Numerical estimation of radiative corrections to e^+e^--annihilation processes

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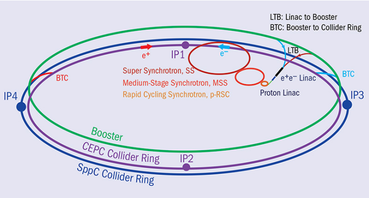
Outline

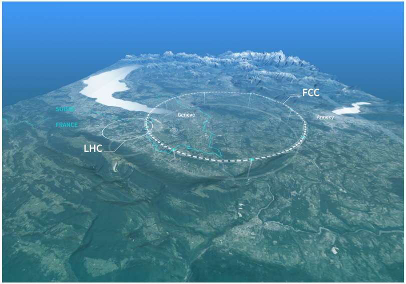
Motivation

- Projects of new e^+e^- colliders for tests of the SM: Z-peak, Higgs boson production, t-quark physics...
- Precise theoretical calculations are needed to predict and describe the results of the experiments with high precision and sensitivity
- Higher order radiative corrections are needed
- Calculation of higher order radiative corrections is a complicated task $\rightarrow$ methods to make it easier
- To compare our theoretical predictions with the experiment we need numerical results

Future: e^+e^- colliders

- FCC-ee (CERN, construction planned in 2030s) - max 350 GeV
- CEPC (China, construction planned in 2027-2035) - max 240 GeV
- CLIC (CERN, start construction in 2026) - max 3 TeV
- ILC - (?) 500-1000 GeV





Parton distribution functions approach

- Based on perturbation theory and R. Feynman parton theory
- Came from QCD to QED
- Allows to calculate the most significant (logarithmic) corrections
- Expansion in powers of coupling constant and the large logarithm

Large logarithm

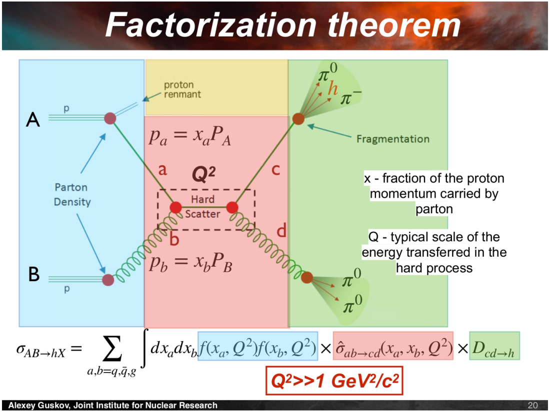
$$L = \ln \frac{\mu^2}{\mu_0^2}$$

μ - factorization scale, μ_0 - renormalization scale (in QED $\mu_0 = m_e$)

- Leading logarithmic approximation (LL or leading order - LO) - $\alpha^n L^n$
- Next-to-leading logarithmic approximation (NLL or next-to-leading order - NLO) - $\alpha^n L^{n-1}$

Parton distribution functions approach

- Partons in QED - electron, positron or photon
- Solve evolution equation of PDFs by iterations to get the desired approximation
- PDF evolution equation is an analog to DGLAP equation in QCD
- Make convolution of process independent PDFs with the functions determining the process



From presentaton of A. Guskov

PDF evolution equation in QED

$$D_{ba}(x,\mu^2,\mu_0^2) = \delta(1-x)\delta_{ba} + \sum_{i=e,\bar{e},\gamma} \int_{\mu_0^2}^{\mu^2} \frac{dt\alpha(t)}{2\pi t} \int_x^1 \frac{dy}{y} D_{ia}(y,t,\mu_0) P_{bi}\left(\frac{x}{y}\right)$$

Equations are solved using iterative method:

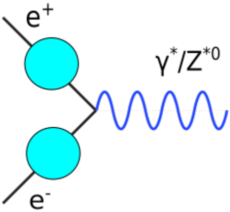
$$D_{ee}^{(k)} = D_{ee}^{(0)} + \frac{\alpha}{2\pi}\left(P_{ee}\otimes D_{ee}^{(k-1)} + P_{e\gamma}\otimes D_{\gamma e}^{(k-1)} + P_{e\bar{e}}\otimes D_{\bar{e}e}^{(k-1)}\right)$$
$$D_{\gamma e}^{(k)} = D_{\gamma e}^{(0)} + \frac{\alpha}{2\pi}\left(P_{\gamma e}\otimes D_{ee}^{(k-1)} + P_{\gamma\bar{e}}\otimes D_{\bar{e}e}^{(k-1)} + P_{\gamma\gamma}\otimes D_{\gamma e}^{(k-1)}\right)$$
$$P_{ji}(x) = P_{ji}^{(0)}(x) + \frac{\alpha}{2\pi}P_{ji}^{(1)}(x) + \mathcal{O}(\alpha^2)$$

Initial conditions:

$$D_{ee}^{(0)}(x,\mu^2) = \delta(1-x) + \frac{\alpha}{2\pi}d_{ee}^{(1)}, \quad D_{\bar{e}\gamma}^{(0)}(x,\mu^2) = 0, \quad D_{\gamma\gamma}^{(0)}(x,\mu^2) = \delta(1-x)\frac{\beta}{3}$$
$$D_{e\gamma}^{(0)}(x,\mu^2) = \frac{\alpha}{2\pi}d_{e\gamma}^{(1)}(x), \quad D_{e\bar{e}}^{(0)}(x,\mu^2) = 0$$

e^+e^--annihilation

$$e^+e^- \to Z^*/\gamma^*$$



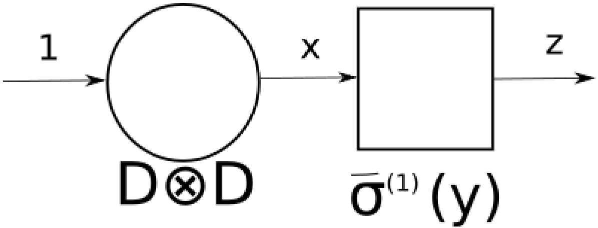
Correction up to the order $\alpha^5 L^6$ were calculated in *Ablinger at al., Nucl. Phys. B, 955, 115045, 2020*

$$\sigma_{\bar{e}e}^{\rm NLO}(s') = \sum_{i,j=e,\bar{e},\gamma} \int\limits_{\bar{z}_1}^{1} \int\limits_{\bar{z}_2}^{1} dz_1 dz_2 D_{ie}^{\rm str}\left(z_1, \frac{\mu_R^2}{\mu_F^2}\right) D_{j\bar{e}}^{\rm str}\left(z_2, \frac{\mu_R^2}{\mu_F^2}\right) \times$$

$$\times \left(\sigma_{ij}^{(0)}(sz_1z_2) + \bar{\sigma}_{ij}^{(1)}(sz_1z_2) + \mathcal{O}(\alpha^2 L^0)\right) \delta(s' - sz) + \mathcal{O}\left(\frac{\mu_R^2}{\mu_F^2}\right)$$

$$D_{\bar{e}\bar{e}} = D_{ee}, \; D_{\gamma\bar{e}} = D_{\gamma e}$$

Cross-section



$$\frac{d\sigma_{\bar{e}e}}{ds'} = \sigma^{(0)}\Big[D_{\bar{e}\bar{e}} \otimes D_{ee} \otimes \sigma_{\bar{e}e} + D_{\gamma e} \otimes D_{ee} \otimes \sigma_{e\gamma} + D_{ee} \otimes D_{e\bar{e}} \otimes \sigma_{ee} +$$
$$+D_{\gamma e} \otimes D_{\bar{e}\bar{e}} \otimes \sigma_{\bar{e}\gamma} + D_{\gamma e} \otimes D_{\gamma\bar{e}} \otimes \sigma_{\gamma\gamma} + D_{\gamma e} \otimes D_{e\bar{e}} \otimes \sigma_{e\gamma}$$
$$+D_{\bar{e}e} \otimes D_{\bar{e}\bar{e}} \otimes \sigma_{\bar{e}\bar{e}} + D_{\bar{e}e} \otimes D_{\gamma\bar{e}} \otimes \sigma_{\bar{e}\gamma} + D_{\bar{e}e} \otimes D_{e\bar{e}} \otimes \sigma_{\bar{e}e}\Big]$$

Contributions to cross-section of different orders

i \ j	$\bar{e}$	γ	e
e	$D_{ee}D_{\bar{e}\bar{e}}\sigma_{e\bar{e}}$ LO (1)	$D_{ee}D_{\gamma\bar{e}}\sigma_{e\gamma}$ NLO $(\alpha^2 L)$	$D_{ee}D_{e\bar{e}}\sigma_{ee}$ NNLO $(\alpha^4 L^2)$
γ	$D_{\gamma e}D_{\bar{e}\bar{e}}\sigma_{\gamma\bar{e}}$ NLO $(\alpha^2 L)$	$D_{\gamma e}D_{\gamma\bar{e}}\sigma_{\gamma\gamma}$ NNLO $(\alpha^4 L^2)$	$D_{\gamma e}D_{e\bar{e}}\sigma_{\gamma e}$ NLO $(\alpha^4 L^3)$
$\bar{e}$	$D_{\bar{e}e}D_{\bar{e}\bar{e}}\sigma_{\bar{e}\bar{e}}$ NNLO $(\alpha^4 L^2)$	$D_{\bar{e}e}D_{\gamma\bar{e}}\sigma_{\bar{e}\gamma}$ NLO $(\alpha^4 L^3)$	$D_{\bar{e}e}D_{e\bar{e}}\sigma_{\bar{e}e}$ LO $(\alpha^4 L^4)$

$$d\sigma^{\mathrm{NLO}}_{ab\to cd} = d\sigma^{(0)}_{ab\to cd}\left\{1 + \sum_{k=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^k \sum_{l=k-1}^{k} c_{kl}L^l + \mathcal{O}(\alpha^k L^{k-2})\right\}$$

PDF evolution equation in QED

Convolution operation

$$(f \otimes g)(x) \equiv \int_0^1 dz \int_0^1 dy \; f(z)g(y)\delta(x - yz) = \int_x^1 \frac{dz}{z} f(z) g(\frac{x}{z})$$

$$f(x) = \lim_{\Delta \to 0} \Big(f_\Theta(x)\Theta(1 - x - \Delta) + f_\Delta \delta(1 - x) \Big)$$

$$\int_z^1 dx [f(x)]_+ g(x) = \int_0^1 dx f(x) \Big[g(x)\Theta(x - z) - g(1) \Big]$$

Delta and Theta parts:

$$f_\Delta = - \int_0^{1-\Delta} f_\Theta(z) dz$$

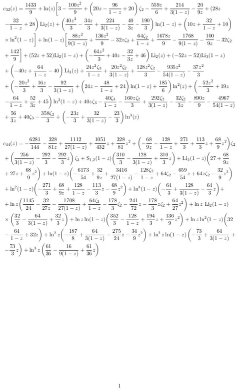
$$\Big(f \otimes g\Big)_\Theta(z) = \lim_{\Delta \to 0} \Big\{ \int_{z/(1-\Delta)}^{1-\Delta} \frac{dx}{x} f_\Theta(x) g_\Theta \left(\frac{z}{x}\right) + f_\Delta g_\Theta(z) + f_\Theta(z) g_\Delta \Big\}$$

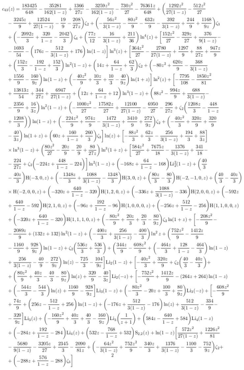
Analytical results

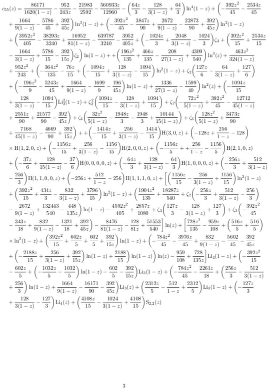
$$d\sigma^{\mathrm{NLO}}_{ab\to cd} \quad = \quad d\sigma^{(0)}_{ab\to cd}\left\{1+\sum_{k=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^k\sum_{l=k-1}^{k}c_{kl}L^l+\mathcal{O}(\alpha^k L^{k-2})\right\}$$

$$c^{\Delta}_{32}=\left(-\frac{2951}{27}+48\zeta_3+104\zeta_2\right)\ln\Delta+\left(32\zeta_2-\frac{280}{3}\right)\ln^2\Delta-\frac{64}{3}\ln^3\Delta-$$
$$-\frac{1387}{36}-80\zeta_4+\frac{4}{3}\zeta_3+\frac{928}{9}\zeta_2$$
$$c^{\Delta}_{43}=-\frac{13969}{216}+\frac{1504\zeta_2\zeta_3}{3}+\frac{1465\zeta_2}{27}+\frac{314\zeta_3}{9}-\frac{488\zeta_4}{3}-1024\zeta_5-\frac{64}{3}\ln^4\Delta+$$
$$+\left(\frac{896\zeta_2}{3}-\frac{1376}{9}\right)\ln^3\Delta+\left(\frac{1616\zeta_2}{3}-672\zeta_3-\frac{8234}{27}\right)\ln^2\Delta+\left(\frac{16408\zeta_2}{27}-\right.$$
$$\left.-\frac{1856\zeta_3}{3}+320\zeta_4-\frac{19270}{81}\right)\ln\Delta$$
$$c^{\Delta}_{55}=\ln\Delta\left(\frac{44243}{810}+64\zeta_4+\frac{3328}{9}\zeta_3-\frac{2672}{9}\zeta_2\right)+\ln^2\Delta\left(\frac{8476}{81}+\frac{512}{3}\zeta_3-\right.$$
$$\left.-\frac{832}{3}\zeta_2\right)+\ln^3\Delta\left(\frac{2672}{27}-\frac{256}{3}\zeta_2\right)+\frac{416}{9}\ln^4\Delta+\frac{128}{15}\ln^5\Delta+\frac{2431}{216}+$$
$$+\frac{1024}{5}\zeta_5+\frac{208}{3}\zeta_4+\frac{5344}{27}\zeta_3-\frac{8476}{81}\zeta_2-\frac{512}{3}\zeta_2\zeta_3$$

Analytical results







arXiv:2405.03443 [hep-ph]

Numerical results

$$h_{ij} = \left(\frac{\alpha}{2\pi}\right)^i L^j \int\limits_{z_{min}}^{1-\Delta} dz(\sigma^{(0)}(z)c_{ij}^{\theta}(z) + c_{ij}^{\Delta}\sigma^{(0)}(1))$$

$$\sigma_{e^+e^-} = \sum_{i,j}\left(\frac{\alpha}{2\pi}\right)^i L^j \int\limits_{z_{min}}^{1-\Delta} dz(\sigma^{(0)}(z)c_{ij}^{\theta}(z) + c_{ij}^{\Delta}\sigma^{(0)}(1))$$

Corrections of different orders, %

Beam energy: $\frac{M_z-10}{2}$, $\frac{M_z-1}{2}$, $\frac{M_z}{2}$, $\frac{M_z+1}{2}$, $\frac{M_z+10}{2}$

$Z_{min} = 0.1, 0.3, 0.5, 0.7, 0.9, 0.99$

$$E_{beam} = \frac{M_z-1}{2},\ Z_{min} = 0.5,\ \Delta = 10^{-7}$$

h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
				γ				
-37.66044	2.2067	7.28480	-0.82656	-0.95286	0.15745	0.09393	-0.02019	-0.00737
				Pairs				
0	0	-0.35204	0.2038	0.13181	-0.08358	0.00234	0.01642	0,00296
				Full				
-37.66044	2.2067	6.93272	-0.62276	-0.82106	0.07387	0.09627	-0.00377	-0.00441

Numerical results

Corrections of different orders, %

$$E_{beam} = \frac{M_z+1}{2},\ Z_{min} = 0.5,\ \Delta = 10^{-7}$$

h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
				γ				
-10.88572	1.09775	-3.20974	0.21972	1.04591	-0.15571	-0.14950	0.03139	0.01349
				Pairs				
0	0	-0.10159	-0.00742	-0.06162	0.06034	-0.00103	-0.02194	-0,00499
				Full				
-10.88572	1.09775	-3.31133	0.21230	0.98429	-0.09537	-0.15054	0.00945	0.00849

$$E_{beam} = \frac{M_z}{2},\ Z_{min} = 0.5,\ \Delta = 10^{-7}$$

h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
				γ				
-32.75622	2.00249	4.88313	-0.59516	-0.37751	0.07101	0.00345	-0.00185	0.00315
				Pairs				
0	0	-0.30650	0.15857	0.08753	-0.04598	0.00157	0.00375	-0,00008
				Full				
-32.75622	2.00249	4.57663	-0.43660	-0.28998	0.02502	0.00502	0.00190	0.00307

Numerical results

$$E_{beam} = \frac{M_z - 10}{2},\ Z_{min} = 0.5,\ \Delta = 10^{-7}$$

h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
				γ				
-19.12788	1.44663	2.17046	-0.27903	-0.18658	0.03160	0.01273	-0.00274	-0.00071
				Pairs				
0	0	-0.17376	0.08432	0.03718	-0.02072	0.00068	0.00265	0,00026
				Full				
-19.12788	1.44663	1.99669	-0.19471	-0.14941	0.01088	0.01342	-0.00009	-0.00045

$$E_{beam} = \frac{M_z + 10}{2},\ Z_{min} = 0.5,\ \Delta = 10^{-7}$$

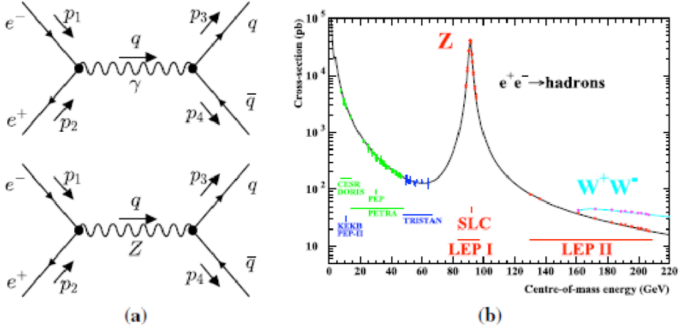
h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
				γ				
184.22291	-6.90449	-22.10853	3.15237	-0.35227	0.07992	0.21928	-0.04591	-0.01688
				Pairs				
0	0	1.80239	-0.82578	0.41544	0.15144	-0.00723	0.01948	0,00573
				Full				
184.22291	-6.90449	-20.30614	2.32659	-0.76771	0.07152	0.21206	-0.02642	-0.01114

Numerical results

$$E_{beam} = \frac{M_z + 10}{2}$$

$Z_{min} = 0.7$								
h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
γ								
		-22.04865	3.12323	-0.32062	-0.08455	0.21710	-0.04521	-0.01688
Pairs								
		1.76420	-0.81231	-0.41363	0.15146	-0.00721	0.01896	0,00585
Full								
180.78750	-6.76365	-20.28446	2.31091	-0.73425	0.06692	0.20989	-0.02642	-0.01104
$Z_{min} = 0.9$								
γ								
		-3.53033	0.30905	0.57429	-0.09861	0.03891	-0.00988	-0.00048
Pairs								
		-0.01564	-0.03548	-0.06738	-0.04443	-0.00115	-0.00880	-0,00117
Full								
-1.75754	0.72009	-3.54597	0.27357	0.50691	-0.05418	-0.04007	0.00108	-0.00068
$Z_{min} = 0.99$								
γ								
		7.96127	-0.91066	-0.73201	0.13316	0.00337	-0.00364	0.00907
Pairs								
		-0.40337	0.22508	0.14528	-0.07880	0.00260	0.00711	-0,00029
Full								
-42.71585	2.39924	7.55790	-0.68559	-0.58674	0.05435	0.00597	0.00347	0.00878

Cross-section



(a) (b)

From M. Stoeltzner, 2022

Conclusion

- Higher-order radiative corrections to e^+e^--annihilation are necessary for prediction of results of the experiments at e^+e^- colliders
- We derived analytical expressions for the coefficients
- We made numerical estimation at different energies
- Plans - numerical results for future colliders energies
- Results will be implemented into ZFITTER program

Thank you for your attention!

Harmonic polylogarithms

$H(a_1, ..., a_k; z)$ – harmonic polylogarithms (HPLs) To determine HPLs, the following functions are introdused:

$$f_1(z) = \frac{1}{1-z}, \quad f_0(z) = \frac{1}{z}, \quad f_{-1}(z) = \frac{1}{1+z},$$

HPLs can be obtained recursively integrating these functons:

$$H(1;z) = \int\limits_0^z f_1(y)dy = -\ln(1-z), \quad H(0;z) = \ln z,$$

$$H(-1;z) = \int\limits_0^z f_{-1}(y)dy = -\ln(1+z),$$

$$H(^n0;z) = \frac{1}{n!}\ln^n z,$$

$$H(a, a_1, ..., a_k; z) = \int\limits_0^z f_a(y) H(a_1, ..., a_k; y)dy,$$

where

$$^n0 = \underbrace{0, ..., 0}_{n}$$

PDF evolution equation in QED

Convolution operation

$$(f \otimes g)(x) \equiv \int_0^1 dz \int_0^1 dy \; f(z) g(y) \delta(x - yz) = \int_x^1 \frac{dz}{z} f(z) g(\frac{x}{z})$$

$$f(x) = \lim_{\Delta \to 0} \Big(f_\Theta(x) \Theta(1 - x - \Delta) + f_\Delta \delta(1 - x) \Big)$$

$$\int_z^1 dx [f(x)]_+ g(x) = \int_0^1 dx f(x) \Big[g(x) \Theta(x - z) - g(1) \Big]$$

$$f_\Delta = - \int_0^{1-\Delta} f_\Theta(z) dz$$

$$\Big(f \otimes g \Big)_\Theta (z) = \lim_{\Delta \to 0} \Bigg\{ \int_{z/(1-\Delta)}^{1-\Delta} \frac{dx}{x} f_\Theta(x) g_\Theta \left(\frac{z}{x} \right) + f_\Delta g_\Theta(z) + f_\Theta(z) g_\Delta \Bigg\}$$

PDF evolution equation in QED

$$\alpha(q^2) = \frac{\alpha_0}{1 + \overline{\Pi}(\frac{-q^2}{\mu^2}, \frac{\overline{m}}{\mu}, \alpha_0)}$$

$$\overline{\Pi} = 2\alpha_0 \left(\left(\frac{5}{9} - \frac{L}{3} \right) + 4\alpha_0^2 \left(\frac{55}{48} - \zeta_3 - \frac{L}{4} \right) + 8\alpha_0^3 \left(\frac{-L^2}{24} \right) \right) + \ldots$$

P. A. Baikov, K. G. Chetyrkin, J. H. Kuhn and C. Sturm, Nucl. Phys. B **867** *(2013), 182-202*
$\alpha_0 = \frac{1}{137}$

Contribution of functions of positron in electron type

$$\Delta c_{44} = \frac{1}{3}\sigma^{(0)}_{e\bar{e}} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma e} \otimes P^{(0)}_{\gamma\bar{e}} \otimes P^{(0)}_{\bar{e}\gamma}$$

From evolution equation:

$$\frac{1}{12}\sigma^{(0)}_{e\bar{e}} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma e} \otimes P^{(0)}_{\gamma\bar{e}} \otimes P^{(0)}_{\bar{e}\gamma}$$

From including $D_{e\bar{e}} \otimes D_{\bar{e}e}$ into cross-section:

$$\frac{1}{4}\sigma^{(0)}_{e\bar{e}} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma e} \otimes P^{(0)}_{\gamma\bar{e}} \otimes P^{(0)}_{\bar{e}\gamma}$$

$$\Delta c_{44}(z) = \frac{1}{3}\Bigg[\ln z\left(-21 - \frac{16}{9z} - 21z - \frac{16}{9}z^2\right) + \ln^2 z\,(-2+2z) + \\ + \ln^3 z\left(-\frac{2}{3} - \frac{2}{3}z\right) - 26 - \frac{176}{27z} + 26z + \frac{176}{27}z^2\Bigg]$$