Two and three color QCD phase diagram with various imbalances and speed of sound

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Group and Acknowlegements

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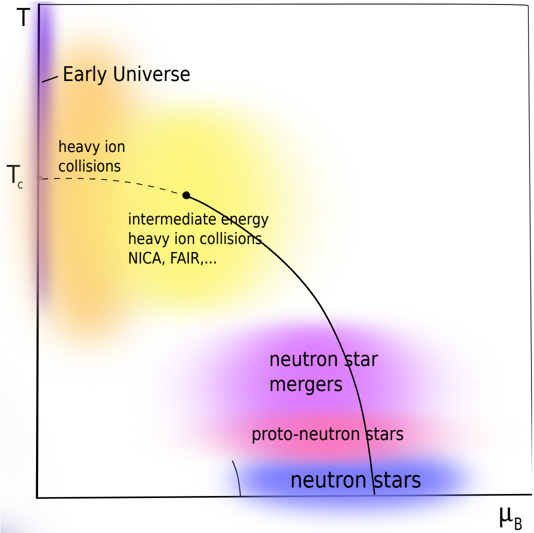
QCD Dhase Diagram

QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



lattice QCD at non-zero baryon chemical potential μ_B

$$Z = \int D[\text{gluons}]\, D[\text{quarks}]\, e^{-S^E_{QCD}}$$

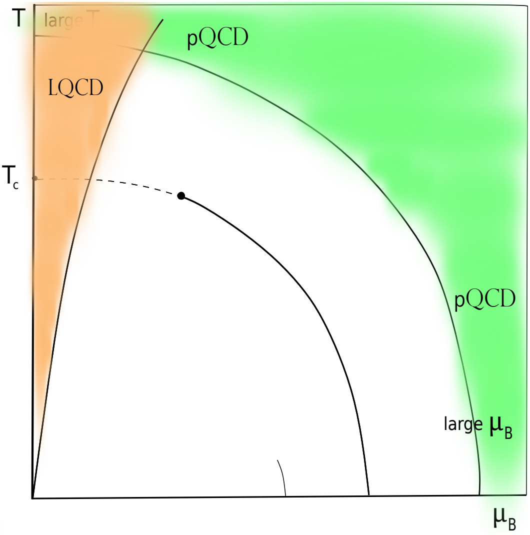
$$Z = \int D[\text{gluons}]\, \mathrm{Det}\, \mathbb{D}(\mu)\, e^{-S^E_{\text{gluons}}}$$

It is well known that **at non-zero baryon chemical potential μ_B lattice simulation** is quite challenging due to the **sign problem**
complex determinant

$$(Det(D(\mu)))^\dagger = Det(D(-\mu^\dagger))$$

Methods of dealing with QCD

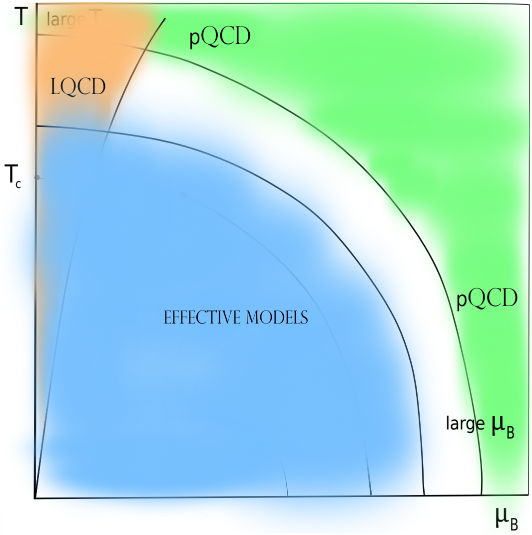
- Perturbative QCD
- First principle calculation – lattice QCD



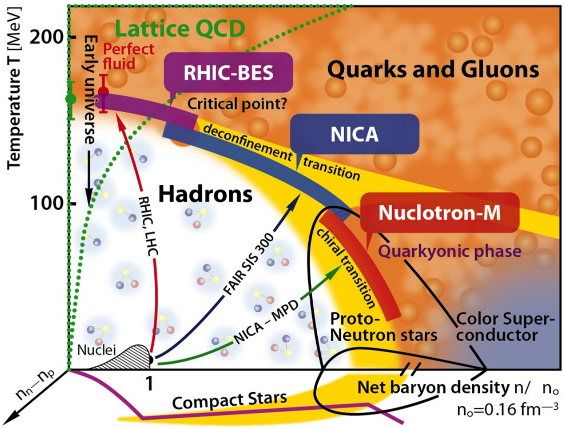
QCD Dhase Diagram and Methods

Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- Gauge/Gravity duality
- .....



QCD Phase Diagram



- **Isotopic chemical potential μ_I**

  Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

- Neutron stars, intermediate energy heavy-ion collisions, neutron star mergers

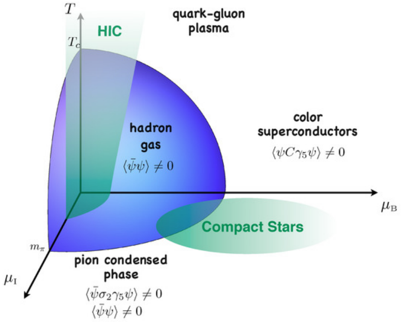


Figure: *taken from Massimo Mannarelli*

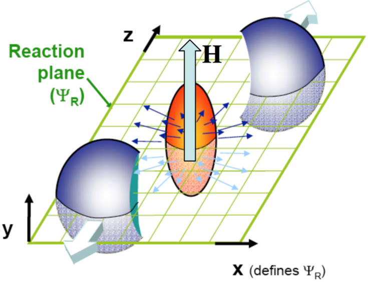
$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right) \qquad n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

Chiral imbalance

- **Chiral (axial) chemical potential**

Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

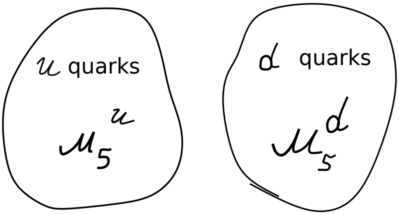
$$n_5 = n_R - n_L$$
$$\mu_5 = \mu_R - \mu_L$$



$$\vec{J} \sim \mu_5 \vec{B},$$

The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

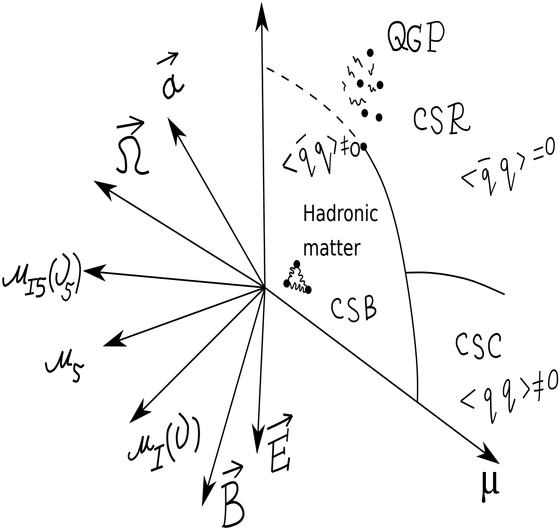
Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \quad \longleftrightarrow \quad \nu_5$$

More external conditions to QCD

More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields
- rotation of the system $\vec{\Omega}$ *(see talk of A. Roenko and D. Sychev)*
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



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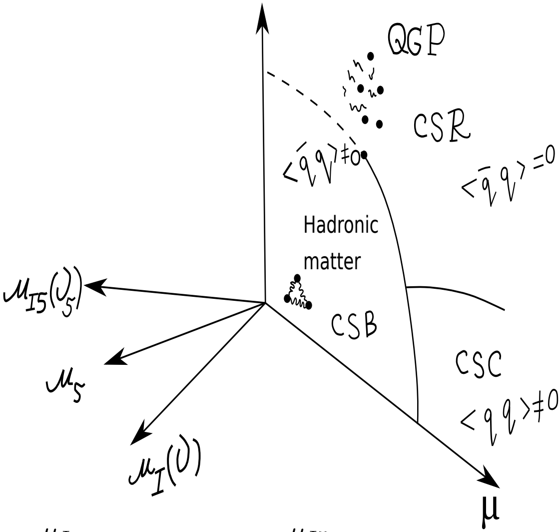

$$\mu = \frac{\mu_B}{3}, \quad \nu = \frac{\mu_I}{2}, \quad \mu_5, \quad \nu_5 = \frac{\mu_{I5}}{2}$$

Recall that in NJL model **in** $1/N_c$ **approximation or in the mean field** there have been found **dualities**

(*It is not related to holography or gauge/gravity duality*)

Chiral symmetry breaking $\Longleftrightarrow$ pion condensation

Isospin imbalance $\Longleftrightarrow$ Chiral imbalance

$$\sigma(x) = -2H(\bar{q}q), \qquad \Delta(x) = -2H\left[\overline{q^c} i\gamma^5 \tau_2 q\right]$$

$$\vec{\pi}(x) = -2H(\bar{q}i\gamma^5\vec{\tau}q), \qquad \Delta^*(x) = -2H\left[\bar{q}i\gamma^5\tau_2 q^c\right]$$

Condensates and phases

$M = \langle\sigma(x)\rangle \sim \langle\bar{q}q\rangle,$ CSB phase: $M \neq 0,$

$\pi_1 = \langle\pi_1(x)\rangle = \langle\bar{q}\gamma^5\tau_1 q\rangle,$ PC phase: $\pi_1 \neq 0,$

$\Delta = \langle\Delta(x)\rangle = \langle qq\rangle = \langle q^T C\gamma^5\tau_2 q\rangle,$ CSC phase: $\Delta \neq 0.$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP

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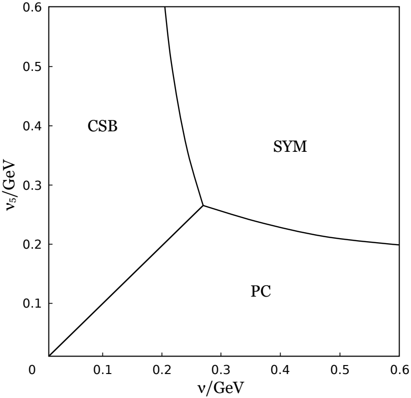


$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

- A lot of densities and imbalances
  baryon, isospin, chiral, chiral isospin imbalances

- Finite temperature $T \neq 0$

- Physical pion mass $m_\pi \approx 140$ MeV

- Inhomogeneous phases (case)

$$\langle \sigma(x) \rangle = M(x), \quad \langle \pi_\pm(x) \rangle = \pi(x), \quad \langle \pi_3(x) \rangle = 0.$$

- Inclusion of color superconductivity phenomenon

Dualities in QC_2D

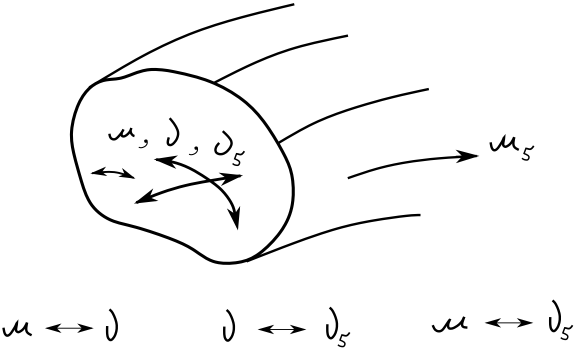
Similarity of SU(2) and SU(3)

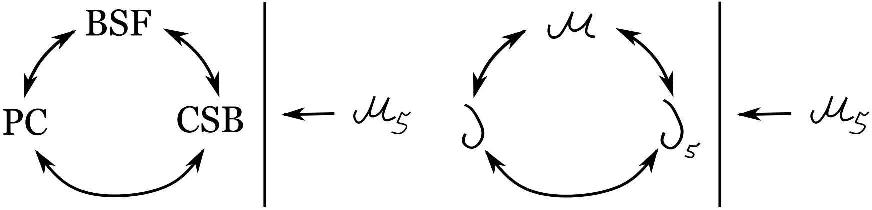
- similar phase transitions:
  confinement/deconfinement, chiral symmetry breaking/restoration
- A lot of physical quantities coincide with some accuracy
  Critical temperature, shear viscosity etc.
- There is **no sign problem** in SU(2) case and lattice simulations at non-zero baryon density are possible — $(Det(D(\mu)))^\dagger = Det(D(\mu))$

It is a great playground for studying dense matter

The phase diagram of (μ, ν, μ_5, ν_5)

The phase diagram is foliation of dually connected cross-section of (μ, ν, ν_5) along the μ_5 direction





Chiral imbalance μ_5 does not participate in dual transformations

Lagrangian of two colour QCD can be written in the form

$$\mathcal{L} = i\bar{\Psi}\gamma^{\mu}D_{\mu}\Psi$$

where $D_{\mu} = \partial_{\mu} + igA_{\mu} = \partial_{\mu} + ie\sigma_a A_{\mu}^a$

$$\Psi^T = \left(\ \psi_L^u,\ \ \psi_L^d,\ \ \sigma_2(\psi_R^C)^u,\ \ \sigma_2(\psi_R^C)^d\ \right)$$

Flavour symmetry is $SU(4)$

Pauli-Gursoy symmetry

Lagrangian of two colour NJL model

$$\frac{\mu_B}{3}\overline{\psi}\gamma^0\psi + \frac{\mu_I}{2}\overline{\psi}\gamma^0\tau_3\psi + \frac{\mu_{I5}}{2}\overline{\psi}\gamma^0\gamma^5\tau_3\psi + \mu_5\overline{\psi}\gamma^0\gamma^5\psi$$

$$\mathcal{M} = \mu\Psi^\dagger \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Psi + \frac{\mu_I}{2}\Psi^\dagger \begin{pmatrix} \tau_3 & 0 \\ 0 & -\tau_3 \end{pmatrix} \Psi +$$

$$\frac{\mu_{I5}}{2}\Psi^\dagger \begin{pmatrix} \tau_3 & 0 \\ 0 & \tau_3 \end{pmatrix} \Psi + \mu_5\Psi^\dagger \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Psi$$

Dualities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ were found in

- In the framework of effective NJL model

  Without any approximation

- From first principles QC_2D

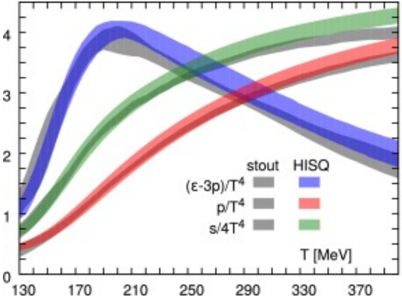
$$\mathcal{D}_{\mathrm{I}}: \quad \langle\bar{\psi}\psi\rangle \longleftrightarrow \langle i\bar{\psi}\gamma^5\tau_1\psi\rangle, \quad M \longleftrightarrow \pi, \quad \nu \leftrightarrow \nu_5$$

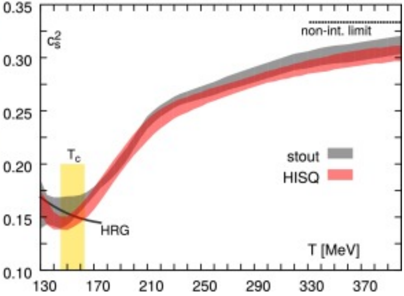
- In the framework of effective NJL model

  Without any approximation

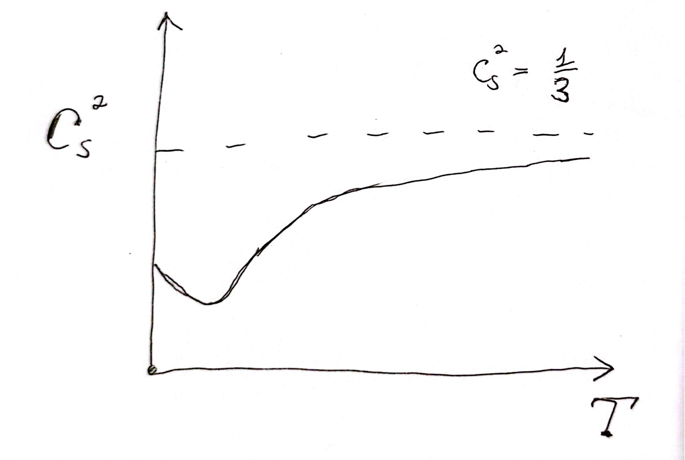
- From first principles QCD

Thermodynamic properties could be calculated in lattice QCD





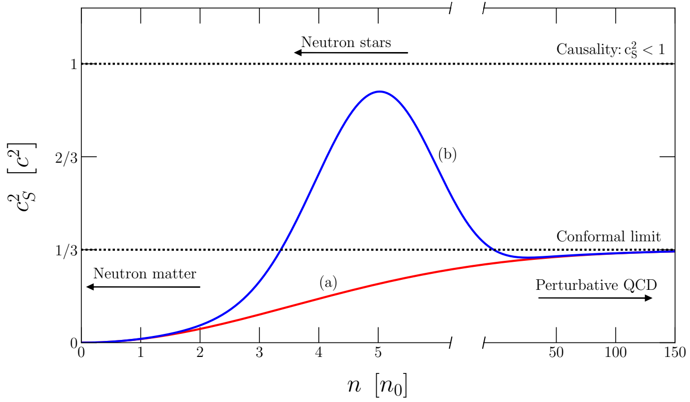
*A. Bazavov et al. [HotQCD], Phys. Rev. D **90** (2014), 094503*



There was discussed bound from holography

A. Cherman, T. D. Cohen and A. Nellore, Phys. Rev. D 80 (2009), 066003

Two possible scenario of speed of sound at non-zero baryon density



taken from S. Reddy et al, Astrophys. J. ***860*** *(2018) no.2, 149*

Sign problem

$$Z = \int D[\text{gluons}]\, D[\text{quarks}]\, e^{-S^E_{QCD}}$$

$$Z = \int D[\text{gluons}]\, Det\, D(\mu)\, e^{-S^E_{\text{gluons}}}$$

It is well known that **at non-zero baryon chemical potential μ_B lattice simulation** is quite challenging due to the **sign problem**

complex determinant

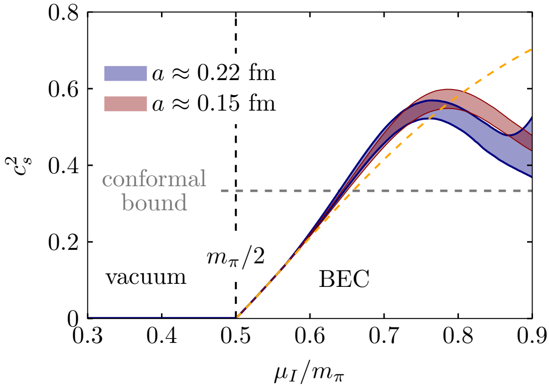
$$Det(D(\mu))^\dagger = Det(D(-\mu))$$

For isospin chemical potential μ_I

$$Det(D(\mu_I))^\dagger = Det(D(\mu_I))$$

- **Sound speed** squared has been obtained from **lattice QCD simulations** for **QCD with non-zero isospin** μ_I

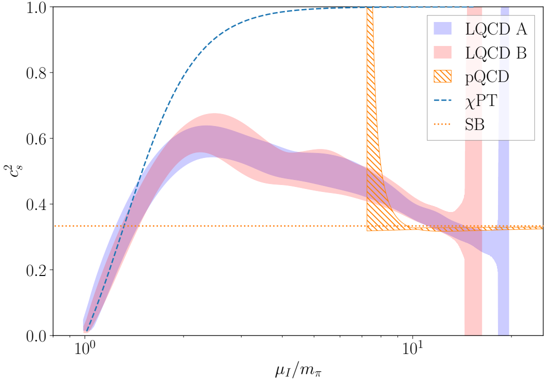
  B. B. Brandt, F. Cuteri and G. Endrodi, JHEP 07, 055 (2023)



Sound speed in QCD with non-zero isospin density

- **Sound speed** squared has been obtained from **lattice QCD simulations** for **QCD with non-zero isospin** μ_I for values of μ_I up to $10m_\pi$

  R. Abbott et al. [NPLQCD], Phys. Rev. D 108, no.11, 114506 (2023)



Duality between chiral symmetry breaking and pion condensation

$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

The TDP of the quark matter

$$\Omega(T,\, \mu,\, \nu,\, \nu_5,\, \mu_5,\, |\, M,\, \pi) = \text{inv}$$

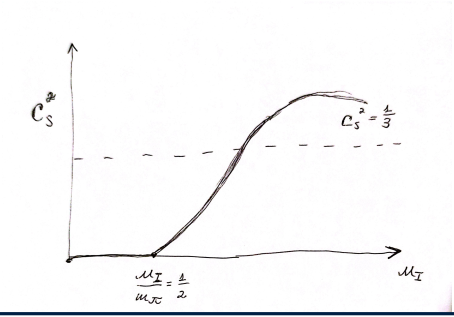
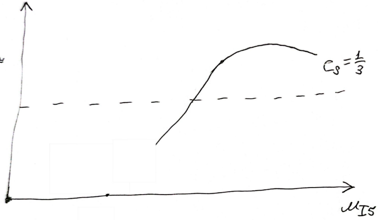
The speed of sound $\qquad c_s^2 = \dfrac{dp}{d\epsilon}$

$$\Omega(T, ...) \Longrightarrow c_s^2(T, ...)$$

Duality: speed of sound

The speed of sound $c_s^2 = \dfrac{dp}{d\epsilon}$, $\Omega(T, ...) \Longrightarrow c_s^2(T, ...)$

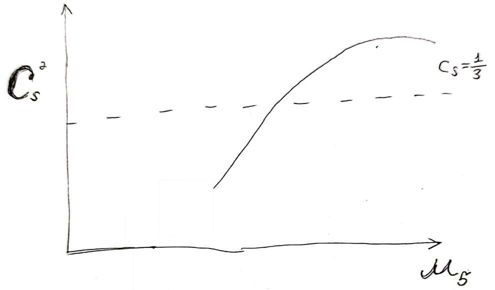
$$\Omega(T, ..., \nu) = \Omega(T, ..., \nu_5) \Longrightarrow c_s^2(T, ..., \nu) = c_s^2(T, ..., \nu_5)$$

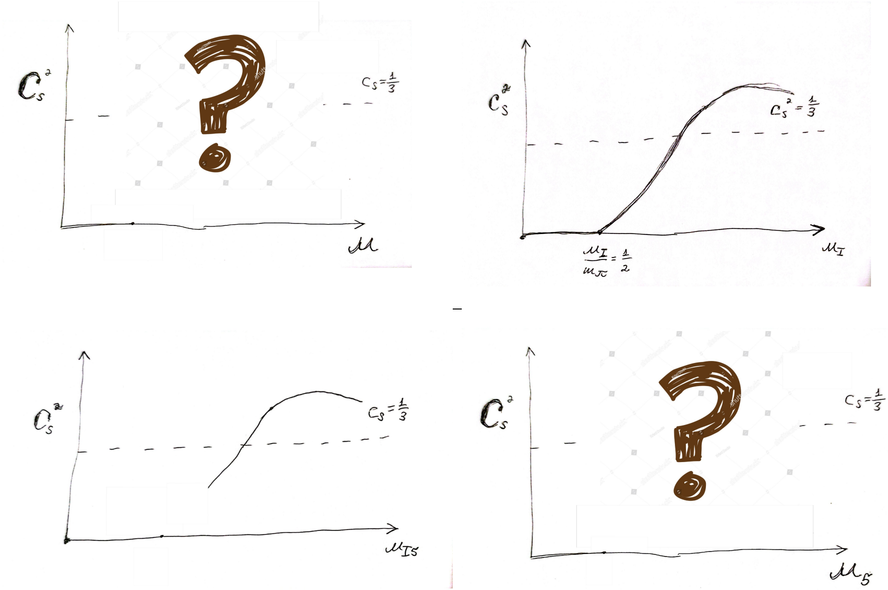



Duality

$$\nu_5 \longleftrightarrow \mu_5, \qquad M \neq 0, \quad \langle \pi \rangle = \langle \Delta \rangle = 0$$

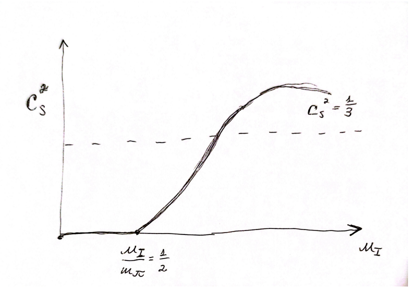
- **Sound speed** squared for **QCD with non-zero chiral imbalance** μ_5 **only in the framewwork of effective model**

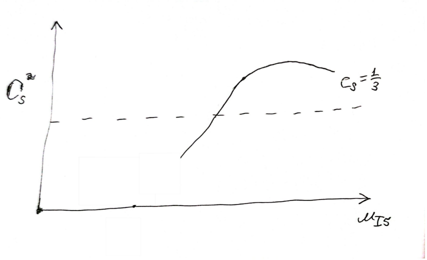


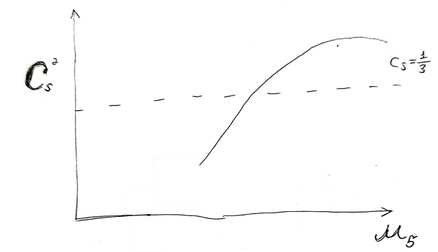


C_s^2
$C_s = \frac{1}{3}$
μ
C_s^2
$C_s^2 = \frac{1}{3}$
$\frac{\mu_I}{m_\pi} = \frac{1}{2}$
μ_I
C_s^2
$C_s = \frac{1}{3}$
μ_{I5}
C_s^2
$C_s = \frac{1}{3}$
μ_5


C_s^2
$c_s = \frac{1}{3}$
μ


C_s^2
$c_s^2 = \frac{1}{3}$
$\frac{\mu_I}{m_\pi} = \frac{1}{2}$
μ_I


C_s^2
$c_s = \frac{1}{3}$
μ_{I5}


C_s^2
$c_s = \frac{1}{3}$
μ_5

Two colour QCD case

QC_2D

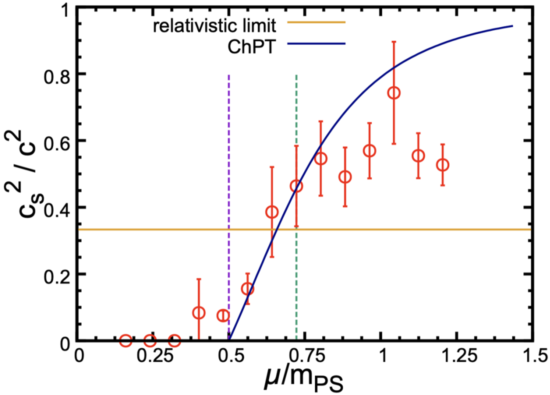
No sign problem in SU(2) case at $\mu_B \neq 0$

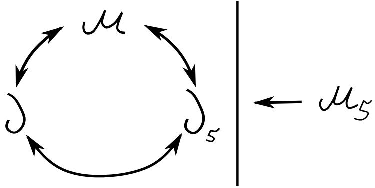
$$(Det(D(\mu)))^\dagger = Det(D(\mu))$$

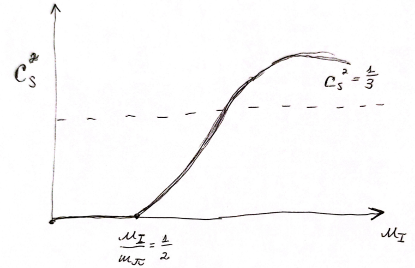
Sound speed in two color QCD

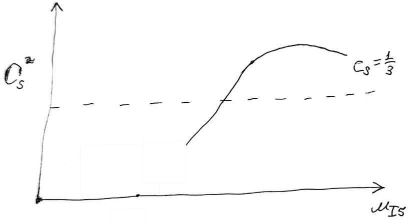
- **Sound speed** squared has been obtained from **lattice QCD simulations** for **two color QCD**

E. Itou and K. Iida, PoS LATTICE2023, 111 (2024); PTEP 2022 (2022) no.11, 111B01




μ
j
j5
μ5

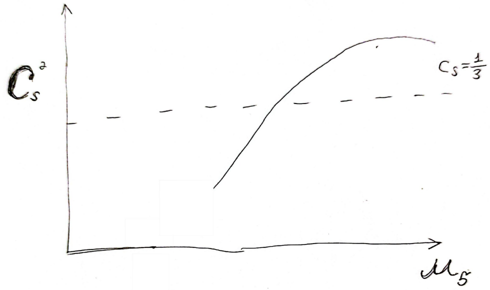

Cs²
Cs² = 1/3
μI/mπ = 1/2
μI


Cs²
Cs = 1/3
μI5

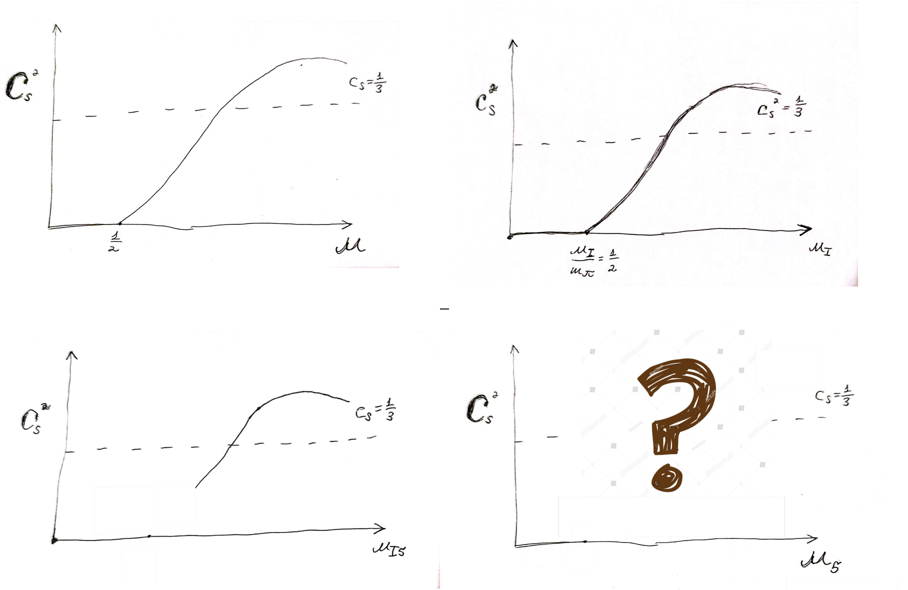
Duality $\nu_5 \longleftrightarrow \mu_5$

was shown in two color effective model as well

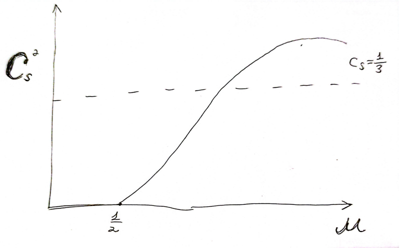
- **Sound speed** squared for **QCD with non-zero chiral imbalance** μ_5 **only in the framewwork of effective model**

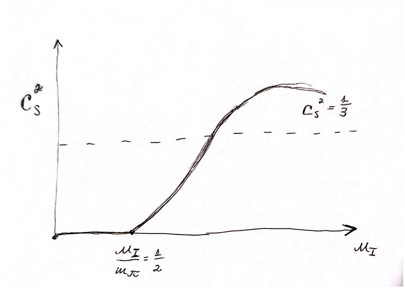


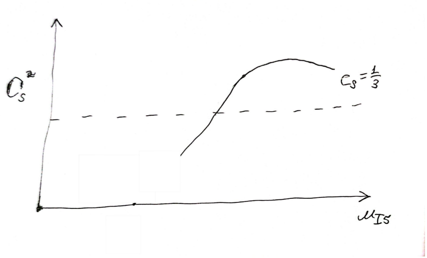
Speed of sound in QC_2D: First principle

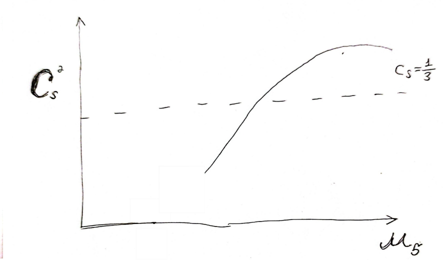


Speed of sound in QC_2D: Effective models









Dualities has been proven from first principles

Speed of sound exceeding the conformal limit is rather **natural** and taking place in a lot of systems, **with various chemical potentials**

And it is natural if it has similar structure in QCD at non-zero baryon density, the most interesting case