**
- **Thus, CVE is essential for both approaches, which can explain the same polarization behaviour in them.**

**There is a connection between the two approaches to the calculation of the polarization of hyperons.**



*G. Prokhorov and O. Teryaev, Phys. Rev. D97, no. 7, 076013 (2018) doi:10.1103/PhysRevD.97.076013 [arXiv:1707.02491 [hep-th]].*

# Conclusions

- **It is shown that CVE follows from the covariant Wigner function for the medium with thermal vorticity** *(the same result was given previously in M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091.)*.
- **A formula for the axial current in a rotating and accelerated medium is derived outside the limits of perturbation theory from the Wigner function. It includes higher order corrections to CVE.**
- **It is shown that the angular velocity plays the role of a chemical potential, while the acceleration appears as an imaginary chemical potential** *(in particular case when* $\boldsymbol{\Omega} || \boldsymbol{a}$ *, in general case scalars* $g_\omega$ *and* $g_a$ *appear)*.
- **At zero temperature, the axial current, as a function of the angular velocity and chemical potential, vanishes in a two-dimensional plane region** $\Omega < 2(m - |\mu|)$ **.**
- **Instabilities in axial current below** $\widetilde{T}_U = \frac{g_a}{2\pi}$ **, indicating the Unruh effect.**
- **The assumption that the "effective" Unruh temperature** $\widetilde{T}_U = \frac{g_a}{2\pi}$ **, depending on acceleration and angular velocity, is the lower boundary temperature** *(based on the requirement of no instabilities)*.

# Thank you for attention!

# The second approach: the equilibrium density operator

- Thermodynamics can be derived from field theory, constructed in terms of the path integral in imaginary time
- The central role is played by the density operator
- **General covariant form of the density operator for a medium in local thermodynamic equilibrium**

$$\widehat{\rho} = \frac{1}{Z} \exp\left[ - \int_{\Sigma} \mathrm{d}\Sigma_\mu \left( \widehat{T}^{\mu\nu}(x)\beta_\nu(x) - \zeta(x)\,\widehat{j}^{\mu}(x) \right) \right] \Rightarrow$$

**Maximum**

$$S = -\mathrm{tr}(\widehat{\rho}\log\widehat{\rho})$$

*D.N. Zubarev, A.V. Prozorkevich and S.A. Smolyanskii, Derivation of nonlinear generalized equations of quantum relativistic hydrodynamics, Theor. Math. Phys. 40 (1979) 821.*

⇓

Provided that $\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \qquad \nabla_\mu \zeta = 0$ ⇒

$$\beta_\mu(x) = b_\mu + \varpi_{\mu\nu} x^\nu \qquad \zeta = \mathrm{const.}$$

$$b_\mu = \mathrm{const} \qquad \varpi_{\mu\nu} = \mathrm{const}$$

⇓

$\widehat{\rho}$ does not depend on the choice of a hypersurface $\mathrm{d}\Sigma_\mu$

⇓

**global thermodynamic equilibrium**



*M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].*

# The density operator for a medium with a thermal vorticity tensor

**Global equilibrium conditions**

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \qquad \nabla_\mu \zeta = 0$$

**The form of the density operator for a medium with rotation and acceleration**

$$\widehat{\rho} = \frac{1}{Z} \exp\left[ -b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

Thermal vorticity tensor

4-momentum operator

Charge operator

**Generators of Lorentz transformations**

$$\widehat{J}^{\mu\nu} = \int_\Sigma \mathrm{d}\Sigma_\lambda \left( x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right)$$

- *F. Becattini and E. Grossi, Phys. Rev. D 92 (2015) 045037 [arXiv:1505.07760] [INSPIRE].*
- *F. Becattini, arXiv:1712.08031 [gr-qc], to appear in Phys. Rev. D.*

*M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].*

# Предельные случаи

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu} + \zeta\widehat{Q}\right]$$

$\varpi = 0$

**Предельные случаи**

$\alpha_\mu = 0$

$$\widehat{\rho} = \frac{1}{Z}\exp[-b\cdot\widehat{P} + \zeta\widehat{Q}]$$

**Великое каноническое распределение**

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\widehat{H}/T_0 + \omega_0\widehat{J}_z/T_0\right]$$

*V.E. Ambrus and E. Winstanley, Rotating fermions inside a cylindrical boundary, Phys. Rev. D 93 (2016) 104014 [arXiv:1512.05239] [INSPIRE].*



*M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].*

# Mean value of the physical quantity operator

*Mean in terms of the path integral*

$$\langle \widehat{O}(x) \rangle = \mathrm{tr}(\hat{\rho}\, \hat{O}(x))_{\mathrm{ren}}$$

Statistical sum: reduction of disconnected correlators

**Perturbation theory in the <u>third order</u>**

$$\langle \widehat{O}(x) \rangle = \langle \widehat{O}(x) \rangle_{\beta(x)} + \frac{\varpi_{\mu\nu}}{2|\beta|} \int_0^{|\beta|} d\tau \langle T_\tau J^{\mu\nu}_{-i\tau u} \widehat{O}(0) \rangle_{\beta(x),c}$$

$$+ \frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}}{8|\beta|^2} \int_0^{|\beta|} d\tau_x d\tau_y \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u} \widehat{O}(0) \rangle_{\beta(x),c}$$

$$+ \frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}\varpi_{\alpha\beta}}{48|\beta|^3} \int_0^{|\beta|} d\tau_x d\tau_y d\tau_z \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u} J^{\alpha\beta}_{-i\tau_z u} \widehat{O}(0) \rangle_{\beta(x),c} + \ldots$$

*Ordering by imaginary time*

**Connected correlators**

$$\langle \widehat{J}\widehat{O} \rangle_c = \langle \widehat{J}\widehat{O} \rangle - \langle \widehat{J} \rangle \langle \widehat{O} \rangle$$

- Reduction of the members by vorticity, starting with order 4 in the approach with the Wigner function and in

***A. Vilenkin, Phys. Rev. D 21 (1980) 2260. doi:10.1103/PhysRevD.21.2260***



*M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].*

# Axial current in the third order of perturbation theory

**Three types of admissible parity:**

$$\langle \widehat{j}_5^{\lambda}(x)\rangle_3 = A_1 w^2 w^{\lambda} + A_2 \alpha^2 w^{\lambda} + A_3 (w\alpha)\alpha^{\lambda}$$

Violates the conservation of axial charge

**The main points of the technique of computing of hydrodynamic coefficients**

- Representation of composite operators in a split form *(point splitting)*

$$\widehat{T}_{\mu\nu}(X) = \lim_{X_1, X_2 \to X} \mathcal{D}_{\mu\nu}(\partial_{X_1}, \partial_{X_2}) \bar{\Psi}(X_1)\Psi(X_2)$$

$$\mathcal{D}_{\mu\nu}(\partial_{X_1}, \partial_{X_2}) = \frac{\mathrm{i}^{\delta_{0\mu}+\delta_{0\mu}}}{4}\left[\tilde{\gamma}_\mu(\partial_{X_2} - \partial_{X_1})_\nu + \tilde{\gamma}_\nu(\partial_{X_2} - \partial_{X_1})_\mu\right]$$

*M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].*

- Summation over the Matsubara frequencies

$$\frac{1}{\beta}\sum_{\{\omega_n\}} \frac{(\omega_n \pm \mathrm{i}\mu)^k \mathrm{e}^{\mathrm{i}(\omega_n \pm \mathrm{i}\mu)\tau}}{(\omega_n \pm \mathrm{i}\mu)^2 + E^2}$$

$$= \frac{1}{2E}\left[(-\mathrm{i}E)^k \mathrm{e}^{\tau E}(\theta(-\tau) - n_F(E \pm \mu)) + (\mathrm{i}E)^k \mathrm{e}^{-E\tau}(\theta(\tau) - n_F(E \mp \mu))\right]$$

$$= \frac{1}{2E}\sum_{s=\pm 1} (-\mathrm{i}sE)^k \mathrm{e}^{\tau s E}[\theta(-s\tau) - n_F(E \pm s\mu)]$$

Fermi distribution

# Детали вычислений

**Термальный пропагатор фермионов:**

*M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].*

$$\langle \mathrm{T}_\tau \Psi_a(X)\bar{\Psi}_b(Y)\rangle = \sum\!\!\!\!\!\!\!\int_{\{P\}} \mathrm{e}^{\mathrm{i}P^+\cdot(X-Y)}\frac{(-\mathrm{i}\not{P}^+ + m)_{ab}}{(P^+)^2+m^2} = \sum\!\!\!\!\!\!\!\int_{\{P\}} \mathrm{e}^{\mathrm{i}P^+\cdot(X-Y)}(-\mathrm{i}\not{P}^+ + m)_{ab}\Delta(P^+)$$

$$\Delta(P^\pm) = \frac{1}{P^{\pm 2}+m^2}$$

Интегрирование по фазовому пространству и суммирование по мацубаровским частотам

**Явная зависимость от координат переписывается в виде производных по импульсу:**

$$\int d^3x d^3y d^3z\, f(\mathbf{p},\mathbf{q},\mathbf{k},\mathbf{r})e^{-i\mathbf{p}(\mathbf{x}-\mathbf{y})-i\mathbf{q}(\mathbf{x}-\mathbf{z})-i\mathbf{k}\mathbf{y}-i\mathbf{r}\mathbf{z}}x^i y^j z^k =$$

$$i\left(\frac{\partial^3}{\partial r^k\partial k^j\partial p^i}+\frac{\partial^3}{\partial r^k\partial k^j\partial k^i}\right)f(\mathbf{p},\mathbf{q},\mathbf{k},\mathbf{r})\Big|_{\substack{\mathbf{q}=-\mathbf{p}\\ \mathbf{k}=\mathbf{p}\\ \mathbf{r}=-\mathbf{p}}}$$

# Axial current in the third order of perturbation theory

**Axial current in the third order of perturbation theory**

$$\langle \widehat{j}_5^{\lambda}(x)\rangle_3 = A_1 w^2 w^{\lambda} + A_2 \alpha^2 w^{\lambda} + A_3 (w\alpha)\alpha^{\lambda}$$

**The results of calculating the coefficients**

$$A_1 = -\frac{1}{24|\beta|^3\pi^2}$$

$$A_2 = -\frac{1}{8|\beta|^3\pi^2}$$

$$A_3 = 0$$

**The axial charge is conserved**

# Comparison with the result obtained using the Wigner function

**Calculation based on the density operator in the framework of a QFT at a finite temperature**

$$j_5^\lambda(x) = (\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2})\omega^\lambda$$

**Calculation based on the Wigner function**

$$\langle : j^5_\mu : \rangle = \left(\frac{1}{6}\left[T^2 + \frac{a^2 - \omega^2}{4\pi^2}\right] + \frac{\mu^2}{2\pi^2}\right)\omega_\mu + \frac{1}{12\pi^2}(\omega \cdot a)\, a_\mu$$

- Cubic terms coincide.
- Coincide linear terms corresponding to CVE.
- The second current has a term along the acceleration vector, which violates the conservation of the axial charge.
- The coefficients that stand before $a^2\omega^\lambda$ are different.

# Запланированные задачи

- Сравнение двух подходов к расчёту поляризации Λ-гиперонов: вывод связи поляризации с аксиальным током.
- Исследование вопроса о неперенормируемости киральных эффектов в теории поля при конечных температурах.
- Исследование влияния эффектов, связанных с гравитаций и, в частности, с гравитационной аномалией, на физику киральных жидкостей.
- Дополнительная проверка сохранения аксиального заряда.

$$\mathrm{tr}\big(X\Sigma^{\nu\beta}\big) = \Big\{\big(\exp\left[(\beta\cdot p - \xi - g_1 + ig_2)\right] + 1\big)^{-1} - \big(\exp\left[(\beta\cdot p - \xi + g_1 - ig_2)\right] + 1\big)^{-1}\Big\}$$
$$\frac{1}{4(g_1 - ig_2)}[\varpi^{\nu\beta} - i\,\mathrm{sgn}(\varpi : \widetilde{\varpi})\widetilde{\varpi}^{\nu\beta}] +$$
$$\Big\{\big(\exp\left[(\beta\cdot p - \xi - g_1 - ig_2)\right] + 1\big)^{-1} - \big(\exp\left[(\beta\cdot p - \xi + g_1 + ig_2)\right] + 1\big)^{-1}\Big\}$$

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = \frac{1}{|\beta|^3}\int d\tau_x d\tau_y d\tau_z d^3x d^3y d^3z \langle T_\tau \widehat{T}^{\alpha_1\alpha_2}(\tau_x, \mathbf{x})$$
$$\widehat{T}^{\alpha_3\alpha_4}(\tau_y, \mathbf{y})\widehat{T}^{\alpha_5\alpha_6}(\tau_z, \mathbf{z})\widehat{j}_5^{\lambda}(0)\rangle_{\beta(x),c} x^i y^j z^k$$

$$A_1 = -\frac{1}{6}(C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} +$$
$$C^{01|01|02|3|221} - C^{01|02|02|3|211} - C^{02|01|02|3|121} - C^{02|02|01|3|112} - C^{01|01|01|3|222})$$

$$\langle T_\tau \widehat{T}^{\alpha_1\alpha_2}(\tau_x,\mathbf{x})\widehat{T}^{\alpha_3\alpha_4}(\tau_y,\mathbf{y})\widehat{T}^{\alpha_5\alpha_6}(\tau_z,\mathbf{z})\widehat{j}_5^{\lambda}(0)\rangle_{\beta(x),c} = \lim_{\substack{X_1,X_2\to X\\ Y_1,Y_2\to Y\\ Z_1,Z_2\to Z\\ F_1,F_2\to F=0}} \mathcal{D}^{\alpha_1\alpha_2}_{a_1a_2}(\partial_{X_1},\partial_{X_2})$$

$$\mathcal{D}^{\alpha_3\alpha_4}_{a_3a_4}(\partial_{Y_1},\partial_{Y_2})\mathcal{D}^{\alpha_5\alpha_6}_{a_5a_6}(\partial_{Z_1},\partial_{Z_2})\mathcal{J}^{\lambda}_{A\,a_7a_8}\langle\bar{\Psi}_{a_1}(X_1)\Psi_{a_2}(X_2)\bar{\Psi}_{a_3}(Y_1)\Psi_{a_4}(Y_2)$$
$$\bar{\Psi}_{a_5}(Z_1)\Psi_{a_6}(Z_2)\bar{\Psi}_{a_7}(F_1)\Psi_{a_8}(F_2)\rangle_{\beta(x),c}$$

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

⇕

**Разложение на генераторы углового момента и сдвига**

$$\widehat{J}^{\mu\nu} = u^\mu \widehat{K}^\nu - u^\nu \widehat{K}^\mu - u_\rho \epsilon^{\rho\mu\nu\sigma}\widehat{J}_\sigma$$

# Аксиальный ток в третьем порядке теории возмущений

Каждый из гидродинамических коэффициентов представляется как сумма величин C

$$A_1 = -\frac{1}{6}(C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} +$$
$$C^{01|01|02|3|221} - C^{01|02|02|3|211} - C^{02|01|02|3|121} - C^{02|02|01|3|112} - C^{01|01|01|3|222})$$

Каждый из этих коэффициентов имеет вид:

$$\frac{i}{{}^3(2\pi)^3} \int [d\tau] d^3p \left( \frac{\partial^3 D_1}{\partial r^k \partial k^j \partial p^i} + \frac{\partial^3 D_1}{\partial r^k \partial k^j \partial k^i} \right) \Bigg|_{\substack{\mathbf{q}=-\mathbf{p} \\ \mathbf{k}=\mathbf{p} \\ \mathbf{r}=-\mathbf{p}}} + \ldots \quad C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = \frac{}{|\beta|^3}$$

$$D_1 = \frac{-1}{16 E_p E_q E_k E_r} \sum_{\substack{s_1, s_2, s_3, \\ s_4 = \pm 1}} e^{(\tau_x - \tau_y) s_1 E_p + (\tau_x - \tau_z) s_2 E_q + \tau_y s_3 E_k + \tau_z s_4 E_r} A^{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_6\lambda}(\widetilde{P}, \widetilde{K}, \widetilde{Q}, \widetilde{R})$$
$$\Big(\Theta(-s_1[\tau_x - \tau_y]) - n_F(E_p - s_1\mu)\Big)\Big(\Theta(-s_2[\tau_x - \tau_z]) - n_F(E_q + s_2\mu)\Big)$$
$$\Big(\Theta(-s_3) - n_F(E_k - s_3\mu)\Big)\Big(\Theta(-s_4) - n_F(E_r + s_4\mu)\Big)$$

Многочлен от аргументов

$$C^{02|02|02|3|111} = -C^{01|01|01|3|222} = \frac{29}{80|\beta|^3\pi^2}$$

$$C^{01|02|02|3|211} = C^{02|01|02|3|121} = C^{02|02|01|3|112} = -C^{02|01|01|3|122} =$$
$$-C^{01|02|01|3|212} = -C^{01|01|02|3|221} = \frac{19}{240|\beta|^3\pi^2}$$

# The covariant Wigner function conclusion

- **The result obtained is in full accordance with all known theoretical calculations of the CVE** ***(the same conclusion in M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].).***
- **The simplicity of the formula obtained.**
- **The cubic term for the vorticity exactly coincides with the other two conclusions for the axial current made by A. Vilenkin.**
- **Symmetry between vorticity and acceleration.**
- **Non-conservation of the axial charge in the case when the acceleration and the rotation speed are not equal to zero and not perpendicular.**
- **An ansatz for the Wigner function is used.**
- **An approximation is used for the Wigner function.**

**AN ADDITIONAL INDEPENDENT APPROACH FOR THE INSPECTION OF THE OBTAINED RESULT SHOULD BE USED**

# Учёт аксиального химического потенциала в первом порядке теории возмущений

**Термальный пропагатор с учётом векторного и аксиального химического потенциалов: случай массы равной нулю**

$$\langle T_\tau \Psi_a(X)\bar{\Psi}_b(Y)\rangle_{\beta(x)} = \sum\!\!\!\!\!\!\int_{\{P\}} \left[ e^{iP^{+-}(X-Y)}\mathcal{P}_+ \frac{\widetilde{\gamma}_\alpha(-iP^{+-}_\alpha)}{(P^{+-})^2} + e^{iP^{++}(X-Y)}\mathcal{P}_- \frac{\widetilde{\gamma}_\alpha(-iP^{++}_\alpha)}{(P^{++})^2} \right]$$

$$\mu_5 \neq 0$$

$$m = 0$$

$$\mathcal{P}_\pm = \frac{1 \pm \gamma_5}{2}$$

$$P^{+-\,\lambda} = (p_n + i\mu - i\mu_5, \mathbf{p})$$

**Аксиальный ток в первом порядке**

$$j_5^\lambda(x) = n_5 u^\lambda + \left(\frac{T^2}{6} + \frac{\mu^2 + \mu_5^2}{2\pi^2}\right)\omega^\lambda$$

**Векторный ток в первом порядке**

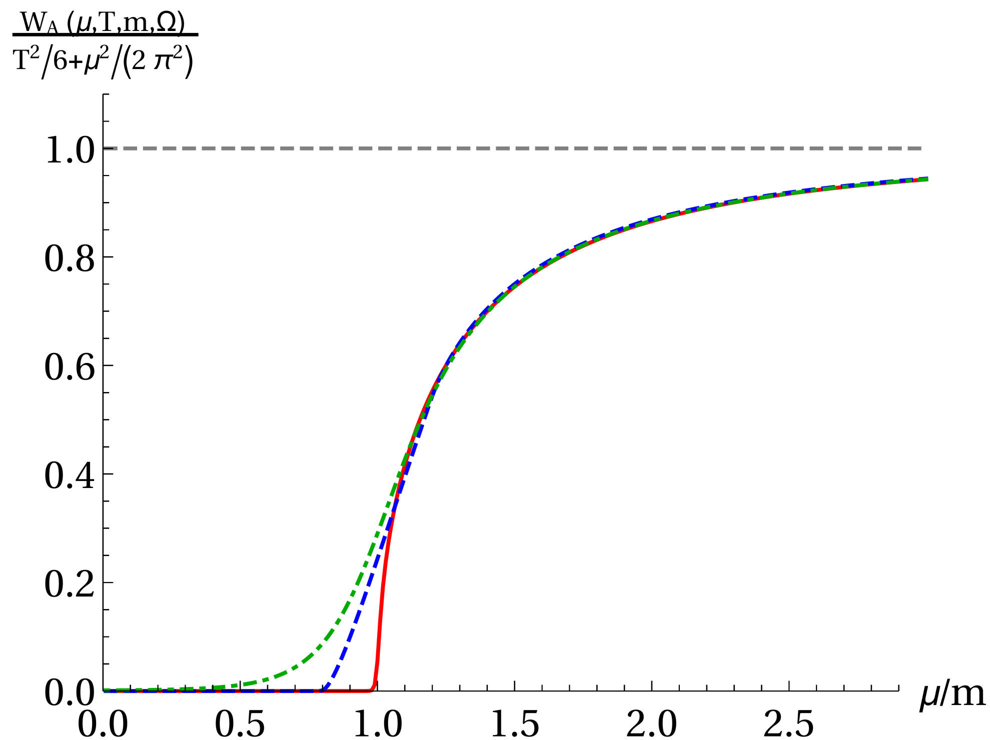
$$j^\lambda = n u^\lambda + \frac{\mu\mu_5}{\pi^2}\omega^\lambda$$

***Соответствует другим подходам: эффективная теория поля с аксиальными аномалиями и кинетический***


- *A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, Phys. Rev. D 83 (2011) 105025 doi:10.1103/PhysRevD.83.105025 [arXiv:1012.1958 [hep-th]].*
- *J. H. Gao, Z. T. Liang, S. Pu, Q. Wang and X. N. Wang, Phys. Rev. Lett. 109 (2012) 232301 doi:10.1103/PhysRevLett.109.232301 [arXiv:1203.0725 [hep-ph]].*

# Axial current, mass is not zero

$\alpha_\mu = 0$




Axial current, as a function of the chemical potential at different values of the rotational speed and temperature

- ***The presence of a step at a chemical potential equal to the mass***
- *The step is smoothed with increasing temperature or speed of rotation*