The current status of studying $Z_c(4200)$ exotic state

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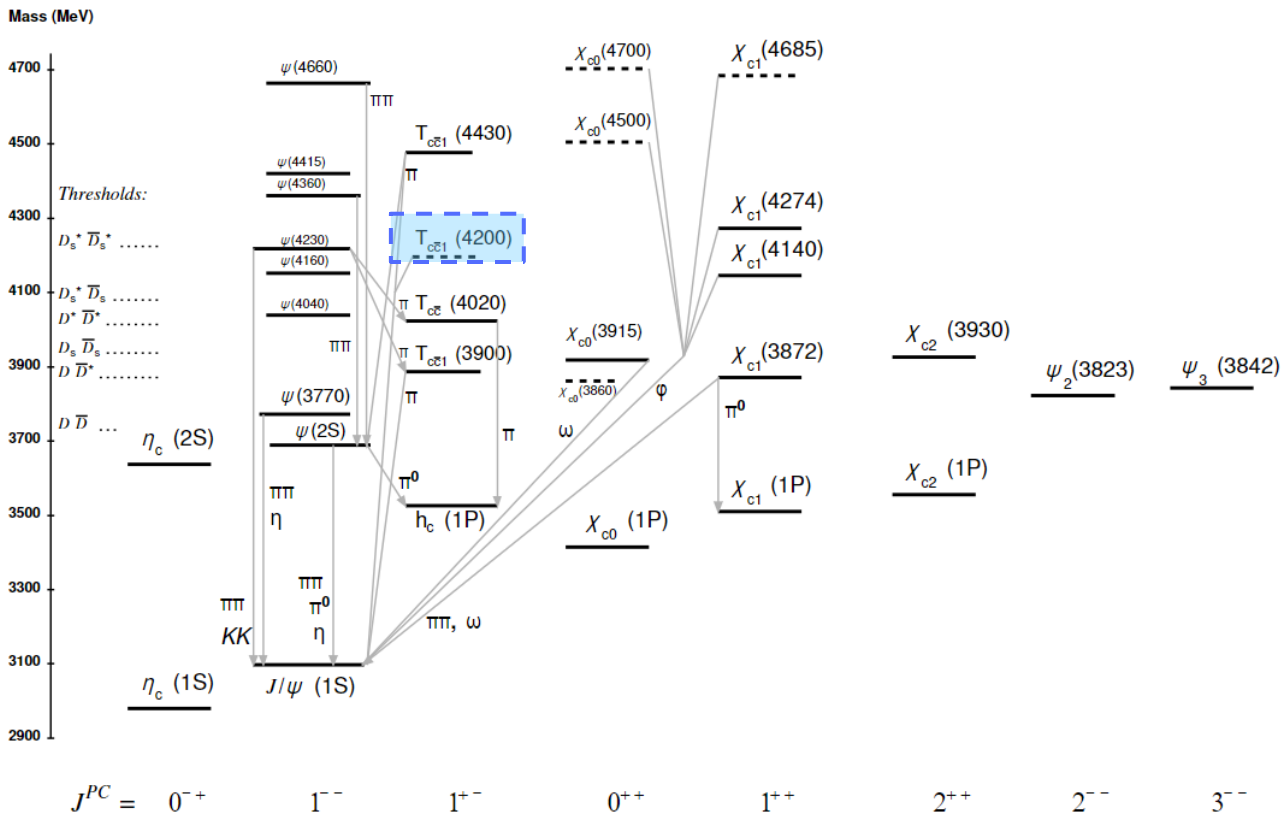
Introduction

- The quark model of hadrons proposed by Murray Gell-Mann assumes the existence of states with structure beyond standard *qq* and *qqq*-models.
- $Z_c(4200)$ was discovered by BELLE collaboration with a significance of 6.2σ in 2014 [1]. A full amplitude analysis was performed to determine the parameters of this state: mass, width, quantum numbers.
- Existence of that state still needs confirmation.
- There are many models describing internal structure of states consisting of 4 valence quarks with hidden charm. The most popular among them are models of compact tetraquark, hadron molecule, hadrocharmonuim. Also there is possibility of interpretation of such states as hybrid mesons and rescattering/cusp effects.



Spectrum of charmonium system (PDG March 2024 [2])

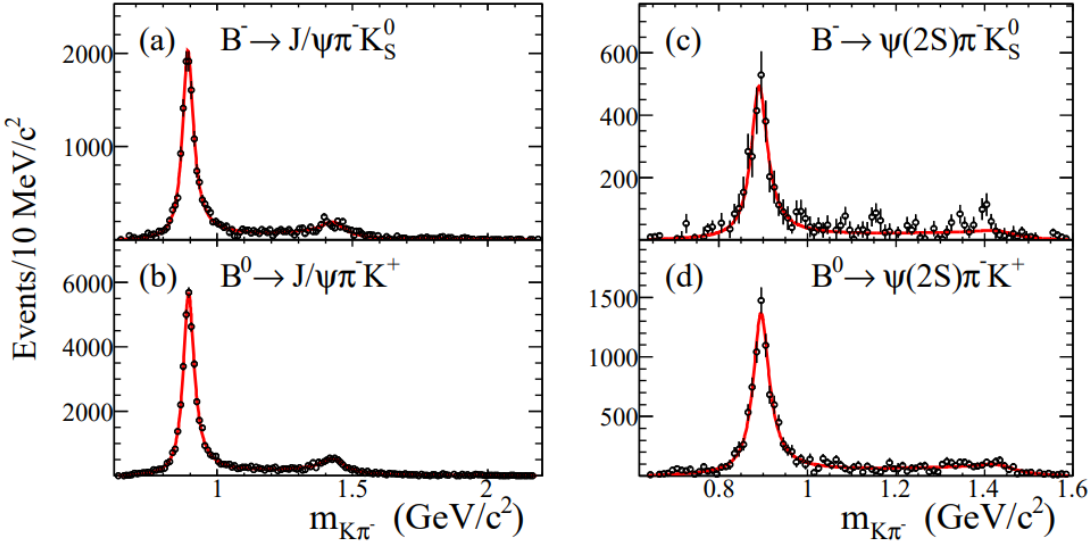
2024 PDG naming scheme for hadrons with minimal quark content $q\bar{q}q\bar{q}$ and $qqqq\bar{q}$

$$T^{(*)}_{\text{quarks}\,J}(\text{mass})^q \quad \text{for tetraquarks,}$$

$$P_{\text{quarks}}(\text{mass})^q\ J^P \quad \text{for pentaquarks,}$$

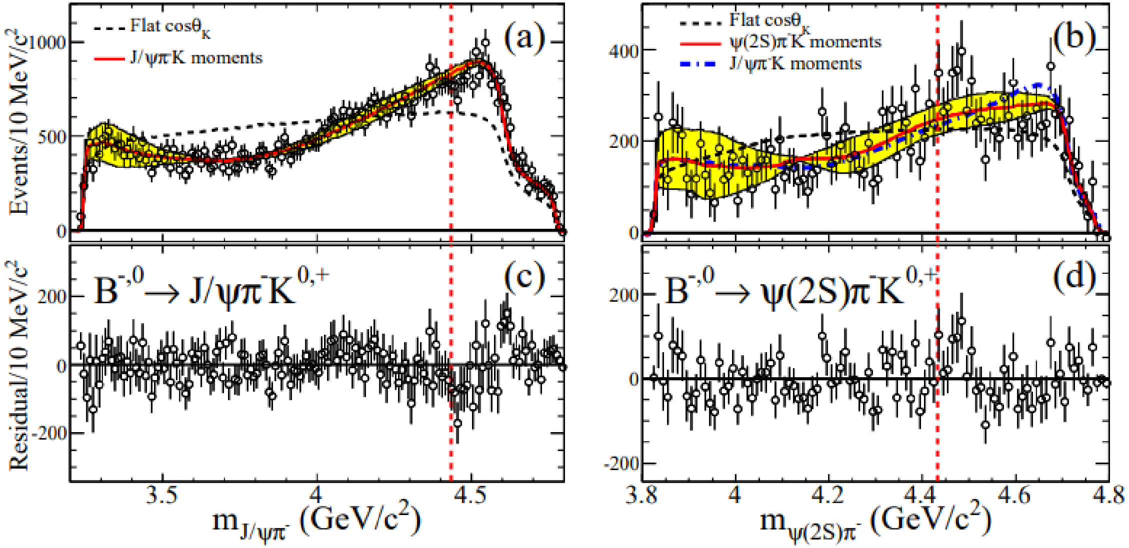
B^0 decays studies before $Z_c(4200)$. BaBar

In 2008 BaBar collaboration performed model independent analysis of $B^{-,0} \to J/\psi K^{0,+}\pi^-$ and $B^{-,0} \to \psi(2S)K^{0,+}\pi^-$ decays to search for $Z_c(4430)$ state. 57231 ± 561/20985 ± 393/5016 ± 292/13237 ± 377 events of $B^0 \to J/\psi K^+\pi^-$/$B^- \to J/\psi K^0_S\pi^-$/$B^- \to \psi(2S)K^0_S\pi^-$/$B^0 \to \psi(2S)K^+\pi^-$ were selected for analysis [3].



m(Kπ) distributions were described by sum of Breit Wigner terms multiplied by Blatt-Weiskopf barrier factors for S,P and D waves.

Kπ angular distribution in a given m(Kπ) value is represented in terms of Legendre polynomial expansion.

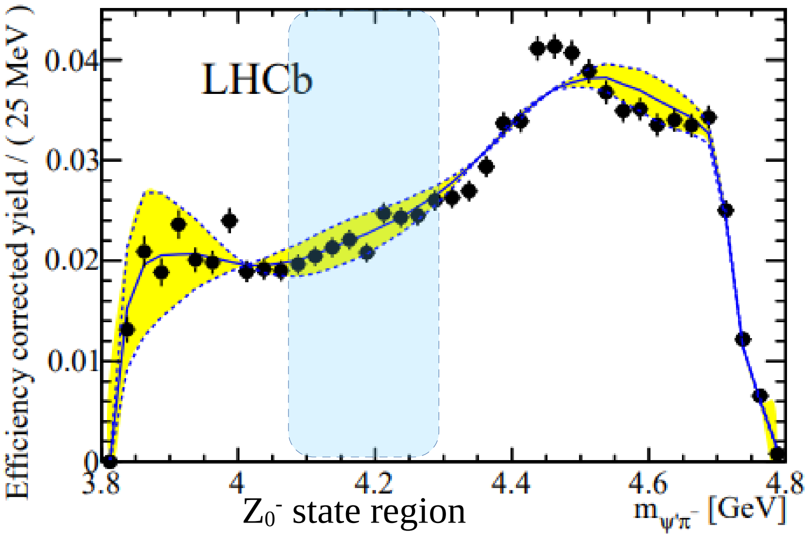


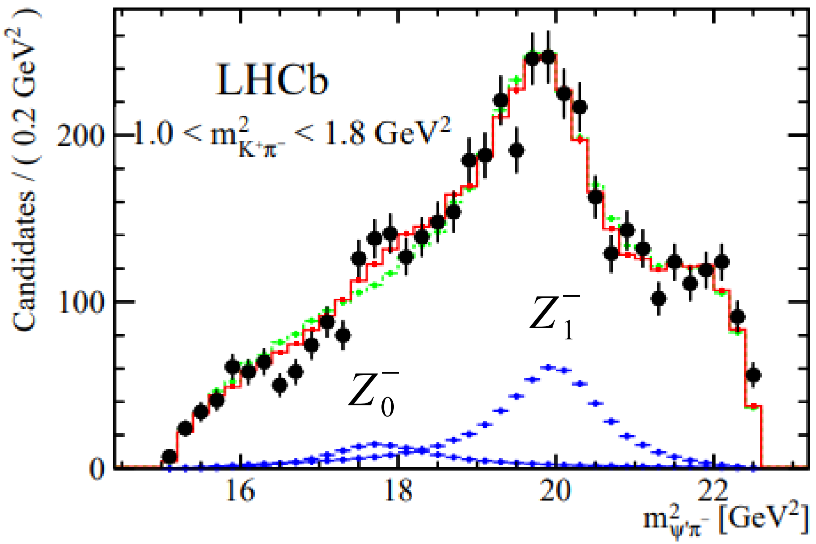
m(J/ψπ) (left) and m(ψ(2S)π) (right) distribution of data events with reflected $K^+\pi^-$ structures after $\cos(\theta_K)$ reweighting.

The model with only Kπ resonances shows satisfactory description of BaBar data. No evidence of exotic contribution to considered decays was found.

B^0 decays studies before $Z_c(4200)$. LHCb

In 2014 LHCb collaboration performed amplitude analysis of 25176 ± 174 $B^0 \to \psi' K^+ \pi^-$ decays [4].





Amplitude analysis was complemented with model independent approach. LHCb data cannot be described via model with only K^* contribition.

Z_0^- state region description is quiet well.

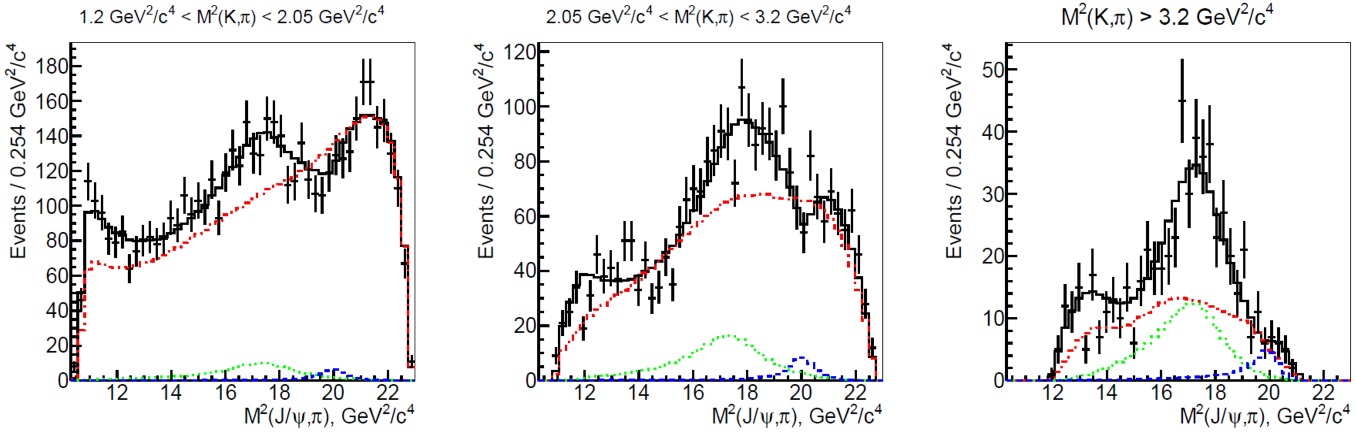
- Central model includes only Z_1^- state in addition to K^* contribution. The significance of Z_1^- is 13.9σ.
- The second Z^- resonance is allowed in the amplitude with $J^P = 0^-$. The preference of $J^P = 0^-$ spin parity hypothesis over 1^+ is only 1σ.
- Z_0^- significance is 6σ.
- Z_0^- parameters: $M = 4239 \pm 18 ^{+45}_{-10}$ MeV, $\Gamma = 220 \pm 47 ^{+108}_{-74}$ MeV

Z_c(4200) discovery

Z_c(4200) was discovered by BELLE collaboration with a significanceof 6.2σ in 2014 [1]. Signal area contains 29990 ± 190 ± 50 events. A full amplitude analysis was performed to determine the parameters of this state:

J^P	0^-	1^-	1^+	2^-	2^+
Mass, MeV/c^2	4318 ± 48	4315 ± 40	4196^{+31}_{-29}	4209 ± 14	4203 ± 24
Width, MeV	720 ± 254	220 ± 80	370 ± 70	64 ± 18	121 ± 53
Significance (Wilks)	3.9σ	2.3σ	8.2σ	3.9σ	1.9σ

$$M = 4196^{+31+17}_{-29-13}\ \text{MeV}$$
$$\Gamma = 370^{+70+70}_{-70-132}\,\text{MeV}$$



Model of $B^0 \to J/\psi K^+\pi^-$ states include

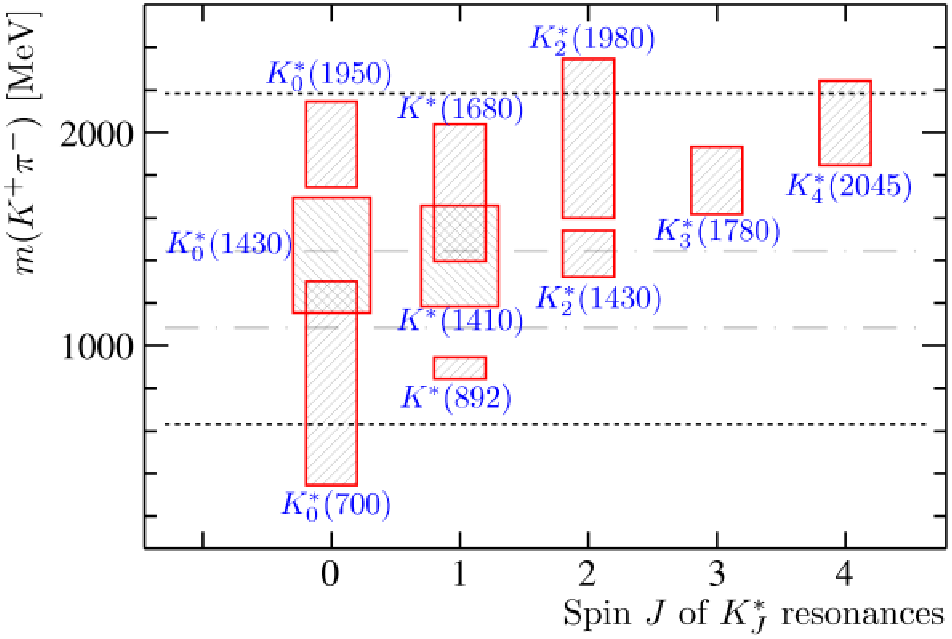
Resonance	Fit fraction	Significance (Wilks)
$K_0^*(800)$	$(7.1^{+0.7}_{-0.5})\%$	22.5σ
$K^*(892)$	$(69.0^{+0.6}_{-0.5})\%$	166.4σ
$K^*(1410)$	$(0.3^{+0.2}_{-0.1})\%$	4.1σ
$K_0^*(1430)$	$(5.9^{+0.6}_{-0.4})\%$	22.0σ
$K_2^*(1430)$	$(6.3^{+0.3}_{-0.4})\%$	23.5σ
$K^*(1680)$	$(0.3^{+0.2}_{-0.1})\%$	2.7σ
$K_3^*(1780)$	$(0.2^{+0.1}_{-0.1})\%$	3.8σ
$K_0^*(1950)$	$(0.1^{+0.1}_{-0.1})\%$	1.2σ
$K_2^*(1980)$	$(0.4^{+0.1}_{-0.1})\%$	5.3σ
$K_4^*(2045)$	$(0.2^{+0.1}_{-0.1})\%$	3.8σ
$Z_c(4430)^+$	$(0.5^{+0.4}_{-0.1})\%$	5.1σ
$Z_c(4200)^+$	$(1.9^{+0.7}_{-0.5})\%$	8.2σ

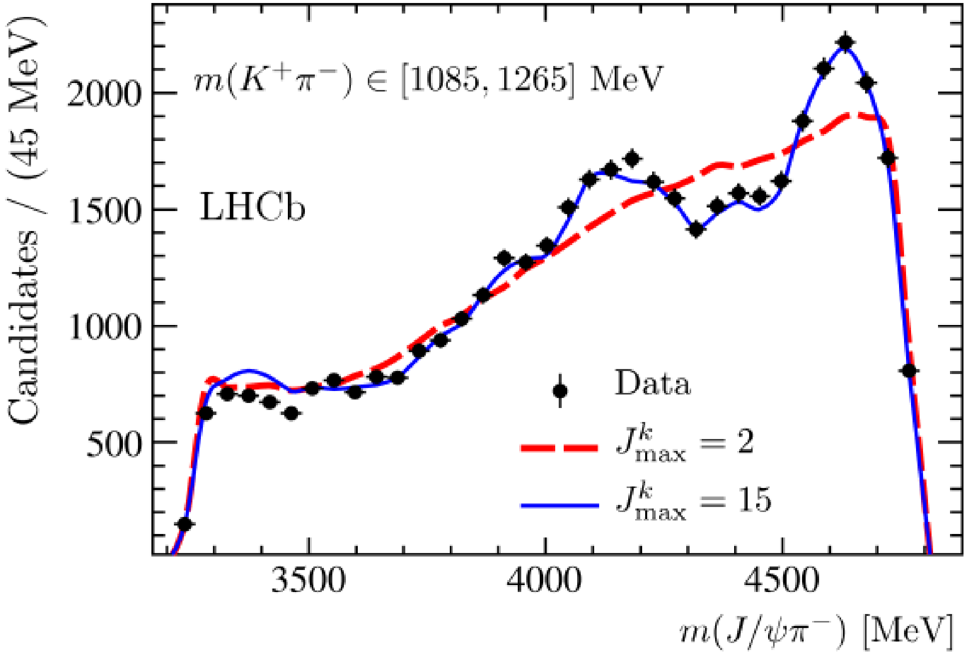
Wilks significance of $Z_{cs} \to J/\psi K$ contribution instead of Z_c(4200) is 4.3σ. The Z_{cs} state become insignificant if Z_c(4200) is added to the model.

A hint of the existence $Z_c(4200)$ in the LHCb data

In 2019 LHCb collaboration performed model independent analysis of $B^0 \to J/\psi K\pi$ decays [5].

- Significance of exotic contribution to $B^0 \to J/\psi K\pi$ decays is more than 10σ.
- They observed an exotic $Z_c(4200)$-like structure near the mass m(J/ψπ) = 4200 MeV.
- Data excess in the mass region m(J/ψπ) ≈ 4600 MeV can not be described with model including only K* contributions.

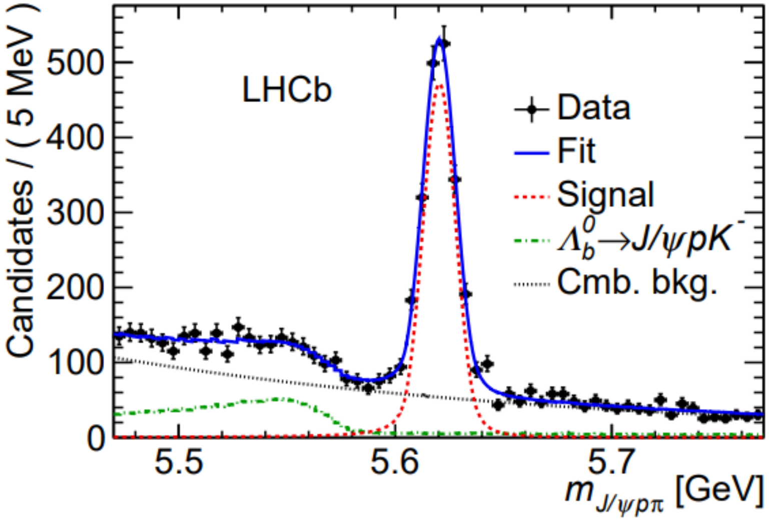




Zc(4200) contribution in $\Lambda_b \to J/\psi p \pi^-$ decays

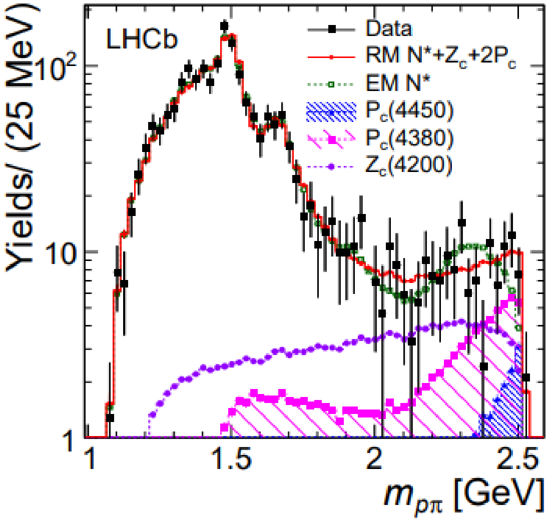
In 2016 LHCb collaboration performed full amplitude analysis of $\Lambda_b \to J/\psi p \pi^-$ decays [6].

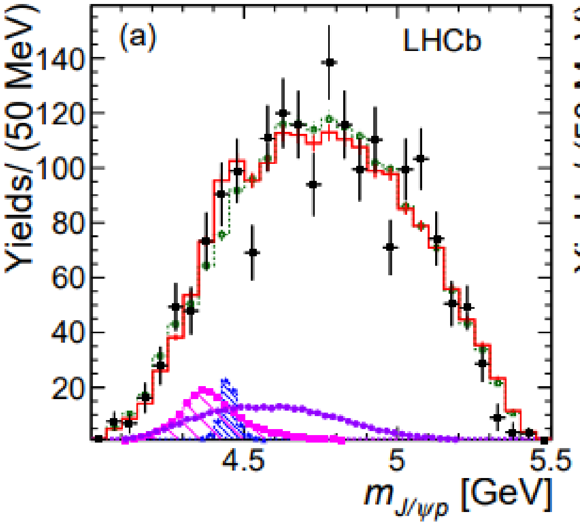
- Model includes: nucleon exitations ($N \to J/\psi p \pi^-$), pentaquark states $P_c(4380)$, $P_c(4450)$ ($P_c \to J/\psi p$) and $Z_c(4200) \to J/\psi \pi^-$.
- The total significance of exotic resonances is 3.1σ.
- The significance of adding $Z_c(4200)$ after 2 P_c ressonances is only 0.4σ.

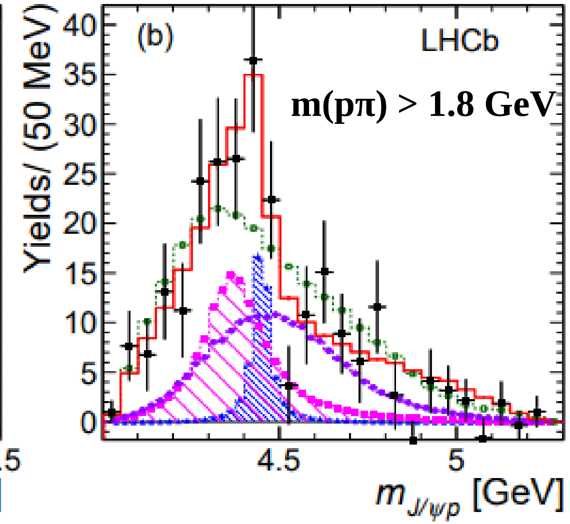


State	RM	EM
NR $p\pi$	18.6 ± 3.2	16.0 ± 3.3
$N(1440)$	34.0 ± 4.9	43.9 ± 5.7
$N(1520)$	7.6 ± 2.2	1.9 ± 3.9
$N(1535)$	25.4 ± 5.9	34.4 ± 6.5
$N(1650)$	10.5 ± 5.1	9.5 ± 4.1
$N(1675)$	$3.4^{+2.2}_{-1.0}$	4.2 ± 1.6
$N(1680)$	-	3.0 ± 1.6
$N(1700)$	-	1.7 ± 3.0
$N(1710)$	-	2.1 ± 1.6
$N(1720)$	$3.9^{+1.8}_{-1.3}$	9.6 ± 3.2
$N(1875)$	-	2.3 ± 1.9
$N(1900)$	-	3.0 ± 1.7
$N(2190)$	-	0.5 ± 0.4
$N(2300)$	-	4.9 ± 2.2
$N(2570)$	-	0.3 ± 0.5
$P_c(4380)$	5.1 ± 1.5	4.1 ± 1.7
$P_c(4450)$	$1.6^{+0.8}_{-0.6}$	$1.5^{+0.8}_{-0.6}$
$Z_c(4200)$	7.7 ± 2.8	$4.1^{+4.3}_{-1.1}$

Fit fractions with stat. uncertainty only

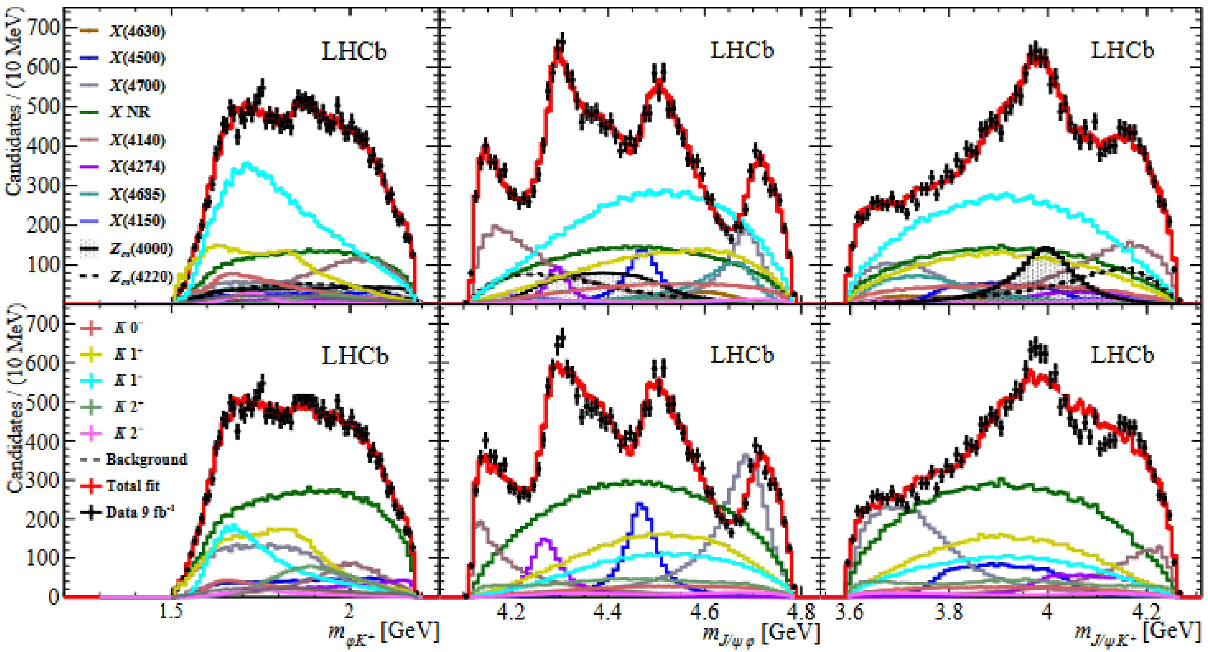




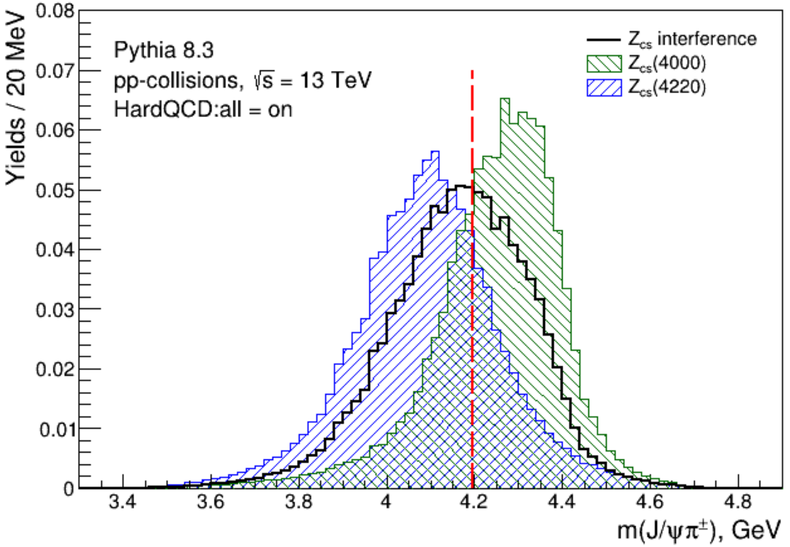


Z_{cs} states discovery by LHCb collaboration

$Z_{CS}(4000)$ and $Z_{CS}(4220)$ were discovered in $B^+ \to J/\psi\varphi K^+$ by the LHCb collaboration in 2021 [8].



Events of $B^0 \to Z_{cs}^+\pi^-$, $Z_{cs}^+ \to J/\psi K^+$ + cc reflected in $m(J/\psi\pi)$



$Z_{cs}(4000)$ significance 15σ:

$M = 4003 \pm 6$ (stat) $^{+4}_{-14}$ (syst) MeV

$\Gamma = 131 \pm 15$ (stat) ± 26 (syst) MeV

$J^P = 1^+$

$Z_{cs}(4220)$ significance 5.9σ:

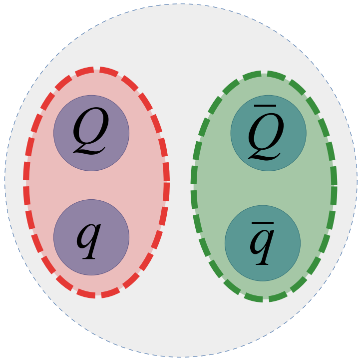
$M = 4216 \pm 24$ (stat) $^{+43}_{-30}$ (syst) MeV

$\Gamma = 233 \pm 52$ (stat) $^{+97}_{-73}$ (syst) MeV

$J^P = 1^+$ or 1^-

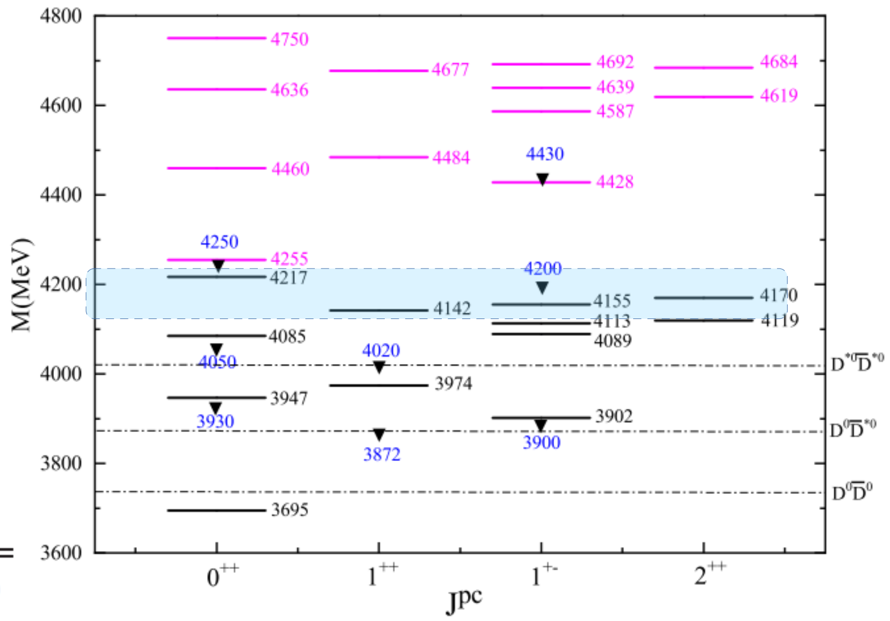
Data excess in $m(J/\psi\pi) \approx 4.2$ region can be described by the interference of Z_{cs} states. $Z_{cs}(4000)$ and $Z_{cs}(4220)$ states should be added in $B^0 \to J/\psi K^+\pi^-$ decay model.

$Z_c(4200)$ as compact tetraquark state



Compact tetraquarks refers to the structure of a strongly interacting diquark pair. The existence of the diquark attraction out to distances as large as ≈ 1 fm is supported by lattice calculations.

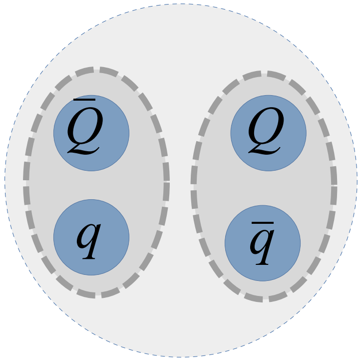
In [8] mass spectra, r.m.s. radii and radial density distributions are investigated within the framework of relativized quark model.



J^{PC}	Configuration	Configuration mixing (I)		$\sqrt{\langle r^2_{12/34}\rangle}$	$\sqrt{\langle r^2\rangle}$	$1_c \otimes 1_c(\%)$	$8_c \otimes 8_c(\%)$
		Eigenvalues	Mixing coefficients(%)				
0^{++}	$\lvert(qc)^{\bar{3}}_1(\bar{q}\bar{c})^{3}_1\rangle_0$	4217	(36.6, 1.1, 0.6, 61.6)	0.523	0.308	54.1	45.9
	$\lvert(qc)^{\bar{3}}_0(\bar{q}\bar{c})^{3}_0\rangle_0$	4085	(7.4, 59.0, 33.4, 0.3)	0.462	0.333	44.6	55.4
	$\lvert(qc)^{6}_1(\bar{q}\bar{c})^{\bar{6}}_1\rangle_0$	3947	(49.0, 15.6, 2.1, 33.3)	0.497	0.333	45.1	54.9
	$\lvert(qc)^{6}_0(\bar{q}\bar{c})^{\bar{6}}_0\rangle_0$	3695	(7.0, 24.4, 63.8, 4.80)	0.475	0.291	56.2	43.8
1^{++}	$\frac{1}{\sqrt{2}}[\lvert(qc)^{\bar{3}}_1(\bar{q}\bar{c})^{3}_0\rangle_1+\lvert(qc)^{\bar{3}}_0(\bar{q}\bar{c})^{3}_1\rangle_1]$	4142	(38.7, 61.3)	0.516	0.325	53.8	46.2
	$\frac{1}{\sqrt{2}}[\lvert(qc)^{6}_1(\bar{q}\bar{c})^{\bar{6}}_0\rangle_1+\lvert(qc)^{6}_0(\bar{q}\bar{c})^{\bar{6}}_1\rangle_1]$	3974	(61.3, 38.7)	0.499	0.344	46.2	53.8
1^{+-}	$\frac{1}{\sqrt{2}}[\lvert(qc)^{\bar{3}}_1(\bar{q}\bar{c})^{3}_0\rangle_1-\lvert(qc)^{\bar{3}}_0(\bar{q}\bar{c})^{3}_1\rangle_1]$	4155	(18.2, 64.8, 16.5, 0.5)	0.521	0.336	39.0	61.0
	$\frac{1}{\sqrt{2}}[\lvert(qc)^{6}_1(\bar{q}\bar{c})^{\bar{6}}_0\rangle_1-\lvert(qc)^{6}_0(\bar{q}\bar{c})^{\bar{6}}_1\rangle_1]$	4113	(28.3, 2.7, 0.7, 68.2)	0.520	0.316	56.3	43.7
	$\lvert(qc)^{\bar{3}}_1(\bar{q}\bar{c})^{3}_1\rangle_1$	4089	(16.8, 3.1, 70.2, 9.9)	0.484	0.418	60.1	39.9
	$\lvert(qc)^{6}_1(\bar{q}\bar{c})^{\bar{6}}_1\rangle_1$	3902	(36.7, 29.4, 12.7, 21.3)	0.508	0.345	44.6	55.4
2^{++}	$\lvert(qc)^{\bar{3}}_1(\bar{q}\bar{c})^{3}_1\rangle_2$	4170	(45.1, 54.9)	0.531	0.333	51.6	48.4
	$\lvert(qc)^{6}_1(\bar{q}\bar{c})^{\bar{6}}_1\rangle_2$	4119	(54.9, 45.1)	0.524	0.344	48.4	51.6

- This approach describe mass spectrum of charmonium-like states.
- It predicts states with $J^P = 0^+, 1^+, 2^+$ in the $Z_c(4200)$ mass region.
- Data excess in the mass region m(J/ψπ) ≈ 4600 MeV observed by LHCb [5] can be interpreted as radial exitation of $Z_c(4200)$.

$Z_c(4200)$ as hadron molecule



In [9] the $Z_c(4200)$ is assigned to be 8×8 molecule-like state:

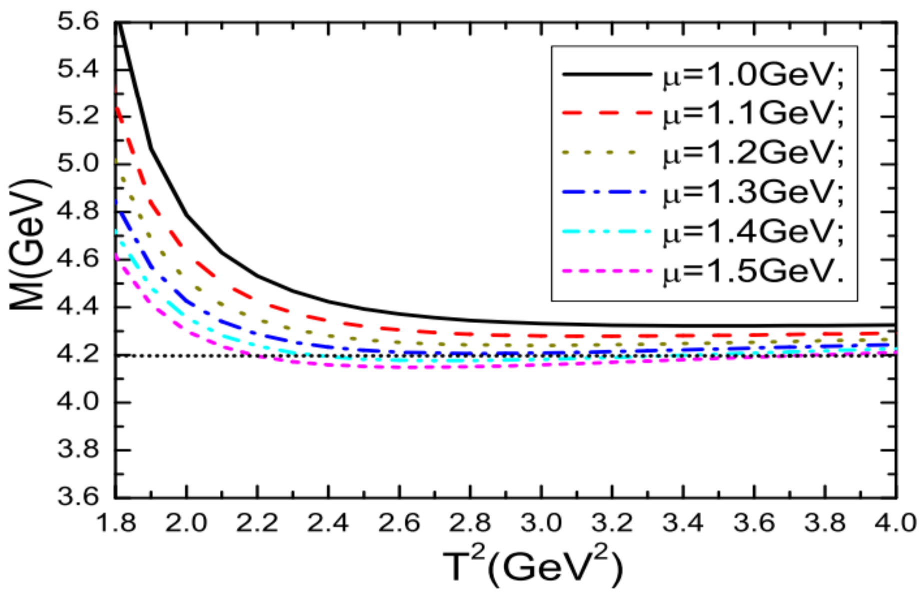
$$Z_c(4200) = \frac{1}{\sqrt{2}}\left(\mathcal{D}\overline{\mathcal{D}}^* + \mathcal{D}^*\overline{\mathcal{D}}\right) \quad (\text{with } 1^{+-})$$

The QCD sum rules were dirived for mass and width of $Z_c(4200)$ state.

$$M_{Z_c(4200)} = 4.19 \pm 0.08\,\text{GeV}\,,$$

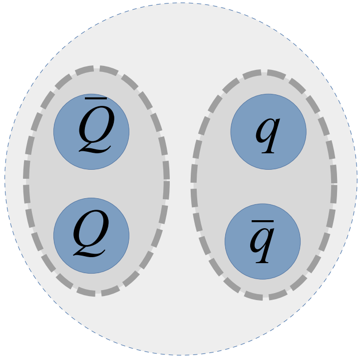
$$\Gamma(Z_c^+(4200) \to J/\psi\pi^+) = \frac{p\left(M_{Z_c}, M_{J/\psi}, M_\pi\right)}{24\pi M_{Z_c}^2} G^2_{Z_c J/\psi\pi}\left[3 + \frac{p\left(M_{Z_c}, M_{J/\psi}, M_\pi\right)^2}{M_{J/\psi}^2}\right] = 24.6\,\text{MeV}\,,$$

$$\Gamma(Z_c^+(4200) \to \eta_c\rho^+) = \frac{p\left(M_{Z_c}, M_{\eta_c}, M_\rho\right)}{24\pi M_{Z_c}^2} G^2_{Z_c\eta_c\rho}\left[3 + \frac{p\left(M_{Z_c}, M_{\eta_c}, M_\rho\right)^2}{M_\rho^2}\right] = 309.1\,\text{MeV}\,,$$



Total width Γ ≈ 344 MeV and mass **are consistent** with Belle result.

$Z_c(4200)$ as hadrocharmonium



In hadrocharmonium model the exotic state possesses a quarqonium core in a specific spin state and with specific radial exitations. This model was originally developed to explain why some exotics prefer to decay to spefific charmonium state.

Hadrocharmonium model consider $Z_c(4200)$ state as heavy quark spin symmetry partner of $Z_c(4100)$ [10].

$$\frac{\mathcal{B}[B^0 \to Z_c(4100)^- K^+]}{\mathcal{B}[B^0 \to Z_c(4200)^- K^+]} \approx \left. \frac{\mathcal{B}[B^0 \to \eta_c \pi^- K^+]}{\mathcal{B}[B^0 \to J/\psi \pi^- K^+]} \right|_{M(c\bar{c}\pi) \approx M(Z_c)}$$

Predictions of decay rates:

$$\Gamma[Z_c(4100) \to J/\psi \rho] \approx 3\, \Gamma[Z_c(4200) \to \eta_c \rho]$$

$$\Gamma[Z_c(4100) \to \eta_c(2S)\pi] \approx \Gamma[Z_c(4200) \to \psi(2S)\pi]$$

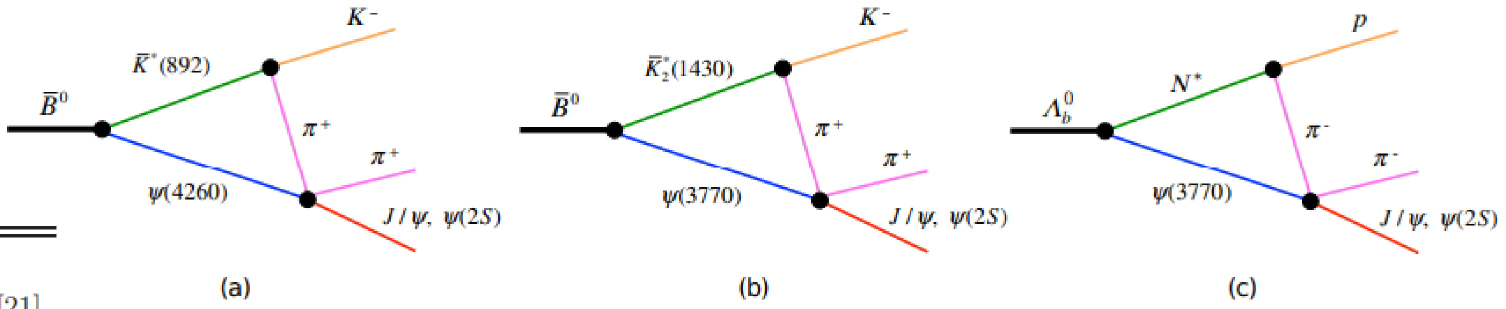
$$\frac{\Gamma[Z_c(4200) \to h_c \pi]}{\Gamma[Z_c(4100) \to \chi_{c1} \pi]} \approx \left(\frac{p_2}{p_1}\right)^3 \approx 1.5$$

$Z_c(4200)$ as triangle singularity

Triangle singularity:

Kinematical effect that arises in triangle diagram for H → abc type decay when a special conditionis reached: three intermediate particles are allowed to be on shell at the same time [11].

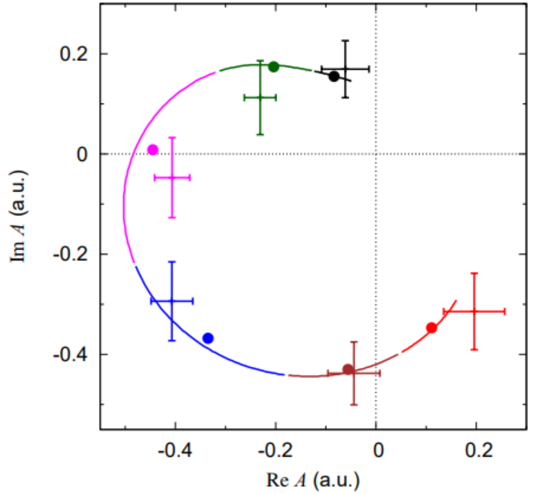
$Z_c(4430)$			$Z_c(4200)$	
(a)	Belle [17]	LHCb [18]	(b)	Belle [21]
4463 ± 13	$4485 \pm 22^{+28}_{-11}$	$4475 \pm 7^{+15}_{-25}$	4233 ± 48	4196^{+31+17}_{-29-13}
195 ± 16	200^{+41+26}_{-46-35}	$172 \pm 13^{+37}_{-34}$	292 ± 56	$370 \pm 70^{+70}_{-132}$



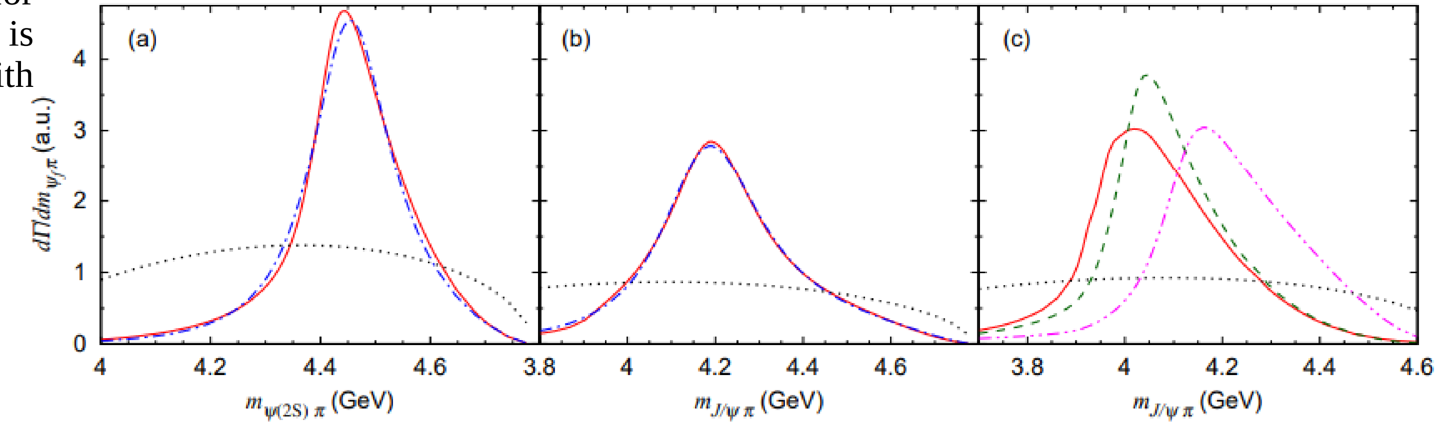
$B^0 \to J/\psi K^+\pi^-$
Generates $Z_c(4430)$-like bump

$B^0 \to J/\psi K^+\pi^-$
Generates $Z_c(4200)$-like bump

$\Lambda_b \to J/\psi p\pi^-$
Generates $Z_c(4200)$-like bump



Argand plot for Zc(4430) state is fitted well with TS amplitude

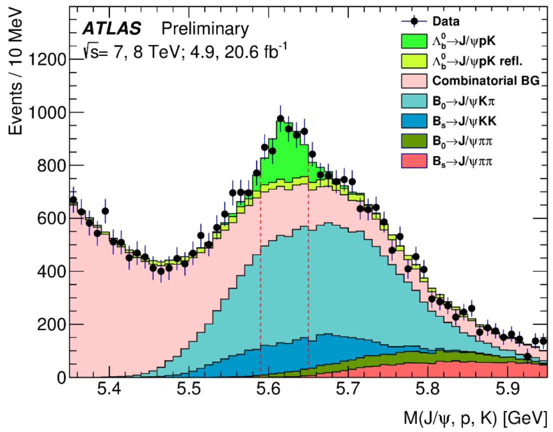


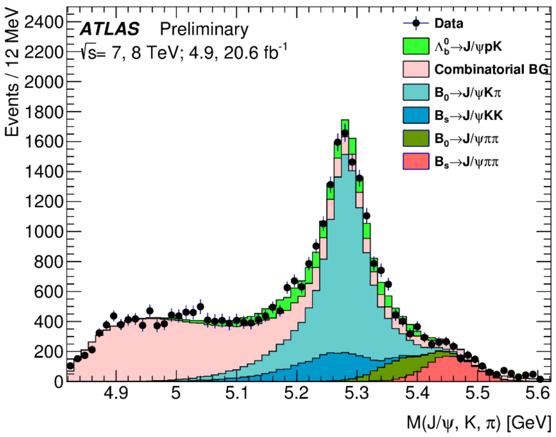
Bumps from TS diagrams

Breit-Wigner amplitudes fit

$Z_c(4200)$ study in ATLAS. Pentaquark analysis

The events of *pp* collisions ($\sqrt{s} = 7, 8$ TeV, L = 4.9, 20.6 fb^{-1}) were selected for analysis by ATLAS [12].





$m(J/\psi h^+h^-)$ distributions of selected B-hadron candidates with fit results.

Decay type	Yield
$B^0 \rightarrow J/\psi K^+\pi^-$	10770
$B^0 \rightarrow J/\psi\pi^+\pi^-$	1070
$B_s \rightarrow J/\psi K^+K^-$	2290
$B_s \rightarrow J/\psi\pi^+\pi^-$	1390
$\Lambda_b \rightarrow J/\psi pK^-$	2270 ± 300

The signal modeling was performed using the following algorithm:

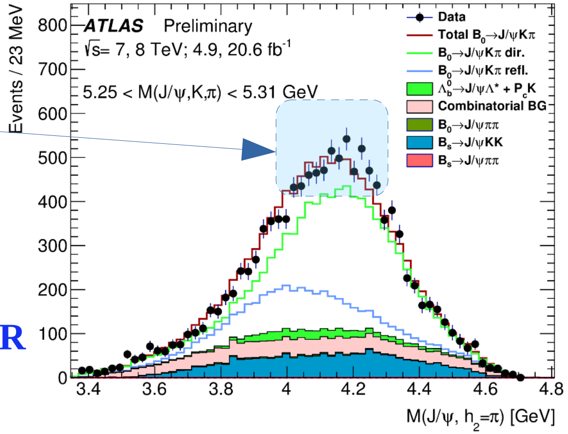
1) Modeling of phase space events was done using Pythia 8 software package.

2) The generated events passed through simulation of ATLAS detector and processed with the same reconstruction software as used for the real data.

3) At each step of fitting model to data each phase space MC event was reweighted by the analiticaly derived matrix element (see Ap. 1). Decay couplings, masses and widths are free during fits.

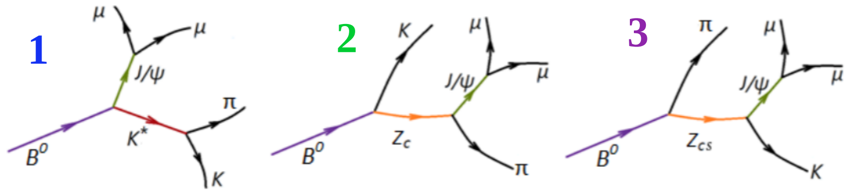
Run1 data shows hint of exotic contribution to $B^0 \rightarrow J/\psi K\pi$ decays

$B^0 \rightarrow J/\psi K\pi$ CR



Z_c(4200) study in ATLAS. Run2

- The search of exotic state in B-hadron decays continues with Run2 data: $\sqrt{s}$ = 13 TeV, L = 139 fb^{-1} => 5 times more statistics. The $B^0 \to J/\psi K^+K^-$, $B_s \to J/\psi K^+\pi^-$ and $\Lambda_b \to J/\psi p\pi^-$ should not be ignored.
- Decay model of $B^0 \to J/\psi K^+\pi^-$ includes:



Decay chain 1 (no exotic model):

state	M, MeV	Γ, GeV	J^P
$K^*(1410)$ [2]	1414	232	1^-
$K^*_0(1430)$ [2]	1425	270	0^+
$K^*_2(1430)$ [2]	1432	109	2^+
$K^*(1680)$ [2]	1718	322	1^-
$K^*(1780)$ [2]	1779	161	1^-
$K^*(1950)$ [2]	1944	100	0^+
$K^*(2045)$ [2]	2048	199	4^+
K^* NR	1975	-	0^+

Decay chain 2 (model with exotic $Z_c \to J/\psi\pi$ contribution):

state	M, MeV	Γ, GeV	J^P
$Z_C(3900)$ [2]	3887	28	1^+
$Z_C(4200)$ [2]	4196	370	1^+
$Z_C(4430)$ [2]	4478	181	1^+

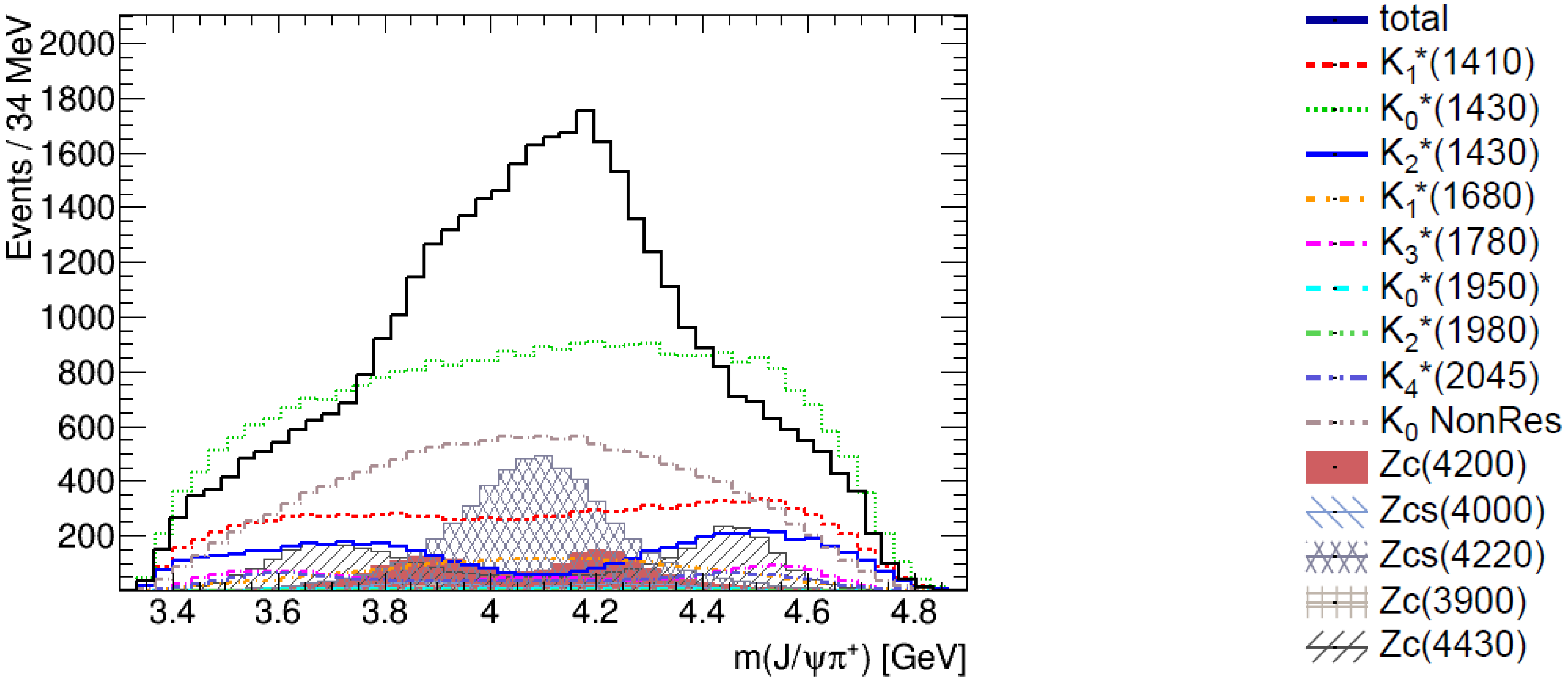
Decay chain 3 (model with exotic $Z_{cs} \to J/\psi K$ contribution):

state	M, MeV	Γ, GeV	J^P
$Z_{CS}(4000)$ [2]	4003	131	1^+
$Z_{CS}(4220)$ [2]	4216	233	1^+

At each fit step phase space events are reweighted by analyticaly derived matrix element in the helicity formalism

$$|M^{B^0}|^2 = \sum_{\Delta\lambda_\mu} \Big| Amp_{K^* chain}(m(K,\pi), anglesA)_{\Delta\lambda_\mu} + e^{i\Delta\lambda_\mu \alpha_\mu^{Z_c}} Amp_{Z_C chain}(m(J/\psi,\pi), anglesB)_{\Delta\lambda_\mu} + e^{i\Delta\lambda_\mu \alpha_\mu^{Z_{cs}}} Amp_{Z_{CS} chain}(m(J/\psi,K), anglesC)_{\Delta\lambda_\mu} \Big|^2$$

Signal modeling example



Conclusion

- Z_c(4200) was discovered by BELLE collaboration with a significance of 6.2σ in 2014. Despite the fact that evidence of its existence has been observed in data from other experiments, it still needs confirmation.

- Different theoretical models (compact tetraquark, hadron molecule, hadrocharmonium) of Z_c(4200) internal structure predict its properties consistent with existing experimental results. Futher studies require precise measurement of Z_c(4200) parameters and serching new decay chanels.

- ATLAS Run1 data shows hint of exotic contribution to $B^0 \to J/\psi K\pi$ decays;

- It is planned to perform amplitude analysis of the $B^0 \to J/\psi K\pi$ decays using the ATLAS Run2 data in order to confirm the contributions of Z_c(4200) and measure its parameters. $B^0 \to J/\psi K\pi$ decay model should include $Z_{cs} \to J/\psi K$ states observed by LHCb collaboration.

- The developed $B^0 \to J/\psi K\pi$ decay modeling procedure allows us to take into account interference effects between different intermediate states and detector effects.

Thank you for your attention!

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Appendix1 Helicity formalism

The problem with using **Spin-orbit** formalism in constructing the decay amplitude is that spin and orbital angular momentum operators of particles are defined in reference frames that are not at rest with respect to one another.

Helicity operator is invariant under both rotations and boosts along momentum direction.

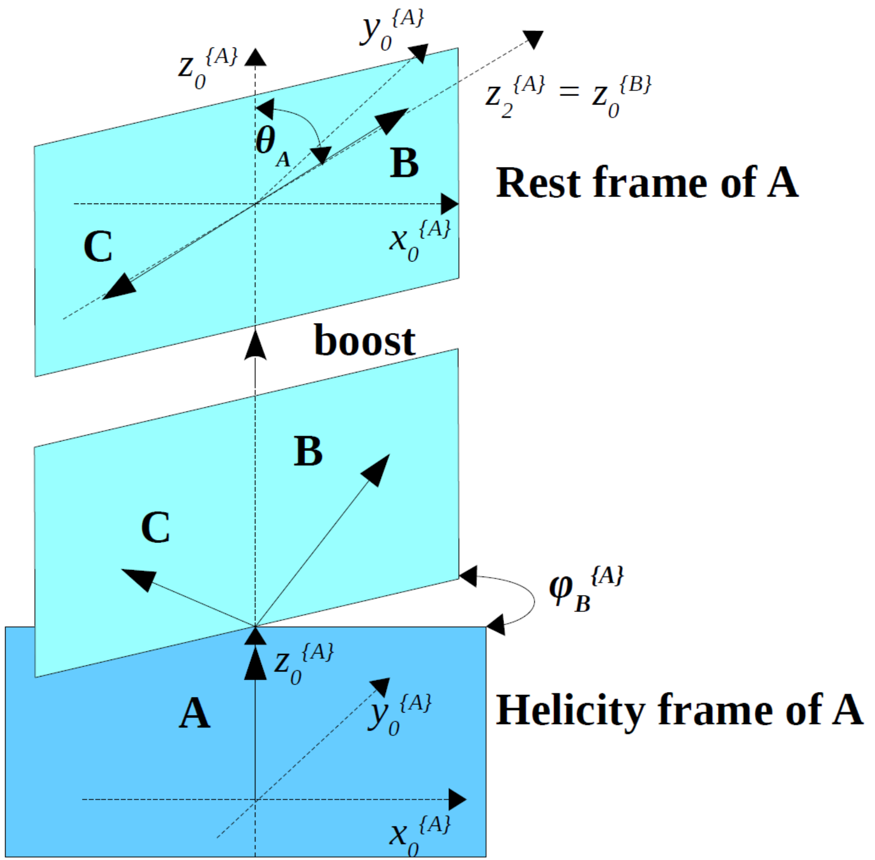
$$Amp_{A\to BC} = H^{A\to BC}_{\lambda_B,\lambda_C} D^{J_A}_{\lambda_A,\lambda_B-\lambda_C}(\phi_B^{\{A\}},\theta_A,0)^*$$

$$D^{J_A}_{\lambda_A,\lambda_B-\lambda_C}(\phi_B^{\{A\}},\theta_A,0)^* = e^{i\lambda_A\phi_B^{\{A\}}} d^{J_A}_{\lambda_A,\lambda_B-\lambda_C}(\theta_A)$$

The behavior of off-shell resonances must be described by the Breit-Wigner term. To describe decays under threshold, Flatté parametrization could be used.

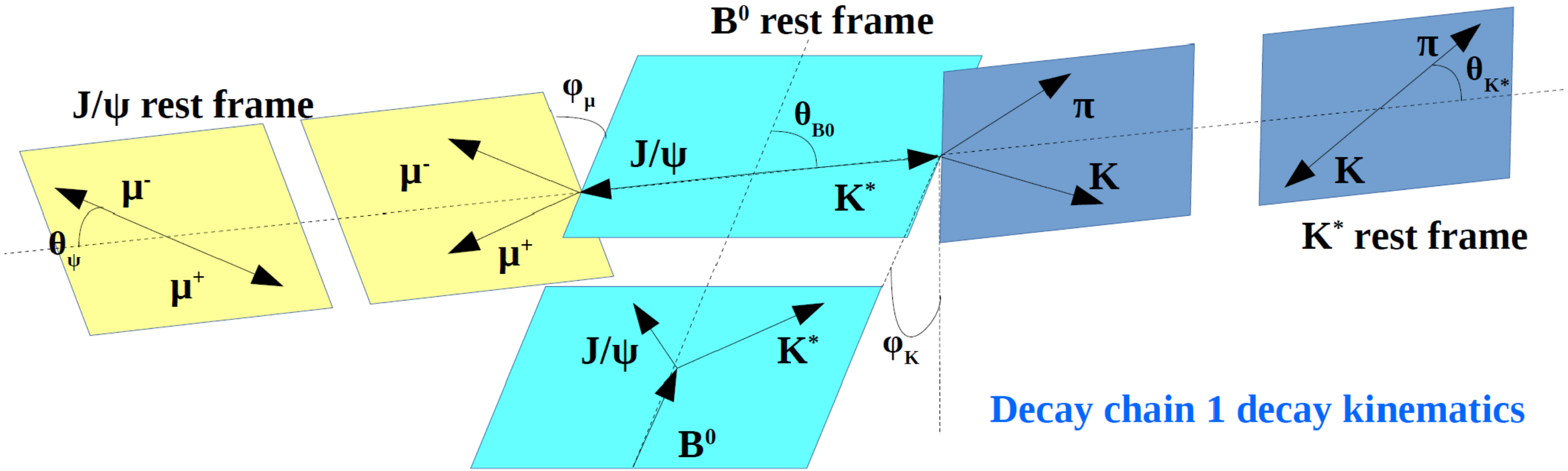
$$BW_R(M_R,m_R,\Gamma_R) = \frac{1}{M_R^2 - m_R^2 - i M_R \Gamma(m_R)}$$

$$\Gamma(m_R) = \Gamma_R \left(\frac{p_R}{p_{R0}}\right)^{2L_R+1} \left(\frac{M_R}{m_R}\right) F^2_{L_R}$$



$$\mathcal{H}^{A\to BC}_{\lambda_B,\lambda_C} = \sum_L \sum_S (-1)^{J_B-J_C+L-S+2\lambda_B-2\lambda_C} \sqrt{(2L+1)(2S+1)} B_{LS} \times \begin{pmatrix} J_B & J_C & S \\ \lambda_B & -\lambda_C & \lambda_C-\lambda_B \end{pmatrix} \begin{pmatrix} L & S & J_A \\ 0 & \lambda_B-\lambda_C & \lambda_C-\lambda_B \end{pmatrix}$$

Appendix1 Helicity formalism



Decay chain 1 decay kinematics

For EM or strong decays:

$$\mathcal{H}^{A\to BC}_{-\lambda_B,-\lambda_C} = P_A P_B P_C (-1)^{J_B+J_C-J_A} \mathcal{H}^{A\to BC}_{\lambda_B,\lambda_C};$$

K* decay chain:

$B^0 \to J/\psi K^*$ (weak decay): $Amp_{B^0\to J/\psi K^*} = H^{B^0\to J/\psi K^*}_{\lambda_\psi,\lambda_{K^*}} d^0_{0,\lambda_{K^*}-\lambda_\psi}(\theta_{B^0})$

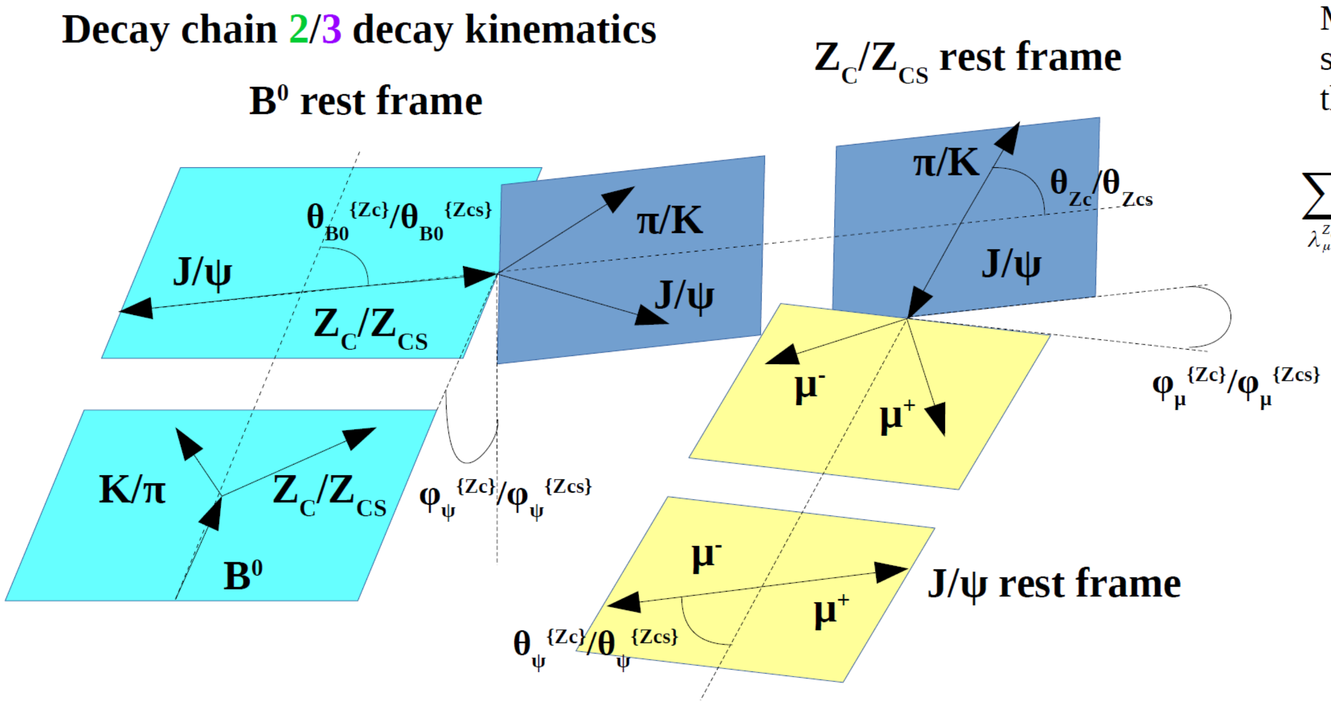
$K^* \to K\pi$ (strong decay): $Amp_{K^*\to K\pi} = H^{K^*\to K\pi}_{0,0} e^{i\phi_K \lambda_{K^*}} d^{J_{K^*}}_{\lambda_{K^*},0}(\theta_{K^*}) * R(m(K\pi), L_{B^0}, L_{K^*})$

$J/\psi \to \mu\mu$ (electromagnetic decay): $Amp_{J/\psi\to\mu\mu} = e^{i\phi_{mu}\lambda_\psi} d^0_{\lambda_\psi,\Delta\lambda_\mu}(\theta_\psi)$

$$R(m(K\pi), L_{B^0}, L_{K^*}) = \left(\frac{p_{B^0}}{m_{B^0}}\right)^{L_{B^0}} F_{L_{B^0}} BW_{K^*}(M_{K^*}, m(K,\pi), \Gamma_{K^*}) \left(\frac{p_{K^*}}{m(K\pi)}\right)^{L_{K^*}} F_{L_{K^*}}$$

$$Amp_{K^*chain}(m(K,\pi), angles\,1)_{\Delta\lambda_\mu} = \\ = \sum_{\lambda\psi,\lambda K^*} \sum_{K^*\,states} Amp_{B^0\to J/\psi K^*} Amp_{K^*\to K\pi} Amp_{J/\psi\to\mu\mu}$$

Appendix1 Helicity formalism



Muons are final-state particles, their helicity states in 2, 3 decay chains need to be rotated to the muon helicity states in K* decay chain.

$$\sum_{\lambda_\mu^{Z_c}} D^{J_\mu}_{\lambda_\mu^{Z_c}\lambda_\mu}(\alpha_\mu,0,0)^* = \sum_{\lambda_\mu^{Z_c}} e^{i\lambda_\mu^{Z_c}\alpha_\mu}\delta_{\lambda_\mu^{Z_c}\lambda_\mu} = e^{i\lambda_\mu^{Z_c}\alpha_\mu}$$

$$\alpha_\mu^{Z_C} = atan2((\hat{p}_\mu^{\{\psi\}} \times \hat{x}_1)\hat{x}_2, \hat{x}_1\hat{x}_2),$$
$$\vec{x}_2 = \hat{p}_{K^*}^{\{\psi\}} - (\hat{p}_{K^*}^{\{\psi\}}\hat{p}_\mu^{\{\psi\}})\hat{p}_\mu^{\{\psi\}},$$
$$\vec{x}_1 = \hat{p}_\pi^{\{\psi\}} - (\hat{p}_\pi^{\{\psi\}}\hat{p}_\mu^{\{\psi\}})\hat{p}_\mu^{\{\psi\}},$$

B^0 decay matrix element:

$$|M^{B^0}|^2 = \sum_{\Delta\lambda_\mu} \Big| Amp_{K^*chain}(m(K,\pi), angles1)_{\Delta\lambda_\mu} + e^{i\Delta\lambda_\mu\alpha_\mu^{Z_c}} Amp_{Z_C chain}(m(J/\psi,\pi), angles2)_{\Delta\lambda_\mu} +$$
$$+ e^{i\Delta\lambda_\mu\alpha_\mu^{Z_{cs}}} Amp_{Z_{CS} chain}(m(J/\psi,K), angles3)_{\Delta\lambda_\mu}\Big|^2$$