# $\pi J/\psi - D\bar{D}^*$ potential described by the quark exchange diagram


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in collaboration with

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"Relativistic Nuclear Physics and Quantum Chromodynamics"

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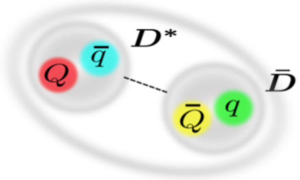
# Outline

- Introduction
  - Exotic hadrons
  - $Z_c(3900)$
- Interaction model
  - Meson exchange model
  - Quark exchange model
- Summary



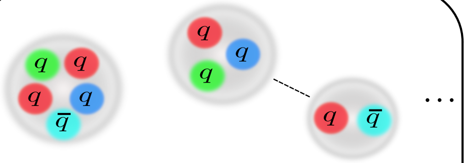

Exotic hadron

- Ordinary Hadrons: Baryon ($qqq$) and Meson ($q\bar{q}$)


Baryon(proton,nucleon,...) Meson ($\pi$,$K$,...)

*$q$: "Constituent quark"

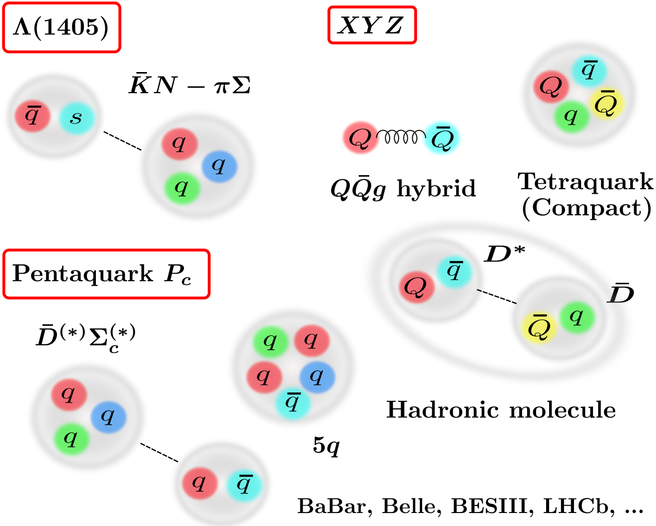
- Exotic Hadrons ($\neq qqq$, $q\bar{q}$): **Multiquark? Multihadron?**




Pentaquark (Compact) Hadronic molecule

# Many exotic candidate!! Many models!!





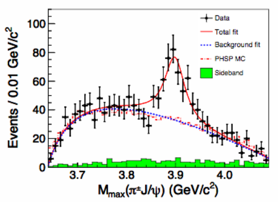

**BaBar, Belle, BESIII, LHCb, ...**

T. Hyodo, D. Jido, PPNP**67**(2012)55, N. Brambilla *et al.*,Eur.Phys.J.C(2011)71,1534

H.X.Chen, et al., Phys.Rept.**639**(2016)1,...

- Charged Charmonium??
- $Y(4260) \to Z_c(3900)\pi \to J/\psi\pi\pi$

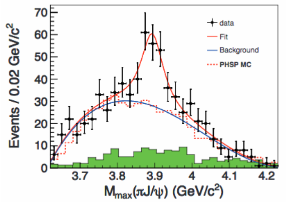
BESIII,PRL**110**(2013)252001




$M = 3899.0 \pm 3.6_{sta} \pm 4.9_{sys}$ MeV

$\Gamma = 46 \pm 10_{sta} \pm 20_{sys}$ MeV

Belle,PRL**110**(2013)252002

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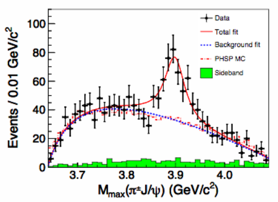


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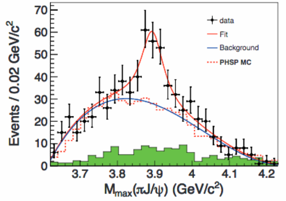


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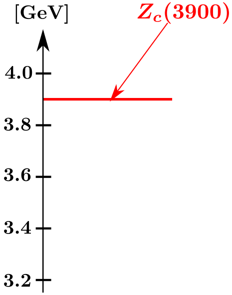
$\Gamma = 63 \pm 24_{sta} \pm 26_{sys}$ MeV

▷ Ordinal Charmonium $c\bar{c}$: no electric charge.
$\Rightarrow Z_c^+(3900)$: **Genuine Exotic State!?** $c\bar{c}u\bar{d}$

- $Z_c(3900)$ close to the hadron-hadron thresholds



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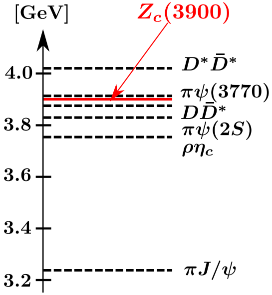
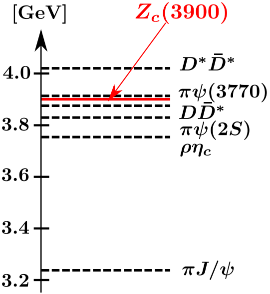
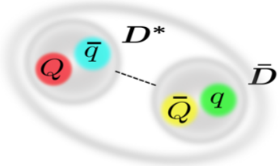


# Structure of $Z_c(3900)$: Hadronic Molecules?
## Introduction

- $Z_c(3900)$ close to the hadron-hadron thresholds




⇒




Hadronic molecule?

- Exotic state may be **a loosely bound state (resonance)** of the meson-meson.
  ⇒ Analogous to atomic nuclei (Deuteron: $B \sim$2.2 MeV)

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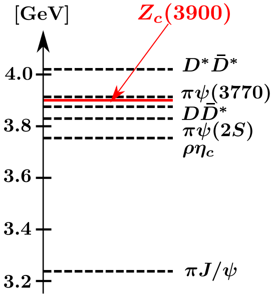
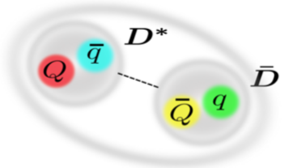



$\Rightarrow$




Hadronic molecule?

- Exotic state may be **a loosely bound state (resonance)** of the meson-meson.
  $\Rightarrow$ Analogous to atomic nuclei (Deuteron: $B \sim$2.2 MeV)
- $D\bar{D}^*$ molecule? **$\pi$ exchange** by $D\bar{D}^* - D^*\bar{D}^*$ (Heavy quark spin symmetry)

ref. A. Hosaka *et al.* PTEP **2016** (2016)062C01, A.Esposito, *et al.*,Phys.Rept.**668**(2016)1,...

- Lattice QCD simulation by HALQCD at $m_\pi = 410 - 700$ MeV

$\Rightarrow$ Coupled-channel $\pi J/\psi - \rho\eta_c - D\bar{D}^*$

PRL **117,** 242001 (2016) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2016

**Fate of the Tetraquark Candidate $Z_c(3900)$ from Lattice QCD**

Yoichi Ikeda,[1,2] Sinya Aoki,[3,4] Takumi Doi,[2] Shinya Gongyo,[3] Tetsuo Hatsuda,[2,5] Takashi Inoue,[6] Takumi Iritani,[7] Noriyoshi Ishii,[1] Keiko Murano,[1] and Kenji Sasaki[3,4]

(HAL QCD Collaboration)

# $Z_c(3900)$: Lattice QCD (Numerical Experiments)

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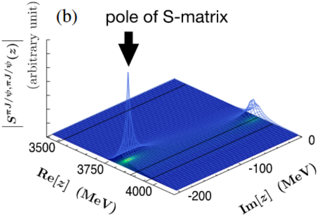


$\Rightarrow$ **Virtual state** is obtained.

- $Z_c(3900)$ is Threshold cusp

Ikeda, *et al.*,PRL**117**(2016)242001

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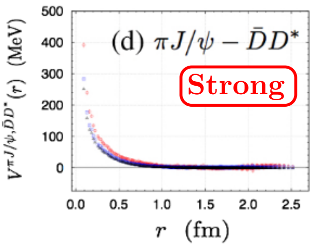



$\Rightarrow$ **Virtual state** is obtained.

- $Z_c(3900)$ is Threshold cusp induced by $\boldsymbol{\pi J/\psi - \bar{D}D^*}$ **potential**

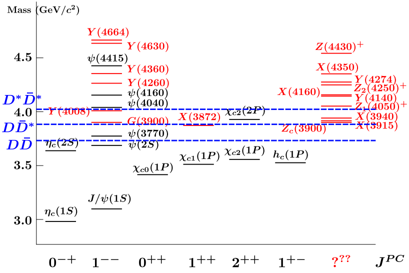
**Charm flavor exchange?**

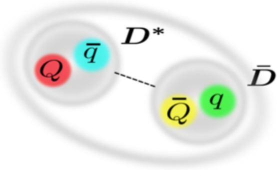
Ikeda, *et al.*,PRL**117**(2016)242001

**Exotic structure:**
**Bound state? Cusp?**

**Exotic structure: Bound state? Cusp?** ⇔ **Hadron-hadron interaction**

- **Hadron-hadron interaction** is important to understand the nature of exotic states! not only $Z_c$ but also others.



- **Long-range force:** one $\pi$ exchange potential (OPEP)
  Lightest meson $\pi$, Importance in the nuclear force,
  **Heavy Quark Spin Symmetry** ($0^- - 1^-$ mixing)

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▷ How can we understand **strong $\pi J/\psi - D\bar{D}^*$ potential?**

- **Long-range force:** one $\pi$ exchange potential (OPEP)
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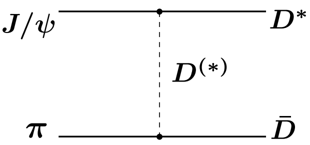
- **Short-range force:** Charm ($c$) exchange

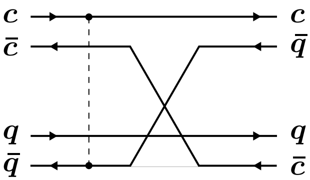
▷ How can we understand **strong $\pi J/\psi - D\bar{D}^*$ potential?**

(a) $D^{(*)}$ meson exchange?




→ next sec.

(b) Quark exchange?

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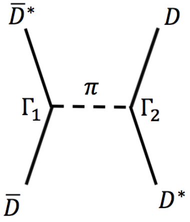
Comparison between **$D$ exchange** and **Quark exchange**

- One boson exchange potential (OBEP)




$D\bar{D}^*$ potential

$DD^*\pi$ vertex induces OPEP
($DD\pi$ vertex violates the parity conservation)

OPEP

$$V^\pi = -\frac{1}{2}\left(\frac{g_\pi}{f_\pi}\right)^2\left[\vec{S}_1\cdot\vec{S}_2 C(r) + S_{12}(\hat{r})T(r)\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

$$C(r) = m_\pi^2\left(\frac{e^{-m_\pi r}}{r} - \frac{e^{-\Lambda r}}{r} - \frac{\Lambda^2 - m_\pi^2}{2\Lambda}e^{-\Lambda r}\right)$$

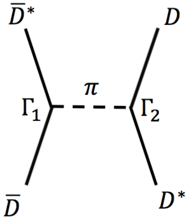
**Comments**

- HQS induces **$D(0^-) - D^*(1^-)$ coupling** $\to$ OPEP works!

# One pion exchange potential in $D^{(*)}\bar{D}^{(*)}$

## Meson exchange model

- One boson exchange potential (OBEP) **with Tensor force!**

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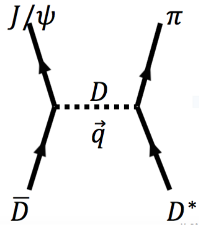
### Comments

- HQS induces $\boldsymbol{D(0^-) - D^*(1^-)}$ **coupling** $\rightarrow$ OPEP works!
- Tensor force $T(r) \Rightarrow$ **the driving force** in atomic nuclei
  $S_{12}(\hat{r}) = 3(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r}) - \vec{S}_1\cdot\vec{S}_2 \rightarrow S\text{-}D$ mixing

# Heavy meson exchange potential
## Meson exchange model

- $D^{(*)}$ meson exchange potential in $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$




$\pi J/\psi - D\bar{D}^*$ potential

**$D$ exchange**

$$V^D = \frac{2}{3}\frac{g_\psi g_\pi}{f_\pi\sqrt{E_\pi}}\left[\vec{S}_1\cdot\vec{S}_2 C(r) + S_{12}(\hat{r})T(r)\right]$$

**$D^*$ exchange**

$$V^{D^*} = \frac{2}{3}\frac{g_\psi g_\pi}{f_\pi\sqrt{E_\pi}}\left[2\vec{S}_1\cdot\vec{S}_2 C(r) - S_{12}(\hat{r})T(r)\right]$$

$g_\psi = 8$

A. Deandrea, G. Nardulli and A. D. Polosa, PRD**68**(2003)034002

### Comments

- $D^{(*)}$ meson exchange gives the $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$ potential.
  Hidden $\leftrightarrow$ Open-Open
- $D^{(*)}$ mass $\sim$ 2 GeV $\Leftrightarrow 1/m_{D^{(*)}} \sim 0.1$ fm
  **Does it work?**

# Numerical results: Phase shift
## Meson exchange model

- We found...

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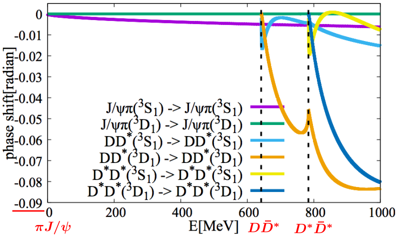
## Meson exchange model

- We found... **No Bound state, No Resonance**
  Very Small phse shift $|\delta| < 0.09$ [rad]




- $D^{(*)}\bar{D}^{(*)}$ channel: **Small** contribution from OPEP

- $\pi J/\psi$ channel: $D^{(*)}$ exchange is **Negligible**

# Numerical results: Phase shift

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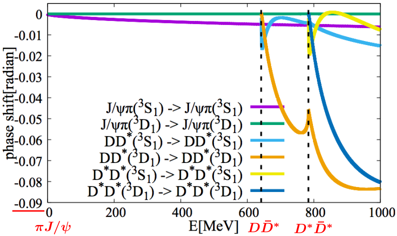



- $D^{(*)}\bar{D}^{(*)}$ channel: **Small** contribution from OPEP
  **Why?**: Isospin factor $\vec{\tau}_1 \cdot \vec{\tau}_2$, $\mathbf{-3}$ $\mathbf{(I = 0)}$, but $\mathbf{Z_c}$**:+1** $\mathbf{(I = 1)}$
- $\pi J/\psi$ channel: $D^{(*)}$ exchange is **Negligible**
  **Why?**: Suppression by the form factor (finite hadron size)

- No resonance $\leftarrow$ agreeing with Lattice QCD result

$\Leftrightarrow$ We cannot explain the strong $\pi J/\psi - D\bar{D}^*$ potential.

- No resonance ← agreeing with Lattice QCD result

⇔ We cannot explain the strong $\pi J/\psi - D\bar{D}^*$ potential.

**Problems in $D^{(*)}$ exchange potential**

- Too large mass, $1/m_{D^{(*)}} \sim 0.1$ fm
- In such short range region, "Hadron" is not good effective d.o.f. ?

⇓

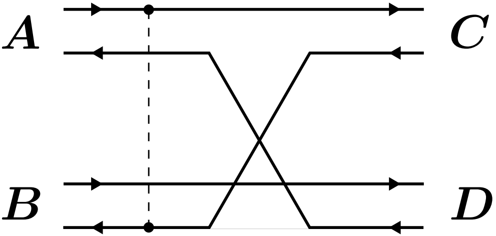
**Quark exchange interaction!**

→next section

- **Born-order quark-exchange diagram**

  T. Barnes and E. S. Swanson, PRD**46**(1992)131. Swanson, Ann. Phys. **220**(1992)73.

- $AB \to CD$ scattering $\mathcal{M}_{fi} \propto \langle C, D|H_I|A, B\rangle$

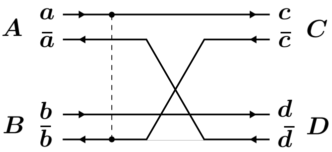



- Ingredients: Meson Wavefunctions($A, B, C, D$)
  Quark interaction (Quark Model)
- Born amplitude ⇒ Meson-meson Potential can be obtained

# Scattering Amplitude

## Quark exchange interaction

capture 1

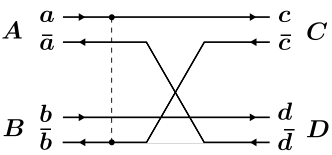



- Meson momenta: $A, B, C, D$
- Quark momenta: $a, \bar{a}, b, \bar{b}, c, \bar{c}, d, \bar{d}$
- Conservation: $A + B = C + D$, $\bar{a} = \bar{d}$, $b = d$

- Amplitude

$$\to \int \int d^3a d^3c \phi_C^*(2\vec{c} - \vec{C})\phi_D^*(2\vec{a} - 2\vec{A} - \vec{C})V(\vec{a} - \vec{c})\phi_A(2\vec{a} - \vec{A})\phi_B(2\vec{a} - \vec{A} - 2\vec{C})$$

# Scattering Amplitude
## Quark exchange interaction

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- Potentials (momentum space)

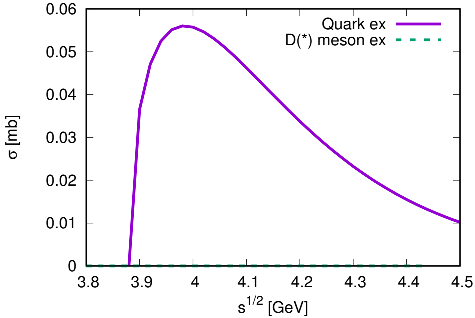
**Coulomb:** $V^{Coul}(q) = -\frac{\alpha_s}{2\pi^2}\frac{1}{\vec{q}^{\,2}}$, **Hyperfine:** $V^{Hyp}(q) = -\frac{8\pi\alpha_h}{3m_i m_j}e^{-\vec{q}^{\,2}/4\sigma^2}$

**Linear (Regularized):**

$$V^{Lin}(r) = br\times e^{-\varepsilon r} \to V^{Lin}(q) = b\left[\frac{-8\pi}{(\vec{q}^{\,2}+\varepsilon^2)^2} + \frac{32\pi\varepsilon^2}{(\vec{q}^{\,2}+\varepsilon^2)^3}\right]$$

- $\pi J/\psi - D\bar{D}^*$: Amplitude $\rightarrow$ Cross section

(i) Quark ex vs $D^{(*)}$ ex



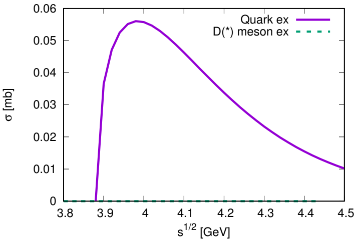

- Comparing results of Quark exchange and $D^{(*)}$ exchange
- Can you see a dashed line ($D^{(*)}$ exchange)?

# Cross Section: Quark exchange vs $D^{(*)}$ exchange

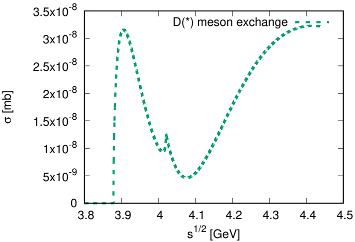
## Quark exchange interaction

- $\pi J/\psi - D\bar{D}^*$: Amplitude → Cross section

(i) Quark ex. vs $D^{(*)}$ ex.

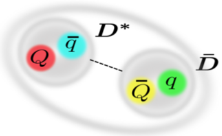


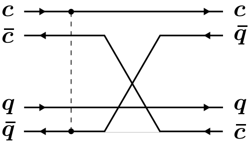

(ii) $D^{(*)}$ ex. (Zoom)




- Comparing results of Quark exchange and $D^{(*)}$ exchange
- Can you see a dashed line ($D^{(*)}$ exchange)? $<$ **$3.5\times10^{-8}$ mb**
- Large difference between Quark exchange and $D^{(*)}$ exchange!

# Summary

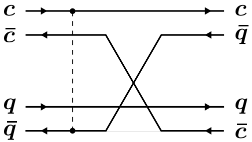



- Many exotic states near the threshold.
  $\rightarrow$ Understanding **the hadron-hadron interaction** is needed.
- Charged charmonium $Z_c(3900)$ has been discussed as the Hadronic molecules or the threshold cusp.
- OPEP contribution is not strong. $D^{(*)}$ meson exchange is **negligible**.
- Quark exchange interaction is introduced as Short range $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$ potential.
  We find **Large difference** between results from Quark exchange and $D^{(*)}$ meson exchange.

# Future Work

$\pi J/\psi - D^{(*)}\bar{D}^{(*)}$ potential




- Beyond Born-order $\rightarrow T = V + VGT$

  $\Rightarrow$ To compare the Lattice result

- Introducing $\rho\eta_c$, $\psi'\pi$,...

- Bottom Sector: $Z_b(10610)$ and $Z_b(10650) \Rightarrow \pi\Upsilon - B\bar{B}^*$

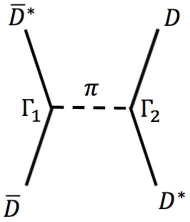
**Thank you for your kind attention.**

# Back Up

# One pion exchange potential in $D^{(*)}\bar{D}^{(*)}$

## Meson exchange model

- One boson exchange potential (OBEP)




**OPEP**

$$V^{\pi} = -\frac{1}{2}\left(\frac{g_\pi}{f_\pi}\right)^2 \left[\vec{S}_1 \cdot \vec{S}_2 C(r) + S_{12}(\hat{r}) T(r)\right] \vec{\tau}_1 \cdot \vec{\tau}_2$$

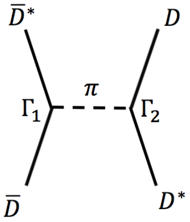
**Vector $(\rho, \omega)$ exchange**

$$V^{v} = -\left(\frac{\lambda g_V}{\sqrt{3}}\right)^2 \left[2\vec{S}_1 \cdot \vec{S}_2 C(r) - S_{12}(\hat{r}) T(r)\right] \vec{\tau}_1 \cdot \vec{\tau}_2$$

### Comments

- Tensor force $T(r) \Rightarrow$ **the driving force** in atomic nuclei
  $S_{12}(\hat{r}) = 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) - \vec{S}_1 \cdot \vec{S}_2 \rightarrow S\text{-}D$ mixing

- One boson exchange potential (OBEP) **with Tensor force!**




**OPEP**

$$V^\pi = -\frac{1}{2}\left(\frac{g_\pi}{f_\pi}\right)^2 \left[\vec{S}_1 \cdot \vec{S}_2 C(r) + \mathbf{S_{12}(\hat{r})T(r)}\right] \vec{\tau}_1 \cdot \vec{\tau}_2$$

**Vector $(\rho, \omega)$ exchange**

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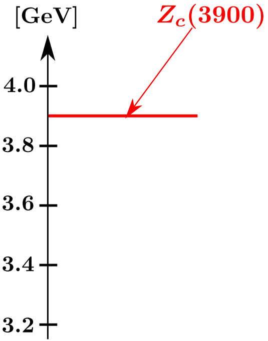
### Comments

- Tensor force $T(r) \Rightarrow$ **the driving force** in atomic nuclei
  $S_{12}(\hat{r}) = 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) - \vec{S}_1 \cdot \vec{S}_2 \to S$-$D$ mixing
- $G-$parity of vector mesons: $\rho$ $(G=-1)$, $\omega$ $(G=+1)$
  $\Rightarrow$ **Working against each other**, $\rho + \omega$ has a minor role...

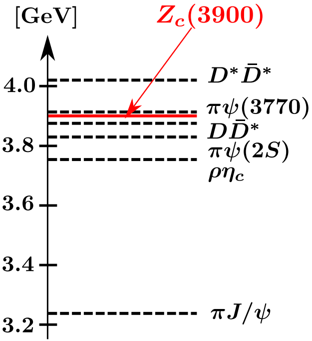
# Meson-meson thresholds
## Model Setup

- Meson-meson thresholds,



# Meson-meson thresholds
## Model Setup

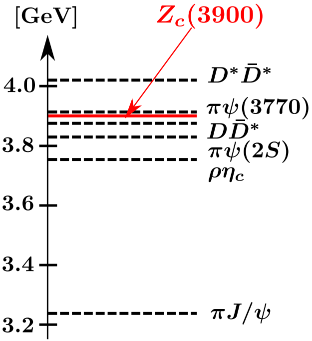
- Meson-meson thresholds, $\pi J/\psi, \rho\eta_c, \pi\psi(2S), D\bar{D}^*, D^*\bar{D}^*, \ldots$




Coupled-Channels

$$\left\{ \begin{array}{c} \pi J/\psi \\ \pi\psi(2S) \\ \rho\eta_c \\ D\bar{D}^* \\ D^*\bar{D}^* \\ \vdots \end{array} \right\} - \left\{ \begin{array}{c} \pi J/\psi \\ \pi\psi(2S) \\ \rho\eta_c \\ D\bar{D}^* \\ D^*\bar{D}^* \\ \vdots \end{array} \right\}$$

- Meson-meson thresholds, $\pi J/\psi, \rho\eta_c, \pi\psi(2S), D\bar{D}^*, D^*\bar{D}^*, \ldots$




Coupled-Channels

$$\begin{Bmatrix} \pi J/\psi \\ \pi\psi(2S) \\ \rho\eta_c \\ D\bar{D}^* \\ D^*\bar{D}^* \\ \vdots \end{Bmatrix} - \begin{Bmatrix} \pi J/\psi \\ \pi\psi(2S) \\ \rho\eta_c \\ D\bar{D}^* \\ D^*\bar{D}^* \\ \vdots \end{Bmatrix}$$

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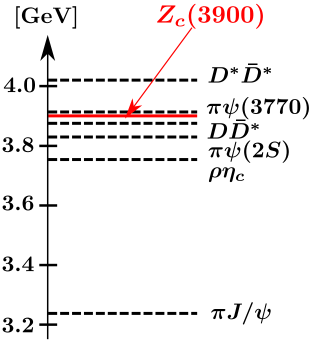
- Born-order quark-exchange
  $\Rightarrow$ Applicable to charm exchange (**Hidden**$\leftrightarrow$**Open-Open**)

# Meson-meson thresholds
## Model Setup

- Meson-meson thresholds, $\pi J/\psi, \rho\eta_c, \pi\psi(2S), D\bar{D}^*, D^*\bar{D}^*, \ldots$




Coupled-Channels

$$\begin{Bmatrix} \pi J/\psi \\ \pi\psi(2S) \\ \rho\eta_c \\ D\bar{D}^* \\ D^*\bar{D}^* \\ \vdots \end{Bmatrix} - \begin{Bmatrix} \pi J/\psi \\ \pi\psi(2S) \\ \rho\eta_c \\ D\bar{D}^* \\ D^*\bar{D}^* \\ \vdots \end{Bmatrix}$$

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- Born-order quark-exchange
  $\Rightarrow$ Applicable to charm exchange (**Hidden**$\leftrightarrow$**Open-Open**)

**Today**: $\pi J/\psi - D\bar{D}^*$ and $\pi J/\psi - D^*\bar{D}^*$ ($S$-wave)

- Quark Hamiltonian (One gluon exchange + Linear potentials)

Barnes and Swanson, PRD**46**(1992)131.; Swanson, Ann. Phys. **220**(1992)73.

$$H_{ij}^{q} = K_q + \left(-\frac{3}{4}br + \frac{\alpha_s}{r} - C\right)\vec{F}_i\cdot\vec{F}_j - \frac{8\pi\alpha_h}{3m_i m_j}\left(\frac{\sigma^3}{\pi^{3/2}}e^{-\sigma^2 r_{ij}^2}\right)\vec{S}_i\cdot\vec{S}_j\vec{F}_i\cdot\vec{F}_j$$

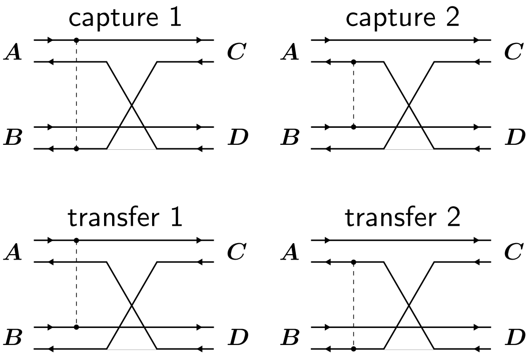
▷ Parameters are fixed to reproduce the mass of mesons

Table: Quark Model Parameters from Ann.Phys.**220**(1992)73.

| | |
|---|---|
| $m_q = 0.375$ GeV | $m_c = 1.9$ GeV |
| $\alpha_s = 0.857$ | $\alpha_h = 0.840$ |
| $b = 0.154$ GeV$^{-2}$ | $C = -0.4358$ GeV |
| $\sigma = 0.70$ GeV | |

# Scattering Amplitude
## Model Setup

- Born quark exchange diagrams T. Barnes and E. S. Swanson, PRD**46**, 131 (1992).
  Quark interaction between Mesons ⇒ Four diagrams




- Scattering Amplitude $\mathcal{M}_{fi} \propto \langle C, D | H^q | A, B \rangle$
  $\mathcal{M}_{fi}^{tot} = \mathcal{M}_{fi}^{capture1} + \mathcal{M}_{fi}^{capture2} + \mathcal{M}_{fi}^{transfer1} + \mathcal{M}_{fi}^{transfer2}$