XXIVth International Baldin Seminar on High Energy Physics Problems "Relativistic Nuclear Physics and Quantum Chromodynamics“, *Dubna*

Classical and quantum dynamics of twisted (vortex) electron beams in electric and magnetic fields

Alexander J. Silenko+++, Pengming Zhang+•, Liping Zou+•

+Research Institute for Nuclear Problems, BSU, Minsk, Belarus

+ Institute for Nuclear Research, Dubna, Russia

+ Institute of Modern Physics CAS, Lanzhou, China

• University of Chinese Academy of Sciences, Beijing, China

17 – 22 September, 2018

OUTLINE

- Twisted (vortex) particles
- From a pointlike Dirac particle to a centroid
- Twisted electrons in external electric and magnetic fields
- Manipulations of twisted electron beams
- Rotation of the intrinsic OAM in crossed electric and magnetic fields as a critical experiment
- Sokolov-Ternov-like effect of a radiative orbital polarization of twisted electron beams in a magnetic field
- Summary

Twisted electrons and their interactions with external fields and matter are considered in detail in the following recent reviews:

K. Y. Bliokh, I. P. Ivanov, G. Guzzinati, L. Clark, R. Van Boxem, A. Beche, R. Juchtmans, M. A. Alonso, P. Schattschneider, F. Nori, and J. Verbeeck, **Theory and applications of free-electron vortex states, Phys. Rep. 690,** 1 (2017).

S. M. Lloyd,M. Babiker, G. Thirunavukkarasu, and J. Yuan, Electron vortices: Beams with orbital angular momentum, Rev. Mod. Phys. 89, 035004 (2017).

H. Larocque, I. Kaminer, V. Grillo, G. Leuchs, M. J. Padgett, R. W. Boyd, M. Segev, E. Karimi, `Twisted' electrons, Contemp. Phys. 59, 126 (2018).

We base our explanations on our recent publications:

A. J. Silenko, Pengming Zhang and Liping Zou, Manipulating Twisted Electron Beams, Phys. Rev. Lett. 119, 243903 (2017);
A. J. Silenko, Pengming Zhang and Liping Zou, Relativistic Quantum Dynamics of Twisted Electron Beams in Arbitrary Electric and Magnetic Fields, Phys. Rev. Lett. 121, 043202 (2018).

Twisted (vortex) particles

Twisted particles are free particles which possess an intrinsic orbital angular momentum (OAM)

cylindrical coordinates $\mathbf{r}(\rho, \phi, z)$:

$$\psi_l(\mathbf{r}, t) = u(\rho, z)\exp(il\phi)\exp(ik_z z)\exp(-i\omega t).$$

$u(\rho, z)$: **a Laguerre-Gaussian function;**
a Bessel function of the first kind

Laguerre-Gaussian beam:

$$\psi_{\ell,n}^{LG} \propto \left(\frac{r}{w(z)}\right)^{|\ell|} L_n^{|\ell|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)} + ik\frac{r^2}{2R(z)}\right) e^{i(\ell\varphi+kz)} e^{-i(2n+|\ell|+1)\zeta(z)},$$

where $L_n^{|\ell|}$ are the generalized Laguerre polynomials, $n = 0, 1, 2, \ldots$ is the radial quantum number, $w(z) = w_0\sqrt{1 + z^2/z_R^2}$ is the beam width, which slowly varies with z due to diffraction, $R(z) = z\left(1 + z_R^2/z^2\right)$ is the radius of curvature of the wavefronts, and $\zeta(z) = \arctan(z/z_R)$. Here, the characteristic transverse and longitudinal scales of the beam are the waist w_0 (the width in the focal plane $z = 0$) and the Rayleigh diffraction length z_R

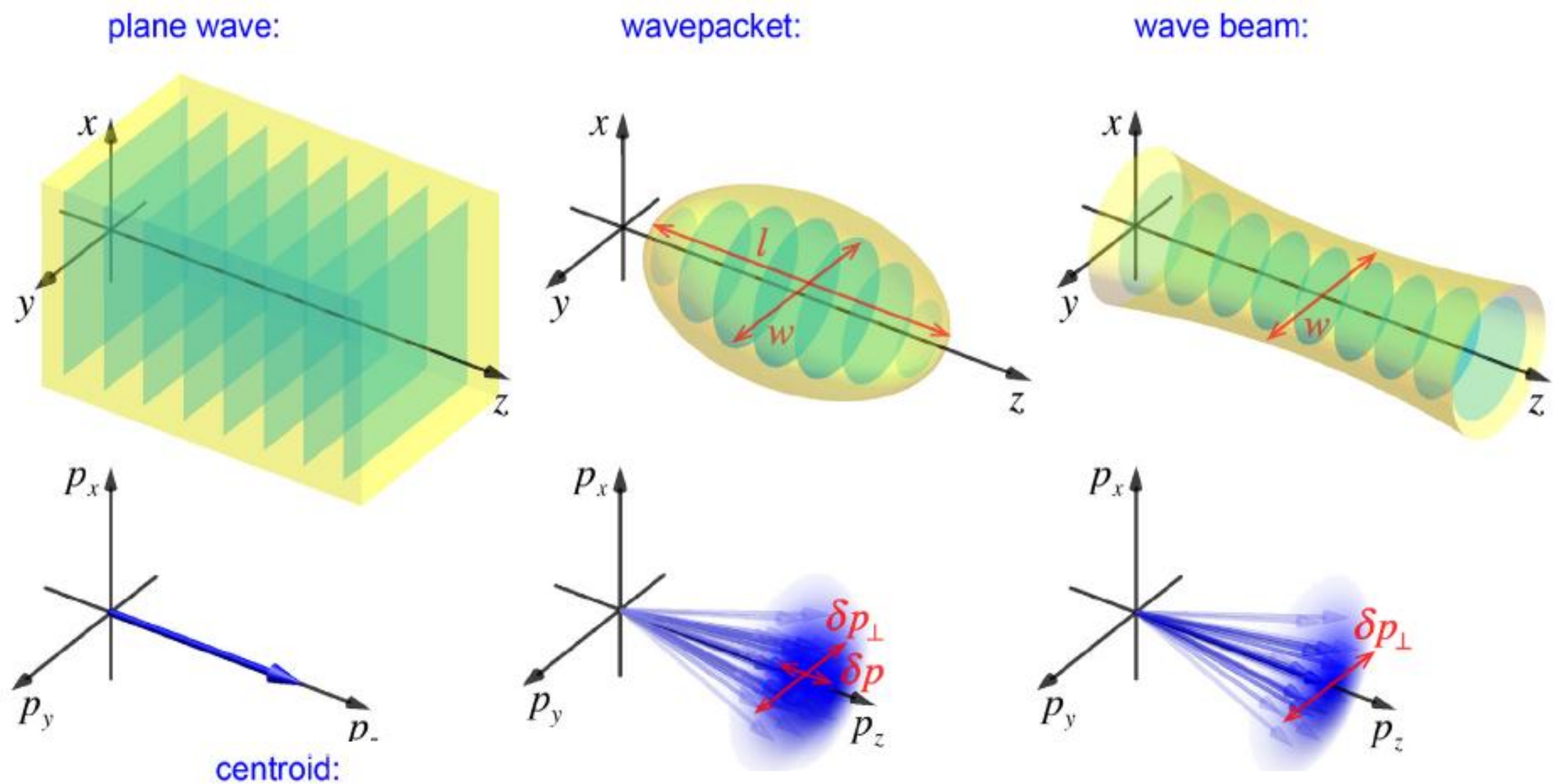
$$w_0 \gg 2\pi/k, \qquad z_R = kw_0^2/2 \gg w_0.$$

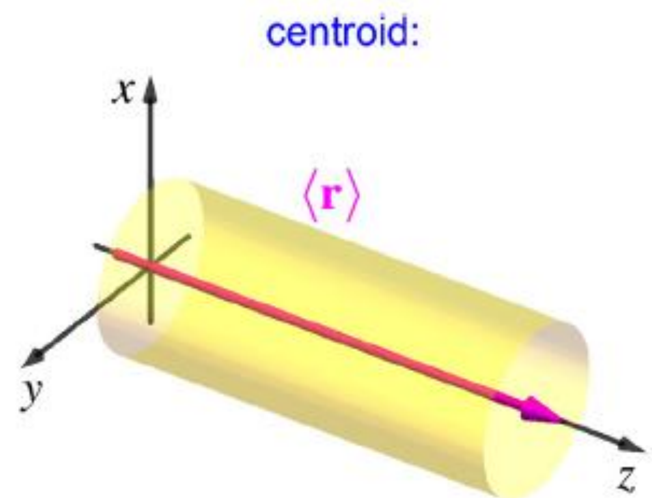
Bessel beam:

$$\psi_\ell^B \propto J_{|\ell|}(\kappa r)\exp[i(\ell\varphi + k_z z)],$$

where J_ℓ is the Bessel function of the first kind, $\ell = 0, \pm 1, \pm 2, \ldots$ is an integer number (azimuthal quantum number), $k_z = p_z/\hbar$ is the longitudinal wave number, and $\kappa = p_\perp/\hbar$ is the transverse (radial) wave number.

The Bessel beams represent the simplest theoretical example of vortex beams. Despite the probability density of Bessel modes decaying as $|\psi_\ell^B| \sim 1/r$ when $r \to \infty$, these solutions are not properly localized in the transverse dimensions. Indeed, the integral $\int_0^\infty |\psi_\ell^B|^2 r\, dr$ diverges, and the function cannot be normalized with respect to the transverse dimensions. The delocalized nature of Bessel beams is reflected in the absence of diffraction and a single transverse quantum number ℓ (instead of two transverse quantum indices in the properly-localized modes).





Wave beams are localized with respect to two dimensions and are described by two discrete transverse quantum numbers

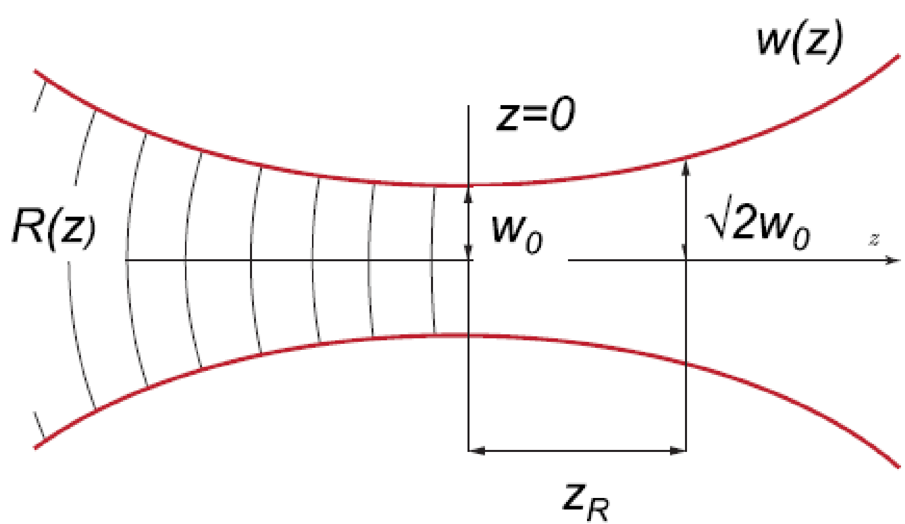
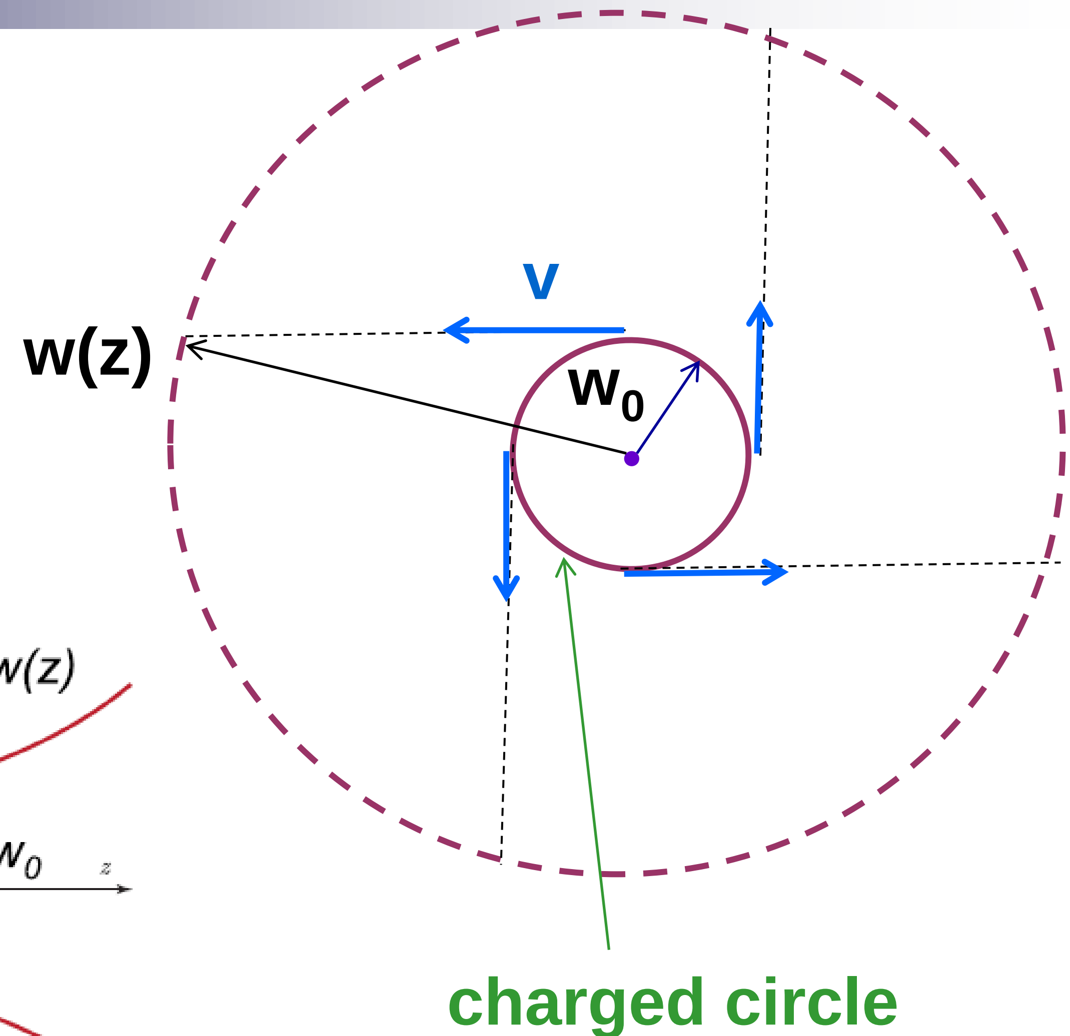


FIG. 2. Schematic representation of the Gaussian profile showing the characteristic parameters, namely, the width $w(z)$, with w_0 the width at the narrowest part of the beam. z_R is the Rayleigh range and $R(z)$ is the in-plane radius of curvature at axial position z



- **Twisted electron beams with large intrinsic OAMs (up to 1000*ħ*) have been recently obtained. At present, much attention is also devoted to interactions of such beams with nuclei and a laser field. A dynamics of the intrinsic OAM in external magnetic and electric fields is also studied. We can disregard the anomalous magnetic moment of the electron because its *g* factor is close to 2.**

The Schrödinger form of the relativistic quantum mechanics is provided by the relativistic Foldy-Wouthuysen transformation. The relativistic Foldy-Wouthuysen Hamiltonians generalize the nonrelativistic Schrödinger ones and are similar to the corresponding classical Hamiltonians.

A.J. Silenko, Foldy-Wouthuysen transformation for relativistic particles in external fields, J. Math. Phys. 44, 2952 (2003); **Foldy-Wouthyusen transformation and semiclassical limit for relativistic particles in strong external fields, Phys. Rev. A 77,** 012116 (2008); **Energy expectation values of a particle in nonstationary fields, Phys. Rev. A 91,** 012111 (2015); **General method of the relativistic Foldy-Wouthuysen transformation and proof of validity of the Foldy-Wouthuysen Hamiltonian, Phys. Rev. A 91,** 022103 (2015).

From a pointlike Dirac particle to a centroid

The exact relativistic Hamiltonian in the FW representation (the FW Hamiltonian) for a Dirac particle in a magnetic field is given by (Case, 1954; Tsai, 1973)

$$\mathcal{H}_{FW} = \beta\sqrt{m^2 + \boldsymbol{\pi}^2 - e\boldsymbol{\Sigma}\cdot\boldsymbol{B}},$$

where $\boldsymbol{\pi} = \boldsymbol{p} - e\boldsymbol{A}$ is the kinetic momentum, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ is the magnetic induction, and β and $\boldsymbol{\Sigma}$ are the Dirac matrices. This Hamiltonian is valid for a twisted and a untwisted particle. The spin angular momentum operator is equal to $\boldsymbol{s} = \hbar\boldsymbol{\Sigma}/2$. The magnetic field is, in general, nonuniform.

It is necessary to take into account that a twisted electron is a charged centroid (Bliokh et al., 2007; 2017). To describe observable quantum-mechanical effects, we need to present the Hamiltonian in terms of the centroid parameters. The centroid as a

whole is characterized by the center-of-charge radius vector $\boldsymbol{R}$ and by the kinetic momentum $\boldsymbol{\pi}' = \boldsymbol{P} - e\boldsymbol{A}(\boldsymbol{R})$, where $\boldsymbol{P} = -i\hbar\partial/(\partial\boldsymbol{R})$. The intrinsic motion is defined by the kinetic momentum $\boldsymbol{\pi}'' = \mathbf{p} - e[\boldsymbol{A}(\boldsymbol{r}) - \boldsymbol{A}(\boldsymbol{R})]$. Here $\mathbf{p} = -i\hbar\partial/(\partial\mathbf{r})$, $\mathbf{r} = \boldsymbol{r} - \boldsymbol{R}$, $\boldsymbol{\pi}' + \boldsymbol{\pi}'' = \boldsymbol{\pi}$, $\boldsymbol{P} + \mathbf{p} = \boldsymbol{p}$. Since

$$\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{A}(\boldsymbol{R}) + \frac{1}{2}\boldsymbol{B}(\boldsymbol{R}) \times \mathbf{r},$$

the operator $\boldsymbol{\pi}^2$ takes the form

$$\boldsymbol{\pi}^2 = \boldsymbol{\pi}'^2 + \mathbf{p}^2 - \frac{e}{2}[\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}] + \boldsymbol{\pi}'\cdot\boldsymbol{\pi}'' + \boldsymbol{\pi}''\cdot\boldsymbol{\pi}'.$$

After summing over partial waves with different momentum directions, $\langle\boldsymbol{\pi}'\cdot\boldsymbol{\pi}'' + \boldsymbol{\pi}''\cdot\boldsymbol{\pi}'\rangle = 0$. More precisely, the operator $\boldsymbol{\pi}'\cdot\boldsymbol{\pi}'' + \boldsymbol{\pi}''\cdot\boldsymbol{\pi}'$ has zero expectation values for any eigenstates of the operator $\boldsymbol{\pi}^2$ and, therefore, it can be omitted. It can be added that this summing can be performed for the squared Hamiltonian $\mathcal{H}_{FW}^2$.

- **The FW Hamiltonian summed over the partial waves takes the form**

$$\mathcal{H}_{\mathrm{FW}} = \beta\epsilon - \beta\frac{e}{4}\left[\frac{1}{\epsilon}\Lambda\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{\Lambda}\frac{1}{\epsilon}\right],$$

$$\epsilon = \sqrt{m^2 + \boldsymbol{\pi}'^2 + \mathbf{p}^2}, \qquad \boldsymbol{\Lambda} = \boldsymbol{L} + \boldsymbol{\Sigma}.$$

kinetic momentum of centroid internal momentum

The momentum and the intrinsic OAM can have different mutual orientations in different Lorentz frames.

As a rule, the intrinsic OAM and the momentum of the twisted electron are collinear in the lab frame. However, it does not take place in other frames. The Lorentz transformation of the OAM from the lab frame ($\boldsymbol{L}=L_z\mathbf{e}_z$) to the rest frame results in $\boldsymbol{L}^{(0)}=\boldsymbol{L}$. The OAM in the frame moving with the arbitrary velocity V relative to the particle rest frame is given by

$$\boldsymbol{L} = \frac{\epsilon}{mc^2}\boldsymbol{L}^{(0)} - \frac{(\boldsymbol{L}^{(0)}\cdot\boldsymbol{p})\boldsymbol{p}}{m(\epsilon+mc^2)}, \qquad \epsilon = \frac{mc^2}{\sqrt{1-\frac{V^2}{c^2}}}$$

Twisted electrons in external electric and magnetic fields

There is a significant difference between the orbital angular momentum (OAM) and the spin. The OAM is formed by the spatial components of the antisymmetric tensor $L^{\mu\nu} \equiv x^\mu p^\nu - x^\nu p^\mu$. Unlike the OAM, the conventional spin ζ is defined by the spatial part of the four-component spin pseudovector a^μ in the particle rest frame.

The spatial components of the spin tensor $\mathfrak{S}^{\mu\nu}$ form the three-component spatial pseudovector $\mathfrak{S}$, which is not equivalent to ζ.

An interaction of the electric and magnetic dipole moments, $\boldsymbol{d}$ and $\boldsymbol{\mu}$, with the external fields is defined by the general Hamiltonian

$$H = -\boldsymbol{d} \cdot \boldsymbol{E} - \boldsymbol{\mu} \cdot \boldsymbol{B},$$

where all quantities are defined in the lab frame.

Now we can take into account that the quantities $\boldsymbol{L}^{(0)}$ and $\boldsymbol{\mu}^{(0)}$ are connected with the nonrotating instantaneous inertial frame and can perform the relativistic transformation of the dipole moments to the lab frame:

$$\mathbf{d} = \boldsymbol{\beta} \times \boldsymbol{\mu}^{(0)} = \boldsymbol{\beta} \times \boldsymbol{\mu}, \qquad \boldsymbol{\mu} = \boldsymbol{\mu}^{(0)} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{\mu}^{(0)}),$$

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c}, \qquad \gamma = (1-\beta^2)^{-1/2},$$

where we have used the fact $\boldsymbol{d}^{(0)}=0$.

A.J. Silenko, Spin precession of a particle with an electric dipole moment: Contributions from classical electrodynamics and from the Thomas effect, Phys. Scripta 90, 065303 (2015).

As a result, the Hamiltonian is given by

$$H = -\frac{e}{2mc}\left[\boldsymbol{B}\cdot\boldsymbol{L}^{(0)} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{B})(\boldsymbol{\beta}\cdot\boldsymbol{L}^{(0)}) - (\boldsymbol{\beta}\times\boldsymbol{E})\cdot\boldsymbol{L}^{(0)}\right]$$
$$= -\frac{e}{2mc\gamma}[\boldsymbol{B}\cdot\boldsymbol{L} - (\boldsymbol{\beta}\times\boldsymbol{E})\cdot\boldsymbol{L}].$$

The corresponding relativistic FW Hamiltonian is similar:

$$\mathcal{H}_{\mathrm{FW}} = \beta\epsilon + e\Phi - \beta\frac{e}{4}\left[\frac{1}{\epsilon}\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}\frac{1}{\epsilon}\right]$$

$$+\frac{e}{4}\left\{\frac{1}{\epsilon^2}\boldsymbol{L}\cdot[\boldsymbol{\pi}'\times\boldsymbol{E}(\boldsymbol{R})] - [\boldsymbol{E}(\boldsymbol{R})\times\boldsymbol{\pi}']\cdot\boldsymbol{L}\frac{1}{\epsilon^2}\right\}.$$

In this equation, spin effects are disregarded because they can be neglected on the condition that $L \gg 1$. The term $e\Phi$ does not include the interaction of the intrinsic OAM with the electric field.

The equation of motion of the intrinsic OAM has the form

$$\frac{d\boldsymbol{L}}{dt} = i[\mathcal{H}_{\mathrm{FW}}, \boldsymbol{L}] = \frac{1}{2}(\boldsymbol{\Omega}\times\boldsymbol{L} - \boldsymbol{L}\times\boldsymbol{\Omega}),$$

$$\boldsymbol{\Omega} = -\beta\frac{e}{4}\left\{\frac{1}{\epsilon}, \boldsymbol{B}(\boldsymbol{R})\right\} + \frac{e}{4}\left[\frac{1}{\epsilon^2}\boldsymbol{\pi}'\times\boldsymbol{E}(\boldsymbol{R}) - \boldsymbol{E}(\boldsymbol{R})\times\boldsymbol{\pi}'\frac{1}{\epsilon^2}\right].$$

In the classical limit,

$$\boldsymbol{\Omega} = -\frac{e}{2mc\gamma}[\boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{E}].$$

The corresponding equation of the spin precession is very different.

Stern-Gerlach-like force

The operator of the total force is given by

$$\boldsymbol{F} = \frac{d\boldsymbol{\pi}'}{dt} = \frac{\partial \boldsymbol{\pi}'}{\partial t} + i[\mathcal{H}_{\mathrm{FW}}, \boldsymbol{\pi}']$$

$$= e\boldsymbol{E}(\boldsymbol{R}) + \beta \frac{e}{4}\left\{\frac{1}{\epsilon}, (\boldsymbol{\pi}' \times \boldsymbol{B}(\boldsymbol{R}) - \boldsymbol{B}(\boldsymbol{R}) \times \boldsymbol{\pi}')\right\} + \boldsymbol{F}_{SGl}.$$

Lorentz force

Beam splitting in nonuniform electric and magnetic fields is conditioned by the Stern-Gerlach-like force operator

$$F_{\mathrm{SGl}} = \beta \frac{e}{4} \left\{ \frac{1}{\epsilon} \nabla [\boldsymbol{L} \cdot \boldsymbol{B}(\boldsymbol{R})] + \nabla [\boldsymbol{B}(\boldsymbol{R}) \cdot \boldsymbol{L}] \frac{1}{\epsilon} \right\}$$

$$- \frac{e}{4} \left\{ \frac{1}{\epsilon^2} \nabla (\boldsymbol{L} \cdot [\boldsymbol{\pi}' \times \boldsymbol{E}(\boldsymbol{R})]) - \nabla ([\boldsymbol{E}(\boldsymbol{R}) \times \boldsymbol{\pi}'] \cdot \boldsymbol{L}) \frac{1}{\epsilon^2} \right\}.$$

The force is exerted to the center of charge of the centroid.

Manipulations of twisted electron beams

The results obtained permit us to develop methods for the manipulation of electron vortex beams

A. J. Silenko, Pengming Zhang and Liping Zou, Manipulating Twisted Electron Beams, Phys. Rev. Lett. 119, 243903 (2017).

Separations of beams with opposite directions of the OAM

The beam separation can be achieved in a longitudinal magnetic field. The nonuniform longitudinal magnetic field leads to a force acting on the OAM. The direction of this force depends on that of the OAM. As a result, accelerations of particles with oppositely directed OAMs have different signs. Therefore, the beam with a given OAM direction can be extracted (e.g., with the Wien filter). We should mention that a transversal magnetic field can destroy a beam coherence as a result of Larmor precession.

Freezing the intrinsic OAM in electromagnetic fields

As in spin physics, it is important to consider a condition which allows one to freeze the intrinsic OAM (i.e., to keep the orbital helicity constant) in electromagnetic fields. These fields deflect the beam. We consider a potential for a beam deflection without a change of the orbital helicity h_{orb}. In this case, the angular velocity of the relativistic Larmor precession should be equal to the angular velocity of the rotation of the momentum direction $\mathbf{N}=\mathbf{p}/p\approx\mathbf{V}/V$:

$$\frac{dN}{dt}=\omega\times N,\qquad \omega=-\frac{e}{mc\gamma}\left(B-\frac{N\times E}{\beta}\right).$$

The standard geometry is $E\perp B\perp V$. The condition $\Omega_L=\omega$ is satisfied when

$$\boldsymbol{B} = \left(\frac{2}{\beta^2} - 1\right)\boldsymbol{\beta} \times \boldsymbol{E}.$$

The device defined this equation is a deflector of the twisted electron beams which freezes the OAM relative to the momentum direction. It rotates the beam direction with the angular velocity

$$\boldsymbol{\Omega}_L = \boldsymbol{\omega} = -\frac{e\boldsymbol{B}}{mc\gamma(\gamma^2 + 1)}.$$

For standard beams with energy ~10^2 keV, the deflection is rather effective.

Flipping the intrinsic OAM

If an electron vortex beam with an upward or a downward orbital polarization is confined in a storage ring, the direction of the intrinsic OAM can be flipped. A flip of the OAM is similar to that of the spin and can be fulfilled by the method of the magnetic resonance. A significant difference between the flips of the OAM and the spin consists in different dependences of the resonance frequencies on the electric and magnetic fields. A spin flip frequency in a storage ring is defined by the Thomas-Bargmann-Mishel-Telegdi equation whose distinction from the corresponding equation for the OAM is evident. The OAM flip can be forced by a longitudinal (azimuthal) magnetic field oscillating with the resonance frequency. A Wien filter with a vertical electric field and a radial magnetic field (when the two fields oscillate with the resonance frequency) can also be used for the OAM flip.

Rotation of the intrinsic OAM in crossed electric and magnetic fields as a critical experiment

We propose a simple critical experiment for a verification of the results obtained. For this purpose, crossed electric and magnetic fields ($E \perp B \perp \beta$) satisfying the relation $E = -\beta \times B$ can be used. Such fields characterizing the Wien filter do not affect a beam trajectory. We suppose the fields E and B to be uniform. In the considered case, the classical limit of the relativistic equation for the angular velocity of precession of the intrinsic OAM is given by

$$\boldsymbol{\Omega}^{(W)} = -\frac{e\left(m^2 + \mathfrak{p}^2\right)}{2\epsilon^3}B.$$

Here $\mathfrak{p}$ is the momentum characterizing the internal motion inside of the centroid, $\varepsilon = m\gamma$. It is necessary to use a single twisted electron beam possessing a standard orbital polarization collinear to the beam momentum. The intrinsic OAM rotates with the angular frequency $\Omega^{(W)}$ and reverses its direction with the angular frequency $2\Omega^{(W)}$. The device is the intrinsic-OAM rotator. As twisted electron beams are relativistic, a quantitative verification of the results obtained can be fulfilled.

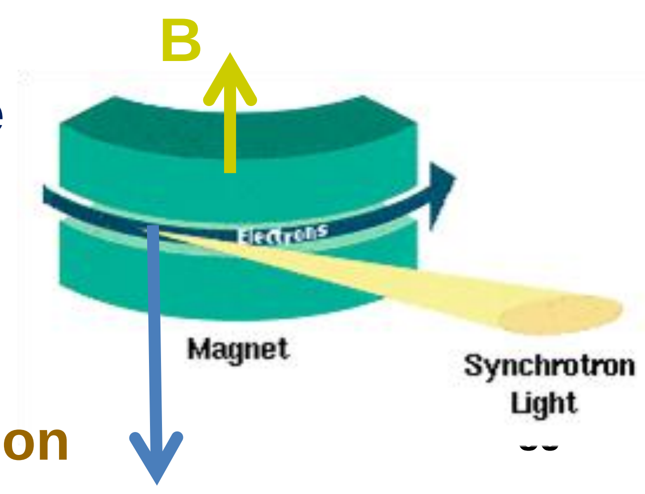
Sokolov-Ternov-like effect of a radiative orbital polarization of twisted electron beams in a magnetic field

The well-known effect is the radiative spin polarization of electron or positron beams in storage rings caused by the synchrotron radiation (Sokolov-Ternov effect). The radiative spin polarization acquired by unpolarized electrons is opposite to the direction of the main magnetic field. The reason for the effect is a dependence of spin-flip transitions from the initial particle polarization. It follows from the previously obtained results (Ivanov, 2012; Ivanov et al., 2016) that quantum-electrodynamics effects are rather similar for twisted and untwisted particles. We can note the evident similarity between interactions of the spin and the intrinsic OAM with the magnetic field. In particular, energies of stationary states depend on projections of the spin and the intrinsic OAM on the field direction. This similarity validates the existence of the effect of the radiative orbital polarization. As well as the radiative spin polarization, the corresponding orbital polarization acquired by unpolarized twisted electrons should be opposite to the direction of the main magnetic field.

The effect is conditioned by transitions with a change of a projection of the intrinsic OAM. The probability of such transitions is large enough if the electron energy is not too small. Similarly to the spin polarization, the orbital one is observable when electrons are accelerated up to the energy of the order of 1 GeV. The acceleration can depolarize twisted electrons but cannot vanish L. During the process of the radiative polarization, the average energy of the electrons should be kept unchanged.

A. J. Silenko, Pengming Zhang and Liping Zou, Relativistic Quantum Dynamics of Twisted Electron Beams in Arbitrary Electric and Magnetic Fields, Phys. Rev. Lett. 121, 043202 (2018).

New Sokolov-Ternov-like effect of a radiative OAM polarization of electron or positron beams in storage rings caused by the synchrotron radiation is predicted.



Final OAM polarization

Summary

- Main distinctive features of the twisted (vortex) particles have been discussed
- General equation defining dynamics of twisted electrons in external electric and magnetic fields has been derived
- Methods of manipulations of twisted electron beams have been considered
- Rotation of the intrinsic OAM in crossed electric and magnetic fields is proposed as a critical experiment for a verification of the results obtained
- Sokolov-Ternov-like effect of a radiative orbital polarization of twisted electron beams in a magnetic field is found


Thank you for your attention

XXIVth International Baldin Seminar on High Energy Physics Problems "Relativistic Nuclear Physics and Quantum Chromodynamics“, *Dubna*

INP

Dynamics of a tensor polarization of particles and nuclei and its influence on the spin motion in external fields

Alexander J. Silenko

Research Institute for Nuclear Problems, BSU, Minsk, Belarus

Institute for Nuclear Research, Dubna, Russia

17 – 22 September, 2018

OUTLINE

- Previous result
- General dynamics of a tensor polarization of particles and nuclei
- Influence of a tensor polarization on the spin motion in external fields
- Summary

The explanations are based on the recent publication:

A. J. Silenko, General dynamics of tensor polarization of particles and nuclei in external fields, J. Phys. G: Nucl. Part. Phys. 42, 075109 (2015).

Previous result

The polarization vector, *P*, and the polarization tensor, P_{ij}

$$P_i = \frac{\langle S_i \rangle}{s}, \quad i = x, y, z$$

$$P_{ij} = \frac{3\langle S_i S_j + S_j S_i \rangle - 2s(s+1)\delta_{ij}}{2s(2s-1)}, \quad i, j = x, y, z.$$

Hamiltonian $\mathcal{H} = \boldsymbol{\Omega} \cdot \boldsymbol{S} + Q_{jk} S_j S_k.$

Commutation relation $\left[S_i, S_j \right] = i e_{ijk} S_k \quad (i, j, k = x, y, z),$

Equation of spin motion

$$\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \frac{i}{\hbar}[\mathcal{H}, \boldsymbol{S}] = \boldsymbol{\Omega} \times \boldsymbol{S}.$$

$$\boldsymbol{\Omega} = -\frac{e}{mc}\left[\left(G+\frac{1}{\gamma}\right)\boldsymbol{B} - \frac{\gamma G}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{B})\boldsymbol{\beta} - \left(G+\frac{1}{\gamma+1}\right)\boldsymbol{\beta}\times\boldsymbol{E} + \frac{\eta}{2}\left(\boldsymbol{E} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{E})\boldsymbol{\beta} + \boldsymbol{\beta}\times\boldsymbol{B}\right)\right].$$

Here $G = (g-2)/2$, $g = 2mc\mu/(e\hbar s)$, $\eta = 2mcd/(e\hbar s)$, $\boldsymbol{\beta} = \mathbf{v}/c$, γ is the Lorentz factor, and μ and d are the magnetic and electric dipole moments, respectively.

Huang–Lee–Ratner approach

$$T_0 = \frac{1}{\sqrt{2}}\left(3S_z^2 - 2\right), \quad T_{\pm1} = \pm\frac{\sqrt{3}}{2}\left(S_\pm S_3 + S_3 S_\pm\right), \quad T_{\pm2} = \frac{\sqrt{3}}{2}S_\pm^2.$$

5X5 matrix was used:

$$\frac{\mathrm{d}T}{\mathrm{d}\phi} = AT, \quad T = \begin{pmatrix} T_{+2} \\ T_{+1} \\ T_0 \\ T_{-1} \\ T_{-2} \end{pmatrix}, \quad A = \begin{pmatrix} 2iG\gamma & -iF_+ & 0 & 0 & 0 \\ -iF_- & iG\gamma & -i\frac{\sqrt{6}}{2}F_+ & 0 & 0 \\ 0 & -i\frac{\sqrt{6}}{2}F_- & 0 & -i\frac{\sqrt{6}}{2}F_+ & 0 \\ 0 & 0 & -i\frac{\sqrt{6}}{2}F_- & -iG\gamma & -iF_+ \\ 0 & 0 & 0 & -iF_- & -2iG\gamma \end{pmatrix}.$$

General dynamics of a tensor polarization of particles and nuclei

- Let us consider the Cartesian coordinate system rotating around the z-axis with the same angular velocity, $\Omega(t)$, *as the spin.* In the general case, this angular velocity depends on time. We may superpose the rotating and nonrotating coordinate systems at the initial moment of time, $t = 0$ ($e_i'(0) = e_i$). The rotating coordinate system is denoted by primes. The spin components in the rotating coordinate system remain unchanged:

$$\frac{dS_i'}{dt} = 0 \quad (i = x, y, z).$$

As a result, all tensor polarization operators and all components of the polarization tensor are also unchanged in the rotating coordinate system ($S_i'S_j' + S_j'S_i' = const$, $P_{ij}' = const$). This important property shows that the tensor polarization of particles/nuclei with spin $s \geq 1$ rotates in external fields similarly to the vector polarization. This is valid not only for electromagnetic interaction, but also for other (weak, gravitational) interactions.

The time dependence of the polarization tensor defined in the laboratory frame can be easily expressed in terms of the basic vectors, e_i'. Since $e_i'(0) = e_i$ and

$$S_i'(t) = \sum_k \left(e_i'(t) \cdot e_k\right) S_k,$$

the polarization tensor is given by

$$P_{ij}(t) = \frac{3}{2s(2s-1)} \sum_{k,l} \left[\left(e_i'(t) \cdot e_k\right)\left(e_j'(t) \cdot e_l\right)\langle S_k S_l + S_l S_k \rangle \right] - \frac{s+1}{2s-1}\delta_{ij}.$$

This simple equation defines the general dynamics of the tensor polarization of particles and nuclei in external fields.

Influence of a tensor polarization on the spin motion in external fields

The quantum-mechanical Hamiltonian describing the interaction of the spin with external fields contains linear and bilinear terms on the spin:

$$\mathcal{H} = \boldsymbol{\Omega} \cdot \boldsymbol{S} + Q_{jk} S_j S_k .$$

The bilinear terms are proportional to the electric and magnetic tensor polarizabilities:

$$W' = -\alpha_T \left(\mathbf{S} \cdot \mathbf{E}'\right)^2 - \beta_T \left(\mathbf{S} \cdot \mathbf{B}'\right)^2 .$$

$\mathbf{E}'$ and $\mathbf{B}'$ are the particle rest frame fields

Methods of measurement of tensor polarizabilities of the deuteron and other nuclei have been proposed by V. Baryshevsky and co-workers:

- **V. Baryshevsky and A. Shirvel, hep-ph/0503214.**
- **V. G. Baryshevsky, STORI 2005 Conference Proceedings, Schriften des Forschungszentrums Jülich, Matter and Materials, Vol. 30 (2005), pp. 227–230; J. Phys. G: Nucl. Part. Phys. 35, 035102 (2008); hep-ph/0504064; hep-ph/0510158; hep-ph/0603191.**
- **V. G. Baryshevsky and A. A. Gurinovich, hep-ph/0506135.**

The results have been summarized in the work:

V. G. Baryshevsky, J. Phys. G: Nucl. Part. Phys. 35, 035102 (2008).

Because of a doubt expressed, the results obtained have been confirmed by the very different matrix method:

A. J. Silenko, Phys. Rev. C 75, 014003 (2007); 77, 021001(R) (2008); 80, 044315 (2009).

Different methods give similar results

The tensor polarizabilities transform the tensor polarization to the vector one and other way round

- The tensor electric polarizability stimulates the buildup of the vertical polarization of a vector-polarized deuteron beam
- The tensor electric polarizability can mimic the presence of the electric dipole moment
- The tensor magnetic polarizability produces the spin rotation with two frequencies instead of one, beating and causes transitions between vector and tensor polarizations

Theoretical data

Tensor electric polarizability of deuteron:

$\alpha_T = -6.2 \times 10^{-41}\ \mathrm{cm}^3$

J.-W. Chen, H. W. Grießhammer, M. J. Savage, and R. P. Springer, Nucl. Phys. A644, 221 (1998).

$\alpha_T = -6.8 \times 10^{-41}\ \mathrm{cm}^3$

X. Ji and Y. Li, Phys. Lett. B591, 76 (2004).

$\alpha_T = 3.2 \times 10^{-41}\ \mathrm{cm}^3$

J. L. Friar and G. L. Payne, Phys. Rev. C 72, 014004 (2005).

Tensor magnetic polarizability of deuteron:

$\beta_T = 1.95 \times 10^{-40}\ \mathrm{cm}^3$

J.-W. Chen, H. W. Grießhammer, M. J. Savage, and R. P. Springer, Nucl. Phys. A644, 221 (1998).

X. Ji and Y. Li, Phys. Lett. B591, 76 (2004).

The best conditions for a measurement of the tensor polarizabilities of the deuteron and other nuclei can be achieved with the use of tensor-polarized initial beams. In this case, we may confine ourselves to the consideration of a zero projection of the deuteron spin onto the preferential direction. When this direction is defined by the spherical angles θ and ψ, the initial polarization is given by

$$P(0) = 0, \quad P_{\rho\rho}(0) = 1 - 3\sin^2\theta\cos^2\psi,$$

$$P_{\phi\phi}(0) = 1 - 3\sin^2\theta\sin^2\psi, \quad P_{zz}(0) = 1 - 3\cos^2\theta,$$

$$P_{\rho\phi}(0) = -\frac{3}{2}\sin^2\theta\sin(2\psi),$$

$$P_{\rho z}(0) = -\frac{3}{2}\sin(2\theta)\cos\psi, \quad P_{\phi z}(0) = -\frac{3}{2}\sin(2\theta)\sin\psi.$$

In this case, the general equation describing the evolution of the polarization vector has the form

$$P_\rho(t) = \sin(2\theta)\left\{\left[\cos(\omega' t)\sin\psi + \frac{\omega_0}{\omega'}\sin(\omega' t)\cos\psi\right]\right.$$

$$\left.\times \sin(bt) + \frac{\mathcal{A}}{\omega'}\sin(\omega' t)\cos(bt)\sin\psi\right\},$$

$$P_\phi(t) = \sin(2\theta)\left\{\left[-\cos(\omega' t)\cos\psi + \frac{\omega_0}{\omega'}\sin(\omega' t)\sin\psi\right]\right.$$

$$\left.\times \sin(bt) + \frac{\mathcal{A}}{\omega'}\sin(\omega' t)\cos(bt)\cos\psi\right\},$$

$$P_z(t) = -\frac{2\mathcal{A}}{\omega'}\sin^2\theta\sin(\omega' t)\left[\cos(\omega' t)\sin(2\psi)\right.$$

$$\left.+ \frac{\omega_0}{\omega'}\sin(\omega' t)\cos(2\psi)\right],$$

$$\mathcal{A} = -\alpha_T\frac{(1+a)^2\beta^2\gamma B_z^2}{2(1-a\beta^2\gamma^2)^2}, \quad \mathcal{B} = -\beta_T\frac{\gamma B_z^2}{(1-a\beta^2\gamma^2)^2}.$$

$$\omega' = \sqrt{\omega_0^2 + \mathcal{A}^2}, b = \mathcal{B} - \mathcal{A}.$$

Summary

- **The tensor polarization of particles and nuclei becomes constant in a coordinate system rotating with the same angular velocity as the spin. It rotates in the laboratory frame with this angular velocity. The general equation defining the time dependence of the tensor polarization is presented.**
- **Influence of the tensor polarization on the spin motion in external fields is defined by the tensor electric and magnetic polarizabilities and manifests in the transformation of the tensor polarization to the vector one and other way round**


Thank you for your attention