Gravitational axial anomaly, cosmological constant and Unruh effect in curved spacetime

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based on

Khakimov, Prokhorov, Teryaev, Zakharov PRD 108, L121701 (2023)
Khakimov, Prokhorov, Teryaev, Zakharov PRD 109, 105001 (2024)

Introduction

What is this work about?

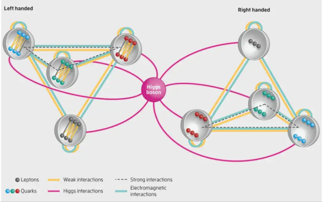
The work is based on the connection between the UV chiral anomaly and relativistic hydrodynamics. In particular,it shows the duality of kinematic effects in a relativistic medium and the curvature of the manifold in which this medium is considered (hydrodynamic/gravity duality).

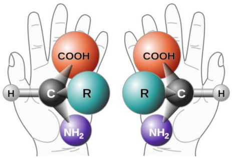
Why is this study important?

Perhaps it points to the nature of the quantum anomaly, whose understanding is very important for modern physics.

Our world has an inherent chirality

Our world has an inherent chirality and it manifests itself on different scales





Chiral anomalies

Anomalies are violations of some symmetries of the theory, preserved in the classical case, due to quantum effects.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi + A_{\mu}j^{\mu}, \quad \psi = \begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}, \quad m = 0.$$

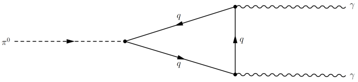
There are some symmetries on classical level, which involves conserving currents

$$\begin{cases} \psi \to \psi e^{ie\alpha} \implies & \partial_{\mu}j^{\mu} = 0, \\ \psi \to \psi e^{ie\alpha\gamma^{5}} \implies & \partial_{\mu}j^{\mu}_{A} = 0 \end{cases}$$

But in external fields due to loop corrections the axial current is not conserved
$\partial_{\mu}j^{\mu}_{A} \neq 0 \to N_R - N_L \neq const.$

It is experimental fact (decay $\pi_0 \to 2\gamma$ restricted on a classical level)

$\partial_{\mu}j^{\mu}_{A} = \frac{e^2}{16\pi^2}F_{\mu\nu}\widetilde{F}^{\mu\nu}$

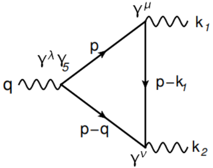


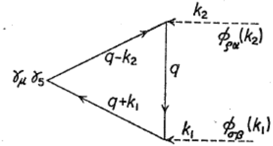
Chiral anomalies in pure QFT

Chiral anomalies is the UV effects in QFT

Feynman diagrams of electromagnetic and gravitational anomalies

$$\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu} \widetilde{F}^{\mu\nu} \qquad \nabla_\mu j^\mu_A = \frac{1}{384\pi^2} R_{\mu\nu\lambda\rho} \widetilde{R}^{\mu\nu\lambda\rho}$$





Hydrodynamics

Fluid dynamics is an IR theory

Hydrodynamics is based on conservation laws

$$\partial_\mu j^\mu = 0, \qquad \partial_\mu T^{\mu\nu} = 0$$

We can express hydrodynamical current or stress energy tensor as a series of derivatives acting on different parameters such as 4-velocity of fluid u_μ, temperature T, space-time metric $g_{\mu\nu}$.

Kinematical variables is the first order in gradients

$$\omega_\mu = \frac{1}{2}\varepsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho, \qquad a^\mu = u_\nu \partial^\nu u^\mu$$

Connection between hydrodynamics and EM chiral anomaly

Current of relativistic fluid with the new vorticity term

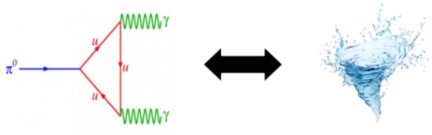
$$j^\mu_A = \underbrace{nu^\mu - \sigma T\left(g^{\mu\nu} + u^\mu u^\nu\right)\partial_\nu\left(\frac{\mu}{T}\right)}_{\text{Landau and Lifshitz}} + \underbrace{\xi\omega^\mu}_{\text{New term}}$$

Electromagnetic chiral anomaly

$$\underbrace{\partial_\mu j^\mu_A = -\frac{1}{8} C \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}}_{\text{Chiral anomaly}}$$

UV anomaly connected with hydrodynamical current: vortical term proportional to the anomaly coefficient [Son, Surowka PRL (2009)]

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{\mu^3 n}{\epsilon + P}\right)$$



In other words, in a hot and dense medium quantum UV anomalies are expressed macroscopically. Such a connection is only possible for an electromagnetic anomaly?

Hydrodynamics and gravitational chiral anomaly

Hydrodynamics is also related to the gravitational anomaly and its called Kinematical Vortical Effect (KVE).

Current in the 3^{rd} order of gradients

$$j_A^\mu = \left(\lambda_1 \omega^2 + \lambda_2 a^2\right) \omega^\mu$$

Gravitational chiral anomaly

$$\nabla_\mu j_A^\mu = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} R^{\sigma\rho}_{\alpha\beta}$$

There is connection between transport coefficients in flat spacetime and coefficient of anomaly, induced by curvature of spacetime [Prokhorov, Teryaev, Zakharov PRL (2022)]

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

Neverthelees, this result was obtained in a case of serious limitation of Ricci-flat spacetime $R_{\mu\nu} = 0$.

Generalization of KVE on Einstein manifolds

Consideration of the chiral medium in Einstein manifolds $R_{\mu\nu} = \Lambda g_{\mu\nu}$

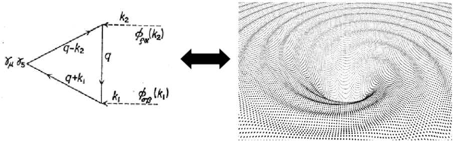
The axial current and the axial anomaly

$$j_A^\mu = \Big(\lambda_1 \omega^2 + \lambda_2 a^2 + \underbrace{\lambda_\Lambda R}_{New\ term}\Big)\,\omega^\mu \qquad\qquad \nabla_\mu j_A^\mu = \mathcal{N}\epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}.$$

Lead to relations

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}, \qquad \lambda_\Lambda = -\frac{\lambda_2}{3}.$$

As can be seen, even in this case KVE still exist. Moreover, the curvature of the spacetime directly connected with the acceleration of the fluid.



Consequence: The Unruh effect in curved spacetime

Hydrodynamic conservation law

$$\nabla_\mu T^{\mu\nu} = 0$$

Stress-Energy Tensors of accelerated fluid in vacuum, embedded in 4-manifolds with scalar curvature R (matches with [Ambrus, Winstanley,Symmetry, 2021])

$$\langle \hat{T}^{\mu\nu} \rangle_{s=0} = \left[\frac{\pi^2}{90} T^4 - \frac{1}{1440\pi^2} \left(|a|^2 + \frac{R}{12} \right)^2 \right] \left(4u^\mu u^\nu - g^{\mu\nu} \right)$$

$$\langle \hat{T}^{\mu\nu} \rangle_{s=1/2} = \left[\frac{7\pi^2}{180} T^4 + \frac{1}{72} \left(|a|^2 + \frac{R}{12} \right) T^2 - \frac{17}{2880\pi^2} \left(|a|^2 + \frac{R}{12} \right)^2 \right] \left(4u^\mu u^\nu - g^{\mu\nu} \right)$$

The condition

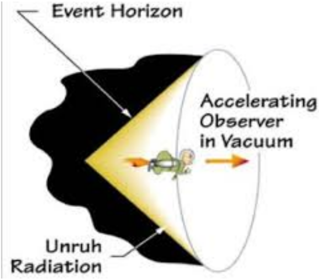
$$\langle \hat{T}^{\mu\nu} \rangle (T = T_U) = 0; \qquad T_U = \frac{a_5}{2\pi} = \frac{\sqrt{|a|^2 + R/12}}{2\pi}$$

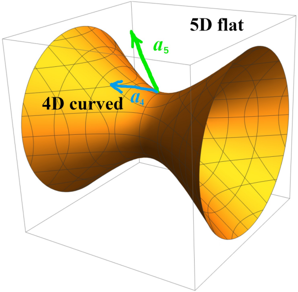
Tells us about the Unruh effect in curved spacetime [Deser, Levin, Classical Quantum Gravity, 1997]

The Unruh effect

From the point of view of an accelerating observer or detector, the empty spacetime contains a gas of particles at a temperature proportional to the acceleration.

$$T_U = \frac{a}{2\pi}\frac{\hbar}{k_B c} \qquad T_U^{AdS} = \frac{a_5}{2\pi} = \frac{\sqrt{|a|^2 + R/12}}{2\pi}$$





Discussion. Kinematic nature of the axial anomaly

The KVE is a purely kinematic effect in **flat** spacetime. The relation holds even without external gravitational field.

$$j_\mu^A = \left(\lambda_1 \omega^2 + \lambda_2 a^2\right) \omega_\mu \qquad \frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}.$$

In other words, the nature of the chiral current violation may be kinematic. External fields only make it "manifest".

Discussion. Extension of the Einstein principle of equivalence

A new result obtained as a result of consideration of the system in the Einstein manifold is the connection of transport coefficients of quadratic acceleration and scalar curvature of spacetime.

This result can be well interpreted as extension of the equivalence principle of inertial effects and gravitation in higher orders on gradients (of velocity and metric)

$$\lambda_\Lambda = -\frac{\lambda_2}{3}$$

The consequence of this connection of transport coefficients is the derivation of mentioned above Unruh temperature for dS/AdS spacetimes

Thank you for your attention!