Spin as an intrinsic form of top angular momentum

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Guide line of my talk

The Standard Model provides an excellent description of what goes on in the subatomic world. But we begin to get into trouble when we ask the question of why the Standard Model has the features that it does. For example, it holds that there are three different types of fermions: the electron, muon and tau. Why three? Why not two, or four, or 15? The Standard Model does not say; we need to explore a deeper level of nature to discover the answer.(I cite Joseph Lykken and Maria Spiropulu, Scientific American, May 2014)

I would like to represent some physical evidences of the existence of this deeper level. The dual mathematical aspects can be found in the references that will be shown at the end.

Spin as the intrinsic top

The idea of Ralph Kronig, George Uhlenbeck and Samuel Goudsmit of particles spinning around their own axis is correct so far as spin obeys the same mathematical laws as quantized angular momentum of symmetrical top do.

The rotational structure of a symmetrical top is defined by the well known commutation relations

$$[T_j,\, T_k] = i\varepsilon_{jkl}T_l, \tag{1}$$

$$[\widetilde{T}_j,\, \widetilde{T}_k] = i\varepsilon_{jkl}\widetilde{T}_l, \quad [T_j,\, \widetilde{T}_k] = 0. \tag{2}$$

An intrinsic form of these commutation relations looks like as follows :

$$[S_j,\, S_k] = i\varepsilon_{jkl}S_l, \quad S_1^2 + S_2^2 + S_3^2 = s(s+1) = \frac{3}{4}, \tag{3}$$

$$[\widetilde{S}_j,\, \widetilde{S}_k] = i\varepsilon_{jkl}\widetilde{S}_l, \quad \widetilde{S}_1^2 + \widetilde{S}_2^2 + \widetilde{S}_3^2 = \frac{3}{4}, \quad [S_j,\, \widetilde{S}_k] = 0. \tag{4}$$

On the fundamental (field - theoretical) level the idea of spin as the intrinsic form of symmetric top angular momentum was not realized during the period of creation of quantum mechanics. And now, in quantum mechanics and particle physics, spin is considered as an intrinsic form of orbital angular momentum (The Pauli-Dirac spin) with commutation relations analogous to those of the orbital angular momentum:

$$[S_j,\, S_k] = i\varepsilon_{jkl}S_l, \quad S_1^2 + S_2^2 + S_3^2 = s(s+1) = \frac{3}{4}.$$

However, It can be shown that internal symmetry of intrinsic top can be realized on a set of the simplest geometrical quantities, which themselves do not exhibit this property. Hence, spin as the intrinsic form of symmetrical top angular momentum is emergent property of the system of fields and as such it will be called emergent spin. The concept of emergent spin is more general than the concept of Pauli-Dirac spin and, hence, it provides the more deeper level which we have possibility to use and learn.

Being realized the Lie algebra (3),(4) of internal symmetry defines intrinsic top. A field which carries emergent spin and appropriate internal symmetry will be called the spin field.

Duality of time

To exhibit the different levels of organization of matter in quantum mechanics from the presented above point of view we also need to say few words about the nature of time.

A natural geometry is defined by a set of real numbers R. A point of this geometry is defined as a 4-tuple of real numbers

$$x = (x^1,\, x^2,\, x^3,\, x^4).$$

More complicated geometries may be constructed on the basis of natural geometry. All points of natural geometry consist a mathematical space R^4. The variables x^1, x^2, x^3, x^4 are considered on equal footing. Hence, it is unclear how to introduce the so-called space coordinates and time coordinate (a space-time structure) in the framework of a mathematical space alone. To do this we need to define a moment of time. Definition: a moment of time is a number that we put in correspondence to any point of mathematical space. More exactly a moment of time is defined by the equation

$$t = f(x) = f(x^1, x^2, x^3, x^4),$$

where function $f(x)$ is a solution to the equations

$$(\nabla f)^2 = \Big(\frac{\partial f}{\partial x^1}\Big)^2 + \Big(\frac{\partial f}{\partial x^2}\Big)^2 + \Big(\frac{\partial f}{\partial x^3}\Big)^2 + \Big(\frac{\partial f}{\partial x^4}\Big)^2 = 1, \quad f(\lambda x^1, \lambda x^2, \lambda x^3, \lambda x^4) = \lambda f(x^1, x^2, x^3, x^4).$$

These equations have two solutions

$$f(x) = a_i x^i = a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4,$$

where $\mathbf{a} = (a^1,\, a^2,\, a^3,\, a^4)$ is a unit constant vector $(\mathbf{a}|\mathbf{a}) = 1$, and

$$f(x) = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2}.$$

All points of the mathematical space that correspond to the same moment of time consist a physical space. From the equations

$$f(x) = a_i x^i = a_1 x^1 +, a_2 x^2 + a_3 x^3 + a_4 x^4 = t = constant,$$

and

$$f(x) = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2} = \tau = constant$$

we see that in one case a physical space is euclidian one and in the another case a physical space is three dimensional sphere S^3. It can be shown that a point particle on the three dimensional sphere S^3 has essential properties of a symmetrical top.

We see that there are two space-time structure (two different times) on the same mathematical space. With this duality of time and physical space, it is natural to put forward idea of dual approach to the world of elementary particles which can explain the existence of leptons and quarks, lepton-quark symmetry and confinement.

Equations of the spin field

A system of field which represent spin as its emergent property consists from scalar field $a(x)$, covariant vector field $a_i(x)$, and antisymmetric covariant tensor fields $a_{ij}(x)$, $a_{ijk}(x)$ and $a_{ijkl}(x)$, which can be arranged as a tuple

$$\mathbf{A} = (a, a_i, a_{ij}, a_{ijk}, a_{ijkl}), \quad i, j, k, l = 1, 2, 3, 4. \tag{5}$$

In the simplest case of usual time and space equations of the spin field can be written in the following form

$$\frac{\partial \kappa}{\partial t} = \mathrm{div}\,\mathbf{K} - m\,\mu; \quad \frac{\partial \lambda}{\partial t} = \mathrm{div}\,\mathbf{L} - m\,\nu$$
$$\frac{\partial \mu}{\partial t} = \mathrm{div}\,\mathbf{M} + m\,\kappa; \quad \frac{\partial \nu}{\partial t} = \mathrm{div}\,\mathbf{N} + m\,\lambda$$

$$\frac{\partial \mathbf{K}}{\partial t} = -\mathrm{rot}\,\mathbf{L} + \mathrm{grad}\,\kappa + m\,\mathbf{M}$$
$$\frac{\partial \mathbf{L}}{\partial t} = \mathrm{rot}\,\mathbf{K} + \mathrm{grad}\,\lambda + m\,\mathbf{N}$$
$$\frac{\partial \mathbf{M}}{\partial t} = \mathrm{rot}\,\mathbf{N} + \mathrm{grad}\,\mu - m\,\mathbf{K}$$
$$\frac{\partial \mathbf{N}}{\partial t} = -\mathrm{rot}\,\mathbf{M} + \mathrm{grad}\,\nu - m\,\mathbf{L}.$$

On different levels of organization of matter in the microphysics

In accordance with the duality of time we distinguish the four level of organization of matter on the quantum level.

I)The real spin field and usual time. Particle of this field carries electric charge and is characterized new interaction, which explains "pairing" of electron in the physics of atoms and molecules.

II) The real spin field and the dual time. The question about physical interpretation under consideration.

III)The complex spin field and usual time. Particle of this field carries pseudocharge, electric charge, neutrino charge. This level of organization corresponds to the electroweak sector of SM.

IV) A complex emergent spin field and dual time. Particle of this field carries pseudocharge, electric charge, neutrino charge. This level of organization corresponds to the physics of strong interactions.

Second time naturally explains the confinement and the baryon number conservation. Indeed, we cannot invoke the artificial concept of force to explain the confinement because for any force there is a more powerful one. But the dual time and compact physical space do this naturally. The baryon number conservation simply expresses in the symbolic form the existence of the second time and nothing else. The quark - lepton symmetry means that leptons live in the area of the usual time and quarks live in the area of the dual time

Algebra of the operators of electric charge Q_e and neutrino charge Q_ν is very simple

$$Q_e^2 = -1, \quad Q_\nu^2 = -1, \quad Q_e Q_\nu + Q_\nu Q_e = 0.$$

From this algebra one can easy derive that the observed generations of quarks and leptons of SM can be considered as different states of the complex emergent spin field and the number of these generations equals four. In deed, let us introduce projection operators

$$P_+ = \frac{1}{2}(1 + iQ_e), \quad P_- = \frac{1}{2}(1 - iQ_e)$$

$$R_+ = \frac{1}{2}(1 + iQ_\nu), \quad R_- = \frac{1}{2}(1 - iQ_\nu)$$

We have

$$1 = P_+ + P_- = P_+R_+ + P_+R_- + P_-R_+ + P_-R_-.$$

This algebra also explains the so-called "pairing" of the electron and neutrino in the SM.

References

[1] I.B. Pestov.(1998). *Spin Properties of Four-Dimentional Space and Confinement.* Physics of Atomic Nuclei, **61(11)**: 1-5.

[2] I.B. Pestov.(2005). *Field Theory and the Essence of Time*, Horizons in World Physics, **248**, Chapter 1, New York: Nova Science.

[3] I.B. Pestov.(1978). *Relativistic equations defined by the operator of the exterior derivative and generalized divergance.* Theor. and Math. Phys. **34(1)**:30-36.

[4] I.B. Pestov.(2011).*Geometrical spin symmetry and spin.* Physics of Atomic Nuclei, **74(7)**: 1055-1061.

[5] I.B. Pestov.(2013). *Geometrization of the electromagnetic field and dark matter.* Physics of Particles and Nuclei, **44(3)**: 442-449.

[6] I.B. Pestov.(2015). *Foundation of a unified physics.*J.Phys.Math. **6:135**: 1-6. doi: 10.4172/2090-0902.1000135.

[7] I.B. Pestov.(2017).*Foundation of a unified physics.* Eur.Phys.J. Web of Conf. **138**, 08008:1-7 . doi: 10.1051/epjconf/201713808008.

[8] Ivanhoe B. Pestov.(2017). *Emergent Spin and New Physics*. J.Phys: Conf.Series **938** 012034:1-4. doi: 10.1088/1742-6596/938/1/012034.

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