Charmonium Radiative Transitions, Meson and Glueball Properties with the Effective Strong Coupling

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Motivation

♣ Many novel behaviors are expected in the region ~ 1 ÷ 9 GeV.
Quark confinement, QCD running coupling, glueball states, etc.
require a correct description of hadron dynamics in this region.

♣ Construct simple, reliable models and apply in different sections (e.g.):

- Strong interaction: mass-dependent strong effective coupling, Fermi coupling, meson spectrum;
- Exotic states: lowest-state glueball mass, radius, etc.;
- Weak: leptonic decay constants;
- Radiative transitions: charmonium excitations' decay widths;

"**Conventional**" (***quark-antiquark***) states: Spin: $\frac{1}{2} \otimes \frac{1}{2}$

Ground state (*P & V mesons*) : $J^{PC} = 0^{-+}, 1^{--}$

Charmonium Excitations (*S, A & T*) : $J^{PC} = 0^{++}, 1^{++}, 2^{++}$

"**Exotic**" (***di-gluon***) states: Spin: $1 \otimes 1$

Ground state (*Scalar Glueball*): $J^{PC} = 0^{++}$

Outline

- *Two-particle bound states within ICM*
- *Infrared regularizations*
- *Effective strong coupling*
- *Mesons: - mass spectrum,*
  - leptonic decay constants
- *Scalar Glueball: mass, radius*
- *Fermi coupling within CCQM*
- *Radiative transitions of charmonium excitations*
- *Summary and Outlook*

Part 1a: Model. Two-particle Bound States

- Consider a QCD-inspired relativistic quantum-field model.

[G.G., PRD79 (2009); PRD81 (2010); Phys.Part.Nucl.(2014)]

$$L = -\frac{1}{4}\left(F^A_{\mu\nu} - g f^{ABC} A^B_\mu A^C_\nu\right)^2 + \sum_f \left(\bar{q}^a_f \left[\gamma_\alpha \partial_\alpha - m_f + g \Gamma^\alpha_C A^C_\alpha\right]^{ab} q^b_f\right)$$

$$F^B_{\mu\nu} \equiv \partial_\mu A^B_\nu - \partial_\nu A^B_\mu \qquad \Gamma^\alpha_C \doteq i\gamma_\alpha t^C$$

- Partition Functional – Path Integral in terms of quark and gluon variables:

$$Z = \iint \delta\bar{q}\,\delta q \int \delta A \;\exp\left\{-\int dx\, L[\bar{q}, q, A]\right\}$$

- **LO** contributions to quark-antiquark and two-gluon bound states:

$$Z_{(\bar{q}q)} = \iint \delta\bar{q}\,\delta q \exp\left\{-\left(\bar{q}\, S^{-1} q\right) + \frac{g^2}{2}\left\langle\left(\bar{q}\,\Gamma A q\right)\left(\bar{q}\,\Gamma A q\right)\right\rangle_D\right\}$$

$$Z_{(AA)} = \left\langle \exp\left\{-\frac{g}{2}\left(f\, AAF\right)\right\}\right\rangle_D \qquad \left\langle(\bullet)\right\rangle_D \doteq \int \delta A \; e^{-\frac{1}{2}\left(AD^{-1}A\right)}(\bullet)$$

- Allocate one-gluon exchange between colored currents

$$L_{qq} = \frac{g^2}{2} \sum_{f_1 f_2} \iint dx_1 dx_2 \, J^B_{\mu\, f_1 f_2}(x_1, x_2) D^{BC}_{\mu\nu}(x_1, x_2) J^C_{\nu\, f_1 f_2}(x_2, x_1),$$

$$J^B_{\mu\, f_1 f_2}(x_1, x_2) \equiv i\, \overline{q}_{f_1}(x_1) \gamma_\mu t^B q_{f_2}(x_2).$$

- Isolate color-singlet combination
- Perform Fierz transformation for spins (J = S, P, A, V, T)
- Introduce orthonormalized system: $\{U_Q\}$ with quantum numbers $Q=\{n,l, \ldots\}$:
- Diagonalization on colorless quark currents $J_N(x)$ with $N=\{Q,J,f_1,f_2\}$
- Gaussian representation: a new path integration over auxiliary fields B_N:

$$e^{\frac{1}{2} g^2 (J_N^+ J_N)} = \iint \delta B_N^+ \, \delta B_N \exp\left\{ -\sum_N (B_N^+ B_N) + g \sum_N [(B_N^+ J_N) + (J_N^+ B_N)] \right\}$$

- Explicit path-integration over quark variables and write the effective action
- Hadronization Ansatz: B_N fields are identified as meson fields with N

• Z_N rewritten in terms of B_N and all quadratic field configurations are isolated:

$$Z_{(\bar{q}q)} \to Z_N = \int \prod_N \delta B_N \exp\left\{ -\frac{1}{2}(B_N [1 + g^2 \, Tr(V_N S_{m1} V_N S_{m2})] B_N) + W_{resid}[B_N] \right\}$$

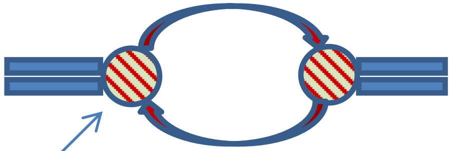
• Diagonalization of the quadratic part is equivalent to the solution of the ladder Bethe-Salpeter equation on the orthonormalized system $\{U_N\}$

$$g^2 \, Tr(V_N S_{m1} \; V_{N'} S_{m2}) = \left(U_N \, \lambda \, U_{N'}\right) = \lambda_N(-p^2) \; \delta^{JJ'} \delta^{QQ'}$$

• Symmetric Bethe-Salpeter kernel is defined:

$$\alpha \cdot \lambda_J(-p^2) = \frac{4g^2 C_J}{9} \int \frac{d^4k}{(2\pi)^4} \{V(k)\}^2 \cdot \Pi_J(k,p)$$

$$\boxed{\Pi_J(p,k) = -\frac{1}{4!} Tr\left\{ \Gamma_J \, \tilde{S}_{m1}\left(\hat{k} + \xi_1 \hat{p}\right) \Gamma_J \, \tilde{S}_{m2}\left(\hat{k} - \xi_2 \hat{p}\right) \right\}}$$



$$\boxed{V_J(k) = \int dx \, \sqrt{D(x)} \, U_J(x) \, e^{ikx}}$$

UV Regularization

- Renormalization: $$U_{REN}(x) \equiv \sqrt{-\alpha \dot{\lambda}_N (M_N^2)} \cdot U_N(x)$$

$$\left\langle U_N \left| 1+\alpha\lambda_N(-p^2) \right| U_N \right\rangle = \left\langle U_N \left| \boxed{1+\alpha\lambda_N(M_N^2)} - \alpha\dot{\lambda}_N(M_N^2)(p^2+M_N^2) \right| U_N \right\rangle$$
$$= \left\langle U_{REN} \left| (p^2+M_N^2) \right| U_{REN} \right\rangle$$

Meson Mass Equation

$$\boxed{1+\alpha\lambda_N(M_N^2) = 0\,, \qquad p^2 = -M_N^2}$$

- Infrared (IR) divergences appear in the loop and vertice functions:

$$\boxed{\Pi_J(p,k) = -\frac{1}{4!} Tr\left\{ \Gamma_J \tilde{S}_{m1}\left(\hat{k}+\xi_1\hat{p}\right) \Gamma_J \tilde{S}_{m2}\left(\hat{k}-\xi_2\hat{p}\right) \right\}}$$

$$\boxed{V_J(k) = \int dx \sqrt{D(x)}\, U_J(x)\, e^{ikx}}$$

IR Regularization

- "***Infrared***" regularizations of ***propagators*** remove these divergencies:

$$\tilde{S}_m(\hat{p}) = \frac{1}{-i\hat{p}+m} = \left(i\hat{p}+m\right)\cdot\int\limits_0^\infty dt\,\exp\left\{-t\cdot(p^2+m^2)\right\}$$

$$\Rightarrow\ \tilde{S}_{IR}(\hat{p}) = \left(i\hat{p}+m\right)\cdot\int\limits_{0}^{1/\Lambda^2} dt\,\exp\left\{-t\cdot(p^2+m^2)\right\}$$

$$D(x) = \frac{1}{4\pi^2 x^2} = \int\limits_0^\infty ds\, e^{-s x^2} \quad \Rightarrow \quad D_{IR}(x) = \int\limits_{\Lambda^2/2}^{\infty} ds\, e^{-s x^2}$$

$\mathbf{\Lambda}$ – mass scale of IR confinement region

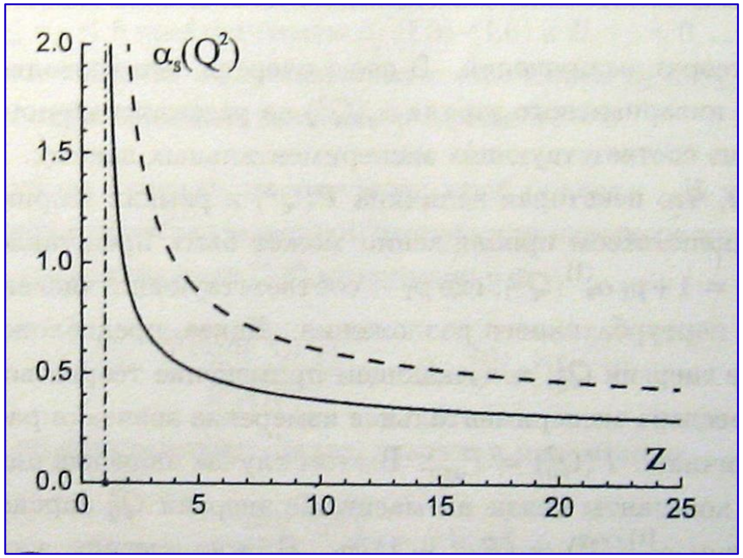
$\mathbf{\Lambda} \to 0$ deconfinement

- Another "***Infrared***" regularization of ***entire q-qbar loop*** is used in the *Covariant Confined Quark Model (see the 2nd part)*

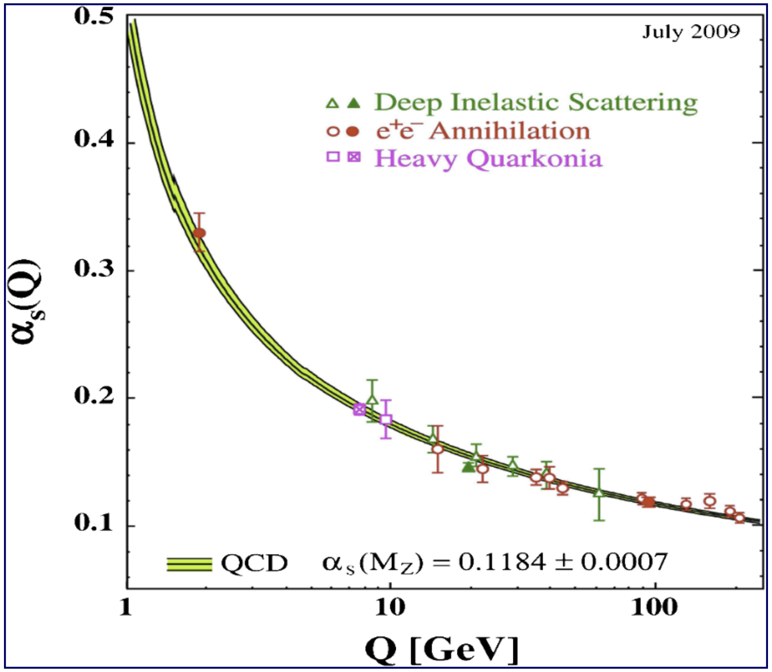
Part 1b: QCD Effective Coupling

THEORY: QCD predicts a dependence of $\alpha_s(Q)$ on energy scale Q.
This ***dependence*** is described theoretically by the Renorm Group equations.

EXPERIMENT: but its ***actual value*** must be obtained from experiment.
It is well determined experimentally at relatively high energies Q >2 GeV.



Perturbation theory: one-loop (dashed) and two-loop (solid) approximations to $\alpha_S(Q^2)$, where $Z=Q^2/\Lambda^2$



$$\alpha_s(Q < 1) \to ?$$

Effective (Mass-dependent) Strong Coupling

$$1+\tilde{\alpha}_s\lambda_N(M_N^2)=0\,,\qquad p^2=-M_N^2$$

$$U_{n\ell\mu}(x,a)\approx T_{\ell\mu}(ax)\cdot L_n^{l+1/2}(a^2x^2)\cdot\sqrt{D(x)}\cdot e^{-ax^2}\,,\qquad \sum_\mu\int dx\left[U_{nl\mu}(x,a)\right]^2=1$$

[GG, J.Phys. CS 938 (2017) 012047]

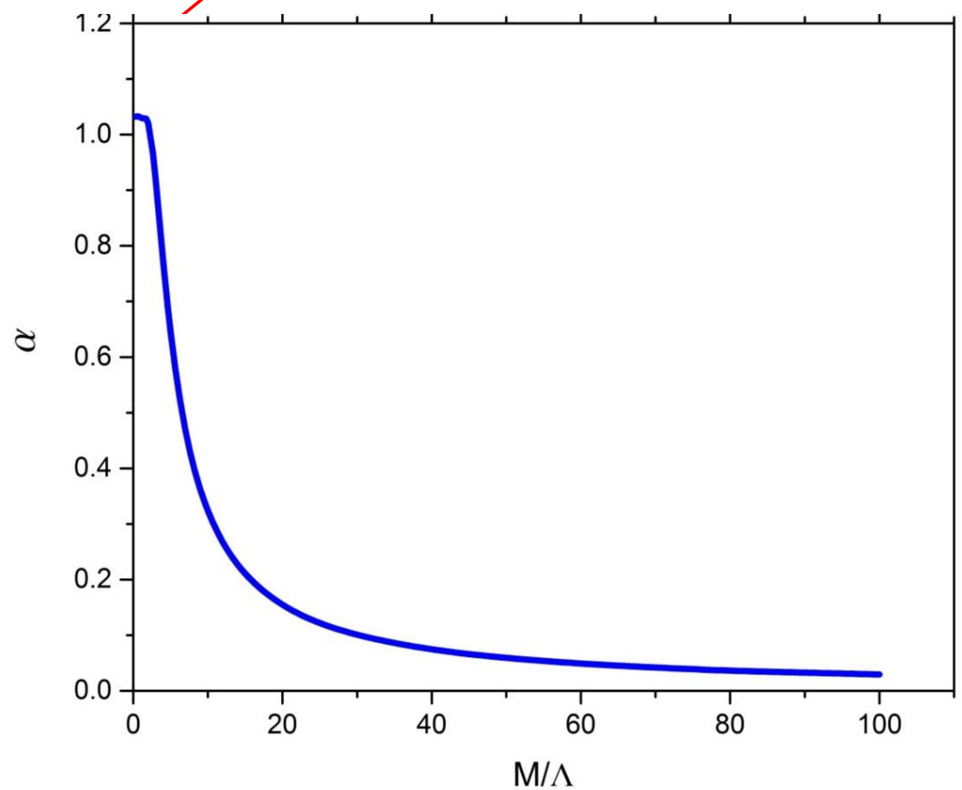
- Particularly, for
  n=l=0,
  $m_1 = m_2 = M/2$:

$$\tilde{\alpha}_s(M)=1/\tilde{\lambda}(M^2)$$

Infra-red fixed point

$$\tilde{\alpha}_s(0)\approx 1.032$$

for any $\Lambda>0$



There exist dispersion relations between time-like and space-like couplings:

α(M) ↔ α(Q)

Part 1c: Meson Spectrum

a) **Analytic results**

- Asymptotical Regge-type behavour:

$$M_{nl}^2 \approx M_{00}^2 + (n+l)\cdot const \qquad for \;\; \{n,l\} \geq 3$$

- Due to spin effect vector mesons are heavier than pseudoscalars at the same quark contents:

$$1 \approx C_J \cdot \exp(M_J^2)\cdot(M_J^2 + const)$$
$$1 = C_P > C_V = 1/2$$

$$\Longrightarrow \quad M_P^2 < M_V^2$$

- The coupling is bounded from above:

$$\alpha_s(M) = 1/\lambda_J(M^2) \leq \alpha_s^{\max}$$

Conventional Meson Mass Estimates

Model parameters fixed by fitting the meson spectrum:

$$\Lambda = 236.0\ MeV,$$
$$m_{ud} = 227.6\,MeV, \qquad m_s = 420.1\,MeV,$$
$$m_c = 1521.6\,MeV, \qquad m_b = 4757.2\,MeV.$$

[GG, J.Phys. CS 938 (2017) 012047]

|relative errors| < 1.8%

$J^{PC} = 0^{-+}$	PDG-2016	Our estim.	$J^{PC} = 1^{--}$	PDG-2016	Our estim.
D (~uc)	1869.62	1893.6	ρ (uu)	775.26	774.3
D_s (~sc)	1968.50	2003.7	K* (us)	891.66	892.9
η_c (~cc)	2983.7	3032.5	D* (uc)	2010.29	2003.8
B (~ub)	5279.26	5215.2	D_s* (~sc)	2112.3	2084.1
B_s (~sb)	5366.77	5323.6	J/ψ (~cc)	3096.92	3077.6
B_c (~cb)	6274.5	6297.0	B* (~ub)	5325.2	5261.5
η_b (~bb)	9398.0	9512.5	Υ (~bb)	9460.30	9526.4

Part 1d: Weak (leptonic) Decay Constants

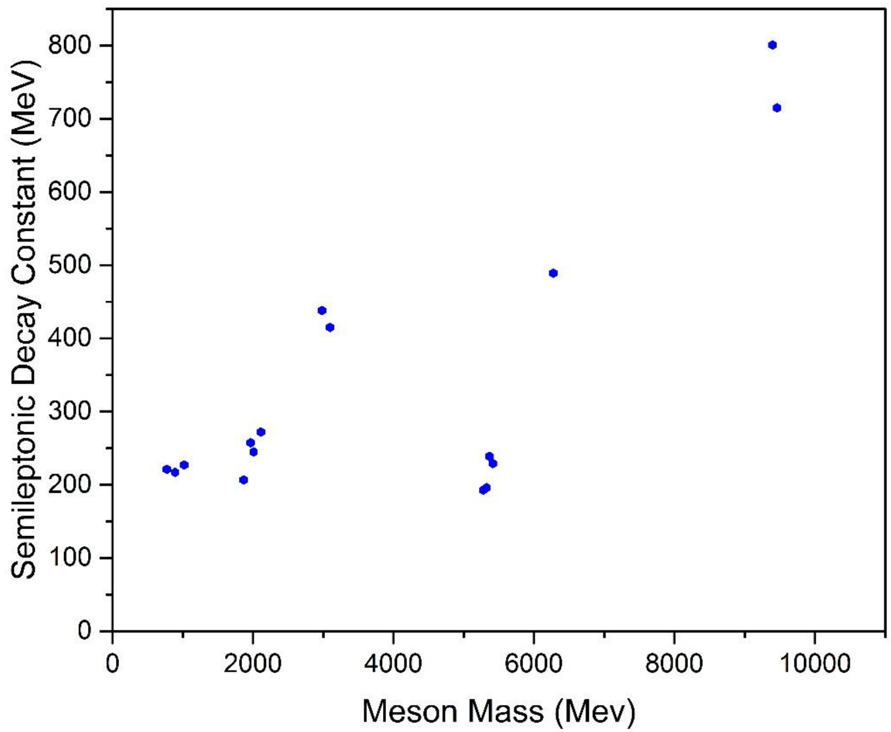
Important value in particle physics:

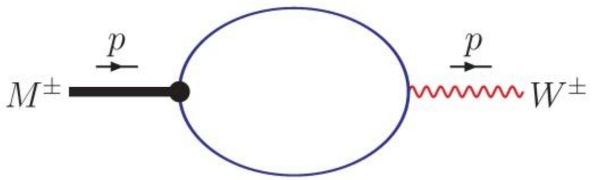
$$\boxed{i\, f_P\, p_\mu = \langle 0 | J_\mu(0) | U_{renorm}(p) \rangle}$$

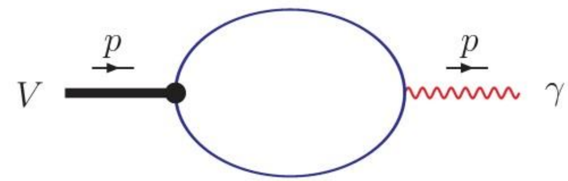
$$\pi^+ \to \mu^+ \nu_\mu + \mu^+ \nu_\mu \gamma$$

$$D_s^+ \to l^+ \nu$$

Mass dependence of weak (leptonic) decay constants (PDG 2017):







$$i f_p p_\mu = \frac{g}{6}\int \frac{d^4k}{(2\pi)^4}\int dx\, e^{ikx}\, U_R(x)\sqrt{D(x)}\, Tr\left\{ i\gamma_5\, \tilde{S}_{m1}\left(\hat{k}+\xi_1\hat{p}\right) i\gamma_5\gamma_\mu\, \tilde{S}_{m2}\left(\hat{k}-\xi_2\hat{p}\right)\right\}$$

R – "size" of meson in mass scale in the Fourier transformation

$$\tilde{U}_R(k) \sim \int_0^1 ds\; f(s)\cdot \exp\left\{-\frac{s\cdot k^2}{R^2}\right\}$$

Model parameters:

$$\Lambda = 236.0\ MeV,$$

$$m_{ud} = 227.65\, MeV, \qquad m_s = 420.07\, MeV,$$

$$m_c = 1521.61\, MeV, \qquad m_b = 4757.20\, MeV.$$

[GG, J.Phys. CS 938 (2017) 012047]

R – estimated to fit experimental data on meson masses and leptonic decay constants (in ***GeV***)

D	D_s	η_c	B	B_s	B_c	η_b
0.93	1.08	1.83	1.73	2.18	3.34	3.80
ρ	K*	D*	D_s*	J/Ψ	B*	ϒ
0.33	0.38	0.78	0.90	2.40	3.34	2.80

• *Estimated values of the leptonic decay constants:*

$J^{PC}=0^{-+}$	Exp.data	Our estim.	$J^{PC}=1^{--}$	Exp.data	Our estim.
f_D	206.7 ± 8.9	207	f_ρ	221 ± 1	221
f_{Ds}	257.5 ± 6.1	257	f_{K*}	217 ± 7	217
$f_{\eta c}$	438 ± 8 (**Lat**)	438	f_{D*}	245 ± 20	245
f_B	192.8 ± 9.9	193	f_{Ds*}	272 ± 26	271
f_{Bs}	238.8 ± 9.5	239	$f_{J/\psi}$	415 ± 7	416
f_{Bc}	489 ± 5	488	f_{B*}	196 ± 44	196
$f_{\eta b}$	801 ± 9	800	f_Υ	715 ± 5	715

[GG, J.Phys. CS 938 (2017) 012047]

+ Agreement between estimated and experimental data is accurate.

+ The meson “size” shrinks **~1/R** as the mass grows.

Part 1e: Glueballs (gg, ggg, …)

Experimental status:

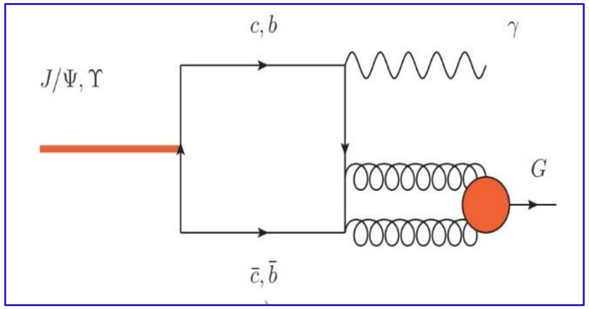
Main signatures expected for glueballs:

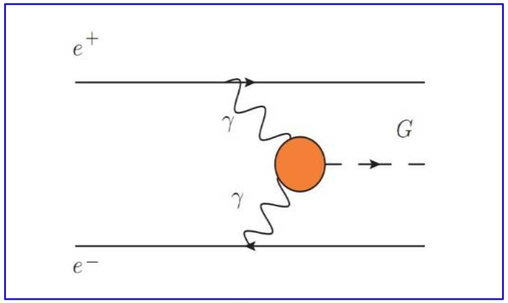
-- enhanced production in gluon-rich channels of radiative decays,

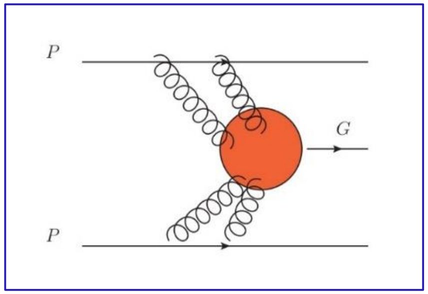
-- decay branching fractions incompatible with (q-qbar) states.

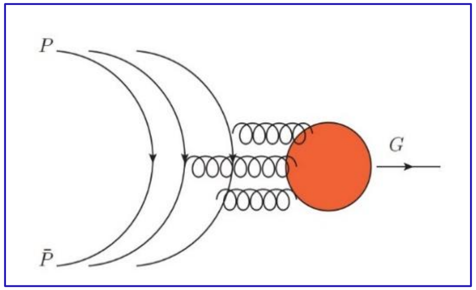
Production expected in:

- heavy quarkonium radiative decays (**BESIII, Belle, BaBar, LHCb**)
- photon-photon fusion (**BESIII, Belle, BaBar**)
- central meson production (**RHIC, LHC**)
- proton-antiproton annihilation (**PANDA**)
- heavy quark (b) weak decay (**BESII, Belle, LHCb**)







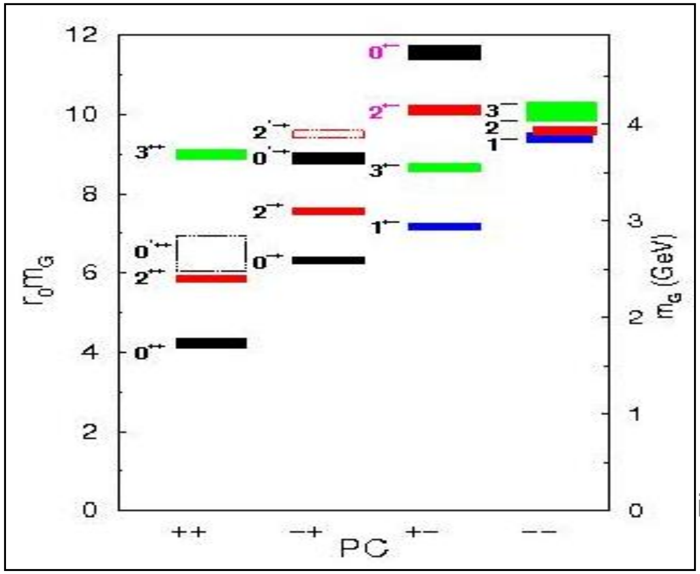


Theoretical status: QCD predicts glueballs due to self-interaction of gluons.

- **MIT bag (gg)** *Jaffe, Johnson*
- **QCD sum rules** *Narison, Forkel, SVZ, Vento, P. Zhang, Pimikov,…*
- **Constituent gluon models** *Simonov, Anisovich, Lubovitskij, …*
- **Instanton-inspired models** *Kochelev, Mathieu, Dong-Pil Min,* **…**
- **Lattice calculations** *Morningstar, Peardon, Weingarten (Q), Gregory, McNeile, Chen (unQ), …*
- **Holographic Ads/QCD models** *Vento, Colangelo, Rinaldi, …*

Identification problems:

- **Possible mixing with quarkonia (q-qbar, q^2-qbar^2)** *[e.g., V.Vento, 2016]*
- **Existence of exotic glueballs - oddballs (ggg, …)** *[e.g., P.Colangelo, 2015]*
- **Many states with the same quantum numbers for M > 1 GeV** *[e.g.,PDG-2016]*



$$J = 0, 1, 2, 3$$

$$PC = ++, -+, +-, --$$

$$M_G \approx 1.6 - 4.9 \text{ GeV}$$

Glueballs (gg)

Scalar Glueball: **$J^{PC} = 0^{++}$** (*field strength square*)

$$O_S = \delta^{\mu\alpha}\delta^{\nu\beta}\left(gG^A_{\mu\nu}\right)\left(gG^A_{\alpha\beta}\right) = \left(gG^A_{\mu\nu}\right)^2$$

Pseudoscalar Glueball: **$J^{PC} = 0^{-+}$** (*topological charge density*)

$$O_P = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\left(gG^A_{\mu\nu}\right)\left(gG^A_{\alpha\beta}\right)$$

Tensor Glueball: **$J^{PC} = 2^{++}$** (*energy density*)

$$O_T = \frac{1}{4}\left(gG^A_{\mu\nu}\right)^2 - \left(gG^A_{0\alpha}\right)\left(gG^A_{0\beta}\right)$$

Scalar Glueball (gg)

$J^{PC} = 0^{++}$

Candidates: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $f_0(1790)$

Signatures:
- Large branching to decay to strange mesons $0^{++} \to K\bar{K}$
- Large branching to decay to η $0^{++} \to \eta\eta,\ \eta'\sigma$
- Weak coupling to photon pairs $0^{++} \to \gamma\gamma$
- Perturbative coupling to heavy quarks (c & b)

Some theoretical estimates:

1750 ± 50 ± 80 MeV	C.J.Morningstar, M.Peardon (2004).
1710 ± 50 ±58 MeV	Anisotropic lattice, infinite volume, continuum limit (Y.Chen 2006)
1710 ± 50 MeV	Analytic confinement model (G.Ganbold PRD79, 2009)
1795 ± 60 MeV	Unquenched Lattice 2+1-flavor m_π =360 MeV (Gregory et al. 2012)
1790 ± 50 ± 20 MeV	UK-QCD Collaboration (2014)
1624 ± 140 MeV	Lattice 2-flavor m_π =580 MeV (Y.Chen et al. 2016)

Scalar glueball mass:

$$\Lambda = 236.0 \ MeV, \quad \alpha(M_G) = 0.451$$

$\Rightarrow$

$$M_G \approx 1738.6 \ MeV$$

[GG, J.Phys. CS 938 (2017) 012047]

Scalar glueball "radius":

$$r_G \cdot M_G = \frac{M_G}{2\Lambda} \sqrt{\frac{\int d^4x \cdot x^2 \cdot W(x) \cdot U^2(x)}{\int d^4x \cdot W(x) \cdot U^2(x)}} = 4.41$$

≈ 4.35 UK-QCD Collaboration (2014)

$$1/r_G = M_G / 4.41 = 394 \, MeV$$

≈ 410 ± 20 M pure $SU_c(3)$ gauge theory (CJM+MP)

Gluon condensate:

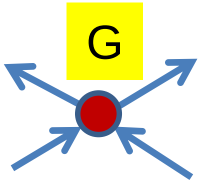
$$\left\langle \frac{\alpha}{\pi} F^2 \right\rangle = \frac{16 N_C}{\pi} \alpha \Lambda^4 = 0.0214 \ GeV^4$$

≈ 0.0223 ± 0.0041 GeV S.Narison, PLB706 (2012)

Part 2a: Model. Fermi Coupling

- **Fermi-type model**

$$L_F = \bar{q}\,(i\hat{\partial} - m)\,q + \frac{G}{2}(\bar{q}\,\Gamma\, q)^2$$



Generating functional

$$\Pi(x-y) \equiv -i \cdot \mathrm{tr}\left\{\Gamma \cdot \Phi \cdot S(x-y) \cdot \Gamma \cdot \Phi \cdot S(y-x)\right\}$$

CCQM used:

Vertex function:

$$\tilde{\Phi}_H(-p^2) = \exp\left(\frac{p^2}{\Lambda_H^2}\right)$$

$1/\Lambda_H$ ~ hadron "size"

Quark propagator:

$$\tilde{S}_{m1}(\hat{p}) = \frac{m_1 + \hat{p}}{m_1^2 - p^2} = \left(m_1 + \hat{p}\right) \cdot \int\limits_0^\infty ds_1 \exp\left[-s\left(m_1^2 - p^2\right)\right]$$

Meson mass equation:

$$1 = G \cdot \tilde{\Pi}(M^2)$$

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

$$\tilde{\Pi}_H(p^2) = \frac{3}{4\pi^2}\int_0^\infty \frac{dt\cdot t}{a_H^2}\int_0^1 ds \exp\left(-t\cdot z_0 + z_H\right)\cdot\left[\frac{n_H}{a_H} + m_1 m_2 + p^2\left(\omega_1 - \frac{b}{a_H}\right)\left(\omega_2 + \frac{b}{a_H}\right)\right]$$

$$\text{Integral} \quad \int_0^\infty \frac{dt\cdot t}{a_H^2}\ldots = \text{diverges and a threshold singularity appears!}$$

- "Infra-red" regularization with parameter **λ** has been used for the ***whole loop.***

$$\text{for } \lambda > 0: \qquad \int_0^{1/\lambda^2} \frac{dt\, t}{a_H^2}\ldots = \text{ converges!}$$

- Model parameters:

fixed earlier by fitting semileptonic decay constants and electromagnetic decay rates of mesons.

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

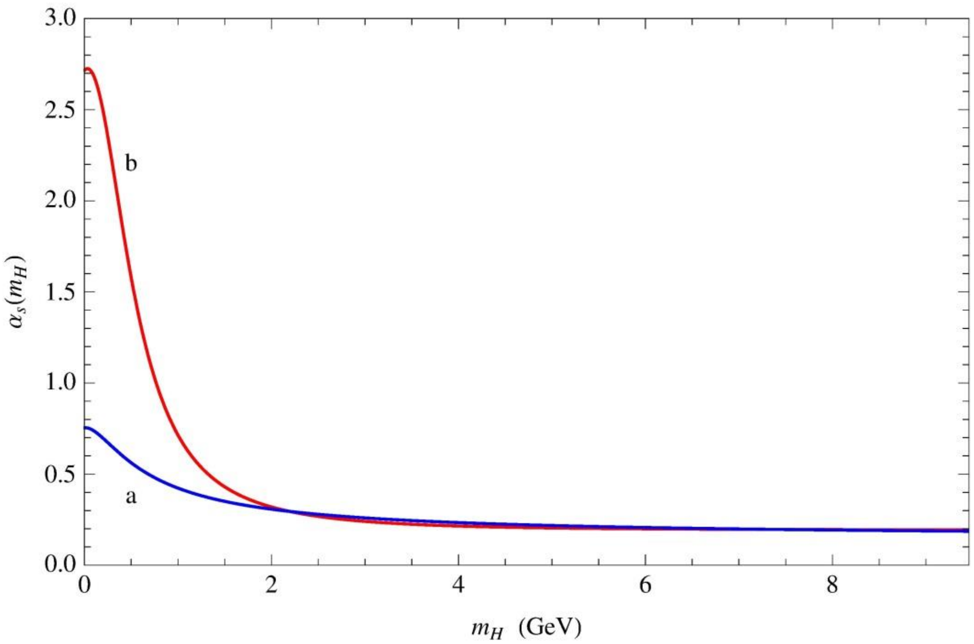
$$\lambda = 0.181\,\text{GeV},$$
$$m_{ud} = 0.235\,\text{GeV},$$
$$m_s = 0.442\,\text{GeV},$$
$$m_c = 1.67\,\text{GeV},$$
$$m_b = 5.07\,\text{GeV}$$

Fermi G coupling: Comparison with QCD Running Coupling

We compare dimensionless Fermi coupling 1.74 **λ^2*G (red curve)**
to QCD effective charge **α_s (blue curve).**

**G.Ganbold, T.Gutsche,
M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).**

**G.Ganbold,
Phys. Rev. D 81, 094008 (2010)
Phys. Part. Nucl. 43, 79, (2012)
Phys. Part. Nucl. 45, 10, (2014).**



[GG, Eur.Phys. J WC 138 (2017) 04004]

- Despite the different model origins, the behaviors of two curves are very similar each other in the region above ~ 2 GeV.
- The values at origin are mostly determined by the different confinement mechanisms. This explains the different behaviors below 2 GeV.

Part 2b: Charmonium Radiative Decays

Motivation:

PRL **119,** 062001 (2017) PHYSICAL REVIEW LETTERS week ending 11 AUGUST 2017

Observation of the Decays $\Lambda_b^0 \to \chi_{c1} p K^-$ and $\Lambda_b^0 \to \chi_{c2} p K^-$

R. Aaij *et al.*[*]
(LHCb Collaboration)
(Received 27 April 2017; published 8 August 2017)

♣ First observation of these decays @LHCb in ***p-p*** collision at energy (c.m.) ***8 TeV***

♣ Measured ratios of branching fractions:

$$\frac{\mathcal{B}(\Lambda_b^0 \to \chi_{c1} p K^-)}{\mathcal{B}(\Lambda_b^0 \to J/\psi p K^-)}$$

$$\frac{\mathcal{B}(\Lambda_b^0 \to \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \to J/\psi p K^-)}$$

$$\frac{\mathcal{B}(\Lambda_b^0 \to \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \to \chi_{c1} p K^-)}$$

$$\frac{\mathcal{B}(\Lambda_b^0 \to \chi_{c1} p K^-)}{\mathcal{B}(\Lambda_b^0 \to J/\psi p K^-)} = 0.242 \pm 0.014 \pm 0.013 \pm 0.009,$$

$$\frac{\mathcal{B}(\Lambda_b^0 \to \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \to J/\psi p K^-)} = 0.248 \pm 0.020 \pm 0.014 \pm 0.009,$$

$$\frac{\mathcal{B}(\Lambda_b^0 \to \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \to \chi_{c1} p K^-)} = 1.02 \pm 0.10 \pm 0.02 \pm 0.05,$$

Comparing with:

$$\boxed{\frac{B(B \to \chi_{c2} K)}{B(B \to \chi_{c1} K)}}$$

Belle (2008), **BaBar** (2009), **LHCb** (2013): ***SUPPRESSED***

M.Beneke NPB811 (2009) Factorization approach: ***SUPPRESSED***

Belle Collaboration (2016): ***SUPPRESSION LESSENED***

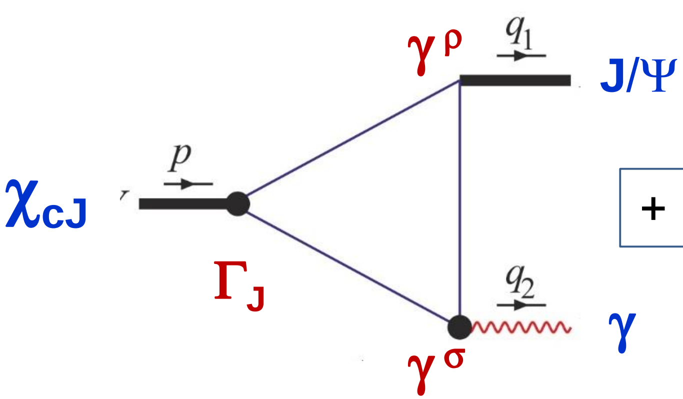
if additional particles are present in the final state

Charmonium orbital (triplet) excitations: χ_{cJ} (J = 0, 1, 2)

	J^{PC}	Spin	L	Mass (MeV)	Current	$\Gamma_{\text{full PDG-2018}}$
η	0^{-+}	0	0	2983.9	*Pseudoscalar*	32.0 ± 0.8 MeV
J/Ψ	1^{--}	1	0	3096.90	*Vector*	92.9 ± 2.8 keV
χ_{c0}	0^{++}	1	1	3414.71	*Scalar*	10.8 ± 0.6 MeV
χ_{c1}	1^{++}	1	1	3510.67	*Axial-vector*	0.84 ± 0.04 MeV
χ_{c2}	2^{++}	1	1	3556.17	*Tensor*	1.97 ± 0.09 MeV

Dominant (one-photon) transitions:

$$\Gamma_{\chi c0} = I,$$
$$\Gamma_{\chi c1} = \gamma_\mu \gamma_5,$$
$$\Gamma_{\chi c2} = \frac{i}{2}\left(\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu\right).$$



+ 'bubble' diagrams

$g_{\chi_{cJ}\to J/\Psi+\gamma}$

Matrix elements and Decay widths of transitions:

in collaboration with T.Gutsche, M.A.Ivanov, V.Lubovitsky

Scalar Excitation: $J^{PC} = 0^{++}$

$X = \chi_{c0}$

Matrix element (taking into accouny gauge inv. conditions):

$$M_{\sigma\rho}^{\chi_{c0}\to J/\Psi+\gamma}(p,q_2) \approx D\cdot\left[p^{\sigma}q_2^{\rho} - g_{\sigma\rho}(p\cdot q_2)\right]$$

Decay width:

$$\Gamma(\chi_{c0} \to J/\Psi + \gamma) = \frac{\alpha}{24} M_{\chi_{c0}}^3 \left(1 - \frac{M_{J/\Psi}^2}{M_{\chi_{c0}}^2}\right)^3 \cdot g_{\chi_{c0}\to J/\Psi+\gamma}^2(D)$$

Axial-vector Excitation: $J^{PC} = 1^{++}$

$X = \chi_{c1}$

Matrix element (Five Lorentz structures):

S.Dubnicka et al, Phys. Rev. D 84, 014006 (2011)

$$M_{\mu\rho\sigma}^{\chi_{c1}\to J/\Psi+\gamma}(q_1,q_2) \approx \varepsilon^{q_2\mu\rho\sigma}(q_1 q_2)\cdot W_1 + \varepsilon^{q_1\mu\rho\sigma}(q_1 q_2)\cdot W_5 + \varepsilon^{q_1 q_2\rho\sigma} q_1^{\mu}\cdot W_2$$
$$+\varepsilon^{q_1 q_2\mu\sigma} q_2^{\rho}\cdot W_3 + \varepsilon^{q_1 q_2\mu\rho} q_1^{\sigma}\cdot W_4, \qquad W_4 + W_5 = 0.$$

♣ ***gauge inv. + relations with*** $T^{\alpha\mu\nu\rho\sigma} = g_{\alpha\mu}\cdot\varepsilon^{\mu\nu\rho\sigma}$ + cyclic$(^{\mu\nu\rho\sigma})$ → *remain two independents:*

Two independent covariants → via Helicity amplitudes:

$$H_L = i\frac{m_X^2}{m_{J/\psi}}|\vec{q}_2|^2\left[W_1 + W_3 - \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}W_4\right],$$

$$H_T = -im_X|\vec{q}_2|^2\left[W_1 + W_2 - \left(1 + \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}\right)W_4\right],$$

$$|\vec{q}_2| = \frac{m_X^2 - m_{J/\psi}^2}{2m_X}.$$

Decay width

$$\boxed{\Gamma(X\to\gamma J/\psi) = \frac{1}{12\pi}\frac{|\vec{q}_2|}{m_X^2}(|H_L|^2 + |H_T|^2)}$$

Tensor Excitation: **$J^{PC} = 0^{++}$**

$X = \chi_{c2}$

Matrix element (taking into account gauge inv. conditions):

$$M^{\chi_{c2}\to J/\Psi+\gamma}_{\mu\nu\rho\sigma}(q_1,q_2) = A\cdot\left\{\begin{array}{l} g^{\mu\rho}\left[g^{\sigma\nu}(q_1q_2)-q_1^{\sigma}q_2^{\nu}\right] \\ +g^{\nu\rho}\left[g^{\sigma\mu}(q_1q_2)-q_1^{\sigma}q_2^{\mu}\right]\end{array}\right\} + B\cdot\left\{\begin{array}{l} g^{\sigma\rho}\left[q_1^{\mu}q_2^{\nu}+q_1^{\nu}q_2^{\mu}\right] \\ -g^{\mu\sigma}q_1^{\nu}q_2^{\rho}-g^{\nu\sigma}q_1^{\mu}q_2^{\rho}\end{array}\right\}$$

Decay width:

$$\boxed{\Gamma(\chi_{c2}\to J/\Psi+\gamma) = \frac{\alpha}{20}M^3_{\chi_{c2}}\left(1-\frac{M^2_{J/\Psi}}{M^2_{\chi_{c2}}}\right)^2\cdot g^2_{\chi_{c2}\to J/\Psi+\gamma}(A,B)}$$

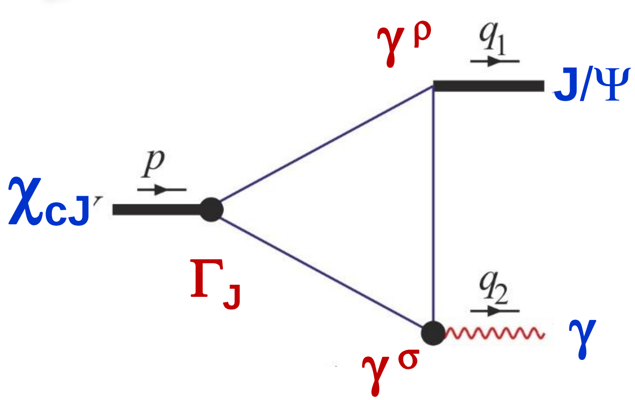
Charmonium excitations: *One-photon Decay Widths*

		$\Lambda_{\chi_{cJ}}$	Mass	$\Gamma^{theor}\left(\chi_{cJ}\to J/\Psi+\gamma\right)$	$\Gamma^{experm}\left(\chi_{cJ}\to J/\Psi+\gamma\right)$
J/Ψ	**1** $^{--}$	**1.68** GeV	3096.90 MeV	**0.157·10**$^{-5}$ GeV	**(0.158±0.041)·10**$^{-5}$ GeV
χ_{c0}	**0** $^{++}$	**3.46** GeV	3414.71 MeV	**0.150·10**$^{-3}$ GeV	**(0.151±0.014)·10**$^{-3}$ GeV
χ_{c1}	**1** $^{++}$	**0.66** GeV	3510.67 MeV	**0.289·10**$^{-3}$ GeV	**(0.288±0.022)·10**$^{-3}$ GeV
χ_{c2}	**2** $^{++}$	**3.18** GeV	3556.17 MeV	**0.373·10**$^{-3}$ GeV	**(0.374±0.026)·10**$^{-3}$ GeV

$$\lambda = 0.181\,MeV$$
$$m_c = 1.681\,GeV$$

PDG-2018

Dominant (one-photon) transitions: (*including* $J/\Psi \to \gamma + \eta_c$)



Summary:

♣ **Analytic** (**Infrared) Confinement** conception combined with QFT methods may serve reasonable frameworks to address simultaneously **different sectors in particle physics, such as:**

- meson ground state spectrum (except light mesons: π, K)

- weak (leptonic) decay constants of conventional mesons

- running (mass dependent) strong effective coupling

- the lowest (scalar) glueball mass and radius

- the gluon condensate

- Fermi coupling's smooth dependence on mass scale

- radiative transition parameters and widths of excited charmonia

Outlook:

Our models can be extended to study:

♣ light mesons (scalar, isoscalar, …).

♣ higher glueball states (**0^{-+}**, **2^{--}**, …)

♣ admixture states (***qq*** + ***gg***, …)

♣ radial excitations of mesons (charmonia and bottomia).

♣ exotic mesons (tetraquark, ***X(3872)*** and ***Z(4430)*** …)

♣ heavy baryon decays ($\Lambda_b \to \Lambda^* + \chi_{cJ}$)