Relativistic description of triply heavy tetraquarks

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The 28th International Scientific Conference of Young Scientists and Specialists,
October 30, 2024

Introduction

- "Ordinary" hadrons:
  - baryons qqq,
  - mesons $\mathrm{q\bar{q}}$.
- Exotic hadrons:
  - tetraquarks $\mathrm{qq\bar{q}\bar{q}}$,
  - pentaquarks $\mathrm{qqqq\bar{q}}$, etc.
- Searches for the $\mathrm{X_{cc\bar{c}\bar{c}}}$, $\mathrm{X_{bb\bar{b}\bar{b}}}$ are conducted on the Large Hadron Collider (LHC) by the LHCb, ATLAS and CMS Collaborations.

Model description I

- All parameters of the model (including the constituent masses of quarks) are fixed from previous studies of the properties of mesons and baryons.
- Quarks under the consideration:
  - $m_{\rm u} = m_{\rm d} = 0.33$ GeV,
  - $m_{\rm s} = 0.50$ GeV,
  - $m_{\rm c} = 1.55$ GeV,
  - $m_{\rm b} = 4.88$ GeV.

Model description II

◇ Quark content:

- $Q, Q' = c, b,\ Q \neq Q'$.
- $q = u, d, s$.
- with one open heavy flavor (without/with strangeness):
    - $QQ\overline{Q}\overline{q}$ (+ c.c.).
- with one open and another hidden heavy flavors (without/with strangeness):
    - $QQ'\overline{Q}\overline{q}$ (+ c.c.).
- with two open heavy flavors (without/with strangeness):
    - $QQ\overline{Q}'\overline{q}$ (+ c.c.).

Model description III

- Diquark–antidiquark bound state:
  - $\{(\mathrm{Q}_1\mathrm{Q}_2)-(\overline{\mathrm{Q}}_3\overline{\mathrm{q}}_4)\}$ (+ c.c.).
- Diquarks under the consideration:
  - nonpoint–like (the internal structure is taken into account)
  - ground state $(1\mathrm{S})$,
  - color-antitriplet $(\overline{3}_c)$,
  - all masses and form factors of diquarks were calculated earlier during analyzing the properties of baryons.

Model description IV

- Ground state diquark spin:
  - $J = 0$ — scalar (S),
  - $J = 1$ — axialvector (A).
- Allowed diquark states:
  - only axialvector (A):
    - QQ.
  - both axialvector and scalar (A, S):
    - QQ',
    - Qq.

Model description V

- Tetraquark's possible configurations:
  - $\mathrm{A\overline{A}}$ — any composition,
  - $\mathrm{A\overline{S}}$ — any composition,
  - $\mathrm{S\overline{A}}$ — $\mathrm{QQ'\overline{Q}\overline{q}}$ (+ c.c.),
  - $\mathrm{S\overline{S}}$ — $\mathrm{QQ'\overline{Q}\overline{q}}$ (+ c.c.).

◇ Relativistic Schrödinger-type quasipotential equation:

$$\left(\frac{b^2(M)}{2\mu_R(M)} - \frac{\mathbf{p}^2}{2\mu_R(M)}\right)\Psi_{d,T}(\mathbf{p}) = \int \frac{d^3 q}{(2\pi)^3} \, V(\mathbf{p},\mathbf{q};M)\Psi_{d,T}(\mathbf{q})$$

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

- Diquark–antidiquark interaction quasipotential:

$$V(\mathbf{p},\mathbf{q};M)=\frac{\langle d(\mathcal{P})|\,J_\mu\,|d(\mathcal{Q})\rangle}{2\sqrt{E_{\rm d}}\sqrt{E_{\rm d}}}\,\frac{4}{3}\alpha_{\rm s}D^{\mu\nu}(\mathbf{k})\frac{\langle d'(\mathcal{P}')|\,J_\nu\,|d'(\mathcal{Q}')\rangle}{2\sqrt{E_{\rm d'}}\sqrt{E_{\rm d'}}}$$

$$+\Psi^*_{\rm d}(\mathcal{P})\Psi^*_{\rm d'}(\mathcal{P}')[J_{{\rm d};\mu}J^\mu_{\rm d'}\,V^{\rm V}_{\rm conf.}(\mathbf{k})+V^{\rm S}_{\rm conf.}(\mathbf{k})]\Psi_{\rm d}(\mathcal{Q})\Psi_{\rm d'}(\mathcal{Q}')$$

Results I

◇ with one open and another hidden heavy flavors (without/with strangeness):

Table 1: Masses $M_{QQ'\bar{Q}\bar{q}}$ of the ground states and orbital excitations of triply heavy tetraquarks with a pair of open and hidden heavy flavors and without/with strangeness ($cb\bar{c}\bar{u}$, $cb\bar{c}\bar{s}$, $bc\bar{b}\bar{u}$, $bc\bar{b}\bar{s}$ + c.c.).

$d\bar{d}'$	nL	n_r	L	S	J	J^P	$M_{cb\bar{c}\bar{u}}$	$M_{cb\bar{c}\bar{s}}$	$M_{bc\bar{b}\bar{u}}$	$M_{bc\bar{b}\bar{s}}$
$A\bar{A}$	1S	0	0	0	0	0^+	8383	8503	11668	11770
				1	1	1^+	8396	8515	11675	11777
				2	2	2^+	8420	8538	11689	11791
	1P		1	1	0	0^-	8723	8838	11961	12061
				0	1	1^-	8724	8838	11963	12063
				1			8728	8843	11965	12065
				2			8734	8847	11966	12065
				1	2	2^-	8731	8845	11969	12068
				2			8739	8853	11970	12069
				2	3	3^-	8742	8856	11974	12074
	1D		2	2	0	0^+	9009	9118	12179	12278
				1	1	1^+	9006	9115	12181	12279
				2			9013	9122	12181	12280
				0	2	2^+	9001	9112	12183	12281
				1			9011	9120	12183	12282
				2			9021	9129	12184	12283
				1	3	3^+	9006	9116	12185	12283
				2			9017	9126	12186	12284
				2	4	4^+	9011	9122	12187	12286

Results II

Table 1: table continues.

$d\bar{d}'$	nL	n_r	L	S	J	J^P	$M_{cb\bar{c}\bar{u}}$	$M_{cb\bar{c}\bar{s}}$	$M_{bc\bar{b}\bar{u}}$	$M_{bc\bar{b}\bar{s}}$
$A\bar{S}$	1S	0	0	1	1	1^+	8344	8460	11660	11764
	1P		1		0	0^-	8666	8777	11943	12045
					1	1^-	8671	8781	11945	12047
					2	2^-	8679	8789	11949	12051
	1D		2		1	1^+	8948	9054	12160	12261
					2	2^+	8952	9058	12162	12262
					3	3^+	8959	9064	12164	12265
$S\bar{A}$	1S		0		1	1^+	8401	8520	11675	11777
	1P		1		0	0^-	8726	8840	11958	12058
					1	1^-	8727	8841	11960	12060
					2	2^-	8728	8842	11964	12063
	1D		2		1	1^+	9010	9118	12176	12274
					2	2^+	9006	9115	12177	12275
					3	3^+	9000	9110	12179	12277
$S\bar{S}$	1S		0	0	0	0^+	8337	8453	11653	11757
	1P		1		1	1^-	8668	8778	11940	12042
	1D		2		2	2^+	8948	9053	12156	12256

Analysis I

- If energetically possible, the tetraquark will fall-apart into a meson pair through the quark rearrangement.

$$\Delta = M_{QQ'\overline{Q}''\overline{q}} - M^{\text{lowest}}_{\text{threshold}}$$

- If $\Delta < 0$, state is stable against fall-apart strong decays.
- The smaller $\Delta > 0$, the narrower is the state.

Analysis II

- Most states lie well above thresholds with $\Delta > 100$ MeV.
- Some states lie above thresholds with $50 < \Delta < 100$ MeV.
- Several states lie slightly above thresholds with $0 < \Delta < 50$ MeV.
- A number of states lie below thresholds with $\Delta < 0$.

Analysis III

- The most promising to be stable states:
  - with one open and another hidden heavy flavors (without/with strangeness):

Table 2: Ground and orbitally excited states of the triply heavy tetraquarks with one open and another hidden heavy flavors and without/with strangeness ($cb\bar{c}\bar{u}$, $cb\bar{c}\bar{s}$, $bc\bar{b}\bar{u}$, $bc\bar{b}\bar{s}$ + c.c.), which lie slightly above or below the meson–meson fall-apart strong decay thresholds.

$QQ'\bar{Q}''\bar{q}$	$d\bar{d}'$	nL	S	J^P	M	M_{thr}	Δ	meson pair
$cb\bar{c}\bar{u}$	$A\bar{A}$	1S	2	2^+	8420	8340	80	$D^*(2007)^0\ B_c(1\,^3S_1)$
		1P	2	3^-	**8742**	8768	**-26**	$D^*(2007)^0\ B_c(1\,^3P_2)$
		1D	2	4^+	**9011**	9036	**-25**	$D^*(2007)^0\ B_c(1\,^3D_3)$
	$A\bar{S}$	1P	1	1^-	8671	8608	63	$D^0\ B_c(1\,P_1)$
				2^-	8679	8626	53	$D^0\ B_c(1\,^3P_2)$
		1D		3^+	8959	8894	65	$D^0\ B_c(1\,^3D_3)$
	$S\bar{S}$	1P	0	1^-	8668	8608	60	$D^0\ B_c(1\,P_1)$
$cb\bar{c}\bar{s}$	$A\bar{S}$	1S	2	2^+	8538	8445	93	$D_s^*\ B_c(1\,^3S_1)$
		1P	2	3^-	**8856**	8873	**-17**	$D_s^*\ B_c(1\,^3P_2)$
		1D	2	4^+	**9122**	9141	**-19**	$D_s^*\ B_c(1\,^3D_3)$
	$A\bar{S}$	1P	1	2^-	8789	8729	60	$D_s^+\ B_c(1\,^3P_2)$
		1D		3^+	9064	8997	67	$D_s^+\ B_c(1\,^3D_3)$

Analysis IV

Table 2: table continues.

$QQ'\bar{Q}''\bar{q}$	$d\bar{d}'$	nL	S	J^P	M	M_{thr}	Δ	meson pair
$bc\bar{b}\bar{u}$	$A\bar{A}$	1P	2	3^-	11974	11919	55	$D^*(2007)^0\ \chi_{b2}(1P)$
	$A\bar{A}$	1D	1	3^+	12185	12162	23	$D_3^*(2750)\ \eta_b(1S)$
			2		12186		24	
			2	4^+	**12187**	12224	**-37**	$D_3^*(2750)\ \Upsilon(1S)$
	$A\bar{S}$		1	3^+	12164	12162	2	$D_3^*(2750)\ \eta_b(1S)$
	$S\bar{A}$			3^+	12179	12162	17	$D_3^*(2750)\ \eta_b(1S)$
$bc\bar{b}\bar{s}$	$A\bar{A}$	1P	2	3^-	12074	12024	50	$D_s^*\ \chi_{b2}(1P)$
	$A\bar{A}$	1D	1	3^+	12283	12259	24	$D_{s3}^*(2860)^+\ \eta_b(1S)$
			2		12284		25	
			2	4^+	**12286**	12321	**-35**	$D_{s3}^*(2860)^+\ \Upsilon(1S)$
	$A\bar{S}$		1	3^+	12265	12259	6	$D_{s3}^*(2860)^+\ \eta_b(1S)$
	$S\bar{A}$			3^+	12277	12259	18	$D_{s3}^*(2860)^+\ \eta_b(1S)$

Experimental data I

◇ In the fully heavy tetraquark sector there are already experimental advancements:

- While studying the double charmonium production, in 2020 the LHCb Collaboration announced the discovery of the narrow resonance X(6900).
- Several other broad structures peaking at about 6.4 and 7.2 GeV were reported.
- In 2022 ATLAS and CMS Collaborations confirmed X(6900) and hinted on a few more states, including structures at 6.4 and 7.2 GeV.

Experimental data II

- Current observation status and our predictions:

Table 3: Exotic X states observed and hinted by the LHCb, ATLAS and CMS Collaborations in di-J/ψ and $J/\psi\,\psi(2S)$ invariant mass spectra and our candidates. All masses M and total widths Γ are given in MeV.

Collaboration		Resonance	M	Γ	Our candidates nL	S	J^{PC}	M
LHCb		X(6600)	$6400 \div 6600$		1S	2	2^{++}	6367
ATLAS	m_0, model A		$6410 \pm 80^{+80}_{-30}$	$590 \pm 350^{+120}_{-200}$				
ATLAS	m_0, model B		$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$				
ATLAS	m_1, model A		$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$				
CMS	BW_1, no interference		$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	2S	0	0^{++}	6782
CMS	BW_1, interference		6638^{+43+16}_{-38-31}	$440^{+230+110}_{-200-240}$				
LHCb	NRSPS, no interference	X(6900)	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$				
LHCb	NRSPS, interference		$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	2S	2	2^{++}	6868
ATLAS	m_2, model A		$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$		0	2^{++}	6921
ATLAS	m_2, model B		$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$	1D	2	0^{++}	6899
ATLAS	m_3, model β		$6960 \pm 50 \pm 30$	$510 \pm 170^{+110}_{-100}$		2	1^{++}	6904
CMS	BW_2, no interference		$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 18$		2	2^{++}	6915
CMS	BW_2, interference		6847^{+44+48}_{-28-20}	191^{+66+25}_{-49-17}				
LHCb		X(7200)	$7200 \div 7400$					
ATLAS	m_3, model α		$7220 \pm 30^{+10}_{-30}$	$90 \pm 60^{+60}_{-30}$	3S	0	0^{++}	7259
CMS	BW_3, no interference		$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$		2	2^{++}	7333
CMS	BW_3, interference		7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}				

Experimental data III

- Plenty of new experimental data are expected in the near future, including regions and mass sectors of our interest.

Conclusion I

- Masses of ground and orbitally excited states of all compositions of the triply heavy tetraquarks were calculated.
- The finite size of a diquark was taken into account.
- Diquarks and antidiquarks were considered to interact as a whole.

Conclusion II

- Triply heavy tetraquark states which are the most convenient for the experimental detection were identified.
- There are already experimental advancements in the fully heavy tetraquark sector, and our previous predictions based on the Relativistic Quark Model are consistent with them.

Publications

◇ Previous publications related to the topic:

- Masses of the $QQ\overline{QQ}$ tetraquarks in the relativistic diquark–antidiquark picture, Physical Review D, 2020, vol. 102, №11, p. 114030;
- Heavy Tetraquarks in the Relativistic Quark Model, Universe, 2021, vol. 7, №4, p. 94;
- Fully Heavy Tetraquark Spectroscopy in the Relativistic Quark Model, Symmetry, 2022, vol. 14, №12, p. 2504;
- Relativistic description of asymmetric fully heavy tetraquarks in the diquark–antidiquark model, The European Physical Journal A, 2024, vol. 60, №96;
- Relativistic Description of Asymmetric Fully Heavy Tetraquarks, Physics of Particles and Nuclei Letters, 2024, vol. 21, №4, p. 597–600.

Thank you for your attention!

This work was supported by the Foundation for the Advancement of Theoretical Physics and Mathematics "BASIS" grant №22-2-10-3-1.