Tensor polarizability of the ρ meson

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Based on:

1) E.L. Bratkovskaya, O.V. Teryaev, V.D. Toneev, Phys. Lett. B 348, 283 (1995)
2) P.V. Buividovich, M.N. Chernodub, D.E. Kharzeev, T.K. Kalaydzhyan, E.V. Luschevskaya, M.I. Polikarpov, Phys. Rev. Lett.105 (2010), arXiv:1003.2180
3) P.V.Buividovich, M.I.Polikarpov, O.V. Teryaev, Lect.Notes Phys. Vol. 871, pp377-385, arXiv: 1211.3014.
4) E.V. Luschevskaya, O.E. Solovjeva, O.A. Kochetkov, O.V. Teryaev, arXiv:1411.4284, Nucl. Phys. B898 (2015) 627- 643, DOI: 10.1016/j.nuclphysb.2015.07.023.
5) E.V. Luschevskaya, O.E. Solovjeva, O.A. Kochetkov, O.V. Teryaev, JHEP, Issue 9 (2017) 142, DOI: 10.1007/JHEP09(2017)142.
6) M.A.Andreichikov, B.O.Kerbikov, E.V.Luschevskaya, Y.A.Simonov, O.E.Solovjeva, JHEP, Issue 5 (2017) 007, DOI: 10.1007/JHEP05(2017)007.

Motivation

The media changes the properties of hadrons including the magnetic field which we investigate.
Magnetic polarizability and hyperpolarizabilities

- are the fundamental quantities describing spin interactions of quarks and the ability to form instantaneous dipoles;
- describe the distribution of quark currents inside a meson in an external field;
- were measured on the lattice and calculated in theoretical models.

  The external magnetic field of hadronic scale can be used as the probe of QCD properties.

Introduction

In lattice quantum chromodynamics we calculate

- the ground state energies of the vector $\rho^{\pm}$ and ρ^0 mesons versus the magnetic field value,
- their dipole polarizabilities and magnetic hyperpolarizabilities ,
- determine the polarization of dileptons which prevails in the collisions.

Motivation

Estimate the polarization of the emitted particles and dilepton asymmetries

- The differential cross section for dilepton production:

$$\frac{d\sigma}{dM^2 d\cos\theta} = A(1 + B\cos^2\theta)$$

- $$B \sim \frac{E_{s_z=+1} + E_{s_z=-1} - 2E_{s_z=0}}{E_{s_z=+1} + E_{s_z=-1} + E_{s_z=0}}$$

Technique for energy calculation

Calculate the propagators:

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m_q}.$$

Calculate the correlation functions of $\rho^\pm$ on the lattice:

$$\langle\bar{\psi}_{d,u}(x)\gamma_i\psi_{u,d}(x)\bar{\psi}_{u,d}(y)\gamma_j\psi_{d,u}(y)\rangle_A = -Tr[\gamma_i D^{-1}_{u,d}(x,y)\gamma_j D^{-1}_{d,u}(y,x)],$$

$D_d^{-1} = D_u^{-1}$, where $x = (\mathbf{n}a, n_t a)$, $y = (\mathbf{n}'a, n_t' a), \mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3) | n_i = 0, 1, ..., N-1\}$

We obtain the correlation functions for different spin projections of the ρ meson on the magnetic field axis:

$$C(s_z = 0) = \langle O_3(x)\bar{O}_3(y)\rangle_A$$
$$C(s_z = \pm 1) = \langle O_1(x)\bar{O}_1(y)\rangle_A + \langle O_2(x)\bar{O}_2(y)\rangle_A$$
$$\pm i(\langle O_1(x)\bar{O}_2(y)\rangle_A - \langle O_2(x)\bar{O}_1(y)\rangle_A)$$

where

$$O_1 = \psi^\dagger_{d,u}(x)\gamma_1\psi_{u,d}(x), \quad O_2 = \psi^\dagger_{d,u}(x)\gamma_2\psi_{u,d}(x),$$

$$O_3 = \psi^\dagger_{d,u}(x)\gamma_3\psi_{u,d}(x).$$

Calculate the correlation functions of ρ^0:

$$\langle\bar{\psi}_d(x)\gamma_i\psi_d(x)\bar{\psi}_d(y)\gamma_j\psi_d(y)+\bar{\psi}_u(x)\gamma_i\psi_u(x)\bar{\psi}_u(y)\gamma_j\psi_u(y)\rangle_A =$$

$$-Tr[\gamma_i D_d^{-1}(x,y)\gamma_j D_d^{-1}(y,x)] - Tr[\gamma_i D_u^{-1}(x,y)\gamma_j D_u^{-1}(y,x)].$$

Correlation functions

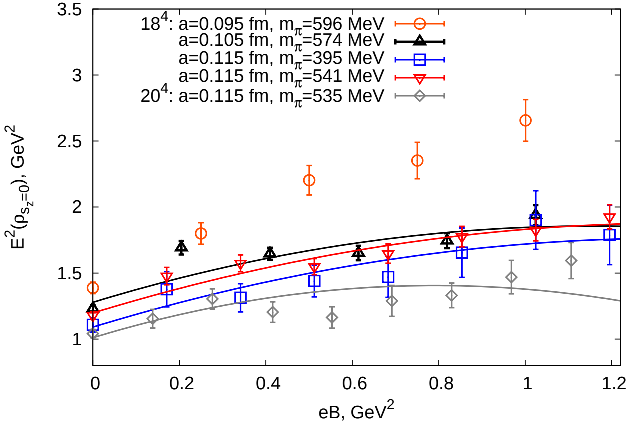
$$G(\vec{p}, n_t) = \frac{1}{N^{3/2}} \sum_{\mathbf{n}\in\Lambda_3} \langle \psi^\dagger(\vec{n}, n_t)\Gamma_1\psi(\vec{n}, n_t)\psi^\dagger(\vec{0}, 0)\Gamma_2\psi(\vec{0}, 0)\rangle e^{-ia\mathbf{np}}$$

$p_i = 2\pi k_i/(aN), \; k_i = -N/2, ..., N/2.$
We obtain the masses from the asymptotic behaviour of correlators

$$\langle \psi^\dagger(\vec{0}, n_t)\Gamma_1\psi(\vec{0}, n_t)\psi^\dagger(\vec{0}, 0)\Gamma_2\psi(\vec{0}, 0)\rangle_A = \sum_k \langle 0|\hat{O}_1|k\rangle\langle k|\hat{O}_2^\dagger|0\rangle e^{-n_t E_k}.$$

The main contribution comes from $\langle 0|\hat{O}_1|k\rangle\langle k|\hat{O}_2^\dagger|0\rangle e^{-n_t E_0}$.
We set $\langle \mathbf{p}\rangle = 0$, so $E_0 = m_0$ because $E^2 - \mathbf{p}^2 = m^2$.

Energy of the $\rho^{\pm}$ meson with $s_z = 0$ (Improved)

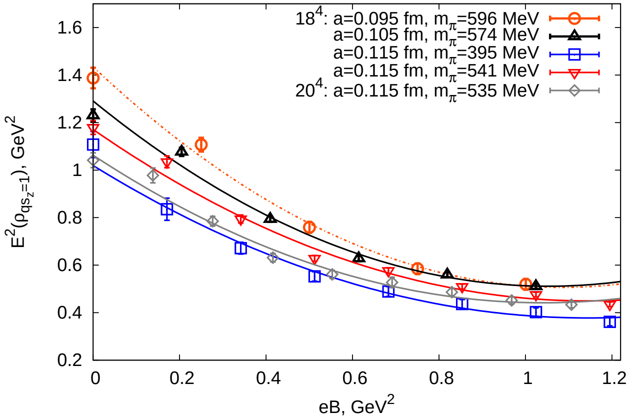


$$E^2 = |eB| + m^2 - 4\pi m \beta_m (eB)^2$$

The dipole magnetic polarizability of the $\rho^{\pm}$ meson for the spin projection $s_z = 0$.

V	m_π(MeV)	a(fm)	β_m(GeV^{-3})	$\chi^2/d.o.f.$
18^4	574 ± 7	0.105	0.03 ± 0.01	6.895
18^4	395 ± 6	0.115	0.028 ± 0.006	0.527
18^4	541 ± 3	0.115	0.027 ± 0.004	1.245
20^4	535 ± 4	0.115	0.050 ± 0.009	2.239

Energy of $\rho^{\pm}$ meson with $qs_z = +1$

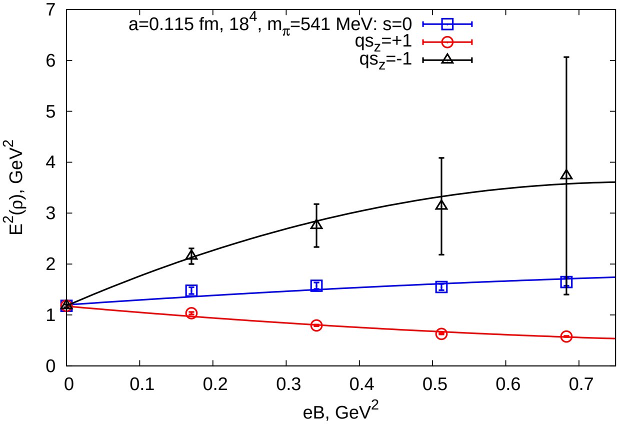


$$E^2 = |eB| - g(eB) + m^2 - 4\pi m \beta_m (eB)^2$$

The dipole magnetic polarizability and g-factor of the $\rho^{\pm}$ meson for the spin projection $|s_z| = 1$.

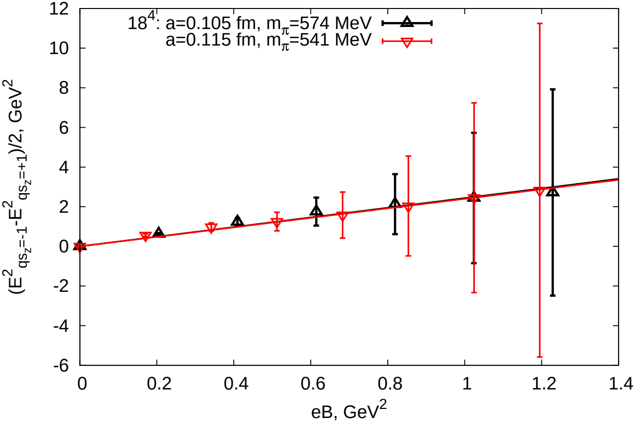
V	m_π(MeV)	a(fm)	g-factor	β_m(GeV^{-3})	$\chi^2/d.o.f.$
18^4	574 ± 7	0.105	2.48 ± 0.19	-0.049 ± 0.010	2.66
18^4	541 ± 3	0.115	2.26 ± 0.14	-0.041 ± 0.006	2.32
20^4	535 ± 4	0.115	2.19 ± 0.12	-0.044 ± 0.006	1.48
18^4	395 ± 6	0.115	2.12 ± 0.13	-0.039 ± 0.006	1.49

Energy of $\rho^{\pm}$ meson for $s_z = 1, \pm 1$.



$$qs_z = -1: \quad E^2 = |eB| + g(eB) + m^2 - 4\pi m \beta_m (eB)^2$$
$$qs_z = 0: \quad E^2 = |eB| + m^2 - 4\pi m \beta_m (eB)^2$$
$$qs_z = +1: \quad E^2 = |eB| - g(eB) + m^2 - 4\pi m \beta_m (eB)^2$$

Energy and magnetic moment of $\rho^{\pm}$ meson



$$E^2(qs_z = -1) - E^2(qs_z = +1) = 2g(eB)$$

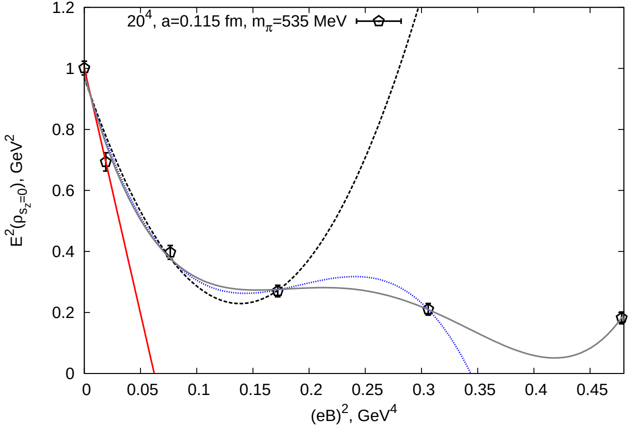
$g = 2.4 \pm 0.1$ for $a = 0.105$ fm and 2.40 ± 0.04 for $a = 0.115$ fm.

The tensor polarizability of the $\rho^{\pm}$ meson.

$$\beta_{tensor} = \frac{\beta_{s_z=+1} + \beta_{s_z=-1} - 2\beta_{s_z=0}}{\beta_{s_z=+1} + \beta_{s_z=-1} + \beta_{s_z=0}}$$

V	m_π(MeV)	a(fm)	β_{tensor}
18^4	574 ± 7	0.105	2.3 ± 0.7
18^4	541 ± 3	0.115	2.5 ± 0.5
20^4	535 ± 4	0.115	5 ± 2
18^4	395 ± 6	0.115	2.7 ± 0.7

Energy of the ρ^0 meson (new)

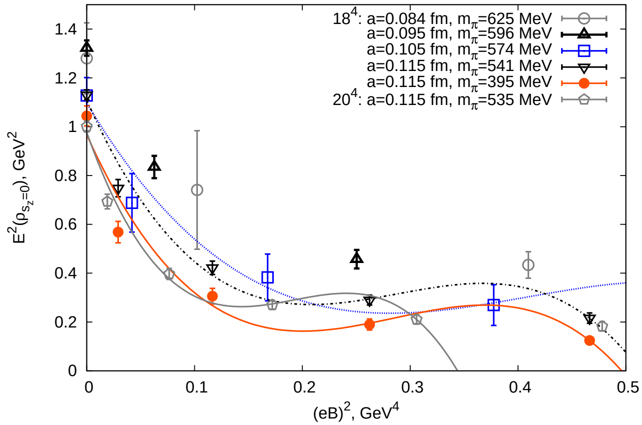


$$E^2 = m^2 - 4\pi m\beta_m(eB)^2 - 4\pi m\beta_m^{h2}(eB)^4 - 4\pi m\beta_m^{h4}(eB)^6 - 4\pi m\beta_m^{h6}(eB)^8$$

Determination of β_m value from different fits (new)

power of field	$(eB)^2$ GeV^4	β_m(GeV^{-3})	β_m^{2h}(GeV^{-7})	$\chi^2/d.o.f.$
2	[0 : 0.05]	1.28	–	–
4	[0 : 0.2]	0.86 ± 0.16	-3.12 ± 0.94	10.9
6	[0 : 0.4]	1.00 ± 0.18	-5.56 ± 1.64	7.5
8	[0 : 0.6]	1.09 ± 0.19	-7.38 ± 2.16	5.7

Energy of the ρ^0 meson with $s_z = 0$ (new)

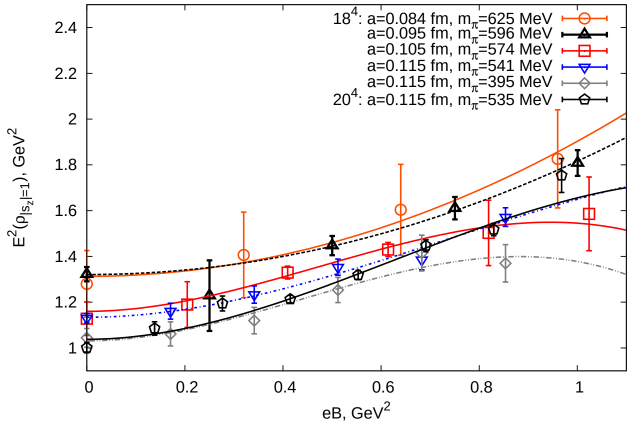


$$E^2 = m^2 - 4\pi m \beta_m (eB)^2 - 4\pi m \beta_m^{h1} (eB)^4 - 4\pi m \beta_m^{h1} (eB)^6$$

The values of the magnetic dipole polarizability of ρ^0 for the spin projection $s_z = 0$

V	m_π	a(fm)	$(eB)^2$, GeV^4	β_m(GeV^{-3})	β_m^{2h}(GeV^{-7})	χ^2/n
18^4	574	0.105	[0 : 0.7]	0.56 ± 0.14	-1.5 ± 0.6	1.6
18^4	541	0.115	[0 : 0.5]	0.75 ± 0.15	-2.8 ± 0.9	3.3
20^4	535	0.115	[0 : 0.4]	1.00 ± 0.18	-5.6 ± 1.6	7.5
18^4	395	0.115	[0 : 0.5]	0.79 ± 0.25	-3.1 ± 1.5	15.9

Energy of the ρ^0 meson with $|s_z| = 1$



$$E^2 = m^2 - 4\pi m \beta_m (eB)^2 - 4\pi m \beta_m^{h1} (eB)^3$$

We obtain
1) The energy of the ρ^0 meson with $s_z = \pm 1$ increases versus the magnetic field value.
2) The energy of the ρ^0 meson with $s_z = 0$ diminishes quickly versus the magnetic field value.

The small energy is more profitable than high energy, so the longitudinal polarization of the ρ^0 meson corresponding to $s_z = 0$ has to dominate in collisions.

Therefore the magnetic field favors longitudinal polarization of the ρ_0 meson with respect to the direction of the external magnetic field (Robust prediction).

1) P.V.Buividovich, M.I.Polikarpov, O.V. Teryaev, Lect.Notes Phys. Vol. 871, pp377-385, "Lattice studies of magnetic phenomena in heavy-ion collisions", arXiv: 1211.3014.

2) P.V. Buividovich, M.N. Chernodub, D.E. Kharzeev, T.K. K alaydzhyan, E.V. Luschevskaya, M.I. Polikarpov, Phys. Rev. Lett.105 (2010), ArXiv:1003.2180

3)E.L. Bratkovskaya, O.V. Teryaev, V.D. Toneev, Phys. Lett. B 348, 283 (1995)

The values of the magnetic dipole polarizability of ρ^0 for the spin projection $|s_z| = 1$

V	m_π	a(fm)	eB, GeV^2	$\beta_m(\mathrm{GeV}^{-3})$	$\beta_m^{1h}(\mathrm{GeV}^{-5})$	χ^2/n
18^4	574	0.105	[0 : 1.1]	-0.097 ± 0.017	0.07 ± 0.02	0.22
18^4	541	0.115	[0 : 1.1]	-0.071 ± 0.021	0.03 ± 0.03	
20^4	535	0.115	[0 : 1.1]	-0.103 ± 0.024	0.06 ± 0.03	2.36
18^4	395	0.115	[0 : 1.1]	-0.111 ± 0.028	0.08 ± 0.03	0.42

The tensor polarizability of the ρ^0 meson.

$$\beta_{tensor} = \frac{\beta_{s_z=+1} + \beta_{s_z=-1} - 2\beta_{s_z=0}}{\beta_{s_z=+1} + \beta_{s_z=-1} + \beta_{s_z=0}}$$

V	m_π(MeV)	a(fm)	β_{tensor}
18^4	574 ± 7	0.105	-3.59 ± 0.74
18^4	541 ± 3	0.115	-2.70 ± 0.31
20^4	535 ± 4	0.115	-2.79 ± 0.29
18^4	395 ± 6	0.115	-3.17 ± 0.66

Conclusions

1. The energy of the ρ^0 meson with $s_z = \pm 1$ increases versus the magnetic field value.
2. The energy of the ρ^0 meson with $s_z = 0$ diminishes quickly versus the magnetic field value.
3. The magnetic dipole polarizability β_m was obtained for the ρ^0 meson with $s_z = 0$.
4. The β_m value was improved for the $\rho^\pm$ meson with $s_z = 0$.
5. Magnetic field favors longitudinal polarization of the ρ_0 meson with respect to the direction of the external magnetic field (Robust prediction)

We plan to consider vorticity which has a relation to NICA experiment.
Work in progress.

Thank you for your attention!