28th International Scientific Conference of Young Scientists and Specialists

T-even hadronic structure functions in the Drell-Yan process

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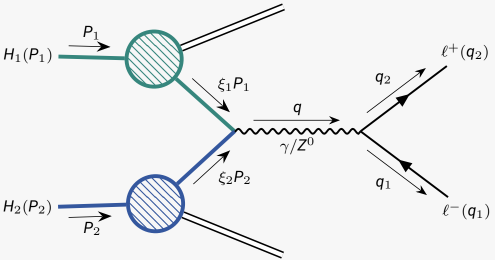
Introduction



$$H_1(P_1) + H_2(P_2) \to \gamma/Z^0 + X \to \ell^-(q_1) + \ell^+(q_2) + X$$

Contents

01

Drell-Yan process fundamentals

Factorization of hadronic and leptonic processes,
collinear factorization and γ/Z^0 interference

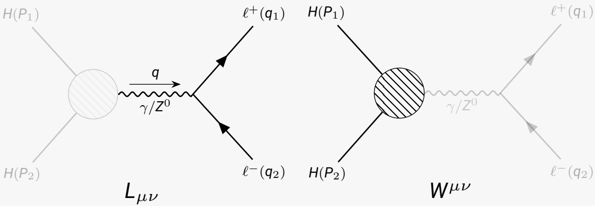
Cross section factorization

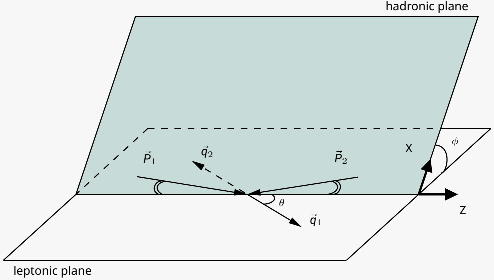
Using the dilepton center-of-mass frame, the cross section is factorized into leptonic and hadronic tensors

$$\frac{d\sigma}{dQ^2\, dQ_T\, dy\, d\Omega} = \frac{\alpha^2}{4(2\pi)^4 Q^4 s^2} L_{\mu\nu}(q_1, q) W^{\mu\nu}(P_1, P_2, q)$$

Hadronic tensor in terms of structure functions

$$\begin{aligned} W^{\mu\nu} &= (X^\mu X^\nu + Y^\mu Y^\nu) W_T + i(X^\mu Y^\nu - X^\nu Y^\mu) W_{T_P} \\ &+ (Y^\mu Y^\nu - X^\mu X^\nu) W_{\Delta\Delta} - (X^\nu Y^\mu + X^\mu Y^\nu) W_{\Delta\Delta_P} \\ &- (X^\nu Z^\mu + X^\mu Z^\nu) W_{\Delta} - (Y^\nu Z^\mu + Y^\mu Z^\nu) W_{\Delta_P} \\ &+ i(X^\nu Z^\mu - X^\mu Z^\nu) W_{\nabla} + i(Y^\mu Z^\nu - Y^\nu Z^\mu) W_{\nabla_P} \\ &+ Z^\mu Z^\nu W_L \end{aligned}$$





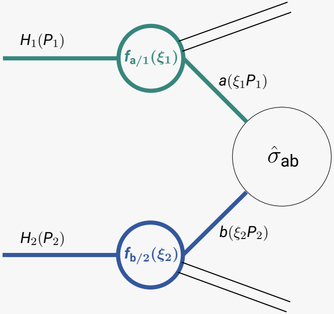
Basis of unit vectors:

$$\begin{aligned} T^\mu &= \frac{q^\mu}{Q} = (1,0,0,0) \\ X^\mu &= (0,1,0,0) \\ Z^\mu &= (0,0,0,1) \\ Y^\mu &= \epsilon^{\mu\nu\alpha\beta} T_\nu Z_\alpha X_\beta = (0,0,1,0) \end{aligned}$$

Relationship with hadron momenta

$$P^\mu_{1/2} = e^{\mp y} \frac{\sqrt{s}}{2} \left(T^\mu \sqrt{1+\rho^2} \pm Z^\mu - \rho X^\mu \right), \quad \rho = \frac{Q_T}{Q}$$

Collinear factorization



Collinear factorization in the Drell-Yan process

$$\frac{d\sigma}{dQ^2\, dQ_T\, dy\, d\Omega} = \sum_{a,b} \int_{x_1}^{1} \frac{d\xi_1}{\xi_1} \int_{x_2}^{1} \frac{d\xi_2}{\xi_2} f_{a/1}(\xi_1) \hat{\sigma}_{ab}(\xi_1, \xi_2) f_{b/2}(\xi_2)$$

In particular, for structure functions

$$W_{\mathrm{I}} = \frac{1}{x_1 x_2} \sum_{a,b} \int_{x_1}^{1} dz_1 \int_{x_2}^{1} dz_2\, f_{a/1}\left(\frac{x_1}{z_1}\right) \tilde{\omega}_{\mathrm{I}}^{ab}(z_1, z_2) \delta\Big((\hat{s} + \hat{t} + \hat{u} - Q^2)/\hat{s}\Big) f_{b/2}\left(\frac{x_2}{z_2}\right), \qquad z_i = \frac{x_i}{\xi_i}$$

Interference of γ and Z^0

For consideration of the γ/Z^0 interference, we require the following specific electroweak couplings

$$g^{Z\gamma}_{\mathrm{EW};1} = 1 + 2\, g^V_{Zq}\, g^V_{Zl}\, \mathrm{Re}[D_Z(Q^2)] + \left((g^V_{Zq})^2 + (g^A_{Zq})^2\right)\left((g^V_{Z\ell})^2 + (g^A_{Z\ell})^2\right) |D_Z(Q^2)|^2$$

$$g^{Z\gamma}_{\mathrm{EW};2} = 2\, g^A_{Zq} \left[2\, g^V_{Zq} \left(g^A_{Z\ell}\, g^V_{Z\ell}\right) |D_Z(Q^2)|^2 + g^A_{Z\ell}\, \mathrm{Re}[D_Z(Q^2)]\right]$$

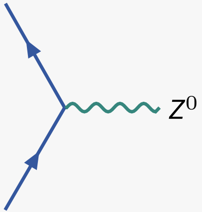
Here, couplings of the Z^0 boson with leptons

$$g^V_{Z\ell} = -\frac{1 - 4\sin^2\theta_W}{2\sin 2\theta_W}\,, \qquad g^A_{Z\ell} = -\frac{1}{2\sin 2\theta_W}\,,$$

with up (u), and down (d) quarks

$$g^V_{Zu} = \frac{1 - 8/3\sin^2\theta_W}{2e_q \sin 2\theta_W}\,, \qquad g^V_{Zd} = -\frac{1 - 4/3\sin^2\theta_W}{2e_q \sin 2\theta_W}\,,$$

$$g^A_{Zu} = \frac{1}{2e_q \sin 2\theta_W}\,, \qquad g^A_{Zd} = -\frac{1}{2e_q \sin 2\theta_W}$$



$$= -i\gamma_\mu \left(g^V_{Zf} - g^A_{Zf}\gamma^5\right)$$

and real and imaginary parts of the Breit-Wigner propagator

$$\mathrm{Re}[D_Z(Q^2)] = \frac{(M_Z^2 - Q^2)Q^2}{(M_Z^2 - Q^2)^2 + M_Z^2\Gamma_Z^2}\,, \qquad \mathrm{Im}[D_Z(Q^2)] = \frac{M_Z\Gamma_Z Q^2}{(M_Z^2 - Q^2)^2 + M_Z^2\Gamma_Z^2}$$

02

Next-to leading order pQCD results

Next-to leading order perturbative QCD predictions for parton-level subprocesses contributing to the Drell-Yan process

Quark-antiquark annihilation

Couplings

$$C_{q\bar{q}} = \frac{C_F}{N_c} = \frac{N_c^2 - 1}{2N_c^2} = \frac{4}{9}$$

$$g_{q\bar{q};i} = (8\,\pi^2 e_q^2\,\alpha_s)\,C_{q\bar{q}}\,g_{\mathrm{EW};i}^{Z\gamma} = G_i C_{q\bar{q}}$$

Structure functions

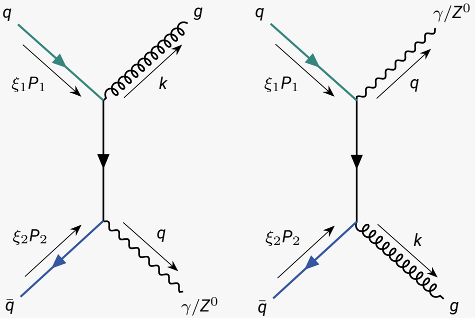
$$\tilde{\omega}_T^{q\bar{q}} = g_{q\bar{q};1}\,\frac{1}{\rho^2}\left(1 + \frac{\rho^2}{2}\right)\frac{z_1^2 + z_2^2}{z_1 z_2}$$

$$\tilde{\omega}_L^{q\bar{q}} = 2\,\tilde{\omega}_{\Delta\Delta}^{q\bar{q}} = g_{q\bar{q};1}\,\frac{z_1^2 + z_2^2}{z_1 z_2}$$

$$\tilde{\omega}_\Delta^{q\bar{q}} = g_{q\bar{q};1}\,\frac{1}{\rho}\,\frac{z_1^2 - z_2^2}{z_1 z_2}$$

$$\tilde{\omega}_{T_P}^{q\bar{q}} = g_{q\bar{q};2}\,\frac{\sqrt{1+\rho^2}}{\rho^2}\,\frac{z_1^2 + z_2^2}{z_1 z_2}$$

$$\tilde{\omega}_{\nabla_P}^{q\bar{q}} = g_{q\bar{q};2}\,\frac{\sqrt{1+\rho^2}}{\rho}\,\frac{z_1^2 - z_2^2}{z_1 z_2}$$



Quark-gluon scattering

Couplings

$$C_{qg} = \frac{T_F}{N_c} = \frac{1}{2N_c} = \frac{1}{6}$$

$$g_{qg;i} = (8\,\pi^2 e_q^2\,\alpha_s)\,C_{qg}\,g^{Z\gamma}_{\mathrm{EW};i} = G_i C_{qg}$$

Structure functions

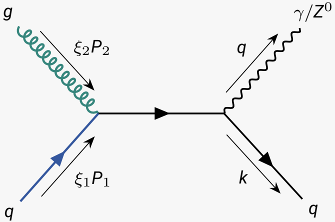
$$\tilde{\omega}^{qg}_T = g_{qg;1}\,\frac{1}{\rho^2}\,\frac{1-z_2}{z_1 z_2}\,\left(z_2^2 + (1-z_1 z_2)^2 + \rho^2\left(1 - \frac{z_1^2}{2} - z_1 z_2(z_1+z_2)\right)\right)$$

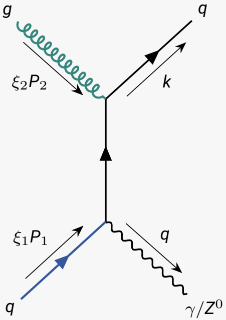
$$\tilde{\omega}^{qg}_L = 2\,\tilde{\omega}^{qg}_{\Delta\Delta} = g_{qg;1}\,\frac{1-z_2}{z_1 z_2}\,\left(z_2^2 + (z_1+z_2)^2\right)$$

$$\tilde{\omega}^{qg}_\Delta = g_{qg;1}\,\frac{1}{\rho}\,\frac{1-z_2}{z_1 z_2}\,\left(z_1^2 - 2z_2^2\right)$$

$$\tilde{\omega}^{qg}_{T_P} = g_{qg;2}\,\frac{\sqrt{1+\rho^2}}{\rho^2}\,\frac{1-z_2}{z_1 z_2}\,\left(z_2^2 + (1-z_2)^2 - (1-z_1)^2\right)$$

$$\tilde{\omega}^{qg}_{\nabla_P} = g_{qg;2}\,\frac{\sqrt{1+\rho^2}}{\rho}\,\frac{1-z_2}{z_1 z_2}\,\left(1 - 2z_2^2 - (1-z_1)^2 + 2z_2(1-z_1)\right)$$





03

Comparison with experimental data

Comparison of angular coefficients and forward-backward asymmetry with experimental data from the ATLAS [JHEP 08 (2016) 159] and CMS [Phys.Lett.B 718 (2013) 752-772, Phys.Lett.B 750 (2015) 154-175] Collaborations

Angular coefficients

The lepton angular distribution, which encodes information about polar and azimuthal asymmetries, can be parameterized by seven angular coefficients

$$
\begin{aligned}
\frac{\mathrm{d}N}{\mathrm{d}\Omega} &= \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Q_T\,\mathrm{d}y\,\mathrm{d}\Omega}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Q_T\,\mathrm{d}y}\right)^{-1} \\
&= \frac{3}{16\pi}\Big[1+\cos^2\theta+\frac{1}{2}A_0(1-3\cos^2\theta)+A_1\sin 2\theta\cos\phi \\
&\quad +\frac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta \\
&\quad + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi\Big]
\end{aligned}
$$

Relationship with structure functions

$$
A_0 = \frac{2W_L}{2W_T+W_L}\,,\quad A_1 = \frac{2W_\Delta}{2W_T+W_L}\,,\quad A_2 = \frac{4W_{\Delta\Delta}}{2W_T+W_L}\,,\quad A_3 = \frac{2W_{\nabla_P}}{2W_T+W_L}\,,\quad A_4 = \frac{2W_{T_P}}{2W_T+W_L}\,,
$$

$$
A_5 = \frac{2W_{\Delta\Delta_P}}{2W_T+W_L}\,,\quad A_6 = \frac{2W_{\Delta_P}}{2W_T+W_L}\,,\quad A_7 = \frac{2W_\nabla}{2W_T+W_L}
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$$

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&\quad \left.+\frac{1}{2}A_2\sin^2\theta\cos 2\phi+A_3\sin\theta\cos\phi+A_4\cos\theta\right]
\end{aligned}
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$$= \frac{3}{16\pi}\Big[1+\cos^2\theta + \frac{1}{2}A_0\left(1-3\cos^2\theta\right) + A_1\sin 2\theta\cos\phi$$
$$+\frac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta\Big]$$

Relationship with structure functions

$$A_0 = \frac{2W_L}{2W_T+W_L}\,, \quad A_1 = \frac{2W_\Delta}{2W_T+W_L}\,, \quad A_2 = \frac{4W_{\Delta\Delta}}{2W_T+W_L}\,, \quad A_3 = \frac{2W_{\nabla_P}}{2W_T+W_L}\,, \quad A_4 = \frac{2W_{T_P}}{2W_T+W_L}$$

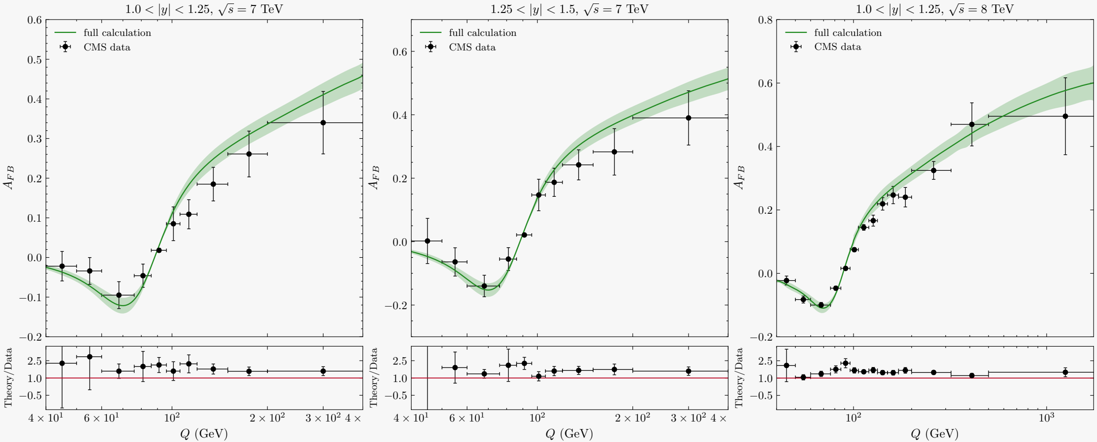
Forward-backward asymmetry

$$A_{\mathrm{FB}} = \frac{\int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} - \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}}{\int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} + \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}} = \frac{3}{8}A_4$$

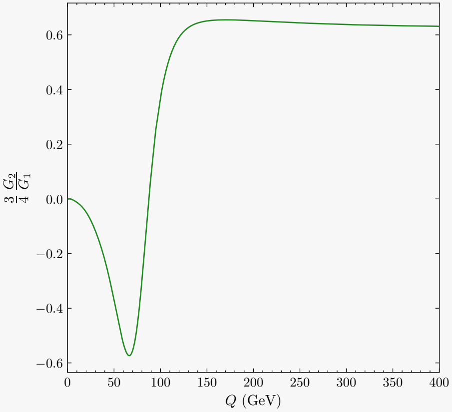
Convexity (asymmetry of the transverse and longitudinal structure functions)

$$A_{\mathrm{conv}} = \frac{3}{8}\left(2-3A_0\right)$$

Forward-backward asymmetry



Forward-backward asymmetry



Leading order in small-Q_T expansion of structure functions

$$W_T^{\mathrm{LP};q\bar q}(x_1^0,x_2^0,L_\rho) = \frac{1}{\rho^2}\, W_L^{\mathrm{LP};q\bar q}(x_1^0,x_2^0,L_\rho) = \frac{g_{q\bar q;1}}{g_{q\bar q;2}}\, W_{T_P}^{\mathrm{LP};q\bar q}(x_1^0,x_2^0,L_\rho)$$
$$= \frac{g_{q\bar q;1}}{\rho^2 x_1^0 x_2^0}\,\frac{1}{C_F}\Big[- C_F(2L_\rho+3)\, q_1(x_1^0)\,\bar q_2(x_2^0)$$
$$+ q_1(x_1^0)\,\Big(P_{qq}\otimes \bar q_2\Big)(x_2^0) + \Big(P_{qq}\otimes q_1\Big)(x_1^0)\,\bar q_2(x_2^0)\Big],$$

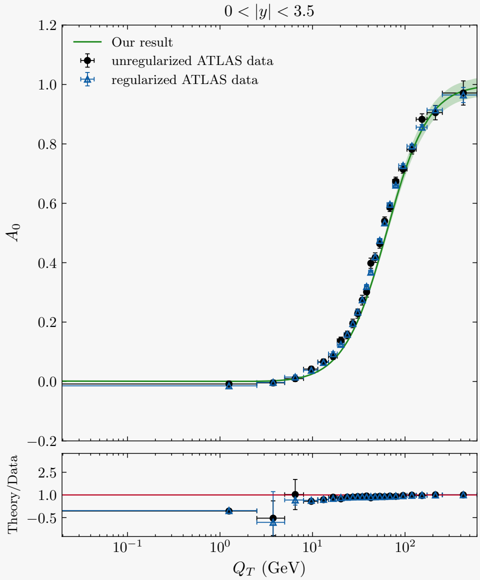
$$W_T^{\mathrm{LP};qg}(x_1^0,x_2^0,L_\rho) = \frac{g_{qg;1}}{g_{qg;2}}\, W_{T_P}^{\mathrm{LP};qg}(x_1^0,x_2^0,L_\rho)$$
$$= \frac{2g_{qg;1}}{\rho^2 x_1^0 x_2^0}\, q_1(x_1^0)\,\Big(P_{qg}^{+}\otimes g_2\Big)(x_2^0)\,,$$

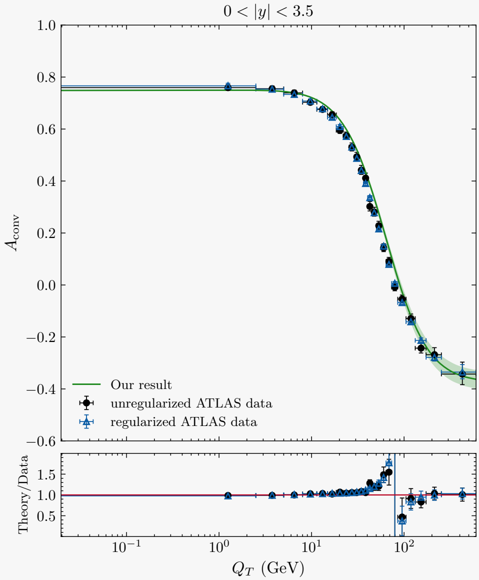
$$W_L^{\mathrm{LP};qg}(x_1^0,x_2^0,L_\rho) = \frac{2g_{qg;1}}{x_1^0 x_2^0}\, q_1(x_1^0)\,\Big(P_{qg}^{-}\otimes g_2\Big)(x_2^0)$$

Angular coefficient A_4 in terms of structure functions

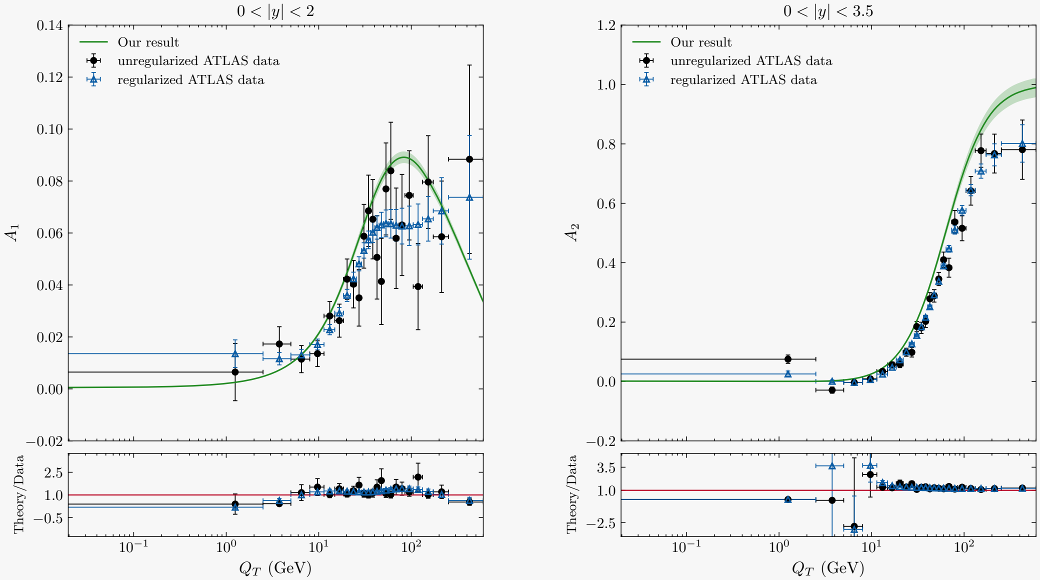
$$A_4^{\mathrm{LP}} = \frac{2W_{T_P}^{\mathrm{LP}}}{2W_T^{\mathrm{LP}} + W_L^{\mathrm{LP}}} \approx \frac{W_{T_P}^{\mathrm{LP}}}{W_T^{\mathrm{LP}}} = \frac{G_2}{G_1}$$

Angular coefficient A_0 and A_{conv}

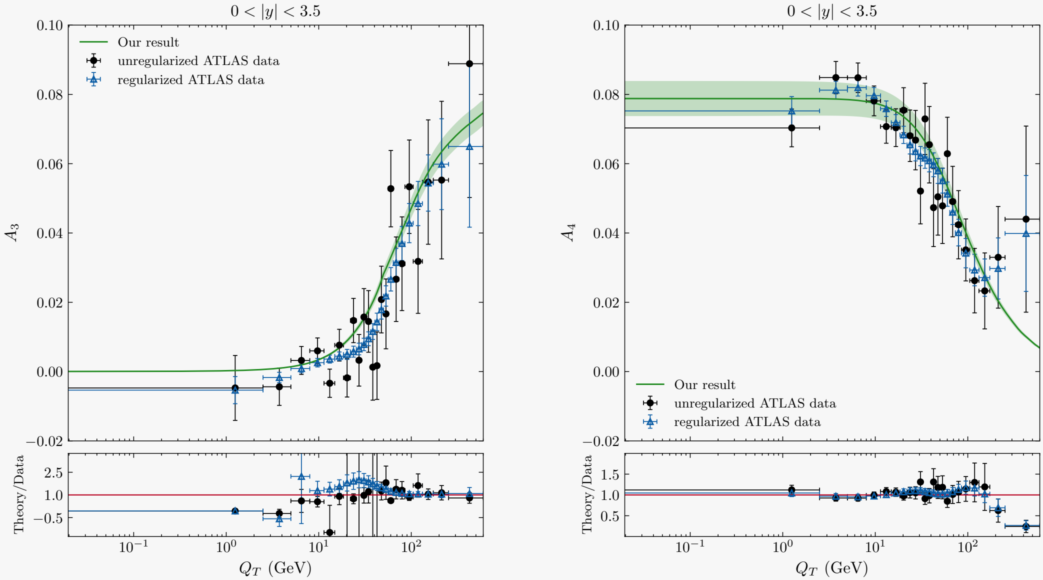




Angular coefficients A_1 and A_2



Angular coefficients A_3 and A_4



Conclusion

We presented analytical results for the Drell-Yan T-even hadronic structure functions in the framework of the pQCD based on the collinear factorization scheme and at the leading order in the α_s expansion

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We demonstrated that our theoretical predictions for angular coefficients and forward-backward asymmetry are in good agreement with data from ATLAS and CMS Collaborations

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We proposed a novel quantity "convexity" in context of Drell-Yan process and found that our prediction for it is in good agreement with data from ATLAS Collaboration

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We proposed systematic and analytic Q_T^2/Q^2 expansion for structure functions in Drell-Yan process up to desired order

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Additionally we pointed out that the small Q_T/Q limit plays an important role for the forward-backward asymmetry

Thanks for your attention!

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