Numerical estimation of radiative corrections to e^+e^- -annihilation processes The XXVIII International Scientific Conference of Young Scientists and Specialists

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Outline



2 PDF approach

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- 4 Numerical results



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Motivation

- Projects of new e^+e^- colliders for tests of the SM: Z-peak, Higgs boson production, t-quark physics...
- Precise theoretical calculations are needed to predict and describe the results of the experiments with high precision and sensitivity
- Higher order radiative corrections are needed
- $\bullet\,$ Calculation of higher order radiative corrections is a complicated task $\to\,$ methods to make it easier
- To compare our theoretical predictions with the experiment we need numerical results

Future: e^+e^- colliders

- FCC-ee (CERN, construction planned in 2030s) max 350 GeV
- CEPC (China, construction planned in 2027-2035) max 240 GeV
- CLIC (CERN, start construction in 2026) max 3 TeV
- ILC (?) 500-1000 GeV





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Parton distribution functions approach

- Based on perturbation theory and R. Feynman parton theory
- Came from QCD to QED
- Allows to calculate the most significant (logarithmic) corrections
- Expansion in powers of coupling constant and the large logarithm Large logarithm

$$L = \ln rac{\mu^2}{\mu_0^2}$$

 μ - factorization scale, μ_0 - renormalization scale (in QED $\mu_0=m_e$)

- Leading logarithmic approximation (LL or leading order LO) $\alpha^n L^n$
- Next-to-leading logarithmic approximation (NLL or next-to-leading order NLO) $\alpha^n L^{n-1}$

Parton distribution functions approach

- Partons in QED electron, positron or photon
- Solve evolution equation of PDFs by iterations to get the desired approximation
- PDF evolution equation is an analog to DGLAP equation in QCD
- Make convolution of process independent PDFs with the functions determining the process



PDF evolution equation in QED

$$D_{ba}(x,\mu^2,\mu_0^2) = \delta(1-x)\delta_{ba} + \sum_{i=e,\bar{e},\gamma} \int_{\mu_0^2}^{\mu^2} \frac{dt\alpha(t)}{2\pi t} \int_x^1 \frac{dy}{y} D_{ia}(y,t,\mu_0) P_{bi}\left(\frac{x}{y}\right)$$

Equations are solved using iterative method:

$$\begin{split} D_{ee}^{(k)} &= D_{ee}^{(0)} + \frac{\alpha}{2\pi} \left(P_{ee} \otimes D_{ee}^{(k-1)} + P_{e\gamma} \otimes D_{\gamma e}^{(k-1)} + P_{e\bar{e}} \otimes D_{\bar{e}e}^{(k-1)} \right) \\ D_{\gamma e}^{(k)} &= D_{\gamma e}^{(0)} + \frac{\alpha}{2\pi} \left(P_{\gamma e} \otimes D_{ee}^{(k-1)} + P_{\gamma \bar{e}} \otimes D_{\bar{e}e}^{(k-1)} + P_{\gamma \gamma} \otimes D_{\gamma e}^{(k-1)} \right) \\ P_{ji}(x) &= P_{ji}^{(0)}(x) + \frac{\alpha}{2\pi} P_{ji}^{(1)}(x) + \mathcal{O}(\alpha^2) \end{split}$$

Initial conditions:

$$D_{ee}^{(0)}(x,\mu^2) = \delta(1-x) + \frac{\alpha}{2\pi} d_{ee}^{(1)}, \quad D_{\bar{e}\gamma}^{(0)}(x,\mu^2) = 0, \quad D_{\gamma\gamma}^{(0)}(x,\mu^2) = \delta(1-x)\frac{\beta}{3}$$
$$D_{e\gamma}^{(0)}(x,\mu^2) = \frac{\alpha}{2\pi} d_{e\gamma}^{(1)}(x), \quad D_{e\bar{e}}^{(0)}(x,\mu^2) = 0$$

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e^+e^- -annihilation



Correction up to the order $\alpha^5 L^6$ were calculated in Ablinger at al., Nucl. Phys. B, 955, 115045, 2020

$$\sigma_{\bar{e}e}^{\mathrm{NLO}}(s') = \sum_{i,j=e,\bar{e},\gamma} \int_{\bar{z}_1}^{1} \int_{\bar{z}_2}^{1} dz_1 dz_2 D_{ie}^{\mathrm{str}}\left(z_1, \frac{\mu_R^2}{\mu_F^2}\right) D_{j\bar{e}}^{\mathrm{str}}\left(z_2, \frac{\mu_R^2}{\mu_F^2}\right) \times \\ \times \left(\sigma_{ij}^{(0)}(sz_1z_2) + \bar{\sigma}_{ij}^{(1)}(sz_1z_2) + \mathcal{O}(\alpha^2 L^0)\right) \delta(s' - sz) + \mathcal{O}\left(\frac{\mu_R^2}{\mu_F^2}\right) \\ D_{\bar{e}\bar{e}} = D_{ee}, D_{\gamma\bar{e}} = D_{\gamma e}$$

Cross-section



$$\begin{aligned} \frac{d\sigma_{\bar{e}e}}{ds'} &= \sigma^{(0)} \Big[D_{\bar{e}\bar{e}} \otimes D_{ee} \otimes \sigma_{\bar{e}e} + D_{\gamma e} \otimes D_{ee} \otimes \sigma_{e\gamma} + D_{ee} \otimes D_{e\bar{e}} \otimes \sigma_{ee} + \\ &+ D_{\gamma e} \otimes D_{\bar{e}\bar{e}} \otimes \sigma_{\bar{e}\gamma} + D_{\gamma e} \otimes D_{\gamma \bar{e}} \otimes \sigma_{\gamma \gamma} + D_{\gamma e} \otimes D_{e\bar{e}} \otimes \sigma_{e\gamma} \\ &+ D_{\bar{e}e} \otimes D_{\bar{e}\bar{e}} \otimes \sigma_{\bar{e}\bar{e}} + D_{\bar{e}e} \otimes D_{\gamma \bar{e}} \otimes \sigma_{\bar{e}\gamma} + D_{\bar{e}e} \otimes D_{e\bar{e}} \otimes \sigma_{\bar{e}e} \Big] \end{aligned}$$

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Contributions to cross-section of different orders



$$d\sigma_{ab\rightarrow cd}^{\mathrm{NLO}} = d\sigma_{ab\rightarrow cd}^{(0)} \left\{ 1 + \sum_{k=1}^{\infty} \left(\frac{\alpha}{2\pi} \right)^k \sum_{l=k-1}^k c_{kl} \mathcal{L}^l + \mathcal{O}(\alpha^k \mathcal{L}^{k-2}) \right\}$$

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PDF evolution equation in QED

Convolution operation

$$(f \otimes g)(x) \equiv \int_{0}^{1} dz \int_{0}^{1} dy \ f(z)g(y)\delta(x - yz) = \int_{x}^{1} \frac{dz}{z}f(z)g(\frac{x}{z})$$
$$f(x) = \lim_{\Delta \to 0} \left(f_{\Theta}(x)\Theta(1 - x - \Delta) + f_{\Delta}\delta(1 - x) \right)$$
$$\int_{z}^{1} dx[f(x)]_{+}g(x) = \int_{0}^{1} dxf(x) \left[g(x)\Theta(x - z) - g(1) \right]$$

Delta and Theta parts:

$$f_{\Delta} = -\int_{0}^{1-\Delta} f_{\Theta}(z) dz$$
$$\left(f \otimes g\right)_{\Theta}(z) = \lim_{\Delta \to 0} \left\{ \int_{z/(1-\Delta)}^{1-\Delta} \frac{dx}{x} f_{\Theta}(x) g_{\Theta}\left(\frac{z}{x}\right) + f_{\Delta}g_{\Theta}(z) + f_{\Theta}(z)g_{\Delta} \right\}$$

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Analytical results

$$d\sigma_{ab\to cd}^{\rm NLO} = d\sigma_{ab\to cd}^{(0)} \left\{ 1 + \sum_{k=1}^{\infty} \left(\frac{\alpha}{2\pi}\right)^k \sum_{l=k-1}^k c_{kl} L^l + \mathcal{O}(\alpha^k L^{k-2}) \right\}$$

$$\begin{split} &c_{32}^{\Delta} = \left(-\frac{2951}{27} + 48\zeta_3 + 104\zeta_2\right)\ln\Delta + \left(32\zeta_2 - \frac{280}{3}\right)\ln^2\Delta - \frac{64}{3}\ln^3\Delta - \\ &-\frac{1387}{36} - 80\zeta_4 + \frac{4}{3}\zeta_3 + \frac{928}{9}\zeta_2 \\ &c_{43}^{\Delta} = -\frac{13969}{216} + \frac{1504\zeta_2\zeta_3}{3} + \frac{1465\zeta_2}{27} + \frac{314\zeta_3}{9} - \frac{488\zeta_4}{3} - 1024\zeta_5 - \frac{64}{3}\ln^4\Delta + \\ &+ \left(\frac{896\zeta_2}{3} - \frac{1376}{9}\right)\ln^3\Delta + \left(\frac{1616\zeta_2}{3} - 672\zeta_3 - \frac{8234}{27}\right)\ln^2\Delta + \left(\frac{16408\zeta_2}{27} - \\ &-\frac{1856\zeta_3}{3} + 320\zeta_4 - \frac{19270}{81}\right)\ln\Delta \\ &c_{55}^{\Delta} = \ln\Delta\left(\frac{44243}{810} + 64\zeta_4 + \frac{3328}{9}\zeta_3 - \frac{2672}{9}\zeta_2\right) + \ln^2\Delta\left(\frac{8476}{81} + \frac{512}{3}\zeta_3 - \\ &-\frac{832}{3}\zeta_2\right) + \ln^3\Delta\left(\frac{2672}{27} - \frac{256}{3}\zeta_2\right) + \frac{416}{9}\ln^4\Delta + \frac{128}{15}\ln^5\Delta + \frac{2431}{216} + \\ &+\frac{1024}{5}\zeta_5 + \frac{208}{3}\zeta_4 + \frac{5344}{27}\zeta_3 - \frac{8476}{81}\zeta_2 - \frac{512}{3}\zeta_2\zeta_3 \end{split}$$

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Analytical results

$$\begin{split} & \alpha(1-\frac{10}{2}+1)\alpha(1)(1-\frac{100}{2}+\frac{100}{2}+(0)(1-\frac{100}{2}+1)\alpha(1-\frac{100}{2$$

$$\begin{split} & u_{01}(-\frac{u_{01}}{2},\frac{u_$$

arXiv:2405.03443 [hep-ph]

 $c_{44}(z) = \frac{183435}{4.66} - \frac{21284}{166(1-z)} + \frac{1366}{277} - \frac{2129z^4}{166(1-z)} - \frac{738z^2}{27} - \frac{78381z}{4.66} + \left(\frac{1290z^4}{257(1-z)} - \frac{512z^2}{27}\right)$ $-\frac{1245_1}{9}+\frac{1231}{27(1-z)}-\frac{19}{9}-\frac{298}{27z}\Big)\zeta_2+\Big(-\frac{50z^2}{3(1-z)}-\frac{80z^2}{9}-\frac{432z}{9}-\frac{332}{2(1-z)}+\frac{244}{9}+\frac{1168}{9z}\Big)\zeta_3+\frac{10}{2}\zeta_4+\frac{10}{9}+\frac{10}{9}\zeta_4+\frac{10}{9}+\frac{10}{9}\zeta_4+\frac{10}{9}+\frac{1$ $+ \left(-\frac{3992}{2} + \frac{330}{1-r} - \frac{3042}{r} \right) \zeta_1 + \left(\frac{77z}{1-r} - \frac{36}{3(1-r)} + \frac{211}{30} \right) \ln^4(z) + \left[\frac{132z^3}{122z^4} - \frac{329z}{122z^4} + \frac{326}{3(1-r)} + \frac{326}{$ $-\frac{1983}{M} - \left(176i - \frac{512}{312 - 2} + 176\right)\ln(1 - z)\right)\ln^2(z) + \left[\frac{364z^2}{27} - \frac{2780}{379(1 - z)} + \frac{1297}{6} + \frac{89}{37} + \frac{947z}{6}\right]$ $+ \left(\frac{152z}{2} - \frac{192}{2} + \frac{112}{2}\right)\ln^3(1-z) + \left(14z + \frac{64}{8-z} + \frac{62}{2}\right)\zeta_2 + \left(-81z^2 + \frac{620z}{8-z} - \frac{388}{38-z}\right)$ $+\frac{1526}{9}-\frac{160}{9c}\Big)\,k_{1}(1-z)+\left(-\frac{46z^{2}}{9}+\frac{18z}{7}+\frac{18}{7}-\frac{40}{9c}\right)\,k_{2}(1+z)\Big]\,k_{1}^{2}(z)+\left[-\frac{7796}{190}-\frac{1850z^{2}}{91}+\frac{18}{9}+\frac{$ $+\frac{13613z}{14}+\frac{344}{97}+\frac{6947}{979(1-z)}+\left(12z+\frac{64}{1-z}+12\right)\ln^3(1-z)+\left(86z^2-\frac{994z}{6}+\frac{685}{97(1-z)}$ $-\frac{3356}{9} + \frac{16}{72}$ $) ls^{2}(1-s) + \left(-\frac{1005^{2}}{37} - \frac{17542z}{37} + \frac{12100}{77(1-s)} - \frac{6950}{37} - \frac{296}{37} + \zeta_{2}\left(\frac{1208z}{9} - \frac{448}{37}\right)\right)$ $+\frac{1206}{2}\Big)\mathbf{k}(1-z) + \left(-\frac{224z^2}{c} + \frac{974z}{c} - \frac{1472}{100-z^2} + \frac{3442}{c} - \frac{272}{20}\right)\zeta_2 + \left(-\frac{40z^4}{c} + \frac{320z}{c} + \frac{320z}{c}$ $-\frac{49}{1+}\Big)\ln(1+z) + \Big(00z + \frac{100}{1+} + \frac{290}{2}\Big)\zeta_0\Big]\ln(z) + \Big[-\frac{88z^2}{3} + \frac{62z}{3} - \frac{256}{22(1+z)^2} + \frac{134}{3} + \frac{88}{4z}\Big]$ $\times\ln^2(1-z) + \left(\frac{84z^2}{27} - \frac{20z}{2} - \frac{20}{2} + \frac{80}{272}\right)\ln^2(1+z) + \left[\frac{184z^2}{27} + \frac{7474z}{14} - \frac{1376}{1472} + \frac{341}{14}\right]$ $-\frac{224}{325}+\zeta_{0}\Big(-224z+\frac{448}{1-z}-224\Big)\Big]\ln^{2}(1-z)+\Big(-108z+\frac{64}{1-z}-108\Big)G_{0}^{2}(1-z)+\Big(\frac{48}{1-z}+108)G_{0}^{2}(1-z)\Big)+G_{0}^{2}(1-z)+G_{0}^{2}(1-z)+G_{0}^{2}(1-z)\Big)$ $-\frac{43c}{3}\left|\theta|-3,0,z\right)+\left(-\frac{1328c}{3}+\frac{1088}{3}-\frac{1328}{3}\right)\theta(3,0,z)+\left(\frac{80c}{3}-\frac{80}{3}\right)\theta(-2,-1,0,z)+\left(\frac{40}{3}-\frac{43c}{3}\right)\theta(-2,-1,0,z)+\left(\frac{40}{3}-\frac{4$ $\times \mathrm{II}(-2, 4, 0, \tau) + \left(-339 \tau + \frac{640}{1 - \tau} - 330\right) \mathrm{II}(1, 2, 4, \tau) + \left(-336 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, \tau) + \left(-582 \tau + \frac{1056}{\pi (1 - \tau)} - 330\right) \mathrm{II}(2, 0, \tau) + \left(-582 \tau + \frac{1056$ $+\frac{640}{1-z}-502\Big]H(2,1,0,z)+\Big(-56z+\frac{192}{1-z}-56\Big]H(1,0,0,0,z)+\Big(-256z+\frac{512}{1-z}-256\Big)H(1,1,0,0,z)$ $+ \left(-228\,i + \frac{648}{1-z} - 328\right) H(1,1,1,0,z) + \left(-\frac{80 c^2}{6} + \frac{28 c}{3} + \frac{29}{9} - \frac{80}{9 c}\right) \zeta_1 \ln(1+z) + \left[-\frac{206 z^2}{9} - \frac{29}{9} + \frac{29}{$ $-\frac{2000i}{2} + (132z + 132)\ln^2(1-z) + \left(-\frac{200i}{2} + \frac{256}{3} - \frac{400}{2}\right)\ln^2 z + \left(\frac{702i^2}{2} - \frac{1112i}{6} + \frac{1112i}{6$ $-\frac{1100}{2}+\frac{928}{2n}\Big)\ln(1-z)+\zeta_0\Big(\frac{536z}{2}+\frac{536}{3}\Big)+\Big(\frac{344z}{2}-\frac{608z^2}{2}+\Big(-\frac{494z}{3}+\frac{128}{2}-\frac{494}{2}\Big)\ln(1-z)$ $+\frac{256}{410-1}-\frac{40}{6}-\frac{272}{2}\Big)\ln(z)-\frac{725}{6}-\frac{104}{62}\Big]Ll_2(1-z)+\Big[-\frac{40z^2}{2}+\frac{328z}{6}+\zeta_2\Big(\frac{43}{6}-\frac{40z}{2}\Big)+$ $+\left(\frac{80t^3}{n}+\frac{40t}{n}-\frac{40}{n}-\frac{80}{n}\right)\ln(z)+\frac{320}{n}-\frac{40}{n}\right)L_2(-z)+\left[-\frac{702t^3}{n}+\frac{1412t}{n}-(264z+264)\ln(1-z)+\frac{1412t}{n}\right]L_2(-z)+\frac{1412t}{n}+\frac{1412t}{n}-(264z+264)\ln(1-z)+\frac{1412t}{n}+\frac{1412t}{n}-(264z+264)\ln(1-z)+\frac{1412t}{n}+\frac{1412t$ $+\left(-\frac{544z}{3}-\frac{544}{3}\right)\ln(z)+\frac{1100}{9}-\frac{928}{9z}\Big]U_2(1-z)+\left(-\frac{80z^2}{3}-28z+\frac{300}{3}+\frac{80}{9z}\right)U_2(-z)+\left[-\frac{608z^2}{9}-\frac{100}{9}+\frac{100}{$ $+\frac{74_2}{9}+\left(256_2-\frac{512}{1-1}+256\right)\ln(1-1)+\left(-176_2+\frac{512}{2(1-1)}-176\right)\ln(1)+\frac{562}{2(1-1)}-\frac{334}{9}$ $-\frac{220}{9\pi}\Big]Li_2(z)+\Big(-\frac{190\pi^2}{9}+\frac{49\pi}{3}+\frac{40}{1}-\frac{364}{9\pi}\Big)Li_2\Big(\frac{1}{\tau+1}\Big)+\Big(384z-\frac{640}{1-\tau}+581\Big)Li_2(1-z)$ $+ \left(-284z + \frac{192}{1-z} - 284\right) (J_4(z) + \left(332z - \frac{708}{1-z} + 332\right) 8_{12}(z) + \ln(1-z) \left[\frac{172z^2}{125(1-z)} + \frac{1226z^2}{81}\right]$ $-\frac{5680}{9(1-z)}-\frac{7285z}{27}+\frac{2345}{3}-\frac{2090}{94z}+\left(-\frac{64z^4}{3(1-z)}+\frac{753z^2}{9}-\frac{340z}{3}+\frac{1370}{3(1-z)}-\frac{1300}{3}-\frac{753}{9z}\right)\zeta_2+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3}-\frac{1300}{3}-\frac{753}{9z}+\frac{1270}{3}-\frac{1300}{3} +\left(-288z+\frac{578}{1-z}-288\right)\zeta_{4}$

 $c_{11}(z) = \frac{86171}{1628(1-z)} - \frac{992}{243z} - \frac{21983}{2202} - \frac{560833z}{12960} - \left(\frac{64z}{3} - \frac{128}{3(1-z)} + \frac{64}{3}\right)\ln^4(1-z) + \left(-\frac{392z^2}{45} - \frac{2534z}{45} + \frac{128}{45} + \frac{128}{45}\right)\ln^4(1-z) + \left(-\frac{392z^2}{45} - \frac{2334z}{45} + \frac{128}{45} + \frac{128}{4$ $+\frac{1994}{401-z^2}-\frac{5790}{49}+\frac{392}{47z}\Big)\,1a^2(1-z)+\left(-\frac{392z^2}{49}-\frac{3447z}{49}+\frac{2072}{49}-\frac{22873}{49}+\frac{392}{49z}\right)\,1a^2(1-z)$ $+\left(\frac{2952z^2}{405}-\frac{38253z}{9100}+\frac{10292}{41(1-z)}-\frac{622737}{2944}-\frac{3962}{405z}+\left(-\frac{1024z}{9}+\frac{2048}{9(1-z)}-\frac{1024}{9}\right)\zeta_3+\left(\frac{202z^2}{15}+\frac{2534z}{15}+\frac{1024}{15}+\frac{1024z}{15}+\frac$ $-\frac{1964}{30(-\tau)}+\frac{5796}{35}-\frac{392}{15c}\Big)\zeta_{2}\Big)\ln(1-\tau)++\Big(\frac{1964^{2}}{156}+\frac{464z}{195}-\frac{208}{27(1-\tau)}+\frac{4309}{500}\Big)\ln^{2}(\tau)+\frac{461z^{2}}{270(1-\tau)}+\frac{461z^{2}}{196}+\frac{461z^$ $+\frac{952z^2}{243} + \left(-\frac{364z^2}{135} - \frac{76z}{15} + \left(-\frac{1094z}{15} + \frac{128}{3(1-z)} - \frac{1094}{15}\right) \ln^2(1-z) + \zeta_2 \left(\frac{127z}{6} - \frac{64}{3(1-z)} + \frac{127}{6}\right)$ $+\left(-\frac{196z^2}{9}-\frac{3243z}{45}+\frac{1064}{9(1-z)}-\frac{9009}{9}-\frac{106}{45z}\right)b((1-z)+\frac{5536}{22((1-z))}-\frac{1009}{40})ba^2(z)+\left(-\frac{1004z}{45z}-\frac{1009}{40}-\frac{1009}{40}\right)ba^2(z)+\left(-\frac{1004z}{45z}-\frac{1009}{40}-\frac{1009}{40}-\frac{1009}{40}\right)ba^2(z)+\frac{1004z}{45z}-\frac{1009}{40}ba^2(z)+\frac{1004z}{45z}-\frac{1009}{40}ba^2(z)+\frac{1004z}{45z}-\frac{1009}{40}ba^2(z)+\frac{1000$ $+\frac{128}{310-z^2}-\frac{1094}{35}\Big)L_{2}^{2}(1-z)+\zeta_{2}^{2}\Big(\frac{1094z}{15}-\frac{128}{9(1-z)}+\frac{1094}{15}\Big)+\zeta_{3}\Big(-\frac{72z^{2}}{512}+\frac{392z^{2}}{55}-\frac{12712}{9(1-z)}\Big)$ $+\frac{2551_2}{99}+\frac{21577}{99}-\frac{392}{97}+\zeta_0\left(\frac{322^2}{9(1-1)}-\frac{2948_2}{9}-\frac{2948}{9}+\frac{10144}{19(1-1)}\right)+\zeta_0\left(\frac{128x^2}{9(1-1)}+\frac{3473x}{99}+\frac{10144}{99}+\frac{101$ $+\frac{7368}{45(1-z)}-\frac{4660}{90}+\frac{202}{35z}\Big)++\Big(-\frac{1414z}{35}+\frac{256}{3(1-z)}-\frac{1414}{55}\Big)H(3,0,z)+\Big(-128z+\frac{256}{1-z}-128\Big)$ $\times \mathrm{H}(1,2,0,z) + \left(-\frac{1156z}{24} + \frac{236}{2(1-z)} - \frac{1156}{24}\right) \mathrm{H}(2,0,0,z) + \left(-\frac{1156z}{4} + \frac{256}{1-z} - \frac{1156}{4}\right) \mathrm{H}(2,1,0,z)$ $+\left(-\frac{37z}{6}+\frac{128}{15(1-z)}-\frac{37}{6}\right)H(0,0,0,z)+\left(-\frac{64z}{2}+\frac{128}{3(1-z)}-\frac{64}{2}\right)H(1,0,0,0,z)+\left(-\frac{250z}{2}+\frac{512}{3(1-z)}-\frac{54}{2}\right)H(1,0,0,0,z)+\left(-\frac{250z}{2}+\frac{512}{3(1-z)}-\frac{54}{2}\right)H(1,0,0,0,z)+\left(-\frac{250z}{2}+\frac{512}{3(1-z)}-\frac{54}{2}\right)H(1,0,0,0,z)+\left(-\frac{250z}{2}+\frac{54}{3(1-z)}-\frac{54}{3($ $+\left(\frac{392z^2}{10}+\frac{434z}{2}-\frac{832}{20(1-z)}+\frac{3796}{10}\right)\ln^2(1-z)+\left(\frac{1964z^2}{100}+\frac{18287z}{100}+\zeta_2\left(-\frac{256z}{2}+\frac{512}{100(1-z)}-\frac{256}{2}\right)\right)\ln^2(1-z)$ $-\frac{2472}{3411-z^4}+\frac{122443}{5446}-\frac{448}{155c}\ln(1-z)-\frac{4590z^2}{435}+\frac{2857z}{1088}+\zeta_3\left(\frac{127z}{3}-\frac{128}{3(1-z)}+\frac{127}{3}\right)+\zeta_3\left(-\frac{290z^2}{45}+\frac{127}{45}+\frac{127}{3}\right)+\zeta_4\left(-\frac{290z^2}{45}+\frac{127}{45}+\frac{127}{3}+\frac{127}{3}\right)+\zeta_4\left(-\frac{290z^2}{45}+\frac{127}{3}+\frac{127}{3}+\frac{127}{3}+\frac{127}{3}\right)+\zeta_4\left(-\frac{127}{45}+\frac{127}{3}+\frac$ $-\frac{343z}{19}+\frac{832}{910-10}-\frac{1321}{19}+\frac{392}{950}-\frac{8476}{910-10}-\frac{128}{910}+\frac{54753}{910}-\frac{10}{910}+\frac{51653}{500}\left[\ln(z)+\left[\frac{728z^2}{195}-\frac{959z}{999}+\left(\frac{516z}{5}+\frac{516}{5}\right)+\frac{10}{910}+\frac{10}{190}\right]\right]$ $\times 1a^2(1-z) + \left(\frac{392z^2}{15} + \frac{602z}{5} + \frac{602}{5} + \frac{392}{15}\right)1a(1-z) + \left(-\frac{284z^2}{45} - \frac{3976z}{45} + \frac{832}{812} - \frac{5602}{45} - \frac{392}{45}\right)a(1-z) + \frac{1602z^2}{45} + \frac{1602z^2}{4$ $+\left(-\frac{2188_2}{15}+\frac{226}{301-z}\right)+\frac{302}{15z}\right)\ln(1-z)+\frac{2188}{15}\right)\ln(1-z)\right)\ln(z)-\frac{929}{108}+\frac{728}{335z}\Big]H_2(1-z)+\left(-\frac{302z^2}{15z}+\frac{302z^2}{15z}\right)\ln(1-z)\right)\ln(z)$ $-\frac{602z}{r} + \left(-\frac{1002z}{r} - \frac{1002}{r}\right)\ln(1-z) - \frac{602}{r} - \frac{302}{14r}\right)\ln_2(1-z) + \left(-\frac{784z^3}{r} - \frac{280z}{16r} + \left(\frac{256z}{r} - \frac{502}{16r} - \frac{502}{16r} + \frac{1002z}{16r} + \frac{1002z}$ $+\frac{296}{3}\left)\ln(1-z)+\frac{1664}{4(1-z)}-\frac{16171}{40}-\frac{392}{45}\right)\ln(z)+\left(\frac{2312z}{5}-\frac{162}{5}-\frac{162}{5}\right)\ln(1-z)+\left(-\frac{127z}{5}-\frac{162}{5}-\frac{162}{5}-\frac{162}{5}\right)\ln(1-z)+\left(-\frac{127z}{5}-\frac{162}{5}-\frac{162}{5}-\frac{162}{5}-\frac{162}{5}\right)\ln(1-z)+\left(-\frac{127z}{5}-\frac{162}{5}$ $+\frac{128}{3(1-z)}-\frac{127}{3}$ $Li_{0}(z)+\left(\frac{4108z}{15}-\frac{9024}{3(1-z)}+\frac{4108}{55}\right)S_{2,2}(z)$

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$$h_{ij} = \left(\frac{\alpha}{2\pi}\right)^{i} L^{j} \int_{z_{min}}^{1-\Delta} dz (\sigma^{(0)}(z) c_{ij}^{\theta}(z) + c_{ij}^{\Delta} \sigma^{(0)}(1))$$

$$\sigma_{e^{+}e^{-}} = \sum_{i,j} \left(\frac{\alpha}{2\pi}\right)^{i} L^{j} \int_{z_{min}}^{1-\Delta} dz (\sigma^{(0)}(z) c_{ij}^{\theta}(z) + c_{ij}^{\Delta} \sigma^{(0)}(1))$$

 $\begin{array}{l} \text{Corrections of different orders, \%} \\ \text{Beam energy: } \frac{M_z-10}{2}, \frac{M_z-1}{2}, \frac{M_z}{2}, \frac{M_z+1}{2}, \frac{M_z+10}{2} \\ Z_{min} = 0.1, 0.3, 0.5, 0.7, 0.9, 0.99 \end{array}$

$$E_{beam}=rac{M_z-1}{2}$$
, $Z_{min}=0.5$, $\Delta=10^{-7}$

h ₁₁	h ₁₀	h ₂₂	h ₂₁	h ₃₃	h ₃₂	h ₄₄	h ₄₃	h ₅₅		
γ										
-37.66044	2.2067	7.28480	-0.82656	-0.95286	0.15745	0.09393	-0.02019	-0.00737		
	Pairs									
0	0	-0.35204	0.2038	0.13181	-0.08358	0.00234	0.01642	0,00296		
Full										
-37.66044	2.2067	6.93272	-0.62276	-0.82106	0.07387	0.09627	-0.00377	-0.00441		

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Corrections of different orders, %

$$E_{beam} = rac{M_z+1}{2}, \ Z_{min} = 0.5, \ \Delta = 10^{-7}$$

$$E_{beam}=rac{M_z}{2}$$
, $Z_{min}=0.5$, $\Delta=10^{-7}$

h ₁₁	h ₁₀	h ₂₂	h ₂₁	h ₃₃	h ₃₂	h ₄₄	h ₄₃	h55	
γ									
-32.75622	2.00249	4.88313	-0.59516	-0.37751	0.07101	0.00345	-0.00185	0.00315	
	Pairs								
0	0	-0.30650	0.15857	0.08753	-0.04598	0.00157	0.00375	-0,00008	
Full									
-32.75622	2.00249	4.57663	-0.43660	-0.28998	0.02502	0.00502	0.00190	0.00307	

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h ₁₁	h ₁₀	h ₂₂	h ₂₁	h ₃₃	h ₃₂	h ₄₄	h ₄₃	h ₅₅	
γ									
-19.12788	1.44663	2.17046	-0.27903	-0.18658	0.03160	0.01273	-0.00274	-0.00071	
				Pairs					
0	0	-0.17376	0.08432	0.03718	-0.02072	0.00068	0.00265	0,00026	
Full									
-19.12788	1.44663	1.99669	-0.19471	-0.14941	0.01088	0.01342	-0.00009	-0.00045	

$$E_{beam} = \frac{M_z - 10}{2}$$
, $Z_{min} = 0.5$, $\Delta = 10^{-7}$

$$E_{beam}=rac{M_z+10}{2}$$
, $Z_{min}=0.5$, $\Delta=10^{-7}$

h ₁₁	h ₁₀	h ₂₂	h ₂₁	h ₃₃	h ₃₂	h ₄₄	h ₄₃	h ₅₅	
γ									
184.22291	-6.90449	-22.10853	3.15237	-0.35227	0.07992	0.21928	-0.04591	-0.01688	
Pairs									
0	0	1.80239	-0.82578	0.41544	0.15144	-0.00723	0.01948	0,00573	
Full									
184.22291	-6.90449	-20.30614	2.32659	-0.76771	0.07152	0.21206	-0.02642	-0.01114	

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$$E_{beam} = \frac{M_z + 10}{2}$$

$Z_{min} = 0.7$										
h ₁₁	h ₁₀	h ₂₂	h ₂₁	h ₃₃	h ₃₂	h ₄₄	h ₄₃	h ₅₅		
		-22.04865	3.12323	-0.32062	-0.08455	0.21710	-0.04521	-0.01688		
Pairs										
		1.76420	-0.81231	-0.41363	0.15146	-0.00721	0.01896	0,00585		
Full										
180.78750	-6.76365	-20.28446	2.31091	-0.73425	0.06692	0.20989	-0.02642	-0.01104		
				$Z_{min} = 0.9$						
	γ									
		-3.53033	0.30905	0.57429	-0.09861	0.03891	-0.00988	-0.00048		
				Pairs						
		-0.01564	-0.03548	-0.06738	-0.04443	-0.00115	-0.00880	-0,00117		
				Full						
-1.75754	0.72009	-3.54597	0.27357	0.50691	-0.05418	-0.04007	0.00108	-0.00068		
			2	$Z_{min} = 0.99$						
γ										
		7.96127	-0.91066	-0.73201	0.13316	0.00337	-0.00364	0.00907		
Pairs										
		-0.40337	0.22508	0.14528	-0.07880	0.00260	0.00711	-0,00029		
Full										
-42.71585	2.39924	7.55790	-0.68559	-0.58674	0.05435	0.00597	0.00347	0.00878		

U.E. Voznaya, A.B. Arbuzov (BLTP JINR, StNumerical estimation of radiative corrections October 31, 2024 17 / 24

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Cross-section



From M. Stoeltzner, 2022

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Conclusion

- Higher-order radiative corrections to e⁺e⁻-annihilation are necessary for prediction of results of the experiments at e⁺e⁻ colliders
- We derived analytical expressions for the coefficients
- We made numerical estimation at different energies
- Plans numerical results for future colliders energies
- Results will be implemented into ZFITTER program

Thank you for your attention!

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Harmonic polylogarithms

 $H(a_1, ..., a_k; z)$ – harmonic polylogarithms (HPLs) To determine HPLs, the following functions are introdused:

$$f_1(z) = \frac{1}{1-z}, \quad f_0(z) = \frac{1}{z}, \quad f_{-1}(z) = \frac{1}{1+z},$$

HPLs can be obtained recursively integrating these functons:

$$H(1; z) = \int_{0}^{z} f_{1}(y) dy = -\ln(1-z), \quad H(0; z) = \ln z$$
$$H(-1; z) = \int_{0}^{z} f_{-1}(y) dy = -\ln(1+z),$$
$$H(^{n}0; z) = \frac{1}{n!} \ln^{n} z,$$
$$H(a, a_{1}, ..., a_{k}; z) = \int_{0}^{z} f_{a}(y) H(a_{1}, ..., a_{k}; y) dy,$$

where

$${}^{n}0 = \underbrace{0, ..., 0}_{n}$$

PDF evolution equation in QED

Convolution operation

$$(f \otimes g)(x) \equiv \int_{0}^{1} dz \int_{0}^{1} dy \ f(z)g(y)\delta(x - yz) = \int_{x}^{1} \frac{dz}{z} f(z)g(\frac{x}{z})$$

$$f(x) = \lim_{\Delta \to 0} \left(f_{\Theta}(x)\Theta(1 - x - \Delta) + f_{\Delta}\delta(1 - x) \right)$$

$$\int_{z}^{1} dx [f(x)]_{+}g(x) = \int_{0}^{1} dx f(x) \left[g(x)\Theta(x - z) - g(1) \right]$$

$$f_{\Delta} = -\int_{0}^{1-\Delta} f_{\Theta}(z) dz$$

$$\left(f \otimes g \right)_{\Theta}(z) = \lim_{\Delta \to 0} \left\{ \int_{z/(1-\Delta)}^{1-\Delta} \frac{dx}{x} f_{\Theta}(x) g_{\Theta}\left(\frac{z}{x}\right) + f_{\Delta}g_{\Theta}(z) + f_{\Theta}(z)g_{\Delta} \right\}$$

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PDF evolution equation in QED

$$\alpha(q^2) = \frac{\alpha_0}{1 + \overline{\Pi}(\frac{-q^2}{\mu^2}, \frac{\overline{m}}{\mu}, \alpha_0)}$$
$$\overline{\Pi} = 2\alpha_0 \left(\left(\frac{5}{9} - \frac{L}{3}\right) + 4\alpha_0^2 \left(\frac{55}{48} - \zeta_3 - \frac{L}{4}\right) + 8\alpha_0^3 \left(\frac{-L^2}{24}\right) \right) + \dots$$

P. A. Baikov, K. G. Chetyrkin, J. H. Kuhn and C. Sturm, Nucl. Phys. B 867 (2013), 182-202 $\alpha_0 = \frac{1}{137}$

Contribution of functions of positron in electron type

$$\Delta c_{44} = \frac{1}{3} \sigma_{e\bar{e}}^{(0)} P_{e\gamma}^{(0)} \otimes P_{\gamma e}^{(0)} \otimes P_{\gamma \bar{e}}^{(0)} \otimes P_{\bar{e}\gamma}^{(0)}$$

From evolution equation:

$$\frac{1}{12}\sigma_{e\bar{e}}^{(0)}P_{e\gamma}^{(0)}\otimes P_{\gamma e}^{(0)}\otimes P_{\gamma \bar{e}}^{(0)}\otimes P_{\bar{e}\gamma}^{(0)}$$

From including $D_{e\bar{e}} \otimes D_{\bar{e}e}$ into cross-section:

$$\frac{1}{4}\sigma_{e\bar{e}}^{(0)}\mathcal{P}_{e\gamma}^{(0)}\otimes\mathcal{P}_{\gamma e}^{(0)}\otimes\mathcal{P}_{\gamma \bar{e}}^{(0)}\otimes\mathcal{P}_{\bar{e}\gamma}^{(0)}$$

$$\Delta c_{44}(z) = \frac{1}{3} \left[\ln z \left(-21 - \frac{16}{9z} - 21z - \frac{16}{9}z^2 \right) + \ln^2 z \left(-2 + 2z \right) + \right. \\ \left. + \ln^3 z \left(-\frac{2}{3} - \frac{2}{3}z \right) - 26 - \frac{176}{27z} + 26z + \frac{176}{27}z^2 \right]$$

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