CLOSED SYSTEM OF EQUATIONS FOR DESCRIPTION OF THE $e^+e^-\gamma$ PLASMA GENERATED FROM VACUUM BY STRONG ELECTRIC FIELD

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Introduction

We consider the Sauter-Schwinger process of electron-positron plasma (EPP) creation from vacuum under the action of a strong electromagnetic ("laser") field with account for side strong nonlinear effects, e.g.:

- radiation from EPP [Blaschke, Dmitriev, Ropke, Smolyansky. PRD (2011)];
- cascade processes [Fedotov, Narozhny, Mourou, Korn. PRL (2010)];
- EPP response to various testing signals;
- spin polarization.

To advance understanding of such phenomena from first principles, we present generalized equations refining nonperturbative and self-consistent kinetic description of EPP production under the action of a strong external electric field.

Besides backreaction (generation of an inner field), they also take into account the accompanying interactions with quantized radiation field. Additional expending of the incoming energy on hard photon emission and successive pair photoproduction should result in faster depletion of the external field.

Partial cases of such kind of kinetic theory have been already considered previously:

- nonperturbative kinetics of spontaneous EPP production from vacuum [Bialynicky-Birula, Gornicki, Rafelski. PRD (1991); Schmidt, Blaschke, Roepke, Smolyansky, Prozorkevich, Toneev. Int. J. Mod. Phys. (1998)];
- backreaction [Bloch, Mizerny, Prozorkevich, Roberts, Schmidt, Smolyansky, Vinnik. PRD (1999)];
- radiation in a single-photon annihilation channel [Blaschke, Dmitriev, Ropke, Smolyansky. PRD (2011)].

Our goal is derivation of a closed self-consistent system of equations of the electron-positron-photon plasma composed of:

- nonperturbative EPP KE, describing spontaneous EPP production from vacuum under the action of a strong linearly polarized time-dependent electric field with account for the interaction with quantized electromagnetic field (vacuum photo effect);
- hard photon KE;
- Maxwell equation, governing the inner (plasma) electric field.

We assume the external field is spatially homogeneous, time dependent and linearly polarized purely electric field; coupling with hard photons is taken into account within a second order of perturbation theory.

Problem statement

Consider an external electric field with 4-potential $A^3_{ex}(t) = A_{ex}(t)$ in the Hamiltonian gauge, the arising semiclassical internal (plasma) field $A^3_{in}(t) = A_{in}(t)$, and the fluctuating quantized field $\hat{A}^\mu(x)$. Both latter fields are created by EPP, modifying in turn the EPP production rate and dynamics.

Evolution of each operator $\mathcal{O}(t)$ is governed by $\dot{\mathcal{O}}(t) = i[H(t), \mathcal{O}(t)]$. We consider so-called quasiparticle representation, where the Hamiltonian $H(t)$ can be splitted as follows:

$$H(t) = H_q(t) + H_{pol}(t) + H_{ph} + H_{int}(t), \quad (1)$$

where $H_q(t)$ describes semiclassical acceleration of quasiparticles by the total semiclassical field $A^\mu(t) = A^\mu_{ex}(t) + A^\mu_{in}(t)$:

$$H_q(t) = \int [dp]\omega(\mathbf{p}, t)[a^+_\alpha(\mathbf{p}, t)a_\alpha(\mathbf{p}, t) + b^+_\alpha(\mathbf{p}, t)b_\alpha(\mathbf{p}, t)], \quad (2)$$

and is diagonal. Here we denote for brevity $[dp] = d^3p/(2\pi)^3$, $\omega(\mathbf{p}, t) = \sqrt{\epsilon^2_\perp + P^2}$ is quasienergy, $\epsilon_\perp = \sqrt{P^2_\perp + m^2}$ is transversal energy, and $P = p_\parallel - eA(t)$ is longitudinal quasimomentum. The next part of the Hamiltonian,

$$H_{pol} = \frac{i}{2}\int [dp]\lambda(\mathbf{p}, t)[a^+_\alpha(\mathbf{p}, t)b^+_\alpha(-\mathbf{p}, t) - b_\alpha(-\mathbf{p}, t)a_\alpha(\mathbf{p}, t)], \quad (3)$$

where $\lambda(\mathbf{p}, t) = \frac{eE(t)\epsilon_\perp(\mathbf{p})}{\omega^2(\mathbf{p},t)}$ and the field strength $E(t) = -\dot{A}(t)$, describes spontaneous pair creation and annihilation from vacuum.

As usually, free quantized radiation in a plane wave basis is described by

$$H_{ph} = \sum_r \int [dk] k A_r^{(+)}(\mathbf{k}, t) A_r^{(-)}(\mathbf{k}, t). \tag{4}$$

Finally, under the assumption that the semiclassical electric field is spatially homogeneous and using the spinor basis of the form

$$\begin{aligned} u_1^+(\mathbf{p}, t) &= B(\mathbf{p})[\omega_+, 0, P^3, P_-], \\ u_2^+(\mathbf{p}, t) &= B(\mathbf{p})[0, \omega_+, P_+, -P^3], \\ \upsilon_1^+(-\mathbf{p}, t) &= B(\mathbf{p})[-P^3, -P_-, \omega_+, 0], \\ \upsilon_2^+(-\mathbf{p}, t) &= B(\mathbf{p})[-P_+, P^3, 0, \omega_+], \end{aligned} \tag{5}$$

from [Blaschke, Dmitriev, Ropke, Smolyansky. PRD (2011)], where $B(\mathbf{p}) = (2\omega\omega_+)^{-1/2}$, $\omega_+ = \omega + m$ and $P_\pm = P^1 \pm iP^2$, the last part describing interaction with the quantized field $\hat{A}^\mu(x)$ reads:

$$\begin{aligned} H_{int}(t) = e(2\pi)^{-3/2} \sum_{\alpha\beta} \int d^3p_1 d^3p_2 \frac{d^3k}{\sqrt{2k}} \delta(\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{k}) \cdot \\ \cdot : ([\overline{u}u]^r_{\beta\alpha}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}; t) a^+_\alpha(\mathbf{p}_1, t) a_\beta(\mathbf{p}_2, t) + \\ +[\overline{u}\upsilon]^r_{\beta\alpha}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}; t) a^+_\alpha(\mathbf{p}_1, t) b^+_\beta(-\mathbf{p}_2, t) + \\ +[\overline{\upsilon}u]^r_{\beta\alpha}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}; t) b_\alpha(-\mathbf{p}_1, t) a_\beta(\mathbf{p}_2, t) + \\ +[\overline{\upsilon}\upsilon]^r_{\beta\alpha}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}; t) b_\alpha(-\mathbf{p}_1, t) b^+_\beta(-\mathbf{p}_2, t)) A_r(\mathbf{k}, t) : \end{aligned} \tag{6}$$

where the convolutions of spinors ξ_α and η_α (vertex functions) are defined by

$$[\overline{\xi}\eta]^r_{\beta\alpha}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}; t) = \overline{\xi}_\alpha(\mathbf{p}_1, t) \gamma^\mu \eta_\beta(\mathbf{p}_2, t) e^r_\mu(\mathbf{k}), \tag{7}$$

$e^r_\mu(\mathbf{k})$, $r = 1, 2$ are the unit photon polarization vectors, and $A_r(\mathbf{k}, t) = A_r^{(+)}(\mathbf{k}, t) + A_r^{(-)}(\mathbf{k}, t)$.

Complete set of equations for $e^{-}e^{+}\gamma$-plasma

Kinetic equation for EPP production - I

Under the assumption of quasineutrality, the distribution function of the quasiparticles

$$f(\mathbf{p},t)=\frac{1}{2}\langle a^{+}_{\alpha}(\mathbf{p},t)a_{\alpha}(\mathbf{p},t)\rangle=\frac{1}{2}\langle b^{+}_{\alpha}(-\mathbf{p},t)b_{\alpha}(-\mathbf{p},t)\rangle \tag{8}$$

obeys the non-Markovian KE with taking into account of interaction with the photon reservoir:

$$\dot{f}(\mathbf{p},t)=\frac{1}{2}\lambda(\mathbf{p},t)\int^{t}dt'\lambda(\mathbf{p},t')[1-2f(\mathbf{p},t')]\cos\left(2\int_{t'}^{t}d\tau\omega(\mathbf{p},\tau)\right)+C^{ann}(\mathbf{p},t)+C^{em}(\mathbf{p},t), \tag{9}$$

where the collision integrals correspond to the one-photon annihilation (pair production) and one-photon emission (absorption) processes, correspondingly:

$$\begin{aligned} C^{ann}(\mathbf{p},t) &= \int[dp_1][dk]\int^{t}dt'\delta(\mathbf{p}-\mathbf{p}_1-\mathbf{k})K^{ann}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t')\{f(\mathbf{p},t')f(\mathbf{p}_1,t')\cdot \\ &\cdot [1+F(\mathbf{k},t')]-[1-f(\mathbf{p},t')][1-f(\mathbf{p}_1,t')]F(\mathbf{k},t')\}, \end{aligned} \tag{10}$$

$$\begin{aligned} C^{em}(\mathbf{p},t) &= \int[dp_1][dk]\int^{t}dt'\delta(\mathbf{p}-\mathbf{p}_1-\mathbf{k})K^{em}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t')\{f(\mathbf{p},t')[1-f(\mathbf{p}_1,t')]\cdot \\ &\cdot [1+F(\mathbf{k},t')]-f(\mathbf{p},t')[1-f(\mathbf{p}_1,t')]F(\mathbf{k},t')\}. \end{aligned} \tag{11}$$

Kinetic equation for EPP production - II

The corresponding kernels are equal:

$$K^{ann}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t') = (2\pi)^3\frac{e^2}{2k}Re\{[\overline{u}\upsilon]^{r+}_{\alpha\beta}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t)[\overline{u}\upsilon]^{r}_{\alpha\beta}(\mathbf{p},\mathbf{p}_1,-\mathbf{k};t')e^{-i\theta^{(+,-)}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t')}\}, \qquad (12)$$

$$K^{em}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t') = (2\pi)^3\frac{e^2}{2k}Re\{[\overline{u}u]^{r+}_{\alpha\beta}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t)[\overline{u}u]^{r}_{\alpha\beta}(\mathbf{p},\mathbf{p}_1,-\mathbf{k};t')e^{-i\theta^{(-,-)}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t')}\} \qquad (13)$$

with the phases:

$$\theta^{(\pm,-)}(\mathbf{p},\mathbf{p}_1,\mathbf{k};t,t') = \int_{t'}^{t} d\tau[\omega(\mathbf{p},\tau)\pm\omega(\mathbf{p}_1,\tau)-k]. \qquad (14)$$

The collision integrals (10), (11) admits interpretation in the terms of "incoming" - "outgoing".

Kinetic coupled equation for hard photon distribution function

Kinetic coupled equation for hard photon distribution function $F(\mathbf{k},t) = \frac{1}{2}\langle A_r^-(\mathbf{k},t)A_r^+(\mathbf{k},t)\rangle$ is of the form:

$$\dot{F}(\mathbf{k},t) = S^{ann}(\mathbf{k},t) + S^{em}(\mathbf{k},t), \tag{15}$$

where the collision integrals in the annihilation and emission channels are equal:

$$S^{ann}(\mathbf{k},t) = \int [dp_1][dp_2] \int^t dt' \delta(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{k}) K^{ann}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t,t') \cdot \tag{16}$$
$$\cdot \{f(\mathbf{p}_1,t')f(\mathbf{p}_2,t')[1+F(\mathbf{k},t')] - [1-f(\mathbf{p}_1,t')][1-f(\mathbf{p}_2,t')]F(\mathbf{k},t')\},$$

$$S^{em}(\mathbf{k},t) = \int [dp_1][dp_2] \int^t dt' \delta(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{k}) K^{em}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k};t,t') \cdot \tag{17}$$
$$\cdot \{f(\mathbf{p}_1,t')[1-f(\mathbf{p}_2,t')][1+F(\mathbf{k},t')] - f(\mathbf{p}_1,t')[1-f(\mathbf{p}_2,t')]F(\mathbf{k},t')\}$$

with the same kernels (12), (13). Thus, these kernels are universal for the fermion and photon sectors.

KE (15) with the collision integrals (16), (17) is a linear integro-differential equation of the non-Markovian type relatively to the photon distribution function $F(\mathbf{k},t)$:

$$\dot{F}(\mathbf{k},t) = I(\mathbf{k},t) + \int^t dt' \Phi(\mathbf{k};t,t')F(\mathbf{k},t') \tag{18}$$

with the source term $I(\mathbf{k},t)$ and the kernel $\Phi(\mathbf{k};t,t')$ that are functionals of the EPP distribution function $f(\mathbf{p},t)$

Maxwell equation for backreaction

Maxwell equation for backreaction has the form [Bloch, Mizerny, Prozorkevich, Roberts, Schmidt, Smolyansky, Vinnik. PRD (1999)]:

$$\boxed{\dot{E}_{in}(t) = -j_{cond}(t) - j_{pol}(t).} \tag{19}$$

Here the conductivity and polarization current densities read:

$$j_{cond}(t) = 2e \int [dp] \frac{P}{\omega(\mathbf{p}, t)} f(\mathbf{p}, t), \tag{20a}$$

$$j_{pol}(t) = e \int [dp] \frac{\epsilon_\perp}{\omega(\mathbf{p}, t)} [u(\mathbf{p}, t) - \frac{e\dot{E}P}{4\omega^4(\mathbf{p}, t)}], \tag{20b}$$

where $u(\mathbf{p}, t)$ is the current polarization function. The second term on the r.h.s. of Eq.(20b) is the counter-term arising after renormalization.

Summary

- A closed-form system of equations describing self-consistently the dynamics of $e^- e^+$-plasma created by a homogeneous time-dependent field is derived. It consists of coupled equations (8), (12) and (15) and accounts for all the relevant first-order processes: one-photon annihilation/photocreation of $e^- e^+$-pairs and radiation/absorption of photons by quasiparticles. Since interaction with quantized field is considered by perturbation theory, the obtained equations can be solved by a suggestive iterative procedure based on decomposition $f = f^{(0)} + f^{(1)}$ of the distribution function of EPP, where $f^{(0)}$ describes nonperturbative evolution of EPP in the total electric field $E(t) = E_{ex}(t) + E_{in}(t)$, while the function $f^{(1)}$ takes into account interaction with photons.

  Detailed analysis of the obtained kinetic equations and their physical implications will be given in a separate follow up work.
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Thank you for attention!