Toolkit in Python for simulation of controllable magnetization reversal in a chain of Phi-0 junctions by an alternating voltage pulse

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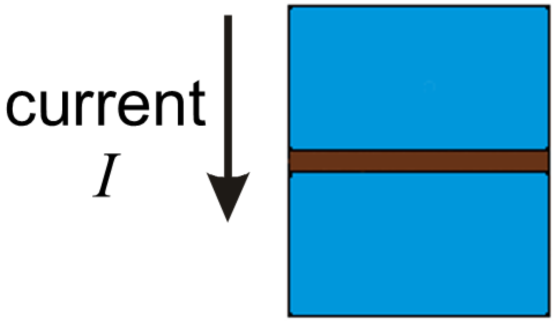
Josephson Effect



Josephson junction

Stationary Josephson effect

$$I < I_c \qquad I_s = I_c \sin\varphi \qquad V = 0$$

Stationary effect: when current is passed below the critical value, there is no voltage in the Josephson junction and superconducting current flows through the junction.

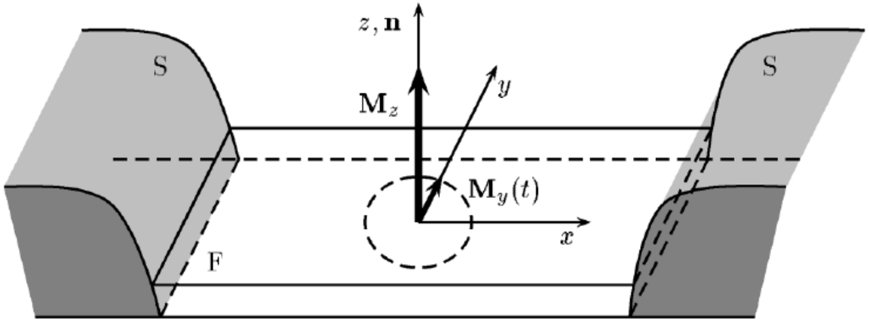
Nonstationary Josephson effect

$$I > I_c \qquad \frac{d\varphi}{dt} = \frac{2eV}{\hbar} \qquad V \neq 0$$

Non-stationary Josephson effect: when current is above the critical value, alternating voltage occurs in the Josephson junction and it is proportional to the phase difference.

Anomalous Josephson Effect

- ► In SFS Josephson junction the spin orbit coupling in Ferromagnetic layer, leads to the direct coupling of magnetization dynamics and superconducting current.



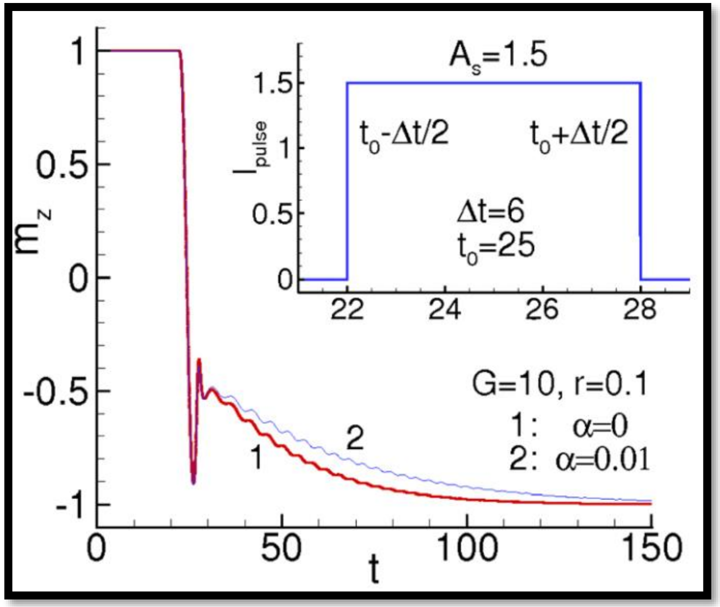
- ► In the **current-phase relation** of such junctions appears the phase shifting Phi-0, and they called Phi-0 junction and observed effect called Anomalous Josephson effect.

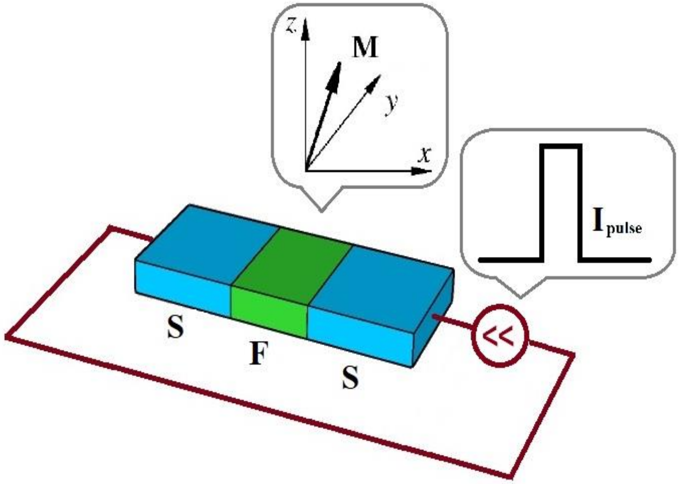
$$I_s = I_c \sin(\varphi - \varphi_0) \qquad \varphi_0 = r\frac{M_y}{M_0}$$

A. Buzdin, Phys. Rev. Lett. 101, 107005 (2008).

F. Konschelle and A. Buzdin, Phys. Rev. Lett. **102**, 017001 (2009)

Magnetization reversal in Phi-0 junction by the pulse of current



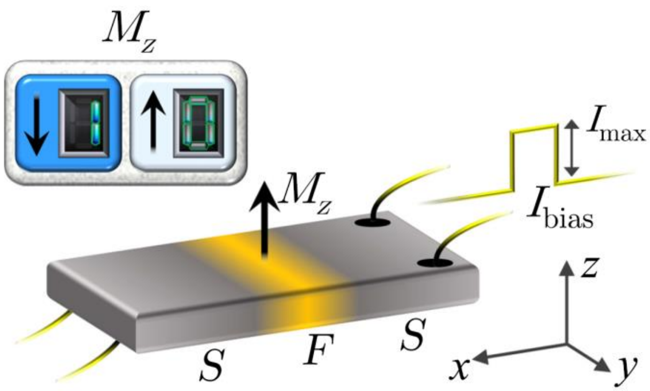


Yu. M. Shukrinov, I. R. Rahmonov, K. Sengupta, and A. Buzdin, Magnetization reversal by superconducting current in u 0 Josephson Junctions, **Appl. Phys. Lett. 110, 182407 (2017)**

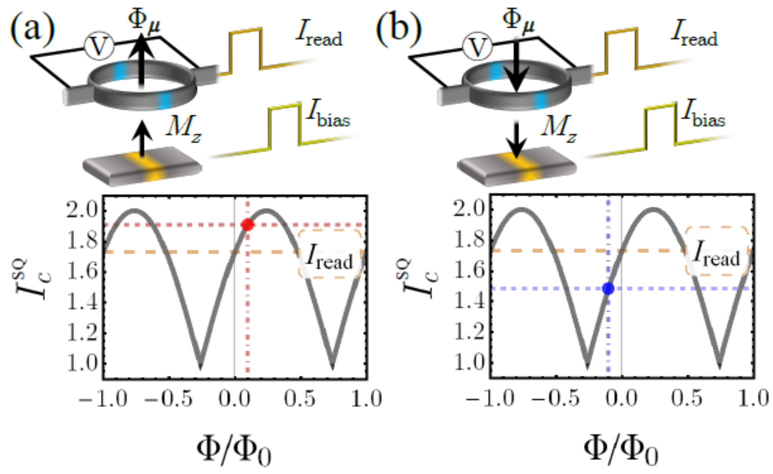
A toolkit for simulation of magnetization reversal in the form of Jupyter Book has been developed: http://studhub.jinr.ru:8080/books/intro.html

Magnetization reversal in Phi-0 junction

Based on the magnetization reversal in Phi-0 junction suggested the cryogenic memory element



S/F/S Josephson junction driven by a rectangular current pulse. The z-component of the magnetization, M_z, is the observable used to define the logic memory states 0 and 1.



SQUID-based memory readout and figure showing the critical current interference pattern of the SQUID, in the cases of both positive and negative orientation along the z-axis of the magnetization, see panel (a) and (b), respectively.

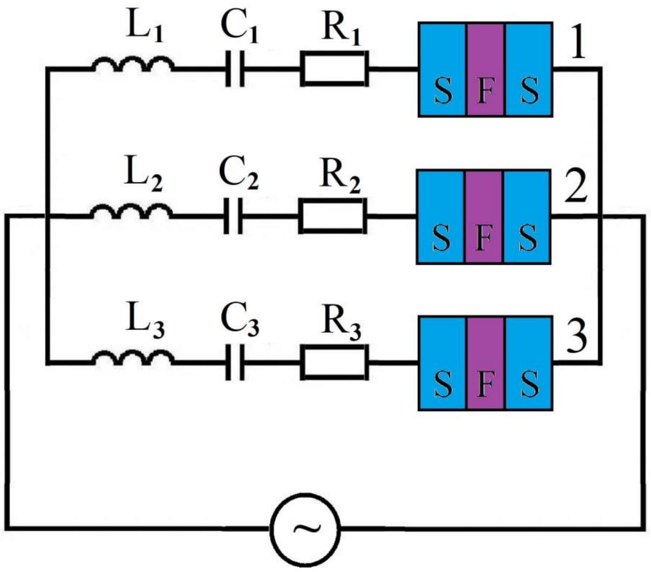
C. Guarcello and F.S. Bergeret, Cryogenic Memory Element Based on an Anomalous Josephson Junction, **Phys. Rev. Appl. 13, 034012 (2020)**

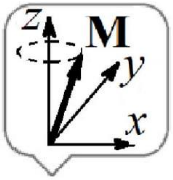
Motivation

An actual question: In case of development of many logic elements on single chip is it possible to realize controllable magnetization reversal in selected junction?

The answer is YES and we will consider this case below.

Equivalent scheme of considered system





F layer

We consider 3 Phi-0 junctions, each of which is connected in series to the LCR circuit.
Each of LCR circuit has its own eigenfrequency, which can be find by expression $\omega_c = \sqrt{\frac{1}{LC}}$

We can apply alternating voltage pulse with fixed frequency $\boldsymbol{\omega_{pulse}}$

By the tuning of $\boldsymbol{\omega_{pulse}}$ to the eigenfrequency of selected circuit branch we can realize magnetization reversal in the selected Ph-0 junction.

Dynamical variables of considered system are: Magnetization vector **M**, Josephson phase difference φ and voltage in the capacitance of the circuit u_c.

Dynamics of **M** is described by the Landau Lifshits Gilbert equations, φ is described by the equation of Resistively capacitively shunted junction model and equation for u_c can be obtained by the Kirchhoff's rules

System of equatons

Finally we can write coupled system of equations, which describes considered system

$$\frac{dm_{xi}}{dt} = -\frac{\omega_F}{1+\alpha^2 m_i^2}\left\{m_{yi}h_{zi} - m_{zi}h_{yi} + \alpha\left[m_{xi}\left(m_{xi}h_{xi} + m_{yi}h_{yi} + m_{zi}h_{zi}\right) - h_{xi}m_i^2\right]\right\}$$

$$\frac{dm_{yi}}{dt} = -\frac{\omega_F}{1+\alpha^2 m_i^2}\left\{m_{zi}h_{xi} - m_{xi}h_{zi} + \alpha\left[m_{yi}\left(m_{xi}h_{xi} + m_{yi}h_{yi} + m_{zi}h_{zi}\right) - h_{yi}m_i^2\right]\right\}$$

$$\frac{dm_{zi}}{dt} = -\frac{\omega_F}{1+\alpha^2 m_i^2}\left\{m_{xi}h_{yi} - m_{yi}h_{xi} + \alpha\left[m_{zi}\left(m_{xi}h_{xi} + m_{yi}h_{yi} + m_{zi}h_{zi}\right) - h_{zi}m_i^2\right]\right\}$$

$$\frac{d\varphi_i}{dt} = C_i U_i - \sin\left(\varphi_i - r m_{yi}\right) + r\frac{dm_{yi}}{dt}$$

$$\frac{du_{ci}}{dt} = U_i\ , \qquad \frac{dU_i}{dt} = \left[V_{pulse} - C_i R_i U_i - u_{ci} - \frac{d\varphi_i}{dt}\right]\omega_{ci}^2$$

Effective field components

$$h_{xi} = 0$$
$$h_{yi} = Gr\sin\left(\varphi_i - r m_{yi}\right)$$
$$h_{zi} = m_{zi}$$

ω_{ci} - eigenfrequency of LCR circuit

C_i - capacitance

R_i - resistance

$i = 1, 2, 3$ - number of circuit branch

m_{xi}, m_{yi}, m_{zi} - components of magnetization

φ_i - phase difference

u_{ci} - voltage in capacitance

ω_F - Ferromagnetic resonance frequency

α - Gilbert damping parameter

r - spin orbit coupling parameter

G - relation of Josephson energy to magnetic energy

Simulation

Here V_{pulse} is the alternating voltage pulse and defined as following

$$V_{pulse} = \begin{cases} A \sin \omega_{pulse}, & (t_0 - \delta t/2) < t < (t_0 + \delta t/2) \\ 0, & \left(t_0 - \frac{\delta t}{2}\right) > t, \quad \left(t_0 + \frac{\delta t}{2}\right) < t \end{cases}$$

t_0 - time interval center, where is applied pulse

δt - duration of time of pulse

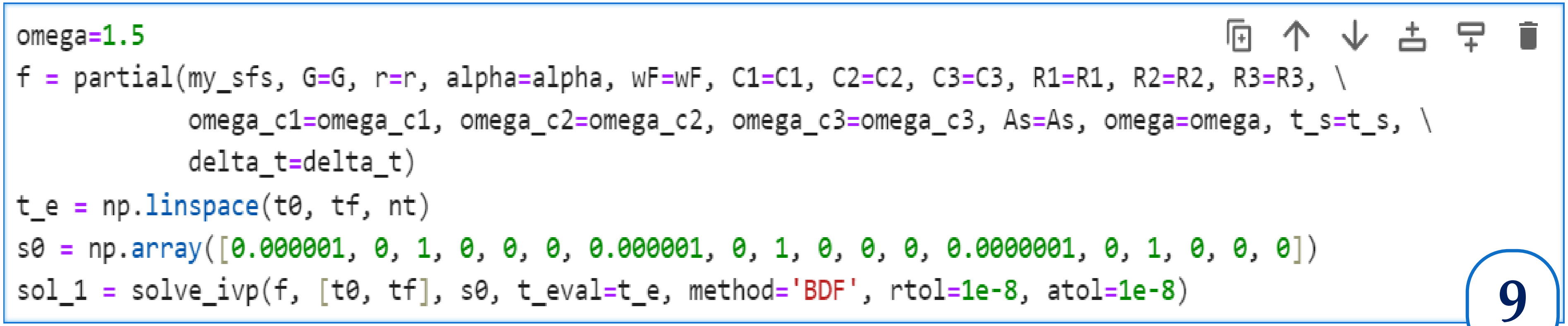
ω_{pulse} - frequency of pulse

A - Amplitude of pulse

Initial condition for system of equations

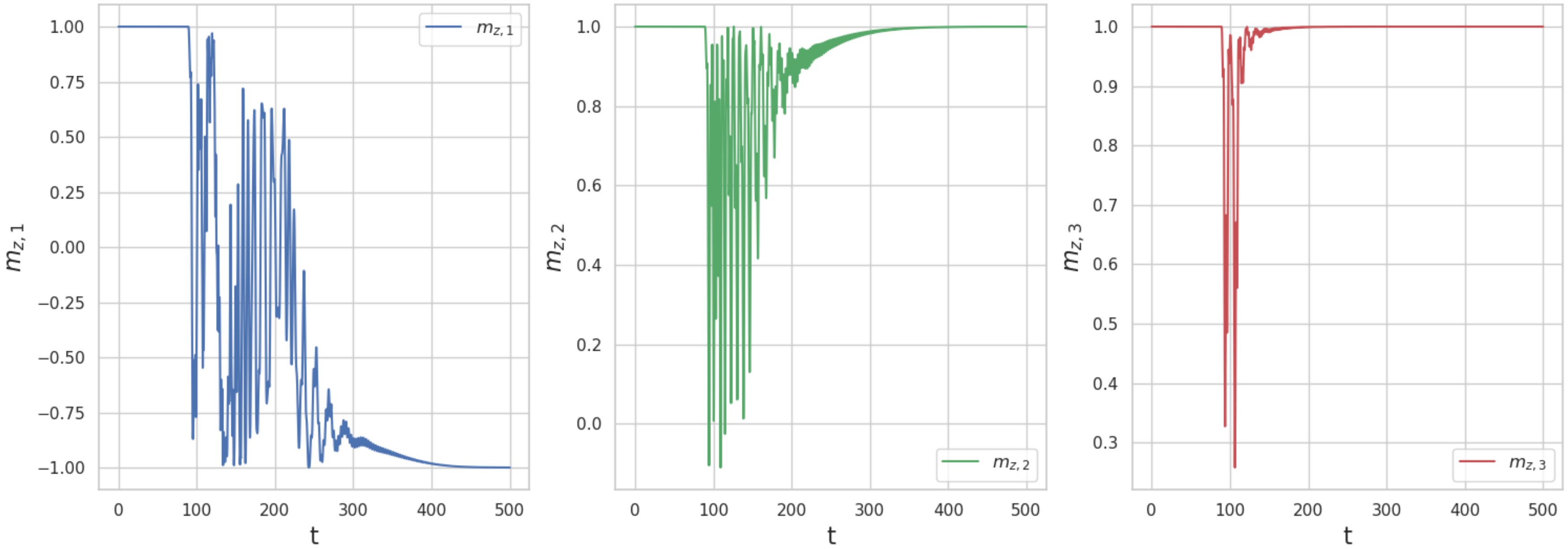
$$m_{xi} = 0,\ m_{yi} = 0,\ m_{zi} = 1, \varphi_i = 0, u_{ci} = 0,\ U_i = 0$$

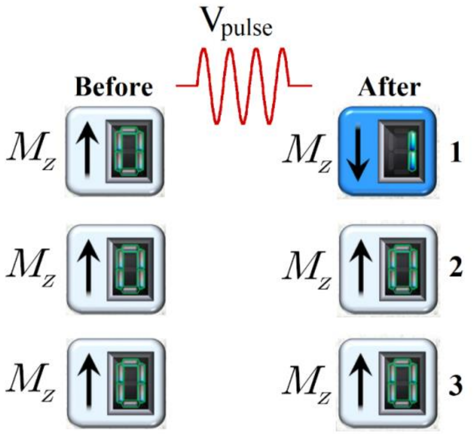
We have solved system of equations using the SOLVE_IVP function from the SCIPY library
Where MY_SFS is the defined function with right side of equations.



```
omega=1.5
f = partial(my_sfs, G=G, r=r, alpha=alpha, wF=wF, C1=C1, C2=C2, C3=C3, R1=R1, R2=R2, R3=R3, \
            omega_c1=omega_c1, omega_c2=omega_c2, omega_c3=omega_c3, As=As, omega=omega, t_s=t_s, \
            delta_t=delta_t)
t_e = np.linspace(t0, tf, nt)
s0 = np.array([0.000001, 0, 1, 0, 0, 0, 0.000001, 0, 1, 0, 0, 0, 0.0000001, 0, 1, 0, 0, 0])
sol_1 = solve_ivp(f, [t0, tf], s0, t_eval=t_e, method='BDF', rtol=1e-8, atol=1e-8)
```

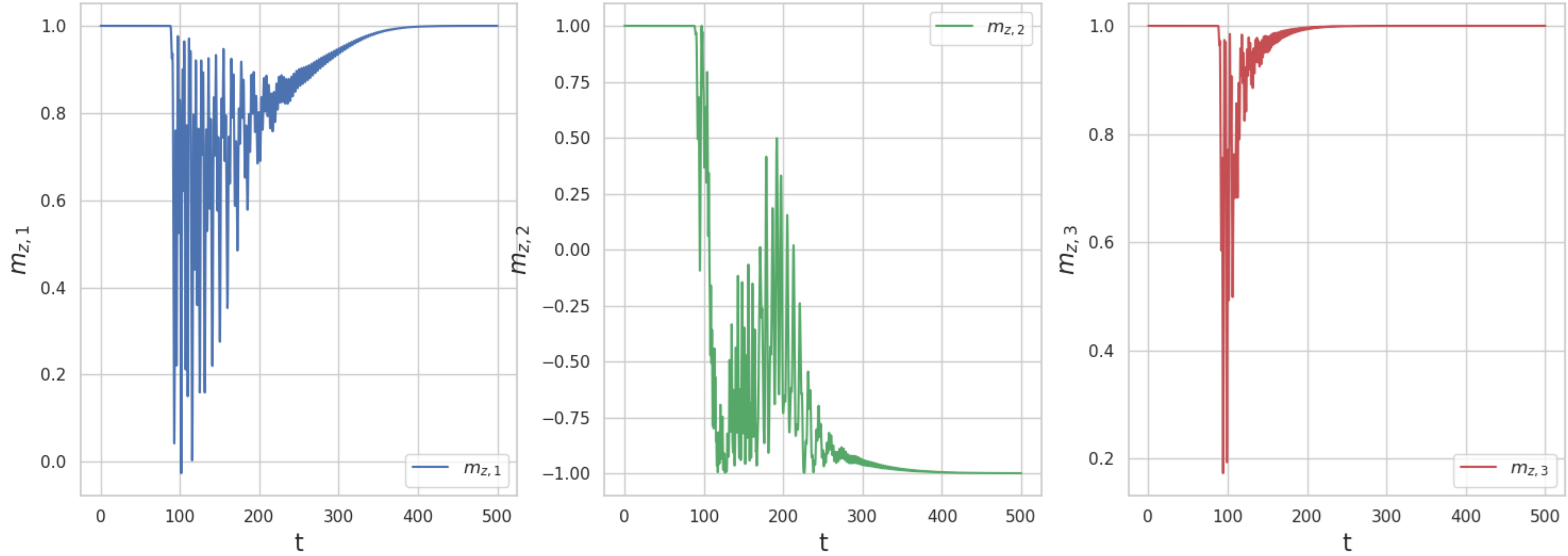
Results

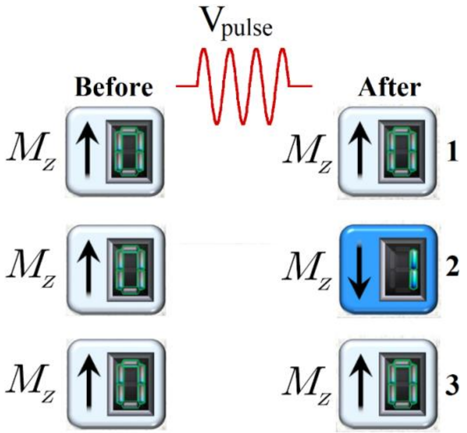




Here we applied voltage pulse with frequency corresponding to the eigenfrequency of first LCR-circuit and obtain magnetization reversal of first Phi-0 junction

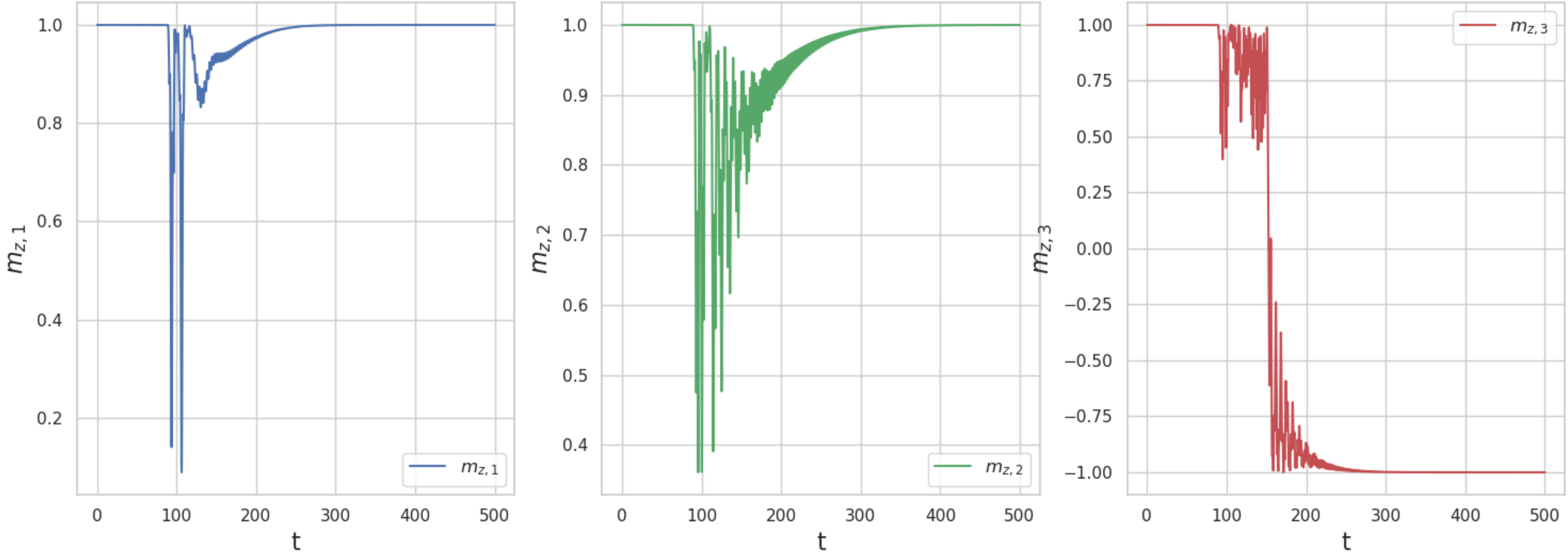
Results

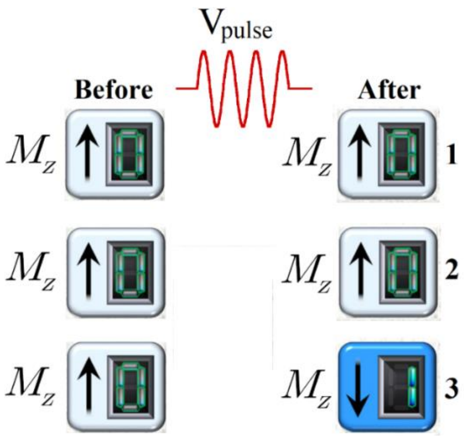




Here we applied voltage pulse with frequency corresponding to the eigenfrequency of second LCR-circuit and obtain magnetization reversal of second Phi-0 junction

Results

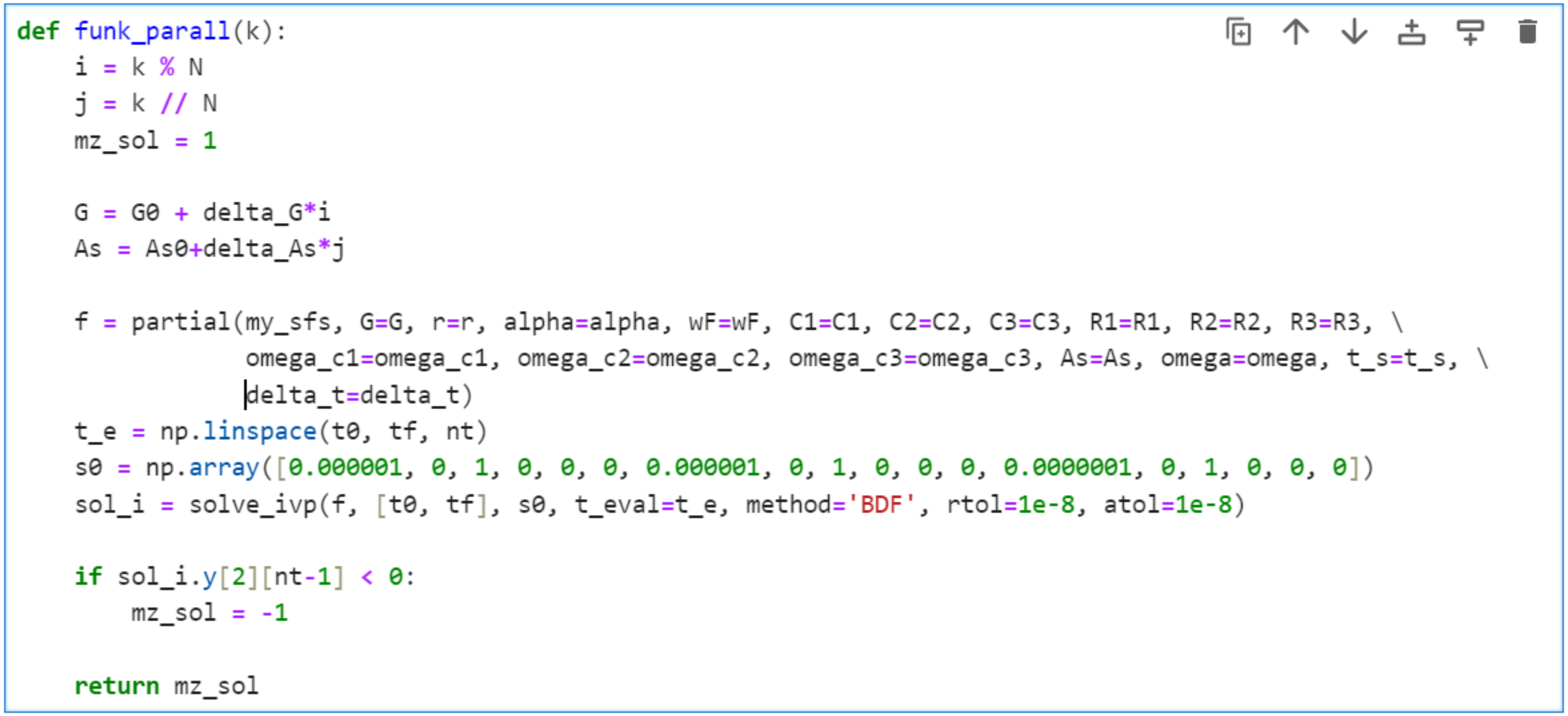




Here we applied voltage pulse with frequency corresponding to the eigenfrequency of third LCR-circuit and obtain magnetization reversal of third Phi-0 junction

Parallel realization

```python
from joblib import Parallel, delayed
import numpy as np
```



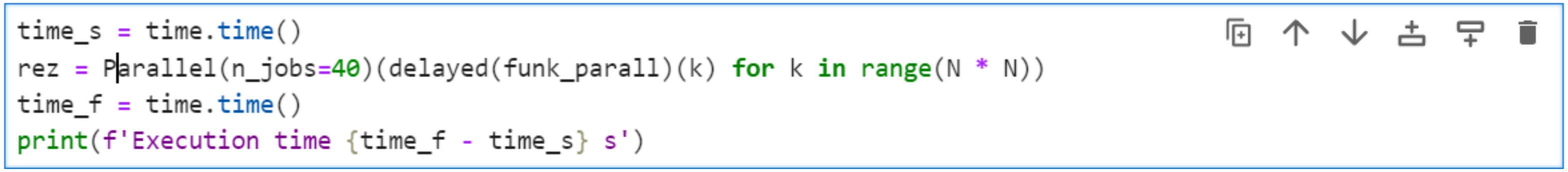
```python
def funk_parall(k):
    i = k % N
    j = k // N
    mz_sol = 1

    G = G0 + delta_G*i
    As = As0+delta_As*j

    f = partial(my_sfs, G=G, r=r, alpha=alpha, wF=wF, C1=C1, C2=C2, C3=C3, R1=R1, R2=R2, R3=R3, \
                omega_c1=omega_c1, omega_c2=omega_c2, omega_c3=omega_c3, As=As, omega=omega, t_s=t_s, \
                delta_t=delta_t)
    t_e = np.linspace(t0, tf, nt)
    s0 = np.array([0.000001, 0, 1, 0, 0, 0, 0.000001, 0, 1, 0, 0, 0, 0.0000001, 0, 1, 0, 0, 0])
    sol_i = solve_ivp(f, [t0, tf], s0, t_eval=t_e, method='BDF', rtol=1e-8, atol=1e-8)

    if sol_i.y[2][nt-1] < 0:
        mz_sol = -1

    return mz_sol
```

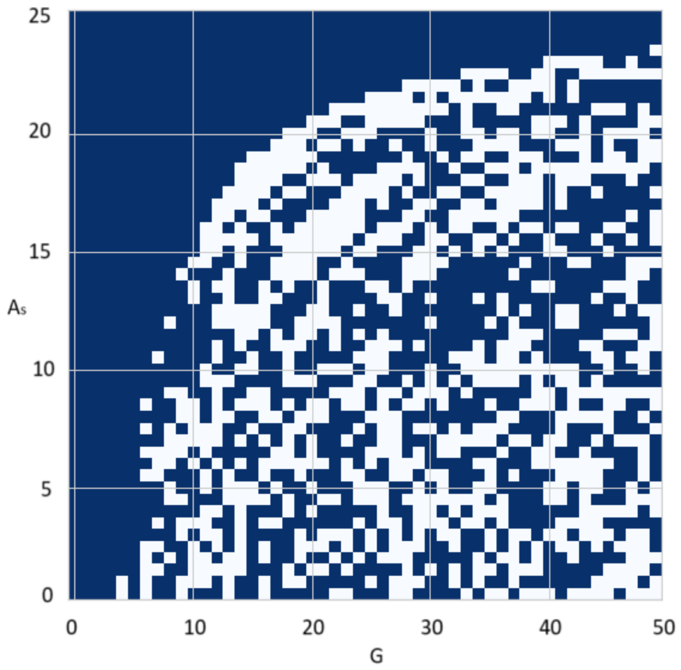


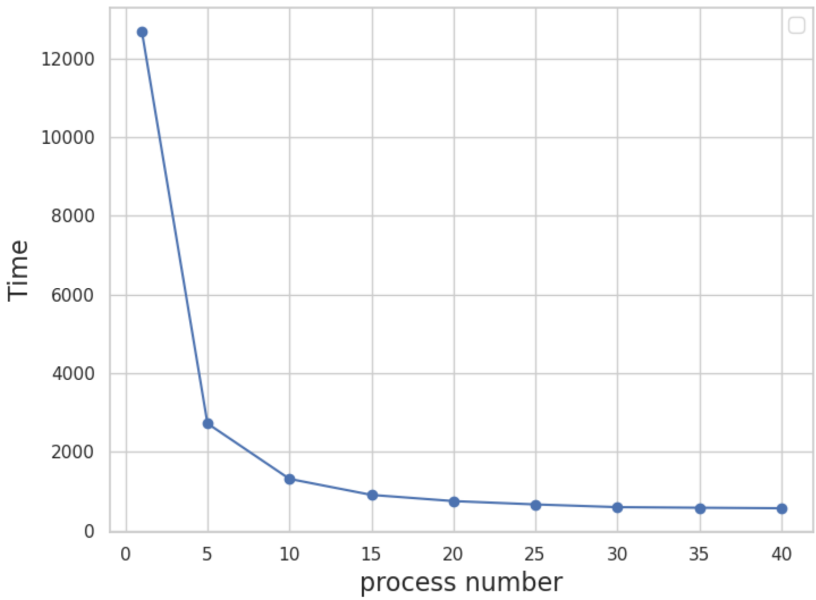
```python
time_s = time.time()
rez = Parallel(n_jobs=40)(delayed(funk_parall)(k) for k in range(N * N))
time_f = time.time()
print(f'Execution time {time_f - time_s} s')
```

We have realized parallel calculations using joblib library.

Calculations are performed for different values of the As and G totally 2500 calculations.

Results of parallelization





Here is demonstrated realization of magnetization reversal as the function of G and As

Dark points corresponds to the magnetization reversal,
White point corresponds to the absence of reversal

Here is demonstrated time of calculation as the function of process numbers.

We obtain up to 22 time acceleration in case of 40 processes

Conclusions

We have realized Toolkit for investigations of system of Phi-0 junction chain with coupled LCR circuits using Jupyter Book environment.

We have demonstrated the controllable magnetization reversal by the pulse of alternating voltage by the tuning of its frequency to the eigenfrequency of LCR circuit.

We have realized parallel calculations for searching of magnetization reversal by changing of parameters and show the acceleration up to 22 time.

We expect that performed investigations and obtained results can be useful for development of cryogenic memory, based on Phi-0 junction.

Thank you for your attention!

Toolkit based on Python libraries and Jupyter ecosystems for solution scientific and applied problems

http://studhub.jinr.ru:8080/books/intro.html

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