Quantum spin dynamics in external classical fields

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Outline

Dynamics of spin and equivalence principle

- High-energy experiments take place in curved space or in noninertial frame (for example, on Earth)
- Equivalence principle (EP) - a cornerstone of gravity
- Newton's theory $\Rightarrow$ Einstein's "falling elevator"
- Colella-Overhauser-Werner (1975) and Bonse-Wroblewski experiments - EP for quantum-mechanical systems:
- Measured phase shift due to inertial and gravitational force
- Gravity on *spin*: EP for relativistic particles?
- Classical theory of spin: Frenkel (1928), Mathisson (1937), Papapetrou (1951), Weyssenhoff-Raabe (1947)
- Compare classical rotator and quantum spin
- Measure spin effects to probe spacetime geometry!

Arbitrary Riemannian geometry in 4 dimensions

- Let t be time, x^a $(a = 1, 2, 3)$ be spatial coordinates:

$$ds^2 = V^2 c^2 dt^2 - \delta_{\widehat{a}\widehat{b}} W^{\widehat{a}}{}_c W^{\widehat{b}}{}_d \left(dx^c - K^c c dt\right)\left(dx^d - K^d c dt\right)$$

V and K^a, and 3×3 matrix $W^{\widehat{a}}{}_b$ depend arbitrarily on t, x^a.

- Their number $1 + 3 + 9 = 13$ but rotation $W^{\widehat{a}}{}_b \longrightarrow L^{\widehat{a}}{}_{\widehat{c}} W^{\widehat{c}}{}_b$ is allowed with arbitrary $L^{\widehat{a}}{}_{\widehat{c}}(t, x) \in SO(3)$: $\Longrightarrow 13 - 3 = 10$
- Coframe e_i^α with $g_{\alpha\beta} e_i^\alpha e_j^\beta = g_{ij}$, $g_{\alpha\beta} = \mathrm{diag}(c^2, -1, -1, -1)$:

$$e_i^{\widehat{0}} = V\, \delta_i^0, \qquad e_i^{\widehat{a}} = W^{\widehat{a}}{}_b \left(\delta_i^b - cK^b\, \delta_i^0\right), \qquad a = 1, 2, 3$$

- Exact metric of flat spacetime in noninertial frame

$$V = 1 + \frac{\boldsymbol{a} \cdot \boldsymbol{r}}{c^2}, \quad W^{\widehat{a}}{}_b = \delta_b^a, \quad K^a = -\frac{1}{c}\left(\boldsymbol{\omega} \times \boldsymbol{r}\right)^a$$

Dirac particle in gravitational & electromagnetic field

- Fermion with moments (AMM $\mu' = \frac{(g-2)e\hbar}{4m}$ & EDM $\delta' = \frac{be\hbar}{2mc}$)

$$\left(i\hbar\gamma^\alpha D_\alpha - mc + \frac{\mu'}{2c}\sigma^{\alpha\beta}F_{\alpha\beta} + \frac{\delta'}{2}\sigma^{\alpha\beta}G_{\alpha\beta}\right)\psi = 0$$

- Spinor covariant derivative (with $\sigma_{\alpha\beta} = i\gamma_{[\alpha}\gamma_{\beta]}$)

$$D_\alpha = e^i_\alpha D_i, \qquad D_i = \partial_i - \frac{ie}{\hbar}A_i + \frac{i}{4}\sigma^{\alpha\beta}\Gamma_{i\,\alpha\beta}$$

- Connection for general spacetime geometry

$$\Gamma_{i\,\widehat{a}\widehat{0}} = \frac{c^2}{V}W^b{}_{\widehat{a}}\,\partial_b V\, e_i{}^{\widehat{0}} - \frac{c}{V}\,\mathcal{Q}_{(\widehat{a}\widehat{b})}\, e_i{}^{\widehat{b}},$$

$$\Gamma_{i\,\widehat{a}\widehat{b}} = \frac{c}{V}\,\mathcal{Q}_{[\widehat{a}\widehat{b}]}\, e_i{}^{\widehat{0}} + \left(\mathcal{C}_{\widehat{a}\widehat{b}\widehat{c}} + \mathcal{C}_{\widehat{a}\widehat{c}\widehat{b}} + \mathcal{C}_{\widehat{c}\widehat{b}\widehat{a}}\right) e_i{}^{\widehat{c}}$$

- Here anholonomity $\mathcal{C}_{\widehat{a}\widehat{b}}{}^{\widehat{c}} = W^d{}_{\widehat{a}}W^e{}_{\widehat{b}}\,\partial_{[d}W^{\widehat{c}}{}_{e]}$ and

$$\mathcal{Q}_{\widehat{a}\widehat{b}} = g_{\widehat{a}\widehat{c}}W^d{}_{\widehat{b}}\left(\frac{1}{c}\dot{W}^{\widehat{c}}{}_d + K^e\partial_e W^{\widehat{c}}{}_d + W^{\widehat{c}}{}_e\partial_d K^e\right)$$

Dirac Hamiltonian

- Naive Hamiltonian is not Hermitian. Rescale wave function $\psi \longrightarrow \left(\sqrt{-g}e_{\widehat{0}}^{0}\right)^{\frac{1}{2}} \psi$ and recast Dirac wave equation into Schrodinger form $i\hbar\frac{\partial\psi}{\partial t} = \mathcal{H}\psi$

Dirac Hamiltonian (with $\mathcal{F}^b{}_a = VW^b{}_{\widehat{a}}$ and $\boldsymbol{\pi} = -i\hbar\boldsymbol{\nabla} - e\boldsymbol{A}$)

$$\mathcal{H} = \beta mc^2 V + e\Phi + \frac{c}{2}\left(\pi_b\,\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b\right) + \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\left(\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma} - \Upsilon\gamma_5\right) - \beta V\left(\boldsymbol{\Sigma}\cdot\boldsymbol{\mathcal{M}} + i\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{P}}\right)$$

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- Here $\beta = \gamma^{\widehat{0}}$, $\alpha^a = \gamma^{\widehat{0}}\gamma^{\widehat{a}}$, $\gamma_5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, $\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$,

$$\Upsilon = V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{a}\widehat{b}\widehat{c}} = -V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\mathcal{C}_{\widehat{a}\widehat{b}\widehat{c}}, \qquad \Xi_{\widehat{a}} = \frac{V}{c}\,\epsilon_{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{0}}{}^{\widehat{b}\widehat{c}} = \epsilon_{\widehat{a}\widehat{b}\widehat{c}}\,\mathcal{Q}^{\widehat{b}\widehat{c}}$$

Electrodynamics in curved spacetime

- Gravity is universal: affects also electromagnetism. How?
- Basic objects: field strength F, excitation H and current J

Maxwell's theory – without coordinates and frames

$$dF = 0, \qquad dH = J, \qquad H = \lambda_0 \star F, \qquad \lambda_0 = \sqrt{\varepsilon_0/\mu_0}$$

- Coordinates x^i: $F = \frac{1}{2}F_{ij}dx^i \wedge dx^j$, $H = \frac{1}{2}H_{ij}dx^i \wedge dx^j$, and $J = \frac{1}{6}J_{ijk}dx^i \wedge dx^j \wedge dx^k$ are $(1+3)$ decomposed:

$$\boldsymbol{E}_a = \{F_{10}, F_{20}, F_{30}\}, \qquad \boldsymbol{B}^a = \{F_{23}, F_{31}, F_{12}\}$$

$$\boldsymbol{H}_a = \{H_{01}, H_{02}, H_{03}\}, \qquad \boldsymbol{D}^a = \{H_{23}, H_{31}, H_{12}\}$$

-
$$\boldsymbol{J}^a = \{-J_{023}, -J_{031}, -J_{012}\}, \qquad \rho = J_{123}$$

- Maxwell equations are recast into standard form

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \dot{\boldsymbol{B}} = 0, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0,$$

$$\boldsymbol{\nabla} \times \boldsymbol{H} - \dot{\boldsymbol{D}} = \boldsymbol{J}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho$$

- Gravity/inertia encoded in *constitutive relation* $H = H(F)$

$$D^a = \frac{\varepsilon_0 w}{V}\,\underline{g}^{ab}E_b - \lambda_0\frac{w}{V}\,\underline{g}^{ad}\epsilon_{bcd}K^c\,B^b,$$

$$H_a = \frac{1}{\mu_0 w V}\left\{(V^2 - K^2)\underline{g}_{ab} + K_aK_b\right\}B^b - \lambda_0\frac{w}{V}\,\epsilon_{adc}K^c\,\underline{g}^{db}E_b$$

Here $K_a = \underline{g}_{ab}K^b$, $K^2 = \underline{g}_{ab}K^aK^b$ and $w = \det W^{\widehat{c}}{}_d$.

- Frame e_i^α needed for fermions $\Longrightarrow F_{\alpha\beta} = e^i_\alpha e^j_\beta F_{ij}$
- Components: $\mathfrak{E}_a = \{F_{\widehat{1}\widehat{0}}, F_{\widehat{2}\widehat{0}}, F_{\widehat{3}\widehat{0}}\}$ & $\mathfrak{B}^a = \{F_{\widehat{2}\widehat{3}}, F_{\widehat{3}\widehat{1}}, F_{\widehat{1}\widehat{2}}\}$
- Relation between holonomic and anholonomic fields

$$\mathfrak{E}_a = \frac{1}{V}\,W^b{}_{\widehat{a}}(\boldsymbol{E} + c\boldsymbol{K}\times\boldsymbol{B})_b, \quad \mathfrak{B}^a = \frac{1}{w}\,W^{\widehat{a}}{}_b\,\boldsymbol{B}^b$$

- Nonminimal coupling $-\beta V\left(\boldsymbol{\Sigma}\cdot\boldsymbol{\mathcal{M}} + i\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{P}}\right)$ governed by

$$\mathcal{M}^a = \mu'\mathfrak{B}^a + \delta'\mathfrak{E}^a, \qquad \mathcal{P}_a = c\delta'\mathfrak{B}_a - \mu'\mathfrak{E}_a/c.$$

- Foldy-Wouthuysen transformation needed to reveal physics (uncouple positive and negative energy states)
- Recast generic Hamiltonian into

$$\mathcal{H} = \beta\mathfrak{M} + \mathcal{E} + \mathcal{O}, \quad \beta\mathfrak{M} = \mathfrak{M}\beta, \quad \beta\mathcal{E} = \mathcal{E}\beta, \quad \beta\mathcal{O} = -\mathcal{O}\beta$$

- Foldy-Wouthuysen unitary transformation

$$\psi_{FW} = U\psi, \qquad \mathcal{H}_{FW} = U\mathcal{H}U^{-1} - i\hbar U\partial_t U^{-1}$$

- In arbitrary external fields (with $\epsilon = \sqrt{\mathfrak{M}^2 + \mathcal{O}^2}$)

$$U = \frac{\beta\epsilon + \beta\mathfrak{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathfrak{M} - \mathcal{O})^2}}\,\beta$$

- For Dirac fermion we have explicitly: $\mathfrak{M} = mc^2 V$ and

$$\begin{aligned}\mathcal{E} &= e\Phi + \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\,\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma} - \beta V\boldsymbol{\Sigma}\cdot\boldsymbol{\mathcal{M}},\\ \mathcal{O} &= \frac{c}{2}\left(\pi_b\,\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b\right) - \frac{\hbar c}{4}\,\Upsilon\gamma_5 - i\beta V\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{P}}\end{aligned}$$

- FW Hamiltonian $\mathcal{H}_{FW} = \mathcal{H}_{FW}^{(1)} + \mathcal{H}_{FW}^{(2)} + \mathcal{H}_{FW}^{(3)} + \mathcal{H}_{FW}^{(4)}$:

$$\mathcal{H}_{FW}^{(1)} = \beta\epsilon' + \frac{\hbar c^2}{16}\left\{\frac{1}{\epsilon'}, \left(2\epsilon^{cae}\Pi_e\{\pi_b, \mathcal{F}^d{}_c\partial_d\mathcal{F}^b{}_a\} + \Pi^a\{\pi_b, \mathcal{F}^b{}_a\Upsilon\}\right)\right\}$$
$$+ \frac{\hbar m c^4}{4}\epsilon^{cae}\Pi_e\left\{\frac{1}{\mathcal{T}}, \left\{\pi_d, \mathcal{F}^d{}_c\mathcal{F}^b{}_a\partial_b V\right\}\right\},$$

$$\mathcal{H}_{FW}^{(2)} = \frac{c}{2}\left(K^a\pi_a + \pi_a K^a\right) + \frac{\hbar c}{4}\Sigma_a\Xi^a + \frac{\hbar c^2}{16}\left\{\frac{1}{\mathcal{T}}, \left\{\Sigma_a\{\pi_e, \mathcal{F}^e{}_b\}, \left\{\pi_f, \left[\epsilon^{abc}\times\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\times\left(\frac{1}{c}\dot{\mathcal{F}}^f{}_c - \mathcal{F}^d{}_c\partial_d K^f + K^d\partial_d\mathcal{F}^f{}_c\right) - \frac{1}{2}\mathcal{F}^f{}_d\left(\delta^{db}\Xi^a - \delta^{da}\Xi^b\right)\right]\right\}\right\}\right\},$$

$$\mathcal{H}_{FW}^{(3)} = e\Phi - \frac{e\hbar c^2}{4}\left\{\frac{1}{\epsilon'}, V^2\Pi^a\mathfrak{B}_a\right\} - \frac{e\hbar c^2}{8}\left\{\frac{1}{\mathcal{T}}, \left[2\hbar\mathcal{F}^b{}_a\partial_b(V^2\mathfrak{E}^a)\right.\right.$$
$$\left.\left.- \Sigma_a\epsilon^{abc}\left(\{\mathcal{F}^d{}_b, \pi_d\}V^2\mathfrak{E}_c - V^2\mathfrak{E}_b\{\mathcal{F}^d{}_c, \pi_d\}\right)\right]\right\},$$

$$\mathcal{H}_{FW}^{(4)} = -\frac{c}{8}\left\{\frac{1}{\epsilon'}, \left[\Sigma_a\epsilon^{abc}\left(\{\mathcal{F}^d{}_b, \pi_d\}V\mathcal{P}_c - V\mathcal{P}_b\{\mathcal{F}^d{}_c, \pi_d\}\right) - 2\hbar\mathcal{F}^b{}_a\partial_b(V\mathcal{P}^a)\right]\right\}$$
$$-V\,\Pi^a\mathcal{M}_a + \frac{c^2}{4}\left\{\frac{1}{\mathcal{T}}, \left(\Pi^a\{\{\mathcal{F}^c{}_a\mathcal{F}^d{}_b V\mathcal{M}^b, \pi_c\}, \pi_d\} + \beta\hbar\left\{\mathcal{F}^b{}_a[\mathcal{J}^a + K^c\partial_c(V\mathcal{P}^a)], \pi_b\right\}\right)\right\}$$

- Here $\{\ ,\ \}$ anticommutators, $\mathcal{T} = 2{\epsilon'}^2 + \{\epsilon', mc^2 V\}$, $\mathbf{\Pi} = \beta\mathbf{\Sigma}$,
$\epsilon' = \sqrt{m^2c^4V^2 + \frac{c^2}{4}\delta^{ac}\{\pi_b, \mathcal{F}^b{}_a\}\{\pi_d, \mathcal{F}^d{}_c\}}$, $\mathcal{J}^a = \epsilon^{abc}\mathcal{F}^d{}_b\partial_d(V\mathcal{M}_c) + \frac{\partial\mathcal{P}^a}{c\partial t}$
- *This result is exact* – no (weak field etc) approximations for $V, W^{\widehat{a}}{}_b, K^a$.

Quantum dynamics of spinning particle

- Evolution of spin (polarization operator $\boldsymbol{\Pi} = \beta\boldsymbol{\Sigma}$)

$$\frac{d\boldsymbol{\Pi}}{dt} = \frac{i}{\hbar}[\mathcal{H}_{FW}, \boldsymbol{\Pi}] = \boldsymbol{\Omega}_{(1)} \times \boldsymbol{\Sigma} + \boldsymbol{\Omega}_{(2)} \times \boldsymbol{\Pi}$$

- Semiclassical precession velocity of spin

$$\begin{aligned} \Omega^a_{(1)} &= \tfrac{c^2}{\epsilon}\mathcal{F}^d{}_c\pi_d\Big(\tfrac{1}{2}\Upsilon\delta^{ac} - \epsilon^{akl}V\mathcal{C}_{kl}{}^c + \tfrac{\epsilon}{\epsilon+mc^2V}\epsilon^{abc}W^k{}_{\widehat{b}}\,\partial_d V \\ &\quad + \tfrac{eV^2}{\epsilon'+mc^2V}\,\epsilon^{acb}\mathfrak{E}_b - \tfrac{2V}{c\hbar}\epsilon^{acb}\mathcal{P}_b\Big) \\ \Omega^a_{(2)} &= \tfrac{c}{2}\,\Xi^a - \tfrac{c^3}{\epsilon(\epsilon+mc^2V)}\,\epsilon^{abc}Q_{(bd)}\delta^{dn}\mathcal{F}^k{}_n\pi_k\mathcal{F}^l{}_c\pi_l \\ &\quad - \tfrac{ec^2V^2}{\epsilon}\mathfrak{B}^a + \tfrac{2V}{\hbar}\Big(-\mathcal{M}^a + \tfrac{c^2}{\epsilon(\epsilon+mc^2V)}\delta^{an}\mathcal{F}^c{}_n\pi_c\mathcal{F}^d{}_b\pi_d\mathcal{M}^b\Big) \end{aligned}$$

- Here $\epsilon = \sqrt{m^2c^4V^2 + c^2\delta^{cd}\mathcal{F}^a{}_c\,\mathcal{F}^b{}_d\,\pi_a\,\pi_b}$

Semiclassical FW Hamiltonian

$$\mathcal{H}_{FW} = \beta\epsilon + e\Phi + \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar}{2}\boldsymbol{\Pi}\cdot\boldsymbol{\Omega}_{(1)} + \frac{\hbar}{2}\boldsymbol{\Sigma}\cdot\boldsymbol{\Omega}_{(2)}$$

- Physical spin $\boldsymbol{s}$ precesses wrt rest frame: $\frac{d\boldsymbol{s}}{dt} = \boldsymbol{\Omega} \times \boldsymbol{s}$

Spin dynamics on Earth (with $\boldsymbol{g} = -\frac{GM}{r^3}\boldsymbol{r}$, $\gamma = 1/\sqrt{1 - v^2/c^2}$)

$$\begin{aligned}\boldsymbol{\Omega} &= \frac{e}{m}\left\{-\frac{1}{\gamma}\boldsymbol{\mathfrak{B}} + \frac{1}{\gamma+1}\frac{\boldsymbol{v}\times\boldsymbol{\mathfrak{E}}}{c^2}\right\} - \boldsymbol{\omega} + \frac{2\gamma+1}{\gamma+1}\frac{\boldsymbol{v}\times\boldsymbol{g}}{c^2} \\ &- \frac{2\mu'}{\hbar}\left\{\boldsymbol{\mathfrak{B}} - \frac{\boldsymbol{v}\times\boldsymbol{\mathfrak{E}}}{c^2} - \frac{\gamma}{\gamma+1}\boldsymbol{v}\frac{\boldsymbol{\mathfrak{B}}\cdot\boldsymbol{v}}{c^2}\right\} \\ &- \frac{2\delta'}{\hbar}\left\{\boldsymbol{\mathfrak{E}} + \boldsymbol{v}\times\boldsymbol{\mathfrak{B}} - \frac{\gamma}{\gamma+1}\boldsymbol{v}\frac{\boldsymbol{\mathfrak{E}}\cdot\boldsymbol{v}}{c^2}\right\}\end{aligned}$$

- Analysis of manifestations of terrestrial rotation and gravity in precision high-energy physics: *influence not negligible*
- E.g.: Earth's gravity produces same effect as deuteron's EDM of $\delta' = 1.5 \times 10^{-29}\ e{\cdot}$cm in planned dEDM experiment with magnetic focusing (AGS proposal EDM Collaboration)

Spin dynamics in a gravitational wave

- Gravitational and electromagnetic fields not superimposed but their action on fermion is combined in a nontrivial way $\Longrightarrow$ new prospects for detection of grav. wave effects?
- In coordinates (t, x, y, z), weak gravitational wave is

$$V = 1, \quad \boldsymbol{K} = 0, \quad W^{\widehat{a}}{}_{b} = \begin{pmatrix} 1 + w_{\oplus} & w_{\otimes} & 0 \\ w_{\otimes} & 1 - w_{\oplus} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

  Functions $w_{\otimes}(\varphi)$, $w_{\oplus}(\varphi)$ of phase $\varphi = \omega(t - \frac{z}{c})$, describe 2 polarizations of a plane wave with frequency ω along z
- Hamiltonian for fermion's spin in this spacetime reduces to

$$\mathcal{H}_{FW} = -\left(\mu_0 + \mu'\right) \boldsymbol{\Pi} \cdot \boldsymbol{\mathfrak{B}}$$

  Here Bohr's magneton $\mu_0 = \frac{e\hbar}{2m}$. Important observation: Anholonomic field $\mathfrak{B}^a = W^{\widehat{a}}{}_{b}\, \boldsymbol{B}^b$ bears "imprint" of the gravitational wave on applied magnetic field $\boldsymbol{B}$!

- Recall particle with magnetic moment in flat space (no gravity) in constant homogeneous magnetic field: spin polarized along/against applied field. Additional rotating (alternating) field in plane perpendicular to original field $\Longrightarrow$ spin flip: magnetic resonance phenomenon occurs
- Suppose $\boldsymbol{B} = (B_0, 0, 0)$ with B_0 =const, and $w_{\oplus} = 0$,

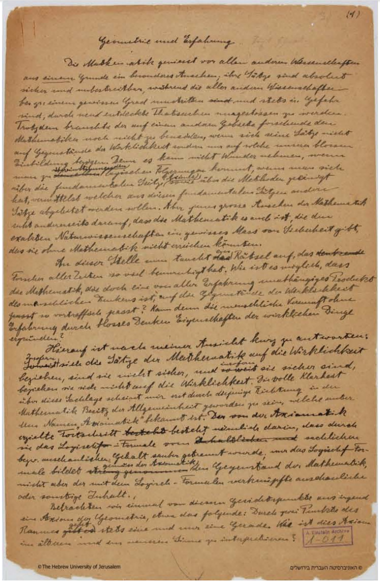
$$w_{\otimes} = g_0 \cos\varphi = g_0 \cos(\omega t - \omega z/c)$$

  describes wave with frequency ω and amplitude g_0 along z
- $\Longrightarrow \mathfrak{B} = (B_0, B_0 w_{\otimes}, 0)$, ie *magnetic resonance conditions*
- Probability to get at t spin oriented oppositely to initial at t_0

$$P_{-\frac{1}{2}} = \frac{\sin^2\{\omega_0 g_0 (t - t_0)\Lambda/4\}}{\Lambda^2}$$

- Here Larmor frequency $\omega_0 = 2(\mu_0 + \mu')B_0/\hbar$, and

$$\Lambda^2 = 1 + \frac{4(1-\xi)^2}{g_0^2}, \quad \xi = \frac{\omega}{\omega_0}\left(1 - \frac{g_0^2}{16\xi^2}\right)$$



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Geometrie und Erfahrung.

Die Mathematik geniesst vor allen anderen Wissenschaften aus einem Grunde ein besonderes Ansehen; ihre Sätze sind absolut sicher und unbestreitbar, während die aller andern Wissenschaften bis zu einem gewissen Grad umstritten ~~sind~~ und stets in Gefahr sind, durch neu entdeckte Thatsachen umgestossen zu werden. Trotzdem brauchte der auf einem andern Gebiete forschende den Mathematiker noch nicht zu beneiden, wenn sich seine Sätze nicht auf Gegenstände der Wirklichkeit sondern nur auf solche unserer blossen Einbildung bezögen. Denn es kann nicht Wunder nehmen, wenn man zu übereinstimmenden logischen Folgerungen kommt, wenn man sich über die fundamentalen Sätze (Axiome) über die Methoden geeinigt hat, vermittelst welcher aus diesen fundamentalen Sätzen andere Sätze abgeleitet werden sollen. Aber jenes grosse Ansehen der Mathematik ruht andererseits darauf, dass die Mathematik es auch ist, die den exakten Naturwissenschaften ein gewisses Mass von Sicherheit gibt, das sie ohne Mathematik nicht erreichen könnten.

An dieser Stelle nun taucht ein Rätsel auf, das Forscher aller Zeiten so viel beunruhigt hat. Wie ist es möglich, dass die Mathematik, die doch ein von aller Erfahrung unabhängiges Produkt des menschlichen Denkens ist, auf die Gegenstände der Wirklichkeit so vortrefflich passt? Kann denn die menschliche Vernunft ohne Erfahrung durch blosses Denken Eigenschaften der wirklichen Dinge ergründen?

Hierauf ist nach meiner Ansicht kurz zu antworten: Insofern sich die Sätze der Mathematik auf die Wirklichkeit beziehen, sind sie nicht sicher, und insofern sie sicher sind, beziehen sie sich nicht auf die Wirklichkeit. Die volle Klarheit über diese Sachlage scheint mir erst durch diejenige Richtung in der Mathematik Besitz der Allgemeinheit geworden zu sein, welche unter dem Namen „Axiomatik" bekannt ist. Der von der Axiomatik erzielte Fortschritt ~~besteht~~ besteht nämlich darin, dass durch sie das Logisch-Formale vom ~~inhaltlichen und~~ sachlichen bezw. anschaulichen Gehalt sauber getrennt wurde; nur das Logisch-Formale bildet, ~~streng genommen~~ gemäss der Axiomatik, den Gegenstand der Mathematik, nicht aber der mit dem Logisch-Formalen verknüpfte anschauliche oder sonstige Inhalt.

Betrachten wir einmal von diesem Gesichtspunkte aus irgend ein Axiom der Geometrie, etwa das folgende: Durch zwei Punkte des Raumes geht stets eine und nur eine Gerade. Wie ist dies Axiom im älteren und im neueren Sinne zu interpretieren?

A. Einstein Archive 1-011

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"...The question whether this continuum has a Euclidean, Riemannian, or any other structure is a question of physics proper which must be answered by experience, and not a question of a convention to be chosen on grounds of mere expediency."

A. Einstein, Geometrie und Erfahrung, Sitzungsber. preuss. Akad. Wiss. **1** (1921) 123-130.

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Experimental bounds on torsion

- To probe spacetime geometry: dynamics of spin

$$\frac{d\mathbf{\Pi}}{dt} = \frac{i}{\hbar}[\mathcal{H}_{FW}, \mathbf{\Pi}] = \mathbf{\Omega} \times \mathbf{\Pi}$$

- *Theory*: spin precession to probe torsion: Adamowicz (1975), Rumpf (1980), Audretsch (1981), Lämmerzahl (1997); review W.T.Ni, Rep.Prog.Phys. 73 (2010) 056901
- *Experiment*: effect of Earth's gravity on nuclear spins Hg
- Spin Hamiltonian (torsion $\check{T}^\alpha = \frac{1}{2}\eta^{\mu\nu\lambda\alpha}T_{\mu\nu\lambda}$, $\check{\boldsymbol{T}} = \{\check{T}^a\}$)

$$\mathcal{H}_{FW} = -g_N\mu_N \boldsymbol{B}\cdot\mathbf{\Pi} - \frac{\hbar}{2}\boldsymbol{\omega}\cdot\mathbf{\Sigma} - \frac{\hbar c}{4}\check{\boldsymbol{T}}\cdot\mathbf{\Sigma}.$$

- B.J. Venema et al, Phys. Rev. Lett. **68** (1992) 135

Limits on torsion from Zeeman energy levels measurements

- $$|\check{\boldsymbol{T}}| < 4.3 \times 10^{-14}\mathrm{m}^{-1}$$

- Recent: C. Gemmel et al, Phys. Rev. **D82** (2010) 111901

Conclusions and Outlook

- Searches for spin effects in gravity is fundamental issue. Overview of relevant laboratory experiments: Wei-Tou Ni, Rep. Prog. Phys. **73** (2010) 056901
- Theoretical framework of fermion spin dynamics developed [based on: Obukhov, Silenko, Teryaev, Phys. Rev. **D90** (2014) 124068; Phys. Rev. **D94** (2016) 044019; Phys. Rev. **D96** (2017) 105005] applicable to arbitrary strong and time-dependent gravitational, inertial *and* electromagnetic fields
- Exact Foldy-Wouthuysen transformation constructed
- Effects of terrestrial gravity and rotation non-negligible
- Influence of gravitational wave on spin possibly detectable in the framework of a magnetic resonance type setup
- Probing spacetime geometry: from nuclear spin dynamics obtained new limits on spacetime torsion $T < 10^{-14}\,\frac{1}{\mathrm{m}}$

Thanks !