National Research Nuclear University MEPhI

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Study of nuclei structure in alpha-cluster model by hyperspherical functions using cubic spline interpolation

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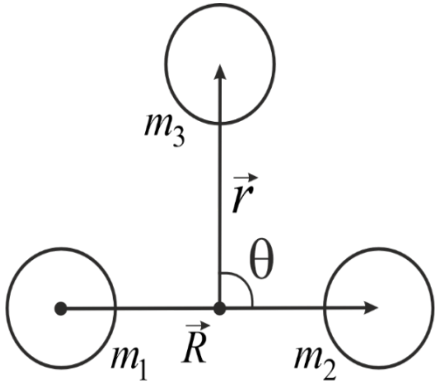
Hyperspherical functions method for three body problem



Jacobi's vectors

$$\vec{R} = \vec{r}_3 - \vec{r}_1, \qquad \vec{x} = \sqrt{\frac{M}{m_0 x_0^2}}\vec{R},$$

$$\vec{r} = \vec{r}_2 - \frac{\vec{r}_1 + \vec{r}_3}{2}, \quad \vec{y} = \sqrt{\frac{\mu}{m_0 x_0^2}}\vec{r}. \qquad (1)$$

$$M = \frac{m_1 m_2}{m_1 + m_2} \qquad \mu = \frac{m_3\left(m_1 + m_2\right)}{m_1 + m_2 + m_3} \qquad (2)$$

x_0 = 1 fm, m_0 = 1 a.u.m.

Hyperspherical coordinates

$x = \rho\cos\alpha, \quad y = \rho\sin\alpha$

Replacing wave function for the ground state ψ_0 by a series of hyperspherical functions,

$l_x = 0,2,4,\ldots;\ n = 0,1,2,\ldots;\ K = 2l_x + 2n;$

$$\psi_0\left(\alpha,\theta,\rho\right) = \sum_{l_x K}\chi_K^{l_x}\left(\rho\right)\rho^{-5/2}\Phi_{K00}^{l_x l_x}\left(\alpha,\theta\right) \quad (3)$$

Functions $\chi_K^{l_x}(\rho)$ are found from system of hyperradial equations $l_{max} = 12,\ n_{max} = 12$

$$\frac{d^2}{d\rho^2}\chi_K^{l_x}\left(\rho\right) + \left[2Eb_0 - \frac{1}{\rho^2}\left(K + 3/2\right)\left(K + 5/2\right)\right]\chi_K^{l_x}\left(\rho\right) = \qquad (4)$$

$$= 2b_0\sum_{K' l_x'}\tilde{U}_{KK'}^{l_x;l_x'}\left(\rho\right).$$

Coefficient m_i of the cubic spline equals to the second derivative of hyperradial wave function

$$\frac{d^2}{d\rho^2}\chi_K^{l_x}(\rho_i) = m_i \quad (5)$$

$$F_i = \chi_K^{l_x}\left(\rho_i\right) \quad (6)$$

The system of equations for the cubic spline

$$\mathbf{Am} = \mathbf{HF} \quad (7)$$

Matrices **A** and **H** can be found in

[Marchuk G.I. Methods of Numerical Mathematics. –Springer NY, 1982]

The problem is reduced to the problem of eigenvalues and eigenvectors of matrix **B**

$$\mathbf{BF} = \lambda\mathbf{F},$$

$$\mathbf{B} = -\mathbf{A}^{-1}\mathbf{HF} + \mathbf{WF},\ \lambda = \frac{2\mu}{\hbar^2}E. \quad (8)$$

[Джибути Р.И., Шитикова К.В. Метод гиперсферических функций в атомной и ядерной физике. 1993 г.]
[V.V. Samarin. Study of spatial structures in α-cluster nuclei, Eur. Phys. J. A (2022) 58,117.]

Interaction potential of alpha-particles

$$V_{\alpha-\alpha}(r) = V^{(N)}_{\alpha-\alpha}(r) + V^{(C)}_{\alpha-\alpha}(r)$$

The potential of strong interaction $V_{\alpha\text{-}\alpha}$ is based on data of alpha-alpha scattering, known as Ali-Bodmer (AB) potential [1]

$$V^{(N)}_{\alpha-\alpha}(r) = v_1 \exp\left(-r^2/a_1^2\right) - v_2 \exp\left(-r^2/a_2^2\right) \quad (1)$$

Coulomb interaction $V^{(C)}_{\alpha-\alpha}(r)$ obtained from [1,2].

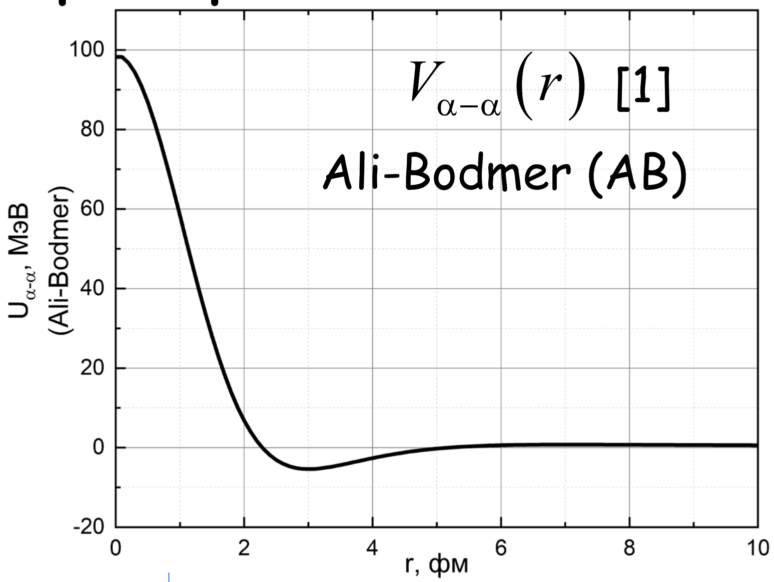
$$V^{(C)}_{\alpha-\alpha}(r, a_c, b_c) = a_c \cdot \mathrm{erf}(b_c r)/r \quad (2)$$

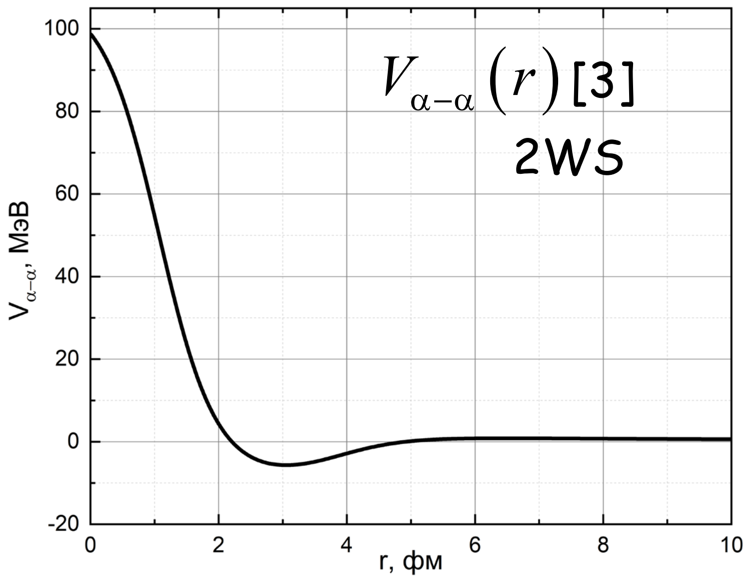
Potential with two Woods-Saxon's functions (2WS) has more parameters. It is important when describing experimental data [3]

$$V^{(N)}_{\alpha-\alpha}(r) = -U_{\alpha 1} f(r, B_{\alpha 1}, a_{\alpha 1}) + U_{\alpha 2} f(r, B_{\alpha 2}, a_{\alpha 2}) \quad (3)$$

Wood-Saxon's type function $f(r,B,a)$

$$f(r,B,a) = \left[1 + \exp\left(\frac{r-B}{a}\right)\right]^{-1} \quad (4)$$





Alpha-alpha scattering potentials AB (left) and 2WS (right) are almost the same.

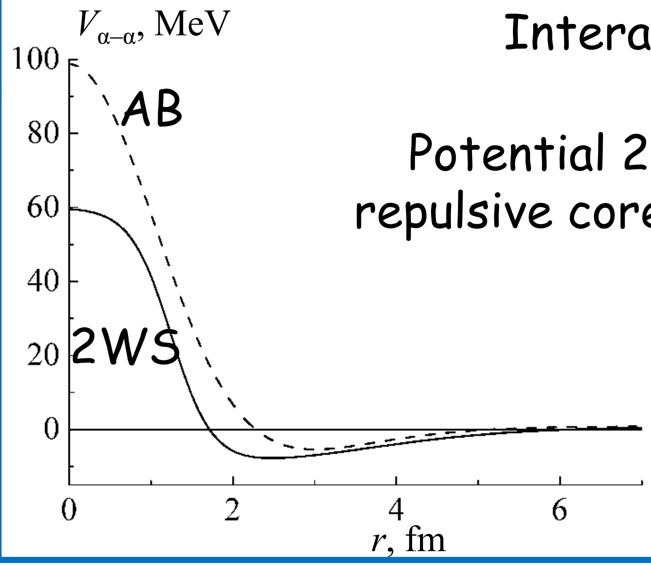
It is known that potential AB doesn't fit for describing bound energy of alpha-cluster nuclei, for example ^{12}C. Because of that, potential 2WS was used for describing interaction of alpha-clusters.

[1] S. Ali, A.R. Bodmer, Nucl. Phys. **80**, 99 (1966).

[2] H. Suno, Y. Suzuki, P. Descouvemont, Phys. Rev. C **91**, 014004 (2015).

[3] V.V. Samarin, Study of spatial structures in α-cluster nuclei, Eur. Phys. J. A, 58, 117 (2022).

Selection of parameters of α–α interaction potential for making an agreement with experimental properties of alpha-cluster nucleus ^{12}C (3α)



Interaction potentials $V_{\alpha-\alpha}$

Potential 2WS has more soft repulsive core unlike AB potential.

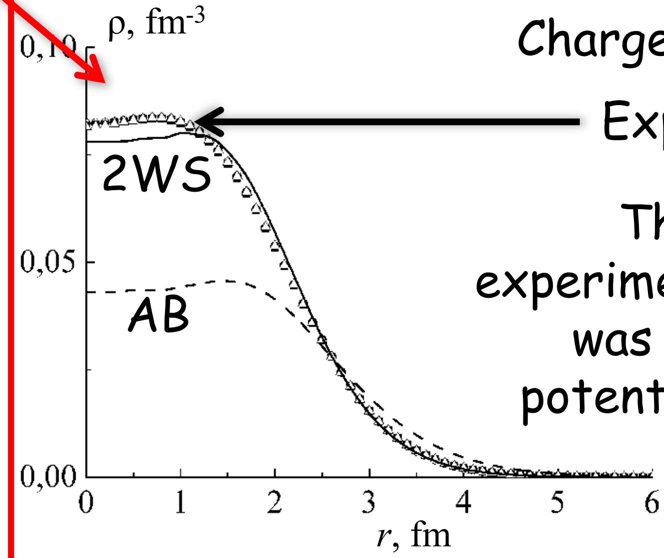
1. To get an agreement with experimental data parameters of potential 2WS of ^{12}C were modified. Obtained charge distribution and separation energy into 3α-particles of ^{12}C are close to the experimental one.

2. After α-particle separation in ^{12}C, unbound nucleus ^{8}Be is formed. Because of that, the ground state energy was obtained E_0=-7.272 MeV that is close to experimental value of α-particle separation of ^{12}C E_s=7.366 MeV [1] ($E_0 \approx -E_s$).

3. Calculated root-mean-square (rms) charge radii is also close to the experimental one

$$\langle r_C^2 \rangle_{\text{theor}}^{1/2} = 2.774 \text{ fm}, \quad \langle r_C^2 \rangle_{\text{exp}}^{1/2} = 2.47 \text{ fm}.$$

The difference occurs because of the errors in calculation the charge distribution at the region of large radii, where the charge density is low.



Charge distribution of ^{12}C.

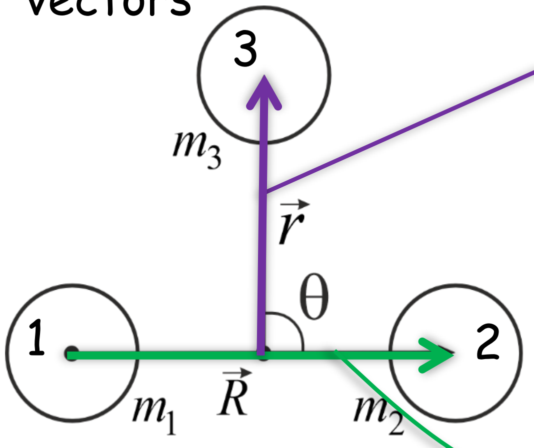
Experimental data

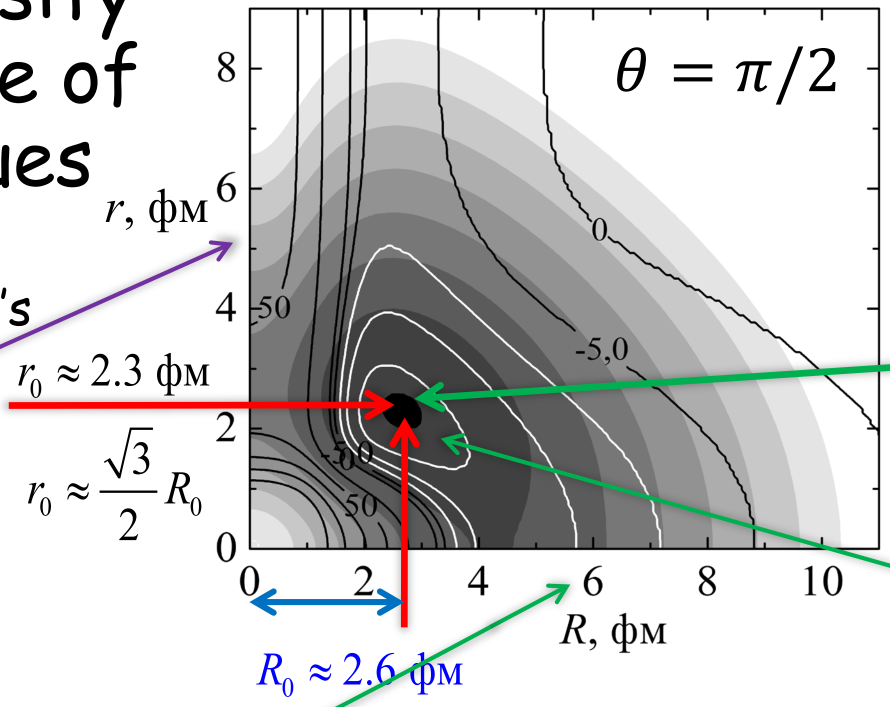
The agreement with experimental charge distribution was obtained using 2WS potential unlike AB potential.

[1] Nuclear Reaction Video. Low Energy Nuclear Knowledge Base. http://nrv.jinr.ru

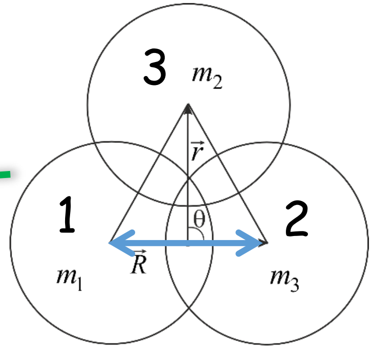
The probability density for the ground state of alpha-cluster nucleues ^{12}C (logarithmic scale)

In different positions of Jacobi's vectors



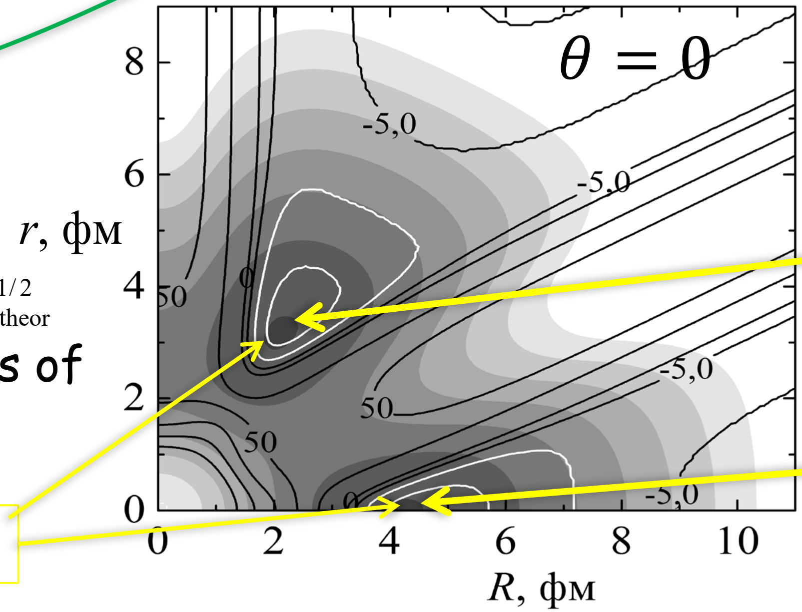


Positions of alpha-clusters, radii of alpha-clusters was chosen as rms radii of ^4He - 1.67 fm.



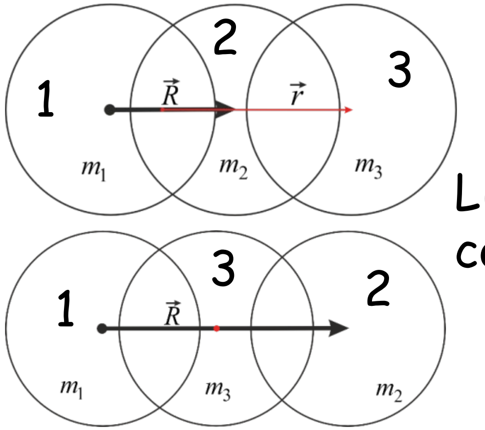
Triangular configuration

The most probable configuration is triangular, when alpha-clusters are located at the vertices of equilateral triangle with length of sides ~2.6 fm.



The softening of the repulsive core leads to overlapping of pure alpha-cluster structure.

Charge distribution and rms radii $\langle r_C^2 \rangle^{1/2}_{theor}$ calculated as average for all positions of vectors $\vec{R}$ and $\vec{r}$.



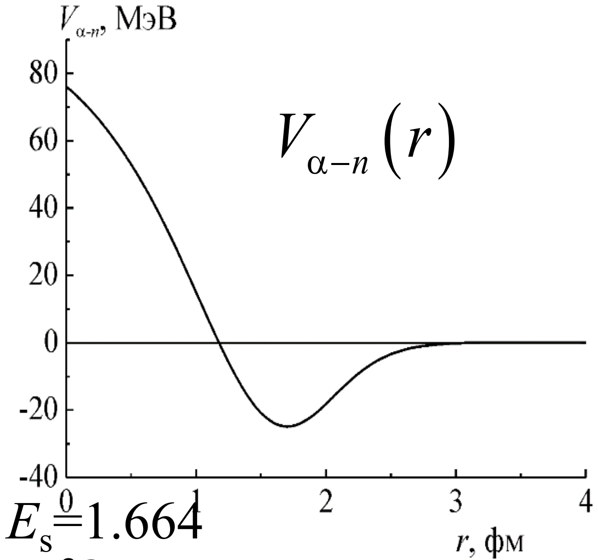
Linear configuration

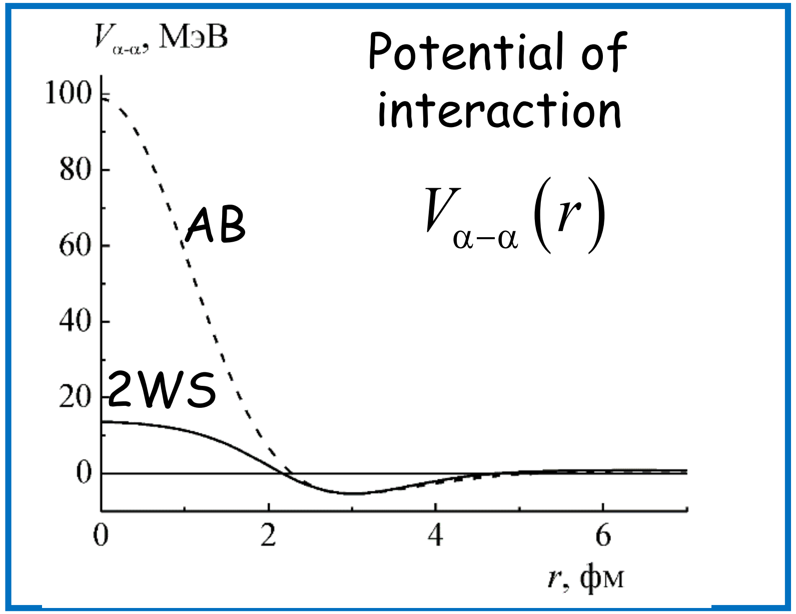
Less probable configurations

Selection of parameters of α–α interaction potential for making an agreement with experimental properties of alpha-cluster nucleus ^{9}Be (α-n-α). Potential $V_{\alpha-n}$

Interaction α-n пis represented by pseudopotential of strong interaction between alpha-cluster and nucleon.
It gives correct description for neutrons outside of alpha-cluster (doesn't take into account neutrons scattering on alpha-clusters)

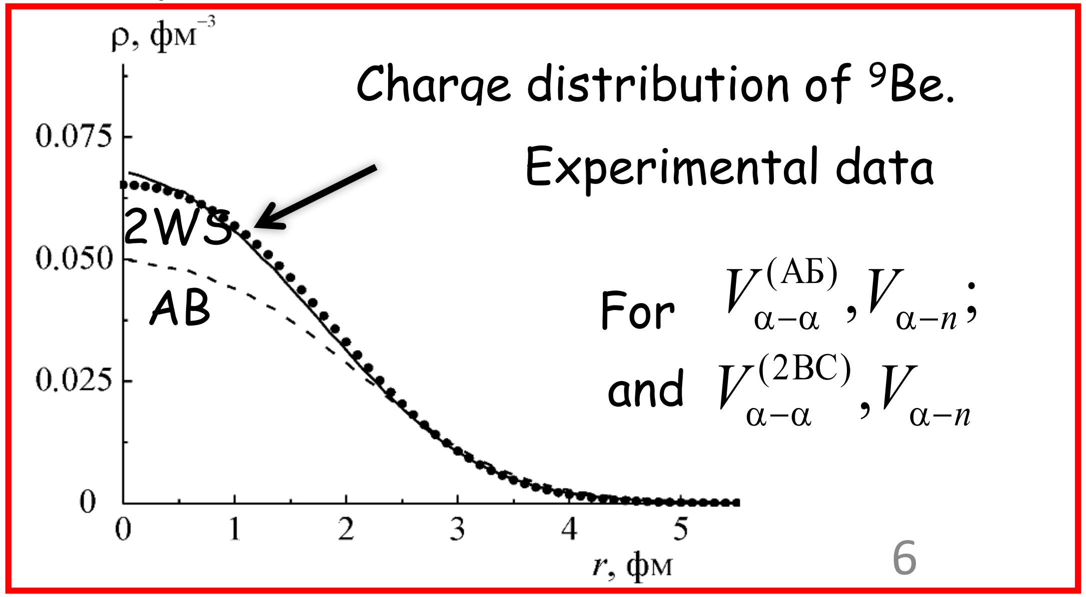
$$V_{\alpha-n}(r) = -U_1 f(r, B_1, a_1) + U_2 f(r, B_2, a_2) - U_3 f(r, B_3, a_3).$$





Experimental value of neutron separation for ^{9}Be E_s=1.664 MeV [1]. After neutron separation, unbound nucleus ^{8}Be is formed.
As a result of the selection, the value of the ground state energy was obtained E_0=-1.663 MeV. Thus, agreement with experimental data was obtained $E_0 \approx -E_s$.
<u>At the same time,</u> charge distribution was obtained, which is close to the experimental data.
Calculated rms charge radii also is close to the experimental value [1].

$$\langle r_C^2 \rangle_{theor}^{1/2} = 2.43 \text{ fm}, \langle r_C^2 \rangle_{exp}^{1/2} = 2.52 \text{ fm}.$$

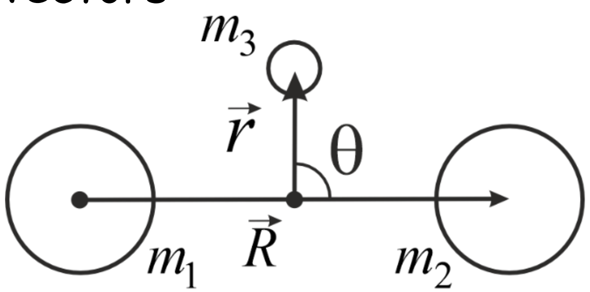


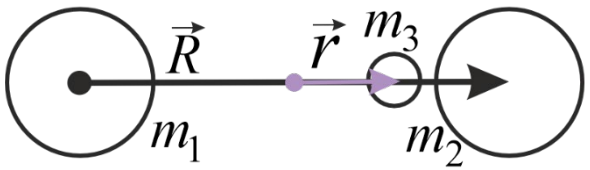
[1] NRV. http://nrv.jinr.ru

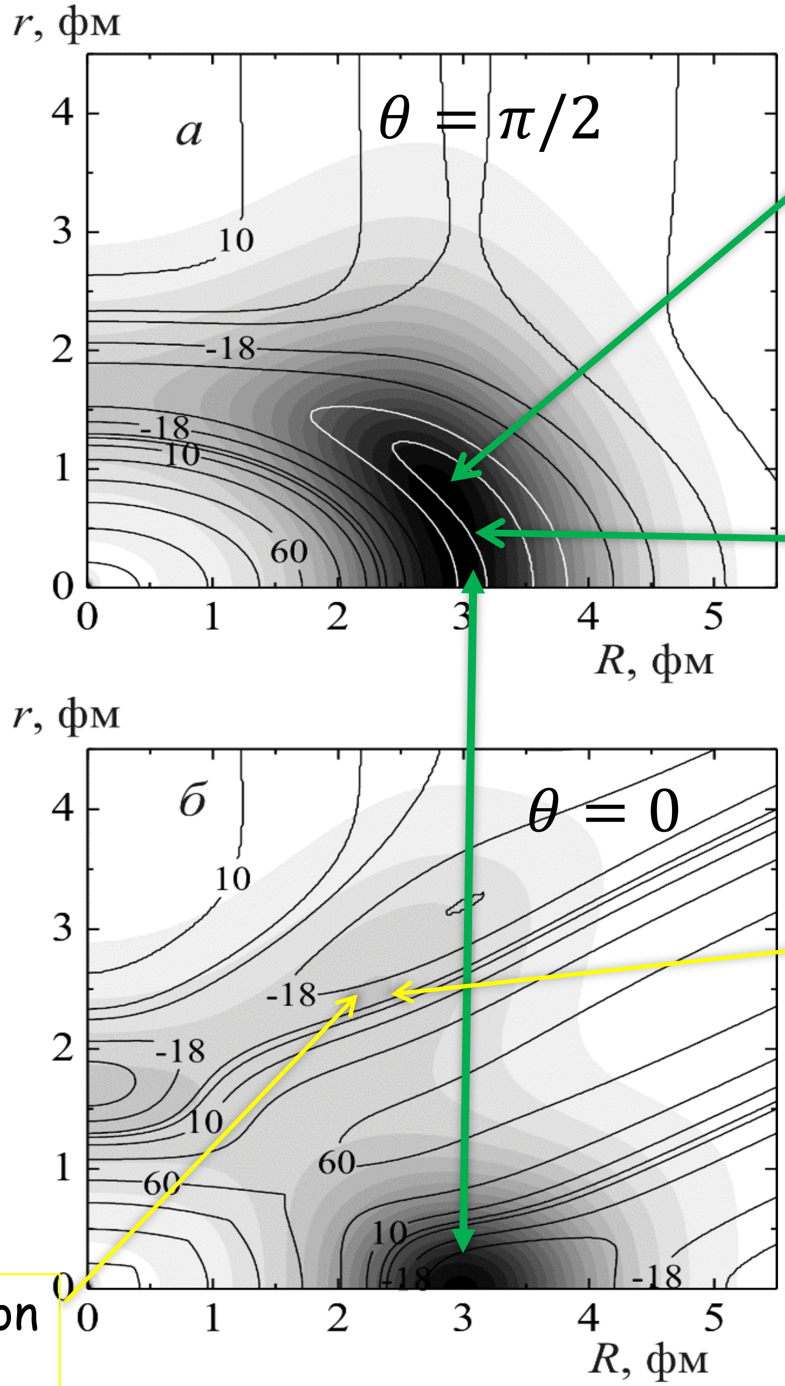
The probability density for the ground state of alpha-cluster nucleues ^{9}Be

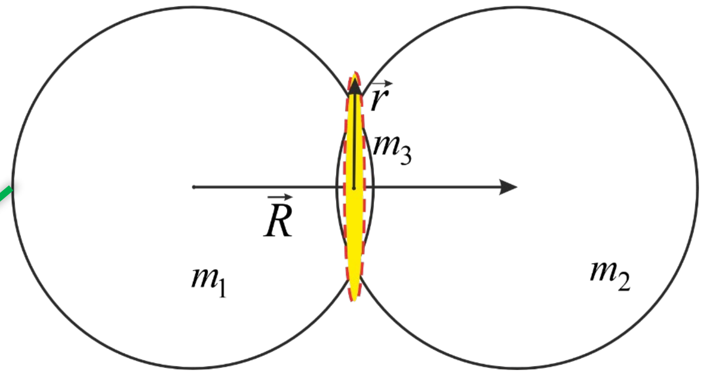
(logarithmic scale)

In different positions of Jacobi vectors

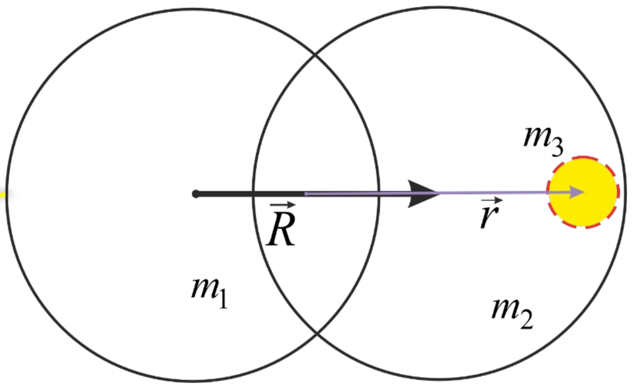








The most probable is "nuclear molecule", when neutron is located beteween alpha-clusters.



Less probable configuration ^{4}He+^{5}He.

Selection of parameters of α–α interaction potential for making an agreement with experimental properties of alpha-cluster nucleus ^{6}Li (α-n-p)

The nuclear part of the nucleon–nucleon interaction may be described by the effective pairwise central soft-core Afnan-Tang (A-T) potential [7] for a triplet state (t) and for a singlet (s) state

$$V_{t,s}(r) = \sum_{i=1}^{3} v_i^{(t,s)} \exp\left(-\beta_i^{(t,s)} r^2\right)$$

The parameters of the SX variant [1] of the A-T-potential

	v_1(MeV)	v_2(MeV)	v_3(MeV)	β_1(fm^{-2})	β_2(fm^{-2})	β_3(fm^{-2})
t	500	-102	-2	11.41	0.625	0.141
s	500	-102	-2	4.15	0.625	0.141

We using effective nucleon-nucleon pseudopotentials $V_{\alpha\text{-n}}$ and $V_{\alpha\text{-p}}$ in calculations [1]. The pseudopotentials do not take into account the data on phase shifts, but their forms are similar to α–α and nucleon–nucleon potentials. The parameters of the pseudopotentials were determined from the condition of equality of the calculated and experimental values of the ground state energies for systems α-cluster + nucleons.

$$V_{\alpha-N}^{(N)}(r) = -u_1 f(r,B_1,a_1) + u_2 f(r,B_2,a_2) - u_3 f(r,B_3,a_3) f(r,B_4,a_4),$$

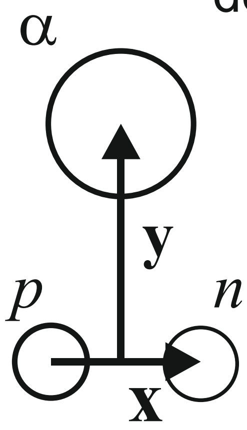
$$f(r,B,a) = \left[1+\exp\left(\frac{r-B}{a}\right)\right]^{-1}, \; V_{\alpha-n}(r) = V_{\alpha-N}^{(N)}(r), \; V_{\alpha-p}(r) = V_{\alpha-N}^{(N)}(r) + V_{\alpha-p}^{(C)}(r)$$

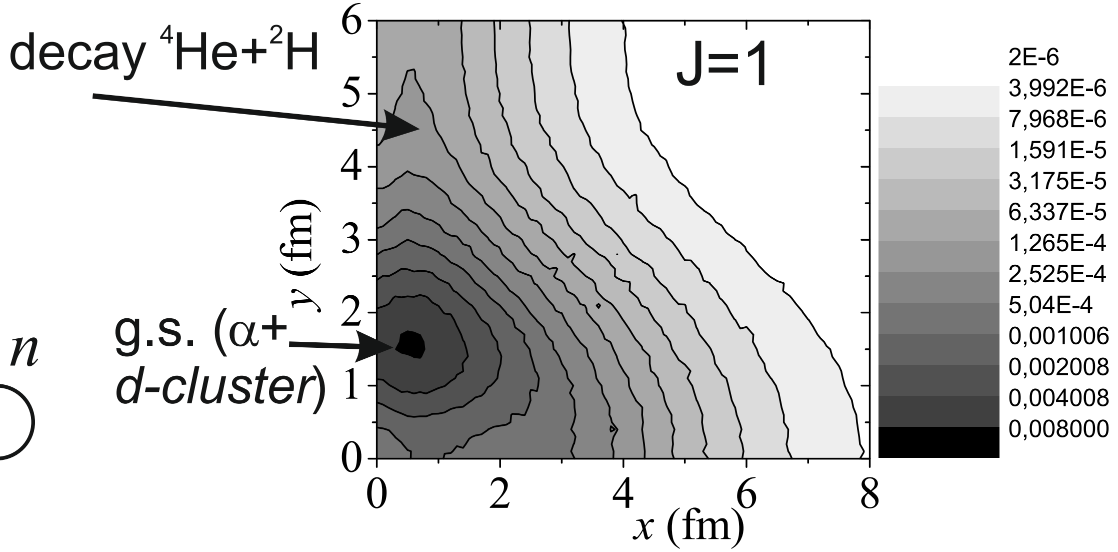
I	u_1 (MeV)	B (fm)	a (fm)
1	64.8	1.95	0.25
2	55.8	1.22	0.3
3	119	0.9	0.5
4	–	2.7	1

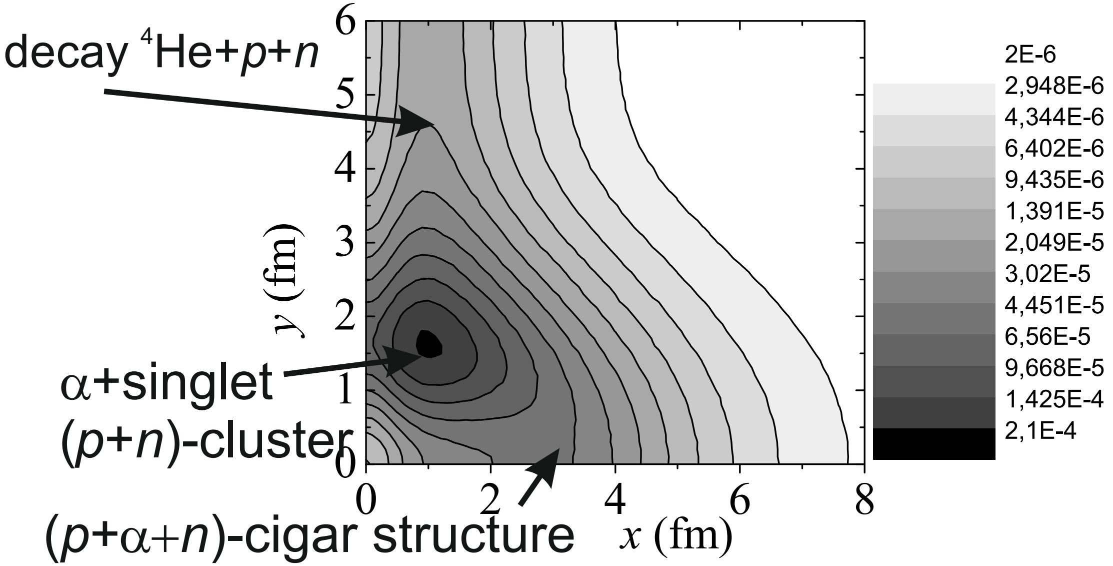
The probability density for the ground state of alpha-cluster nucleues ^{6}Li

The ground state J=1:
Esep =3.725 MeV (to 3 paticles a, p and n).
The calculated energy of the ground state with the triplet state of (p+n) subsystem is -3.7 MeV.

The excited state J=0:
with E_{exc}=3.563 MeV, E_{sep} =0.162 MeV (to 3 paticles a, p and n) may be presented as the ground state with the singlet state of (p+n) subsystem, calculated energy is -0.3 MeV.







Thank you for your attention!