RELATIVISTIC INVESTIGATION OF THE TRITON IN THE BETHE-SALPETER-FADDEEV APPROACH

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object

Three-nucleon bound states systems:

^{3}He(ppn)   T(^{3}H)(nnp)

goal

The study of these systems at **relativistic** energies

The study of the influence of *P* and *D* states on the **binding energy** and **form factors**

method of research

nonrelativistic case

relativistic case

2 particles

Lippmann-Schwinger equation

Bethe-Salpeter equation

$$p = (p_1, p_2)$$

$$t(p,p') = V(p,p') + \int dp'' V(p,p'')G(p'')t(p'',p')$$

3 particles

Faddeev equation

Relativistic Faddeev equation

Bethe-Salpeter-Faddeev

$$p = (p_1, p_2, p_3)$$

$$T^{(i)}(p,p') = t^{(i)}(p,p') + \int dp'' t^{(i)}(p,p'')G(p'')[T^{(j)}(p'',p') + T^{(k)}(p'',p')]$$

Method

Relativistic Faddeev equation

Two-particle t-matrix

$$T^{(i)}(p_i, q_i; p'_i, q'_i; P) = t^{(i)}(p_i, q_i; p'_i, q'_i; P) +$$

$$\int dp''_i dq''_i t^{(i)}(p_i, q_i; p'_i, q'_i; P) G(p'', q'', P) \times$$

$$\times [T^{(j)}(p''_j, q''_j; p'_j, q'_j; P) + T^{(k)}(p''_k, q''_k; p'_k, q'_k; P)]$$

Components of the full three-particle t matrix T = $T^1 + T^2 + T^3$

Two-particle propagator

$$G_i = (k_j^2 - m_n^2 + i\epsilon)^{-1}(k_k^2 - m_n^2 + i\epsilon)^{-1}$$

The Bethe-Salpeter equation

Equation for the relativistic system of two particles

$$p = \frac{1}{2}(k_2 - k_1)$$

$$P = k_2 + k_1$$

Two-particle t-matrix

$$t(p;p';P) = V(p;p';P) + \int dp'' V(p;p'';P) G(p'',P) t(p'';p';P)$$

Potential of NN interaction

Two-particle propagator

Separable potential of nucleon-nucleon interaction

$$\boxed{V(p,p^{'}) = \sum_{ij=1}^{N} \lambda_{ij} g_i(p) g_j(p^{'})}$$

g - form factor of potential

N - rank of potential

Yamaguchi functions for form factor of potential

S state

$$g_Y^{[S]}(p_0,p) = \frac{1}{-p_0^2 + p^2 + \beta_0^2 - i\epsilon}$$

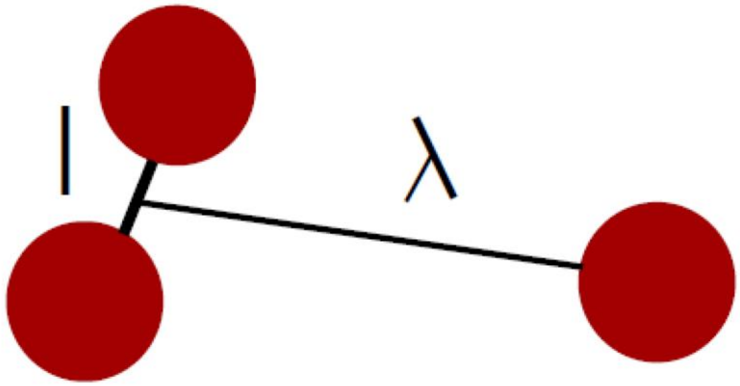
P state

$$g_Y^{[P]}(p_0,p) = \frac{\sqrt{|-p_0^2 + p^2|}}{(-p_0^2 + p^2 + \beta_1^2 - i\epsilon)^2}$$

D state

$$g_Y^{[D]}(p_0,p) = \frac{C(-p_0^2 + p^2)}{(-p_0^2 + p^2 + \beta_2^2 - i\epsilon)^2}$$

Partial-wave decomposition



$$\Psi(\mathbf{p},\mathbf{q};s) = \sum_{l\lambda LM} \Psi_{l\lambda L}(p,q;s)\mathcal{Y}_{l\lambda LM}(\mathbf{p},\mathbf{q})$$

где

$$\mathcal{Y}_{l\lambda LM}(\mathbf{p},\mathbf{q}) = \sum_{m\mu} Y_{lm}(\mathbf{p})Y_{\lambda\mu}(\mathbf{q})$$

$$t(\mathbf{p},\mathbf{p}') = \sum_{lm} t_l(p,p')Y_{lm}(\mathbf{p})Y_{lm}(\mathbf{p}')$$

The system of integral equations for a three-body system

$$t = V + \int VGt$$

$$V(p, p') = \sum_{ij=1}^{N} \lambda_{ij} g_i(p) g_j(p')$$

$$t(p, p') = \sum_{ij=1}^{N} \tau_{ij}(s) g_i(p) g_j(p')$$

Bethe-Salpeter-Faddeev equation

Не удается отобразить рисунок.

after partial-wave decomposition

$$\Psi(p, q; s) = \sum_{ij=1}^{N} g_i(p) \tau_{ij}(s) \Phi(q)$$

The system of integral equations for Φ

$$\boxed{\Phi^a_{jl\lambda L}(q_0,q) = -\frac{1}{4\pi^3}\sum_b\sum_{kn}\sum_{l'\lambda'}\int_{-\infty}^{\infty} dq_0^{'}\int_0^{\infty} q^{'2}dq^{'}\times \\ Z^{ab}_{jkl\lambda l'\lambda' L}(iq_0,q;iq_0^{'},q^{'};s)\frac{\tau^b_{knl'\lambda'}[(\frac{2}{3}\sqrt{s}+iq_0^{'})^2-q^{'2}]}{(\frac{1}{3}\sqrt{s}-iq_0^{'})^2-q^{'2}-m^2}\Phi^b_{jl'\lambda' L}(q_0^{'},q^{'})}$$

$$Z^{ab}_{jkl\lambda l'\lambda' L}(iq_0,q;iq_0^{'},q^{'};s) = \Delta^a_l\Delta^b_{l'}\underline{C^{ab}}\int_{-1}^{1}dx\,\underline{K^L_{l\lambda l'\lambda'}}(q,q^{'},x)\times$$

$$\frac{g^a_{jl}(-\frac{1}{2}q_0-q_0^{'},\sqrt{\frac{1}{4}q^2+q^{'2}+qq^{'}x})\,g^b_{kl'}(q_0+\frac{1}{2}q_0^{'},\sqrt{q^2+\frac{1}{4}q^{'2}+qq^{'}x}}{(\frac{1}{3}\sqrt{s}+q_0+q_0^{'})^2-(q^2+q^{'2}+2qq^{'}x)-m^2}$$

a,b = $^{2S+1}L_J$ $\qquad$ $^1S_0,\ ^3S_1,\ ^3D_1,\ ^3P_0,\ ^1P_1,\ ^3P_1$

Spin-isospin structure of the system

C^{ab} a = (S,I) (S,I) = {(1,0);(1,1);(0,1);(0,0)}

$$C^{ab} = C^{(s_A,i_A)(s_B,i_B)} =$$

$$=< (s_1 s_2)s_A, s_3, S|(s_2 s_3)s_B, s_1, S >< (i_1 i_2)i_A, i_3, I|(i_2 i_3)i_B, i_1, I >$$

$$C^{ab} = \frac{1}{4}\begin{pmatrix} 1 & \sqrt{3} & -3 & -\sqrt{3} \\ \sqrt{3} & -1 & \sqrt{3} & -3 \\ -3 & \sqrt{3} & 1 & -\sqrt{3} \\ -\sqrt{3} & -3 & -\sqrt{3} & -1 \end{pmatrix}$$

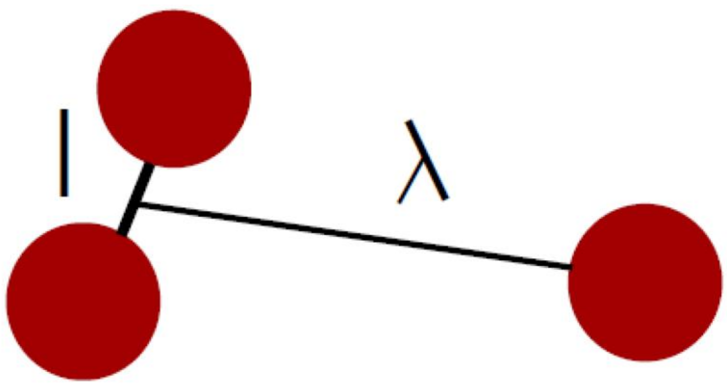
	$^1S_0$	$^3S_1$	$^3D_1$	$^3P_0$	$^1P_1$	$^3P_1$
S	0	1	1	1	0	1
I	1	0	0	1	0	1

The influence of the orbital angular momentum

$$K^{(aa')}_{\lambda\lambda' L}(q, q', \cos\vartheta_{qq'}) = (4\pi)^{3/2} \frac{\sqrt{2\lambda+1}}{2L+1}$$

$$\sum_{mm'} C^{Lm}_{lm\lambda 0} C^{Lm}_{l'm'\lambda' m-m'} Y^*_{lm}(\vartheta, 0) Y_{l'm'}(\vartheta', 0) Y_{\lambda' m-m'}(\vartheta_{qq'}, 0)$$

$$\cos\vartheta = (\frac{q}{2} + q'\cos\vartheta_{qq'})/|\frac{\mathbf{q}}{2} + \mathbf{q}'|, \qquad \cos\vartheta' = (q + \frac{q'}{2}\cos\vartheta_{qq'})/|\mathbf{q} + \frac{\mathbf{q}'}{2}|$$



Iterative method for solving integral equations

$$f(x) = \int_a^b K(x,y,s) f(y) dy$$

$$f_0(x) = 1$$

$$f_1(x) = \int_a^b K(x,y,s) f_0(y) dy$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$f_i(x) = \int_a^b K(x,y,s) f_{i-1}(y) dy$$

bound-state condition

$$\lim_{i\to\infty} \frac{f_i(x,s)}{f_{i+1}(x,s)} = 1$$

$$\sqrt{s} = 3m - E_{bs}$$

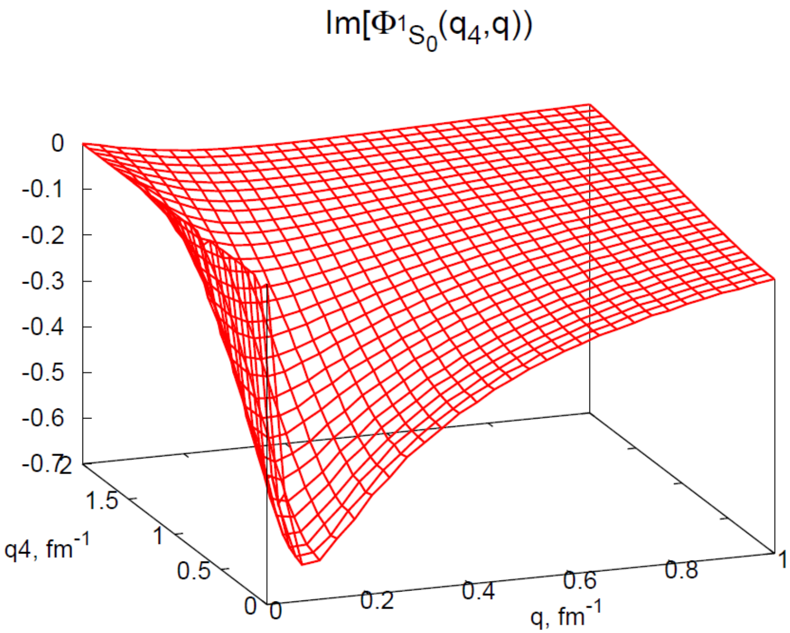
The binding energy of the triton **T (nnp)** in the case of the Yamaguchi potential

Exp.: 8.48 MeV

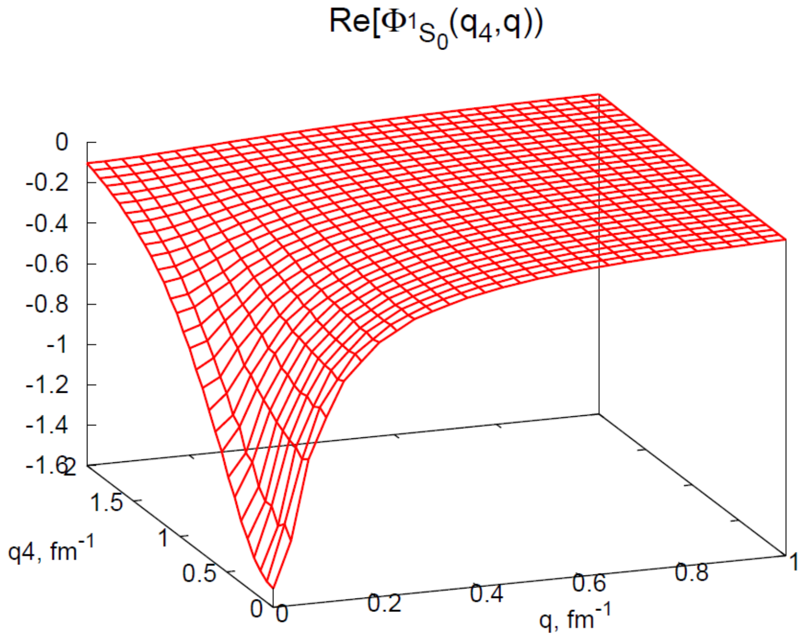
$\mathrm{p_D}$	${}^1S_0 - {}^3S_1$	${}^3D_1$	${}^3P_0$	${}^1P_1$	${}^3P_1$	
4	**9.221**	**9.294**	**9.314**	**9.287**	**9.271**	
	0	0.073	0.020	-0.027	-0.016	0.050
5	**8.819**	**8.909**	**8.928**	**8.903**	**8.889**	
	0	0.090	0.019	-0.025	-0.014	0.070
6	**8.442**	**8.545**	**8.562**	**8.540**	**8.527**	
	0	0.103	0.017	-0.022	-0.013	0.085

1S0 amplitude

Imaginary part

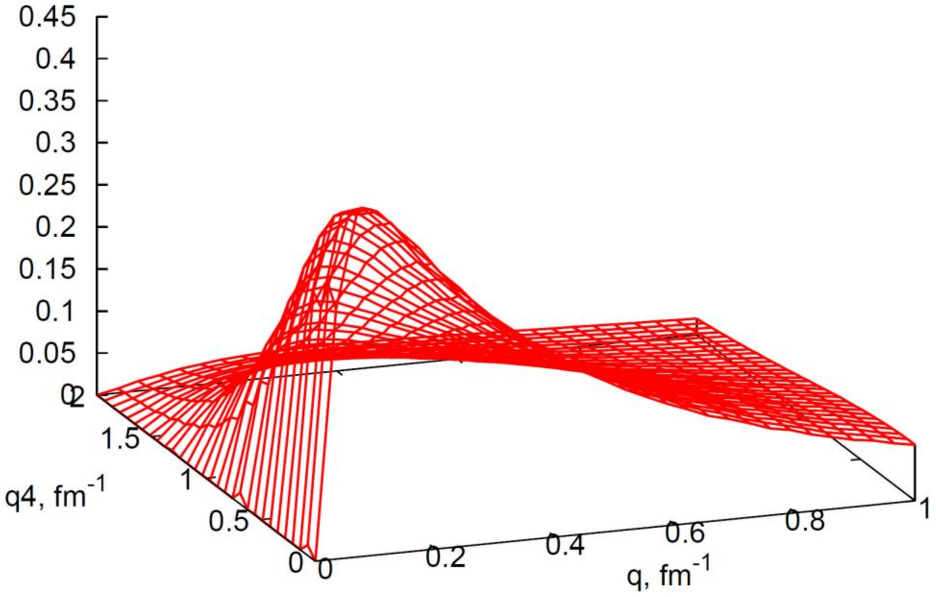


Real part

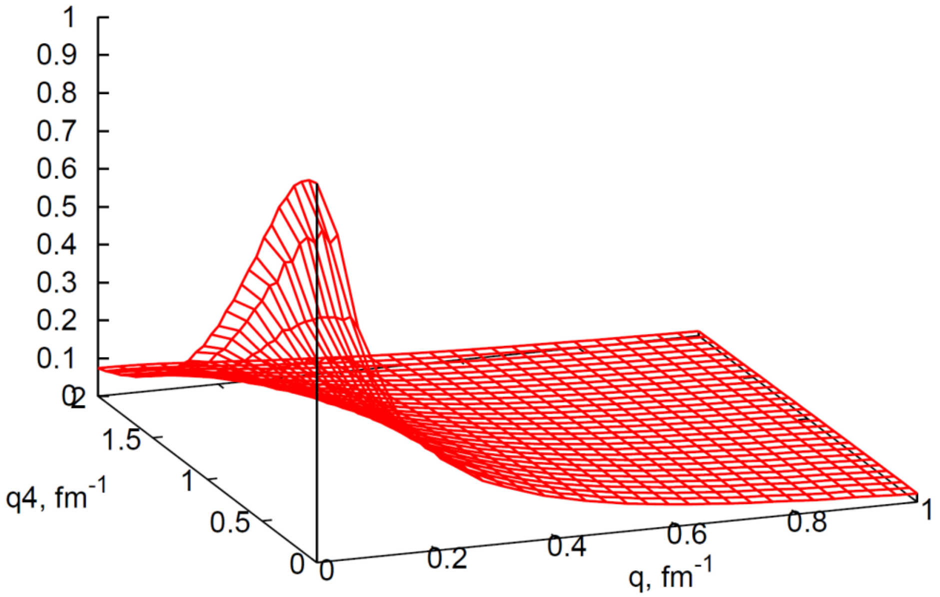


3S1 amplitude

Imaginary part

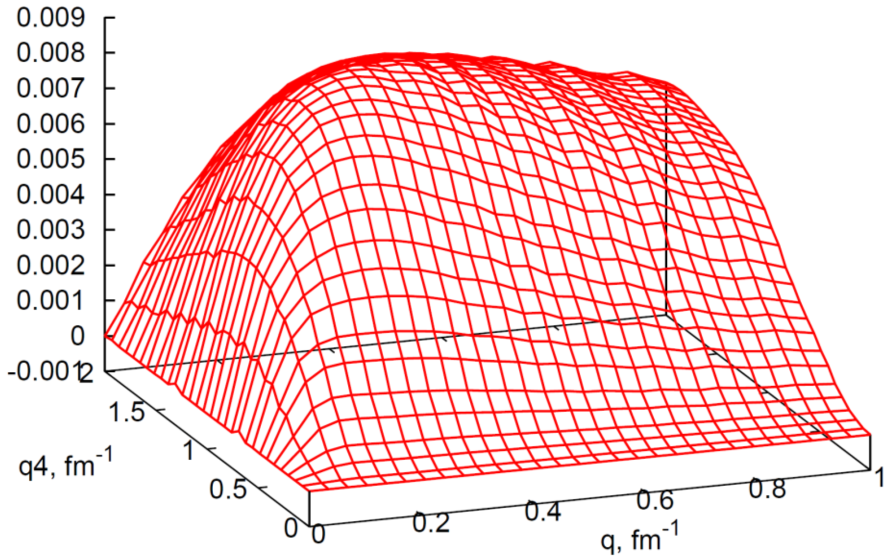


Real part

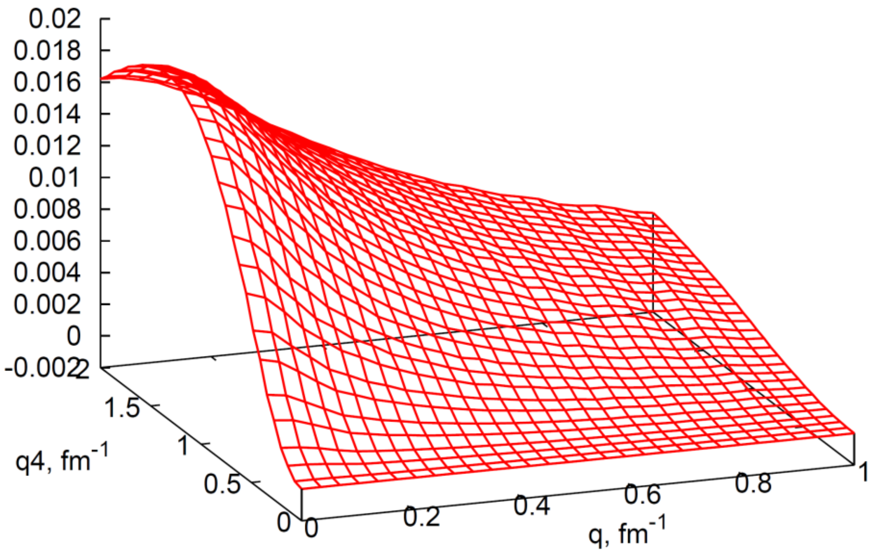


3D1 amplitude

Imaginary part

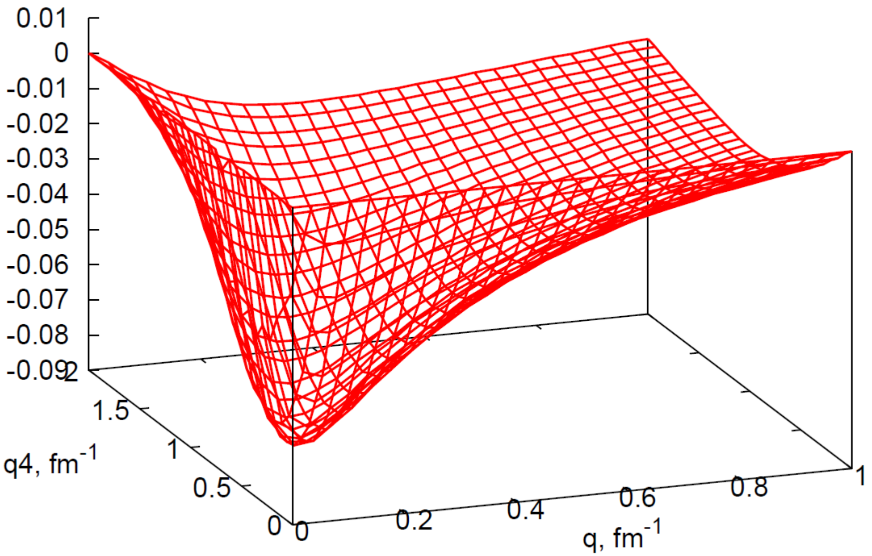


Real part

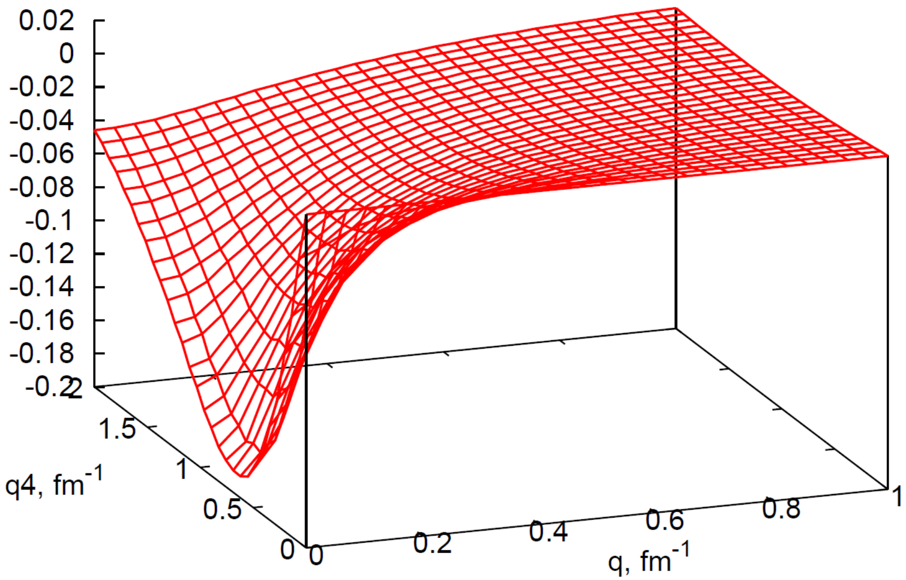


3P0 amplitude

Imaginary part

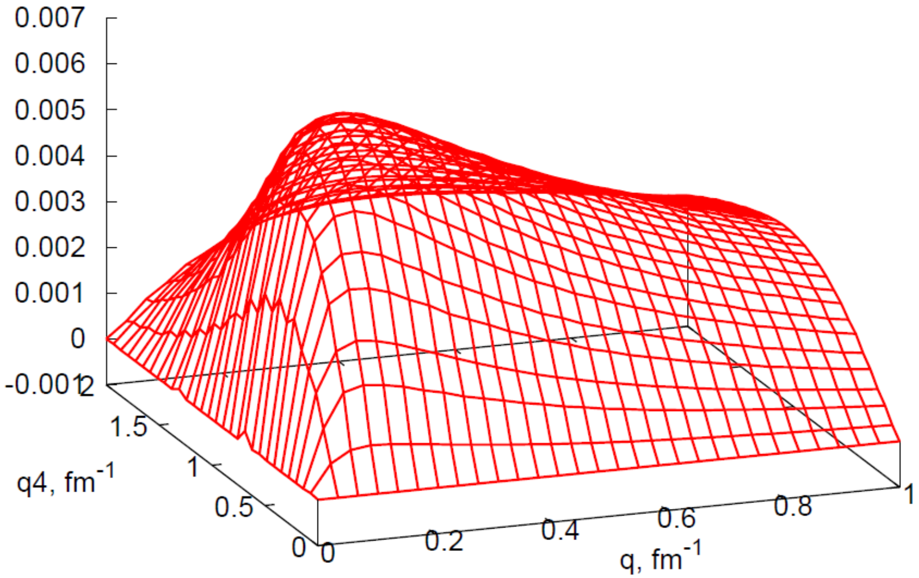


Real part

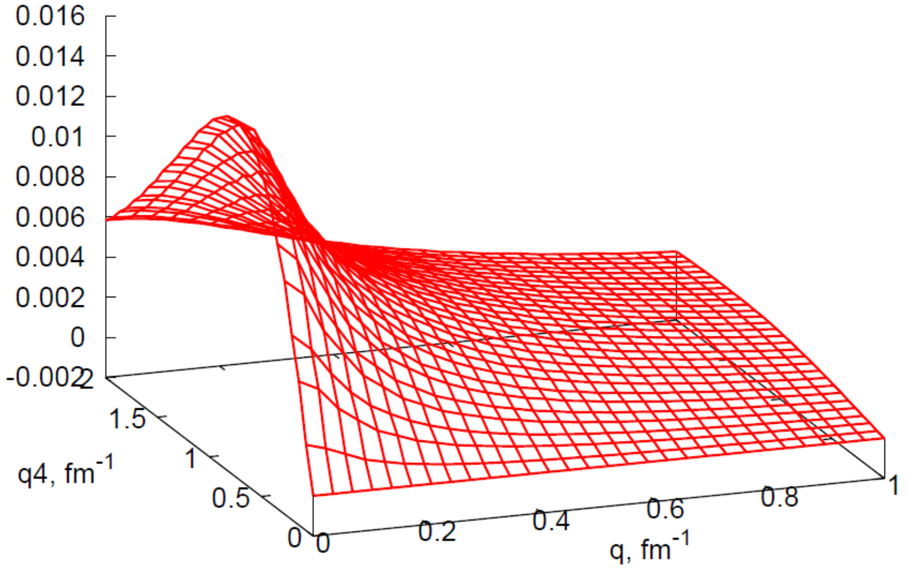


1P1 amplitude

Imaginary part

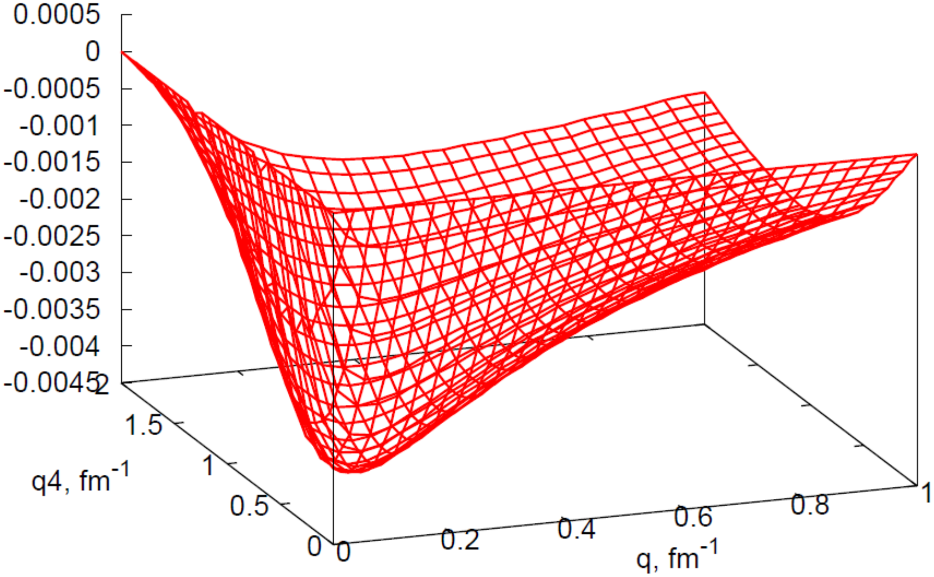


Real part

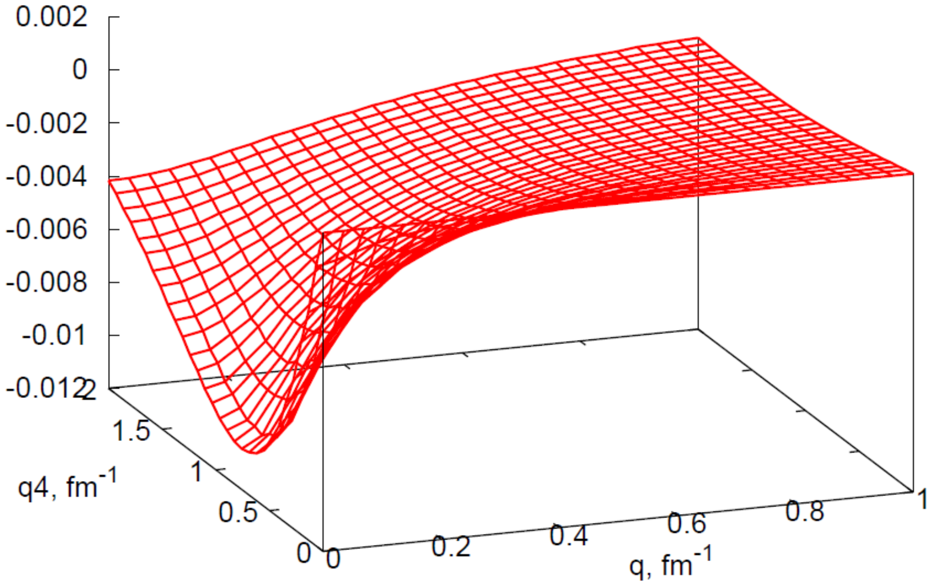


3P1 amplitude

Imaginary part



Real part



Form factors of the three-nucleon nuclei

$$2F_{ch}(^3He) = (2F^p_{ch} + F^n_{ch})F_1 - \frac{2}{3}(F^p_{ch} - F^n_{ch})F_2$$

$$F_{ch}(^3H) = (F^p_{ch} + 2F^n_{ch})F_1 + \frac{2}{3}(F^p_{ch} - F^n_{ch})F_2$$

$$\mu(^3He)F_{mag}(^3He) = G^n F_1 + \frac{2}{3}(G^n + G^p)F_2$$

$$\mu(^3H)F_{mag}(^3H) = G^p F_1 + \frac{2}{3}(G^n + G^p)F_2$$

$F^p_{ch}, F^n_{ch}, G^p, G^n$ electric and magnetic Form Factors of proton and neutron

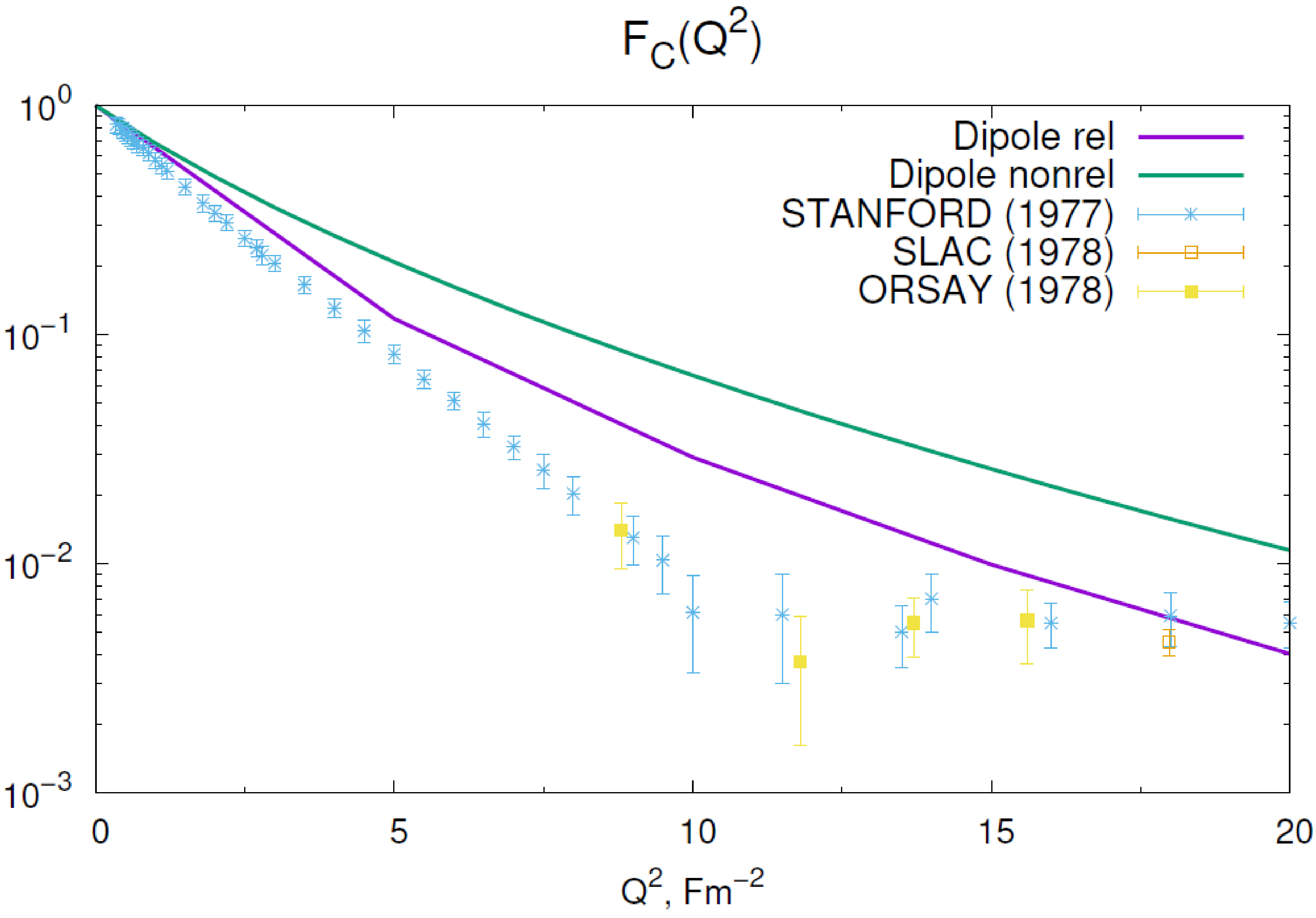
$F_1 \ F_2$ $\longleftarrow$ from amplitudes of states Φ(solution of BSF equation)

Form factors of the three-nucleus nuclei

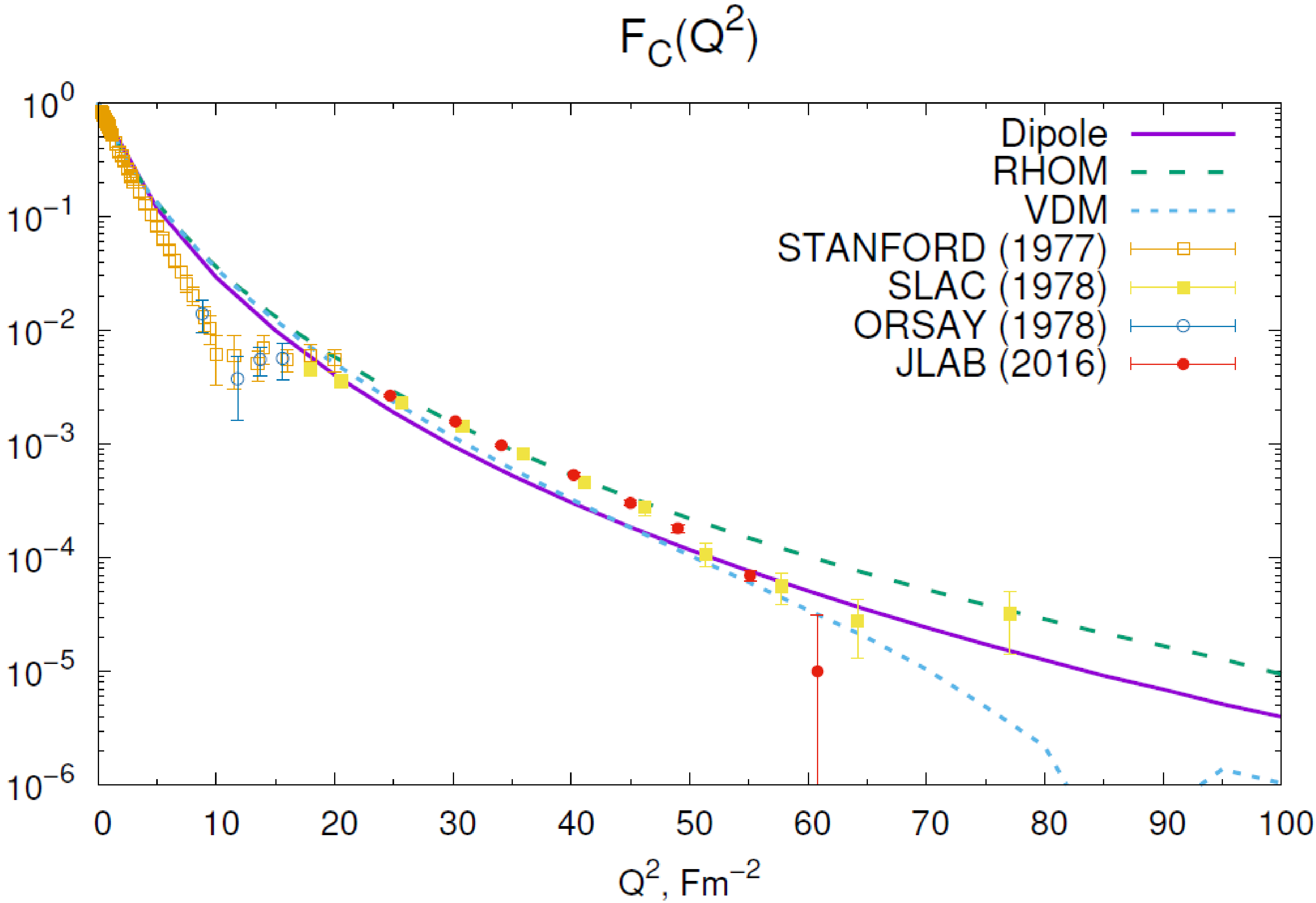
$$F_1(Q) = \int dp_4 \int d\mathbf{p} \int dq_4 \int d\mathbf{q} G_1 G_2 G_3 G_3' \Psi_S^*(p_4, p, q_4, q) \Psi_S(p_4, p, q_4, |\mathbf{q} - \frac{2}{3}\mathbf{Q}|)$$
$$= 4\pi^2 \int dp_4 \int dp \int dq_4 \int dq \int_{-1}^{1} d[Cos(\mathbf{p}, \mathbf{q})] \int_{-1}^{1} d[Cos(\mathbf{q}, \mathbf{Q})] p^2 q^2$$
$$G_1 G_2 G_3 G_3' \Psi_S^*(p_4, p, q_4, q) \Psi_S(p_4, p, q_4, |\mathbf{q} - \frac{2}{3}\mathbf{Q}|)$$

$$F_2(Q) = -3 \int dp_4 \int d\mathbf{p} \int dq_4 \int d\mathbf{q} G_1 G_2 G_3 G_3' \Psi_S^*(p_4, p, q_4, q) \Psi_{S'}(p_4, p, q_4, |\mathbf{q} - \frac{2}{3}\mathbf{Q}|)$$
$$= -12\pi^2 \int dp_4 \int dp \int dq_4 \int dq \int_{-1}^{1} d[Cos(\mathbf{q}, \mathbf{Q})] p^2 q^2$$
$$C G_3 G_3' \Psi_S^*(p_4, p, q_4, q) \Psi_{S'}(p_4, p, q_4, \sqrt{q^2 + \frac{4}{9}Q^2 - \frac{4}{3} q Q Cos(\mathbf{q}, \mathbf{Q})})$$

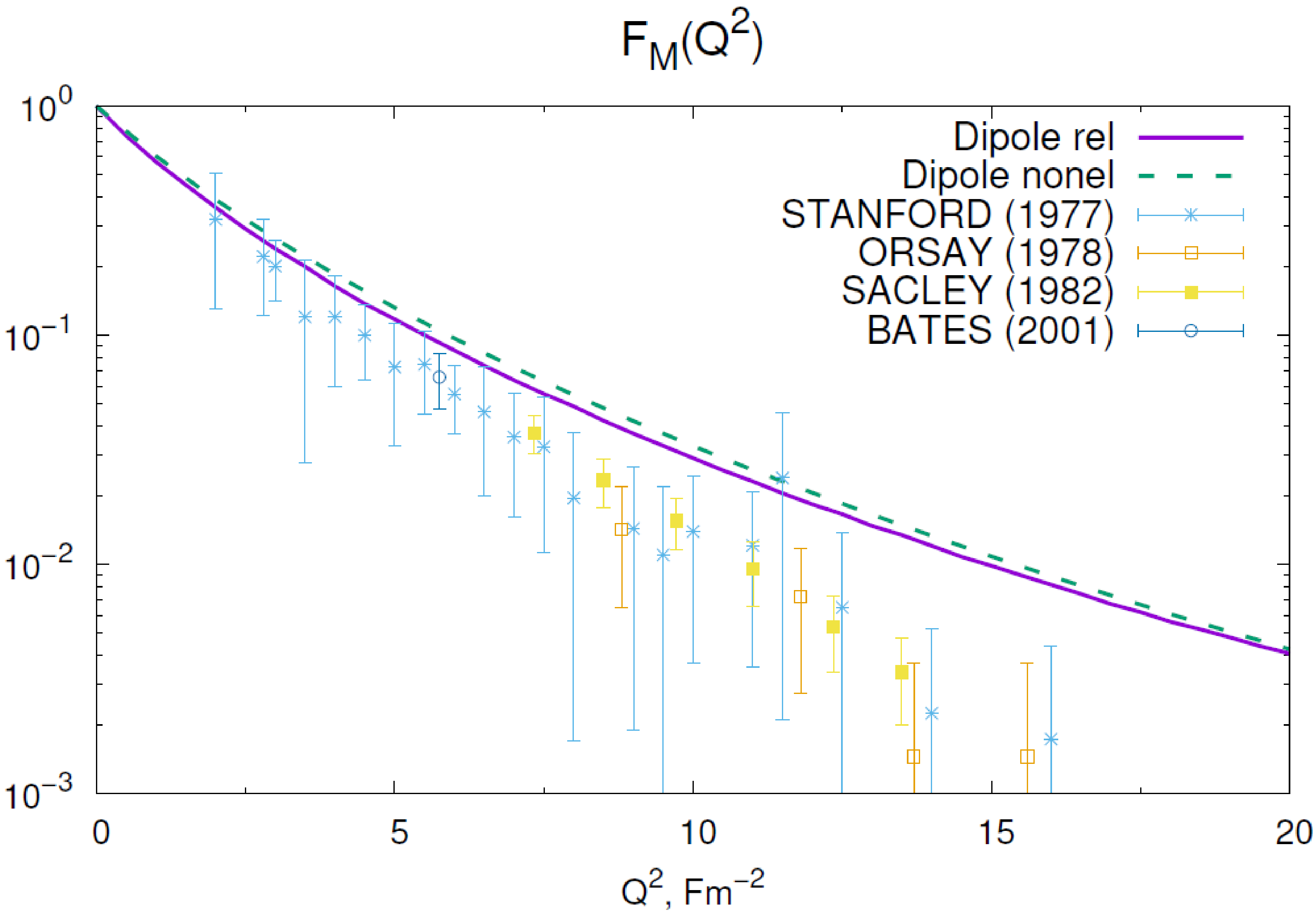
Charge form factors of 3He



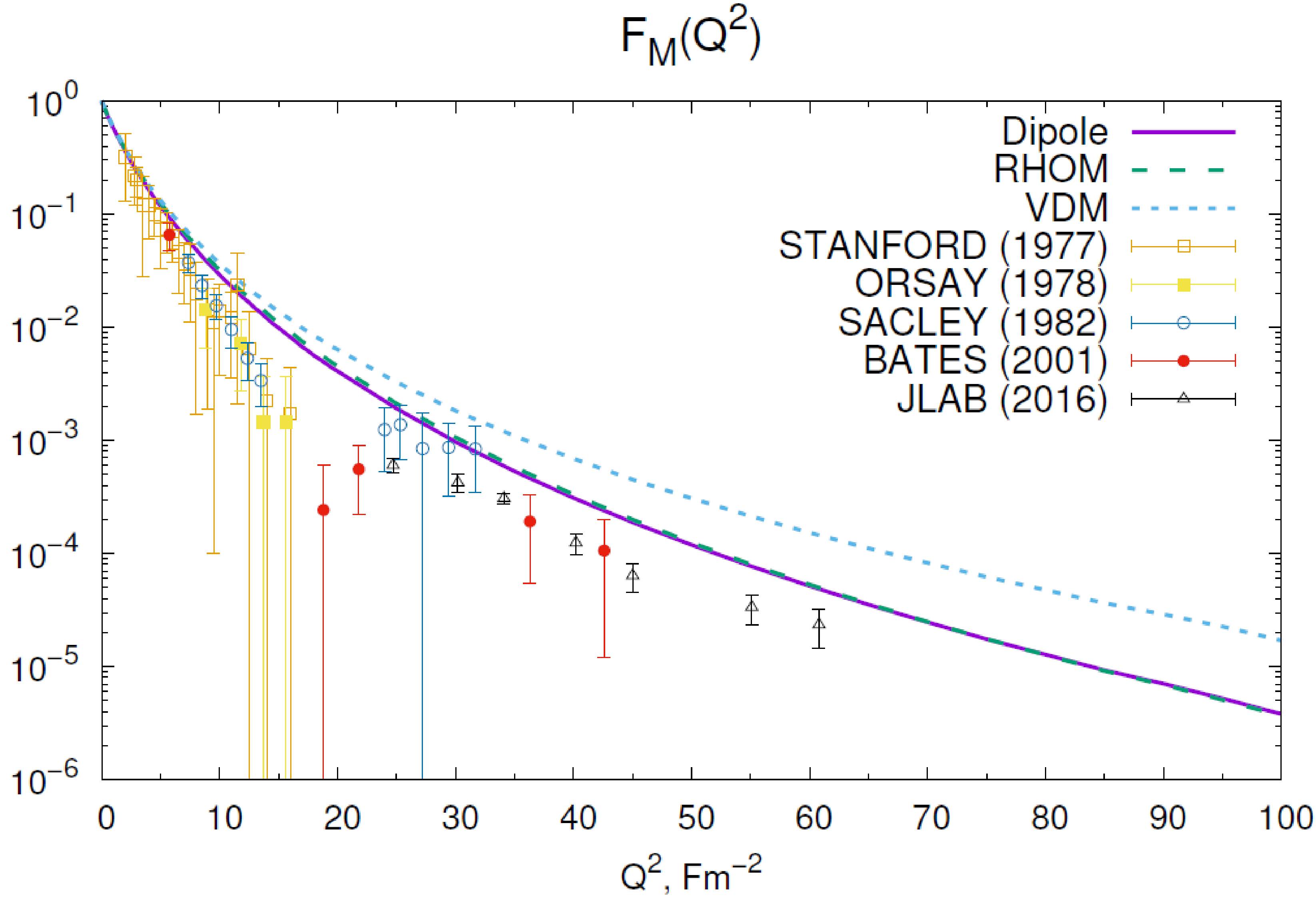
Charge form factors of 3He



Magnetic form factors of 3He



Magnetic form factors of 3He



Summary

[T = 3H = (nnp)] and [3He = (npp)] were investigated.

For this, a relativistic generalization of the Faddeev equation was applied.

As a two-particle t matrix, we used the solution of the Bethe-Salpeter equation.

The potential of NN interaction is taken in a separable form.

The system of integral equations describing T was solved by the iterative method.

The binding energy and the amplitudes of its S, P, and D states were calculated. Amplitudes used to calculation charge and magnetic formfactors of the three-nucleus nuclei.

Bound state energy of Triton

Experiment: $E_{bs} = 8.48 MeV$

relativistic

Potential	only S -state	with D
GRAZ-II(1)	8.716	8.716
GRAZ-II(2)	8.298	8.298
GRAZ-II(3)	7.894	7.894
Paris-I	7.545	7.545

nonrelativistic

Potential	only S -state	with D
GRAZ-II(1)	8.372	8.334
GRAZ-II(2)	7.964	7.934
GRAZ-II(3)	7.569	7.548

Goal: relativictic, l ≠0

Object: H-3, He-3

Method: relativictic **Faddeev** equation

$$T_i = t + \int tG(T_j + T_k)$$

Bethe-Salpeter equation

$$t = V + \int VGt$$

Separabel potential

$$V(p,p^{'}) = \sum_{ij=1}^{N} \lambda_{ij} g_i(p) g_j(p^{'})$$

Partial wave decomposition

Account **spin-isospin** structure

Account **angulat momentum**

Integral equation for amplitudes of states

$$^1S_0,\ ^3S_1,\ ^3D_1,\ ^3P_0,\ ^1P_1,\ ^3P_1$$

Solution by **iteration** method

Bound state energy

Amplitudes

Form factors of He-3 and H-3

Comparison with NR case
Contribution P and D states