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Interaction of Quantum Systems with Envinronment in QCD

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Quantum system, Environment , Decoherence

(Haken, Hake, Peres)

- **Interactions of some quantum system with the environment can be effectively represented by additional stochastic terms in the Hamiltonian of the system.**
- **The density matrix of the system is obtained by averaging of the general density matrix with respect to degrees of freedom of environment**
- **Interactions with the environment result in decoherence and relaxation of quantum superpositions. Information on the initial state of the quantum system is lost after suficiently large time**
- **Quantum decoherence** is the loss of coherence or ordering of the phase angles between the components of a system in a quantum superposition
- **D.** can be viewed as the loss of information from a system due to the environment (often modeled as a heat bath)
- **Dissipation** is a decohering process by which the populations of quantum states are changed due to entanglement with a bath
- **Relaxation** usually means the return of a perturbed system into equilibrium. Each relaxation process can be characterized by a **relaxation time** τ.

Stochastic QCD Vacuum

(Ambjorn; Simonov; Dosch)

- The model of QCD stochastic vacuum is one of the popular phenomenological models which explains quark confinement (WL decreasing) , string tensions and field congurations around static charges

- Only the second correlators are important and the other are negligible (which are important in coherent vacuum where all correlators are important) (Simonov) (Gauss domination) It has been confirmed by lattice calculation Shevchenko, Simonov . The most important evidence for this is Casimir scaling [Ambjrn J, Olesen P, Peterson].

- It is based on the assumption that one can calculate vacuum expectation values of gauge invariant quantities as expectation values with respect to some well-behaved stochastic gauge field

- It is known that such vacuum provides confining properties, giving rise to QCD strings with constant tension at large distances

Stochastic QCD vacuum as environment

(Kuvshinov, Kuzmin, Buividovich)

- We consider QCD stochastic vacuum as the environment for colour quantum particles and average over external QCD stochastic vacuum implementations.
- Instead of considering nonperturbative dynamics of Yang-Mills fields one introduces external environment and average over its implementations

As a consequence we obtain:

- decoherence of quantum superpositions
- information lost and confinement of colour phenomenon
- White objects can be obtained as mixtures of states described by the density matrix as a result of evolution in the QCD stochastic vacuum as environment

Colour decoherence

(Kuvshinov, Kuzmin, Buividovich)

Consider propagation of heavy spinless colour particle along some fixed path γ. The amplitude is obtained by parallel transport

$$|\phi(\gamma)\rangle = \hat{P} \exp\left(i \int_{\gamma} \hat{A}_\mu dx^\mu \right) |\phi_{in}\rangle$$

In order to consider both colour particle and QCD stochastic vacuum (environment) we introduce the colour density matrix

After the averaging over all environment degrees of freedom –decoherence due to interaction with environment, we obtain transition of pure colour states into a mixed white states with expression for density matrix

$$\rho(loop, \gamma\bar{\gamma}) = N_c^{-1} + \left(|\phi_{in}\rangle\langle\phi_{in}| - N_c^{-1}\right) W_{adj}(loop, \gamma\bar{\gamma})$$

$$W_{fund}(loop, \gamma\bar{\gamma}) = \langle T_r \hat{P} \exp\left(\int_{loop, \gamma\bar{\gamma}} i\hat{A}_\mu dx^\mu \right) \rangle$$

$$W_{adj}(L) = \exp(-\sigma_{adj} RT) \qquad \sigma_{fund} = \frac{g^2}{2} l_{corr}^2 F^2$$

g is coupling constant, lcorr – correlation length in the QCD stochastic vacuum, F - average of the second cumulant of curvature (Dosch; Simonov) Density matrix becomes diagonal ρout=diag (1/Nc)

In the case of stochastic (not coherent) QCD vacuum in confinement region (Wilson loop decays exponentially) we have decoherence of pure colour states into a mixed white states

We have decoherence

Decoherence rate, Purity, Von Neumann entropy, Information

The decoherence rate of transition from pure colour states to white mixture can be estimated on the base of purity (Haake)

$P = \mathrm{Tr}\,\rho^2$

$$P = N_c^{-1} + (1 - N_c^{-1}) \exp(-2\sigma_{fund} G_{adj} G_{fund}^{-1} RT)$$

When RT tends to 0, P → 1, that corresponds to pure state. When composition RT tends to infinity the purity tends to 1/Nc, that corresponds to the mixture (equal probability to find any colour)

The rate of purity decrease is

$$T_{dec}^{-1} = -2\sigma_{fund} G_{adj} G_{fund}^{-1}$$

Left side of the equation is the characteristic time of decoherence proportional to QCD string tension and distance R

Von Neumann entropy: $S = -\mathrm{Tr}\left(\hat{\rho} \ln \hat{\rho}\right)$ S = 0 for the initial state and S = ln Nc for large RT-increases

The information of quark colour states I=1-S/lnNc is lost due to interactions between quarks and confining non-Abelian gauge fields, corresponds (no-cloning (Park), no-hiding ((Braunstein, Pati) theorems. For multiparticles (pure separable, mixed separable and nonsepaparable (entangled) when RT→∞ we obtain diagonalization of density matrix, decreasing of purity and fidelity, information, increasing of Von Neumann entropy

Thus mixture (equal probability to find any colour) can be obtained as a result of decoherence process from pure colour states

Interaction of Colour Superposition with QCD Vacuum

(Kuvshinov, Bagashov)

When the initial (pure) colour state is a superposition of colour states

$$|\phi_{in}\rangle = \alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle \qquad |\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$$

The corresponding density m $\hat{\rho}_{in} = |\phi_{in}\rangle\langle\phi_{in}|$

$$\hat{\rho}_{in} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \alpha^*\beta & |\beta|^2 & \beta\gamma^* \\ \alpha^*\gamma & \beta^*\gamma & |\gamma|^2 \end{pmatrix}$$

After integration and averaging

$$\hat{\rho}(y) = \langle\langle\hat{\rho}_1(y)\rangle\rangle = N_c^{-1}\hat{I} + (\hat{\rho}_{in} - N_c^{-1}\hat{I})W_{adj}(L)$$

When RT→∞ $W_{adj}(L) = \exp(-\sigma_{adj}RT)$

$$\begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \alpha^*\beta & |\beta|^2 & \beta\gamma^* \\ \alpha^*\gamma & \beta^*\gamma & |\gamma|^2 \end{pmatrix} \to \begin{pmatrix} N_c^{-1} & 0 & 0 \\ 0 & N_c^{-1} & 0 \\ 0 & 0 & N_c^{-1} \end{pmatrix}$$

Density matrix becomes diagonal ρ_{out}=diag (1/Nc)

Purity, Von Neumann entropy, Information for Colour Superposition

PurityEntrop $P = N_c^{-1} + (1 - N_c^{-1}) W_{adj}^2(L)$

$S=(1-N^{-1}_{C})(1- \ln\frac{Wadj(L)}{Nc})$

- **For the initial stateRT →0: purity P → 1 -pure state, entropy S→O**
- **Asympt**otically **RT→∞: P=N$_C$ $^{-1}$-fully mixed state,entropy S=lNc**

-Interaction of an arbitrary colour superposition with the QCD stochastic vacuum at large distances leads to an emergence of a mixed state

-With equal probabilities for different colours

-Without any non-diagonal terms in the corresponding density matrix ρout=diag (1/Nc)

Evolution of Two Particle States

(Kuvshinov, Bagashov)

- Consider two quark system. Every quark is subsystem wth colour and anticolour.
- Consider the next possible states and their evolution in stochastic vacuum:
- 1) on purity - pure, mixed
- 2) on separability – separable, nonseparable (entangled)
- In the case of mixed separable the system is described by density matrix not by
- Vector state
- Under the same reasoning we obtain new density matrix as the result of evolution (in the basis of corresponding state vectors) and easily obtain information on changes of purity and entropy
- We see the diagonalization of density matrix, increasing of entropy and decreasing of purity

N_p-particle states in QCD Vacuum
Density Vatrix, Purity, Von Neumann entropy

(Kuvshinov, Bagashov)

$$\hat{\rho}(y) = N_c^{-N_p}\hat{I} + \left(\hat{\rho}_{in} - N_c^{-N_p}\hat{I}\right)W_{adj}(L)$$

$$\hat{\rho}(L : RT \to \infty) = N_c^{-N_p}\hat{I}$$

$$\mathrm{P} = N_c^{-N_p} + \left(1 - N_c^{-N_p}\right)W_{adj}^2(L)$$

$$S = -\mathrm{Tr}\left(N_c^{-N_p}\hat{I}\ \ln\left(N_c^{-N_p}\hat{I}\right)\right) = \mathrm{Tr}\left(N_c^{-N_p}\hat{I}N_p \ln N_c\right) = N_c^{N_p}N_c^{-N_p}N_p \ln N_c =$$
$$= N_p \ln N_c$$

Purity, Von Neumann Entropy

RT=0

State:	pure separable	mixed separable	pure entangled
P (purity)	1	$\frac{1}{N_c^{N_p}} \leq \mathrm{P} < 1$	1
S (entropy)	0	$0 < \mathrm{S} \leq N_p \ln N_c$	0

$\downarrow$ **RT→∞**

State:	pure separable	mixed separable	pure entangled
P (purity)	$\frac{1}{N_c^{N_p}}$	$\frac{1}{N_c^{N_p}}$	$\frac{1}{N_c^{N_p}}$
S (entropy)	$N_p \ln N_c$	$N_p \ln N_c$	$N_p \ln N_c$

Purity decreases, Entropy increases

On possibility of squeezed and entangled states in QCD

(Kuvshinov, Marmysh, Shaporov)

= **Nonperturbative (NP)** effects in jets

Role of **NP** effects in particular :

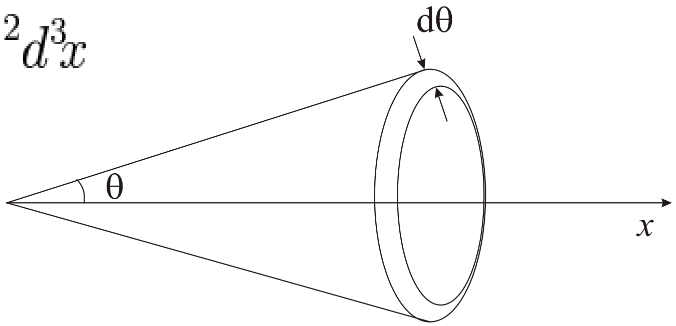
- ❑confinement and hadronization
- ❑exact YM field equations, solutions, ex. instantons, vacuum properties
- ❑long distances, soft collisions, diffraction
- ❑power corrections
- ❑NP evolution
- ❑MC hadronization models, LPHD

Gluon Evolution

Consider gluon self-interaction Hamiltonian of QCD

$$V = -g\int f_{abc}\mathbf{E}_a\mathbf{A}_b A_c^0\, d^3x + \frac{g}{2}\int f_{abc}\mathbf{B}_a[\mathbf{A}_b\mathbf{A}_c]d^3x +$$
$$+\frac{g^2}{2}\int\left(f_{abc}\mathbf{A}_b A_c^0\right)^2 d^3x + \frac{g^2}{8}\int\left(f_{abc}[\mathbf{A}_b\mathbf{A}_c]\right)^2 d^3x$$

Take jet ring with cone angle



In terms of annihilation (creation) operators

$$V = \frac{k_0^4}{4(2\pi)^3}\left(1-\frac{q_0^2}{k_0^2}\right)^{3/2} g^2\,\pi\, f_{abc}f_{aa'f}\left\{\left(2-\frac{q_0^2}{k_0^2}\right)\left[a_{1212}^{bca'f} + a_{1313}^{bca'f}\right] + \right.$$
$$\left. + a_{2323}^{bca'f} + \frac{\sin^2\theta}{2}\left(1-\frac{q_0^2}{k_0^2}\right)\left[2a_{2323}^{bca'f} - a_{1212}^{bca'f} - a_{1313}^{bca'f}\right]\right\}\sin\theta\, d\theta.$$

Here $\quad a_{lmlm}^{bca'f} = a_l^{b+}a_m^{c+}a_l^{d}a_m^{f} + a_l^{b+}a_m^{c}a_l^{d+}a_m^{f} + a_l^{b}a_m^{c+}a_l^{d+}a_m^{f} + h.c.$

Gluon Evolution

Hamiltonian *V* has squares of operators of annihilation and creation.
As it is known from QM and QO such structures in evolution Hamiltonian are nessesary condition of Squeezing States (SS) production because squeezing operator *S(z)* is

$$S(z) = \exp\left\{\frac{z^*}{2}a^2 - \frac{z}{2}(a^+)^2\right\}$$

where *z* = *r exp*(iθ) is an arbitrary complex number, *0 ≤ r < ∞* and *0 ≤* θ *≤ 2π*, *r- squeezing* coefficient

For small time evolution *t* we have final state

$$|\,\mathrm{f}\,\rangle \simeq |\,\mathrm{in}\,\rangle - \,i\,t\,V\,|\,\mathrm{in}\,\rangle$$

task*: to search possibility of Quantum Squeezing of Gluon States by study NP evolution of initial state under *V

Gluon SS production

To check whether final gluon state describes SS we should by analogy to quantum optics introduce operators

$$(\hat{X}_l^b)_1 = \left[\hat{a}_l^b + (\hat{a}_l^b)^+\right]/2 \quad \text{and} \quad (\hat{X}_l^b)_2 = \left[\hat{a}_l^b - (\hat{a}_l^b)^+\right]/2i$$

and to find out that dispersion of one then is smaller than that for coherent (or vacuum) state

Condition of squeezing **for fotons (Walls, 1983),** $\left\langle \left(\triangle (X_l^b)_{\frac{1}{2}} \right)^2 \right\rangle = \left\langle N \left(\triangle (X_l^b)_{\frac{1}{2}} \right)^2 \right\rangle + \frac{1}{4} < \frac{1}{4}$

for gluons (Kuvshinov,Shaporov, Marmysh, 1999) : or $\left\langle N \left(\triangle (X_l^b)_{\frac{1}{2}} \right)^2 \right\rangle < 0.$

In terms of a, a+:

$$\left\langle N \left(\triangle (X_l^h)_{\frac{1}{2}} \right)^2 \right\rangle = \mp \frac{i\,t}{4} \Big\{ \langle \alpha \,|\, [a_l^h\,(k)\,,[a_l^h\,(k)\,,V\,]\,]|\alpha\rangle - \\ - \langle \alpha \,|[\,[V, a_l^{h+}\,(k)], a_l^{h+}\,(k)]|\alpha\rangle \Big\} < 0$$

Here as initial state vector of nonperturbative evolution we use at the end of P evolution (Lupia S., Ochs W. and Wosiek J., Nucl. Phys. B 540, 405 (1999))

NBD distribution is equal to superposition of products of

the gluon coherent states (Poissonian distributions) $|\alpha\rangle = \prod_{c=1}^{8} \prod_{l=1}^{3} |\,\alpha_l^c(0)\,\rangle$

Gluon SS production

➢**Terms ~A^3 (three-gluon self-interaction) don't give contribution to the squeezing condition**

$$\left[a_l^h(k),\left[a_l^h(k),V\sim A^3\right]\right]=0,\quad \left[\left[V\sim A^3,a_l^{h+}(k)\right],a_l^{h+}(k)\right]=0 \qquad f_{hhb}=0$$

Four-gluon selfinteraction is source of the squeezing effect

For example: for colour index *h=1*

$$\left\langle N\left(\triangle(X_l^1)_{\frac{1}{2}}\right)^2\right\rangle=\pm\,4\pi\,u_2\,t\,\sin\theta\,d\theta\Big\{(1+u_1)\Big[\delta_{l1}(Z_{33}+Z_{22})+$$
$$+(1-\delta_{l1})Z_{11}\Big]+\delta_{l2}Z_{33}+\delta_{l3}Z_{22}+$$
$$+u_1\sin^2\theta\left[-\tfrac{1}{2}\delta_{l1}(Z_{22}+Z_{33})+\delta_{l2}(Z_{33}-\tfrac{1}{2}Z_{11})+\delta_{l3}(Z_{22}-\tfrac{1}{2}Z_{11})\right]\Big\}\neq 0$$

Here

$$Z_{mn}=\sum_{k=2}^{7}{}'\langle(X_m^k)_1\rangle\langle(X_n^k)_2\rangle(m,n=1,2,3),$$
$$\sum_{k=2}^{7}{}'(\,)=\sum_{k=2}^{3}(\,)+\frac{1}{4}\sum_{k=4}^{7}(\,),\ u_1=\left(1-\frac{q_0^2}{k_0^2}\right),\ u_2=\frac{k_0^4}{4(2\pi)^3}\frac{g^2}{2}\sqrt{u_1^3}.$$

Gluon SS production

Conditions for gluons (Kuvshinov,Shaporov, Marmysh, 1999)

- **We have gluon phase squeezed state if:**

$k \neq 1, m \neq l$

$$\langle (X_m^k)_1 \rangle < 0, \langle (X_m^k)_2 \rangle < 0$$
or
$$\langle (X_m^k)_1 \rangle > 0, \langle (X_m^k)_2 \rangle > 0$$

- **We have gluon amplitude squeezed state if:**

$$\langle (X_m^k)_1 \rangle > 0, \langle (X_m^k)_2 \rangle < 0$$
or
$$\langle (X_m^k)_1 \rangle < 0, \langle (X_m^k)_2 \rangle > 0$$

- **The conditions cover all possible cases => *gluon SS –exist***
- **The same is true for other colours**
- **Obviously, the larger are both the amplitudes of the initial gluon coherent fields with different colours and polarization indexes and coupling constant, the larger is the two-mode squeezing effect**

Entangled States in QO

- Consider the Superposition state vector for a system with two

  $|\Psi\rangle = \alpha\,|1\rangle + \beta\,|2\rangle$ tes $|1\rangle$ and $|2\rangle$:

$$\rho = |\Psi\rangle\langle\Psi| = |\alpha|^2\,|1\rangle\langle 1| + |\beta|^2\,|2\rangle\langle 2| + \alpha\beta^*\,|1\rangle\langle 2| + \alpha^*\beta\,|2\rangle\langle 1|$$

$\rho_{\mathrm{mix}} = |\alpha|^2\,|1\rangle\langle 1| + |\beta|^2\,|2\rangle\langle 2|$ differs from a mixed state

- As an example of an Entangled state, consider the state of a composite system: two-level atom-field
- After a short period of interaction, the atom and the field become spatially separated
- However, the state of the whole system remains Entangled:

  the State of the Atom is strictly correlated with the State of the Field

$$|\Psi\rangle = |\mathrm{atom}\rangle_1\,|\mathrm{field}\rangle_1 + |\mathrm{atom}\rangle_2\,|\mathrm{field}\rangle_2$$

Entangled States

- **Other example of the Entangled States is two single-photon beams with different wave vectors**

$$|1_1 + 1_2\rangle = C_{\updownarrow\updownarrow} |\updownarrow\rangle_1 |\updownarrow\rangle_2 + C_{\leftrightarrow\leftrightarrow} |\leftrightarrow\rangle_1 |\leftrightarrow\rangle_2 + C_{\updownarrow\leftrightarrow} |\updownarrow\rangle_1 |\leftrightarrow\rangle_2 + C_{\leftrightarrow\updownarrow} |\leftrightarrow\rangle_1 |\updownarrow\rangle_2$$

Bell in 1964 introduced these states in relation to the EPR paradox.

- **Basis of the Bell states**

$$|\Phi^+\rangle = (|\updownarrow\rangle_1 |\updownarrow\rangle_2 + |\leftrightarrow\rangle_1 |\leftrightarrow\rangle_2)/\sqrt{2},$$

$$|\Phi^-\rangle = (|\updownarrow\rangle_1 |\updownarrow\rangle_2 - |\leftrightarrow\rangle_1 |\leftrightarrow\rangle_2)/\sqrt{2},$$

$$|\Psi^+\rangle = (|\updownarrow\rangle_1 |\leftrightarrow\rangle_2 + |\leftrightarrow\rangle_1 |\updownarrow\rangle_2)/\sqrt{2},$$

$$|\Psi^-\rangle = (|\updownarrow\rangle_1 |\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1 |\updownarrow\rangle_2)/\sqrt{2},$$

- **Each of these Entangled States has a remarkable property:**
- **If one photon is registered with definite polarization, the other**
- **photon immediately becomes opposite polarized**
- **Measurement over one particle have an instantaneous effect on**
- **the other, possibly located at a large distance**
- **Two-mode squeezed state is one of the example of the en-**
- **tangled states** ***(De Wolf , 2001)***

Entangled States

- Entangled states have another paradoxical property, which was pointed out by Schrodinger in 1935:

  Complete information about the state of the total system still does not provide complete information about the states of its parts.

  Indeed, for example: Suppose that we are going to find out the state of a particle in one of the pairs two single-photon beams

  Then we have to average the density matrix of the pure state

$$|\Psi^{-}\rangle = (|\updownarrow\rangle_1 |\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1 |\updownarrow\rangle_2)/\sqrt{2}$$

  over the states of the second particle. The resulting density matrix of the first particle

$$\rho^{(1)} = \mathrm{Tr}(|\Psi^{-}\rangle\langle\Psi^{-}|) = \underbrace{(|\updownarrow\rangle_1\, {}_1\langle\updownarrow| + |\leftrightarrow\rangle_1\, {}_1\langle\leftrightarrow|)/2}_{\text{mixture}}$$

  is apparently the density matrix of a mixed state, which is not maximally determinate

Gluon Entangled States

- By **analogy with Index of the correlation (Dodonov 2002)**

we used as the measure of entanglement for gluon states the next coefficient: (Kuvshinov, Marmysh, Shaporov 2004)

$$y = \left[\frac{|\overline{a_l^h a_l^{g+}}|^2 + |\overline{a_l^h a_l^g}|^2}{2(\overline{a_l^{h+} a_l^h} + 1/2)(\overline{a_l^{g+} a_l^g} + 1/2)} \right]^{1/2}$$

$$\overline{a_l^h a_l^{g+}} = \langle a_l^h a_l^{g+} \rangle - \langle a_l^h \rangle \langle a_l^{g+} \rangle$$

$$0 \le y < 1$$

Gluon Entangled States

- Measure of entanglement is proportional to the squeezing coefficient (at small squeezing)

$$\boldsymbol{y} = \sqrt{2}\boldsymbol{r}$$

- Entanglement imposes additional restrictions on the squeezing parameter

- In particular, for the collinear gluons we have $0 < \boldsymbol{r} < \frac{1}{\sqrt{2}}$

$$0 < \left| t\frac{\alpha_s \pi}{2k_0}(f_{ahb}f_{ahc} + f_{agb}f_{agc} + f_{ahb}f_{agc} + f_{agb}f_{ahc})\sum_{l_1 \neq l} |\alpha_{l_1}^b| |\alpha_{l_1}^c| \sin(\gamma_{l_1}^b + \gamma_{l_1}^c) \right| < \frac{1}{\sqrt{2}}$$

- Thus, by analogy with quantum optics as a result of four-gluon self-
- interaction we obtain two-
- mode gluon squeezed states which are also entangled

Stability of Movement of Gauge Fields and Source Fields

Classical level

Order to Chaos transition, critical energy, Higgs mass

SU(2) Yang-Mills field system has unstable movement under any values of parameters, **chaotic solutions of Yang-Mills** (**S.G. Matinyan); G. K. Savvidy**)), **possible chaos onset** (**Kawabe**)

Higgs fields and quantum fluctuations of gauge fields induce regularization of dynamics of system of Yang-Mills, to appear of areas of stable and the regularized motion (**Berman; Matinyan; Salashnich; Muller**); **Kuvshinov, Kuzmin, Petrov**).

It was shown that Higgs bosons and its vacuum quantum fluctuations regularize the system and lead to the emergence of order-chaos transition at some critical energy (Matinyan Kuvshinov, Kuzmin)

$$E_c = \frac{3\mu^4}{64\pi^2}\exp\left(1 - \frac{\lambda}{g^4}\right)$$

$$E_C = \frac{3\mu^4}{32\pi^2}\exp\left(2\alpha_w - \frac{2\lambda}{g^4}\beta_w\right)\left(1 + \frac{1}{2\cos^4\theta_w}\right)\left(1 - 7e^{-2}\right)$$

$$\alpha_w = \frac{2\ln\cos\theta_w}{1 + 2\cos^4\theta_w}; \quad \beta_w = \frac{32\pi^2\cos^4\theta_w}{9\left(1 + 2\cos^4\theta_w\right)}$$

For SU(2) X U(1)

Here μ is mass of Higgs boson, λ is its self interaction coupling constant, g is coupling constant gauge and Higgs fields

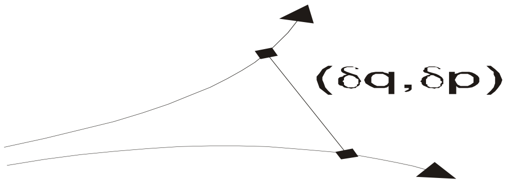
In the region of confinement there exists the point of order -chaos transition where the fidelity decreased exponentially and which is equal to string tension

This connects the properties of stochastic QCD vacuum and Higgs boson mass and self interaction coupling constant

Generalized Toda criterion (N- number degrees of freedom)

(Kuvshinov,Kuzmin)

Hamiltonian:



$$H=\frac{1}{2}\vec{p}^{\,2}+V(\vec{q}),$$

$$\vec{p}=(p_{1,\dots,}p_N),\qquad \vec{q}=(q_{1,\dots,}q_N)$$

Linearized Hamilton equations:

$$\frac{d}{dt}\begin{pmatrix}\delta\vec{q}\\ \delta\vec{p}\end{pmatrix}=G\begin{pmatrix}\delta\vec{q}\\ \delta\vec{p}\end{pmatrix},G\equiv\begin{pmatrix}0 & I\\ -\Sigma & 0\end{pmatrix},\qquad \Sigma\equiv\left(\frac{\partial^2 V}{\partial q_i\partial q_j}\right)_{\vec{q}_0}$$

Let λ_i , $i=\overline{1,2N}$ eigenvalues of G matrix

ξ_i , $i=\overline{1,N}$ eigenvalues of Σ matrix

Generalized Toda criterion

If there exist i, to satisfy $\mathrm{Re}\,\lambda_i \neq 0$

If there exist j, to satisfy $\xi_j \geq 0$, then

we have local instability and chaos

In other case movement is regular and stable

In the case of two degrees jf freedom , the criterium transfer into the known

Toda critetion (M. Toda, Phys.Lett.A vol.48 N5 (1974) p.335)

Quantum level

Fidelity

The stability of quantum motion of the particles is described by fidelity f (Peres, Prosen, Cheng). The definition of fidelity is similar with Wilson loop definition in QCD (Kuvshinov, Kuzmin). Using the analogy between the theory of gauge fields and the theory of holonomic quantum computation (Reineke ; Kuvshinov,Kuzmin, Buividovich) we can define the fidelity of quark motion (the scalar product of state vectors for perturbed and unperturbated motion) (or two density matrices) as an integral over the closed loop, with particle traveling from point x to the point y

$$\boxed{f = \left\langle \left(\langle \phi_{in} | \hat{P} \exp \left(\int i \hat{A}_\mu dx^\mu \right) | \phi_{in} \rangle \right) \right\rangle}$$

The final expression for the fidelity of the particle moving stochastic vacuum is

$$f = \exp\left(-\frac{1}{2} g^2 l_{corr}^2 F^2 S_\gamma \right)$$

Thus, fidelity for colour particle moving along contour decays exponentially with the surface spanned over the contour, the decay rate being equal to the string tension

Motion becomes more and more unstable when $S_\gamma \rightarrow \infty$.

❖ Sometimes fidelity is defined in another way [Hubner], [Uhlmann], [Kuvshinov, Bagashov]

$$F(\omega,\tau) = \mathrm{Tr}\left(\sqrt{\sqrt{\omega}\tau\sqrt{\omega}}\right) \qquad F = \mathrm{Tr}\left(\hat{\rho}_{in}\sqrt{N_c^{-1} + (1 - N_c^{-1})W_{adj}(L)}\right)$$

(Square root of probability of transition from the state with density matrix ω to state with d.m. τ; ϱin to ϱout) The fidelity decreases For two random paths in Minkowski space, which are close to each other, the expression for the fidelity is similar, but now the averaging is performed with respect to all random paths which are close enough. And the final expression is

$$f = \exp\left(-\frac{1}{2}g^2 l_{corr}\int\limits_{\gamma_1} dx^{\upsilon} F_{\chi\alpha}\widetilde{F}_{\upsilon\beta}\upsilon^{\chi}\langle\delta\chi^{\alpha}\delta\chi^{\beta}\rangle\right) \qquad (10)$$

where δχ - is the deviation of the path γ2 from the path γ1, υ is the four-dimensional velocity and lcorr is the correlation length of perturbation of the particle path expressed in units of world line length. If unperturbed path is parallel to the time axis in Minkowski space, the particle moves randomly around some point in three dimensional space. The fidelity in this case decays exponentially with time.

❖ Thus, we have connection between regions of confinement and instability of colour particle motion

Quantum Chaos Criterion

(Kuvshinov, Kuzmin PL 2002)

Two-point connected Green function :

$$G_{ik}(x,y) = -\frac{\delta^2 W[\vec{J}]}{\delta J_i(x)\delta J_k(y)}\bigg|_{\vec{J}=0}$$

Chaos criterion :

- **Chaos:** Green function exponentially (or faster) tends to zero under $|x - y| \to \infty$, $(x - y)^2 > 0$, $(x^0 - y^0) > 0$.
- **Order:** Green function oscilates and slowly decreases in this limit.

Correspondence with classical chaos criterion:

$$G_i(t_1 - t_2) = \frac{i}{2}\mathrm{Re}\left(\frac{\exp\{-\lambda_i(t_1 - t_2)\}}{\lambda_i}\right), \; t_1 > t_2$$

When $\tau q \gg \Delta tcl$ (dynamical localization)

a) If classical motion is locally unstable (chaotic) then according Toda criterion there is real eigenvalue λi. Therefore Green function exponentially goes to zero for some i when $(t_1 - t_2) \to +\infty$. Opposite is also true. If Green function exponentially goes to zero under the condition $(t_1 - t_2) \to +\infty$ for some i, then there exists real eigenvalue of the stability matrix and thus classical motion is locally unstable.

b) If all eigenvalues of the stability matrix G are pure imaginary, that corresponds classically stable motion, then in the limit $(t_1 - t_2) \to +\infty$ Green function oscillates as a sine. Opposite is also true. If for any i Green functions oscillate in the limit $(t_1 - t_2) \to +\infty$ then $\{\lambda_i\}$ are pure imaginary for any i and classical motion is stable and regular. Proposed criterion coincides with Toda criterion in the semi-classical limit (corresponding principle)

Chaos and Gauge Field Theories

- **Classical gauge fields demonstrate chaotic behavior** (Savidi, PL,1977; Kawabe, PRD,1990; Kuvshinov,NPCS,1999)

- **Intermittency and chaos in multiple and branching processes of strong interactions(QCD) , Jets, QGP** (Kittel, Dremin,…, Kuvshinov…seventies, eighties)

- **Decoherence, Confinement of colour, instability,** squeezing, entanglement, decreasing of Putity, Fidelity, Information in Stochastic QCD Vacuum as environment Kuvshinov, 2003-1016…)

- **Mathematical apparatus of gauge field theories (namely QCD) can be used in chaos theory and vice versa.** V. Kuvshinov, A. Kuzmin. Gauge Fields and Theory of Determenistic Chaos (Belorussian Science, Minsk, 2006, p. 1-268 in Russian).
- In particular:

Instanton technique can be applied to investigate chaos assisted tunneling regime (Chaos assisted instanton tunneling (chaotic instanton solutions, dilute instanton gas squeezing, exponential widening of the ground quasi-energy zone, numerical simulations) (Kuvshinov, A.Kuzmin et al, PR, APP, 2003-2006)

- **Mathematical apparatus of the gauge field theory can be applied to investigate the stability of holonomic quantum computations** (The influence of the classical control errors on holonomic quantum computations. Fidelity of HQC, Wilson loop and non-Abelian Stokes theorem, robust Hadamard gate for HQC)

Chaos assisted tunneling. Instanton approach

- Chaos in classical system gives acceleration (Lin, Ballentine, PRL, 1990) or slowing down (Grossman et al, PRL, 1991) the process of tunneling up to several order of magnitude (CAT)
  Ex: (Quantum tunneling and chaos in a driven anharmonic oscillator. The Husimi distribution (quasiprobability distribution commonly used in quantum mechanics to represent the phase space distribution of a quantum state such as light in the phase space formulation) is computed for a particle in a double-well potential and an oscillatory driving force. The extended phase space of the classical system contains two disjoint stable tubes of regular orbits, embedded in a chaotic sea
- For the quantum system we find coherent oscillatory tunneling between these stability tubes, at a rate many orders of magnitude greater than the rate of ordinary undriven tunneling).

➢ Both phenomena are seen at experiments: C. Dembowski et al., Phys. Rev. Lett. 84, 867 (2000),

D. A. Steck et al., Science 293, 274 (2001), W. K. Hensinger et al., Nature 412, 52 (2001).

Approaches to chaos assisted tunneling (CAT)

- **Numerical methods based on Floquet theory (Floquet theory is a branch of the theory of ordinary differential equations relating to the class of solutions to periodic linear differential equations)**
- **Three-level model for chaos assisted tunneling.**
- **Path integral approach for billiard systems.**
- **Quantum mechanical amplitudes in complex configuration space.**
- **Approach based on instanton technique**

V.I. Kuvshinov, A.V. Kuzmin, R.G. Shulyakovsky, Phys.Rev.E vol. 67 (2003) 015201(R)-4;

V.I. Kuvshinov, A.V. Kuzmin, Progr. Theor. Phys. Suppl. No.150 (2003) pp.363-366;

V.I. Kuvshinov, A.V. Kuzmin, R.G. Shulyakovsky, Acta Phys. Pol.B vol. 33 (2002) pp.1721-1728

V.I. Kuvshinov, A.V. Kuzmin, PEPAN vol.36, No.1 (2005) p.183-244 – in Russian.

V. Kuvshinov, A. Kuzmin. Gauge Fields and Theory of Determenistic Chaos (Belorussian Science, Minsk, 2006, p. 1-268 in Russian).

Andrea Addazi Chaotic instantons in scalar field theory arXiv:1607.08360v1 [hep-th] 28 Jul 2016

Chaotic Instantons

(Kuvshinov, Kuzmin, PR, 2003)

- (The influence of chaos on properties of dilute instanton gas in quantum mechanics.is studied. We demonstrate on the example of one-dimensional periodic potential that small perturbation leading to chaos squeezes instanton gas and increases the rate of instanton tunnelling).

- Chaotic instanton is the solution of the Euclidean equations of motion of the perturbed system. This configuration is responsible for the enhancement of tunneling

$$\overline{H} = \frac{1}{2}\overline{p}^2 + \omega_0^2 \cos x - \varepsilon x \sum_{n=-\infty}^{+\infty} \delta(t - n\overline{T})$$

The systems with spatially periodic potential are well-studied in solid-state physics (Kittel, 1959)and instanton physics (Rajaraman,1982) Perturbation used in was widely exploited in the systems exhibiting quantum chaos (Berman and, Zaslavsky,1982)

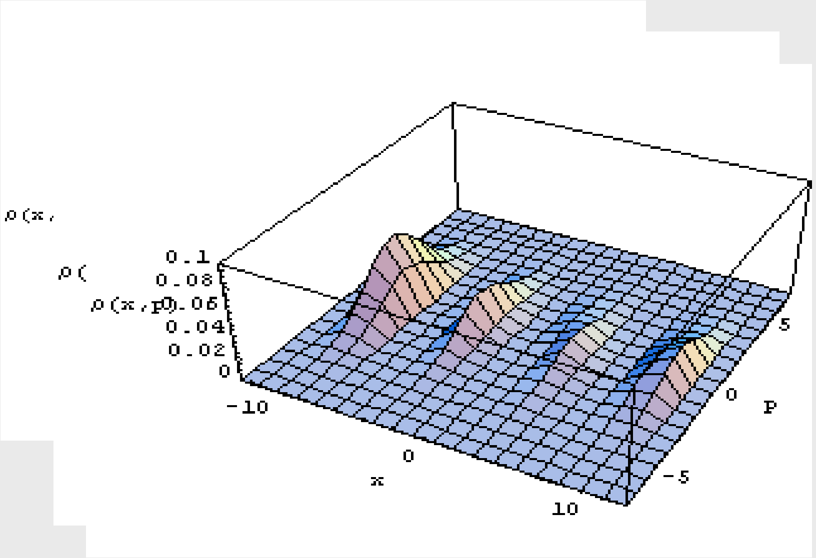
Dynamical tunneling amplitude with the contribution of the chaotic instanton solutions

$$A = N\int_0^{\Delta H} d\xi \int_{-\infty}^{+\infty} dc_0 \sqrt{S[x^{ins}(\tau,\xi)]}\exp(-S[x^{ins}(\tau,\xi)]) \approx N\sqrt{8\omega_0}\Gamma e^{-8\omega_0} \exp\left(\frac{\pi\Delta H}{\omega_0}\right)$$

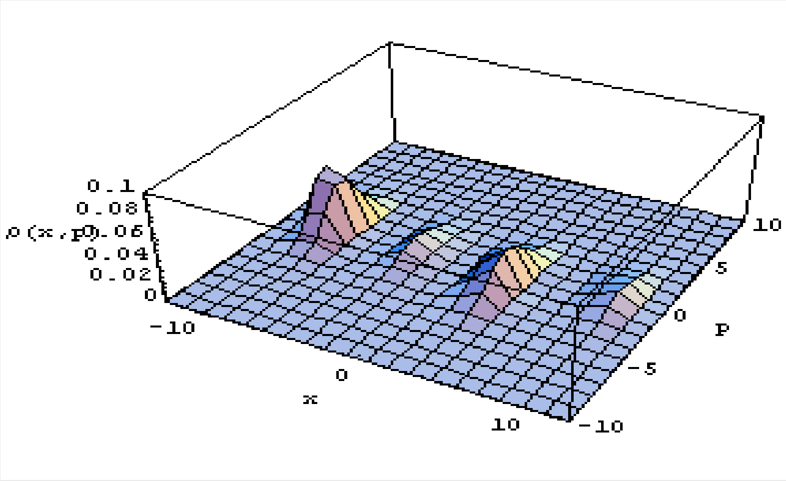
Numerical simulation of dynamical tunneling properties

Small perturbation leading to chaos can essentially enhance the tunnelling rate in comparison with non-perturbed system

Wave packet after the time interval T with no perturbation



Wave packet after the time interval T/13.7 when the perturbation acts



Conclusion

- Interactions of quantum system with the environment can be effectively considered in terms of density when density matrix of the system is obtained by averaging of the joint density matrix with respect to the degrees of freedom of environment
- Vacuum of quantum chromodynamics can be considered as environment for colour particles
- In the case of stochastic (not coherent) QCD vacuum (only correlators of the second order are important) in confinement region (Wilson loop decays exponentially) we have decoherence of pure colour states into a mixed white states with purity which decays exponentially (decay rate =string tension)
- Density matrix, Purity and Fidelity for colour particles are depended on Wilson loop averaged over QCD vacuum degrees of freedom
- For multiparticle states (pure separable, mixed separable and nonsepaparable (entangled) when $RT \to \infty$ we obtain diagonalization of density matrix, decreasing of purity and fidelity, increasing of Von Neumann entropy
- Possibility of Squeezed and Entangled States in QCD
- Instability of Movement in QCD
- Chaotic Instanton and Acceleration of Tunneling by E[ternal Perturbation

Some current and perspective studies on the subject

- Interaction of ms's, environment, apparatus -> hadronization
- Deconfinement, interaction of ms's with quarks, hadrons and QGP as environment at different QGP stages
- Interactions of quarks and ms's with stochastic vacuum in multiquarks
- Clarify connection with "pointer basis"
- Search of SS and ES
- Instability in QCD
- Applications of chaotic instantons to nuclear physics: fission and fusion nuclei and particles under external perturbations

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Thank you for the attention!