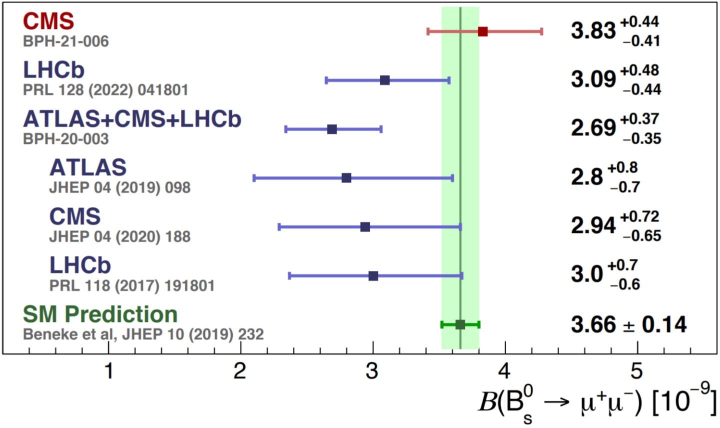
Coulomb interaction in rare decays of B-Mesons

Stepan Ilich Manukhov[1]
Scientific advisor: Nikolai Viktorovich Nikitin[1,2]

[1] Faculty of Physics, Lomonosov Moscow State University
[2] Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University

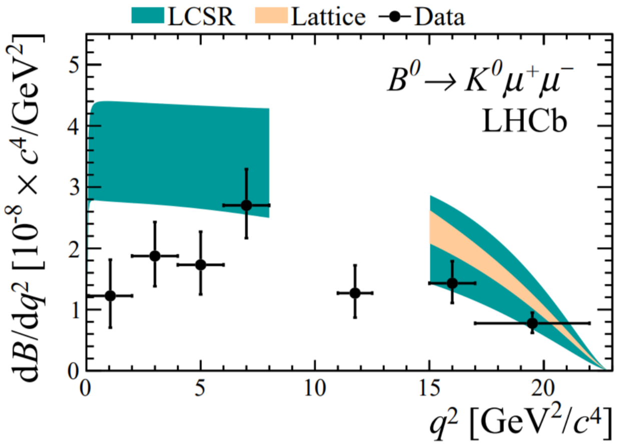
October 30, 2024

Problem statement and relevance of the work



The partial decay width of $B_s^0 \to \mu^+\mu^-$, measured in various experiments, as well as the averaged predictions of the Standard Model

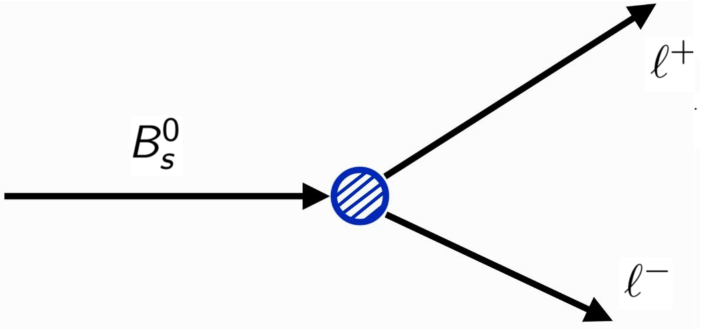
Problem statement and relevance of the work



Differential decay width of $B^0 \to K^0 \mu^+ \mu^-$ - predictions of the Standard Model, experimental values arXiv:1403.8044 [hep-ex]

Part 1. Formulation and justification of method.

Key idea - replacement of the secondary quantization procedure



Leptons cannot be considered free - in the final state, they interact with each other.

Key idea - replacement of the secondary quantization procedure

The standard procedure of secondary quantization - expansion in plane waves:

$$\ell(x) = \sum_{s=1,2} \int \frac{d^3p}{(2\pi)^3} (a_p u(p,s) e^{-ipx} + b_p^\dagger \bar{v}(p,s) e^{+ipx}) \qquad (1)$$

Modified procedure of secondary quantization - expansion in exact solutions of the wave equation with an external potential (Furry's method):

$$\ell(x) = \sum_{s=1,2} \int \frac{d^3p}{(2\pi)^3} (a_p \Psi^{(+)}_{\mathcal{E}\vec{p}}(x) e^{-i\mathcal{E}^{(+)}t} + b_p^\dagger \Psi^{(-)}_{-\mathcal{E}-\vec{p}}(x) e^{+i\mathcal{E}^{(-)}t}) \qquad (2)$$

Justification of the approach using the decay $B \to S^+ S^-$

Three methods for accounting for Coulomb interaction

1. The non-relativistic Gamow-Sommerfeld-Sakharov factor:

$$\mathcal{K}^{(GSS)} = \frac{2\pi\alpha/v}{1 - e^{-2\pi\alpha/v}}, \tag{3}$$

2. The exact relativistic method of Crater (arXiv:hep-ph/9912386) and Sazdjian (PhysRevD 33, 3401):

$$\mathcal{K}^{(CS)} = \left| \frac{\Gamma(\sqrt{\frac{1}{4} - \alpha^2} + \frac{1}{2} + i\frac{\alpha}{v})}{\Gamma(\sqrt{1 - 4\alpha^2} + 1)} \right|^2 \cdot e^{\pi\alpha/v}, \tag{4}$$

3. The approximate relativistic method of Furry (PhysRev 81,115):

$$\mathcal{K}^{(Furry)} = e^{\pi\alpha/v}. \tag{5}$$

Justification of the approach using the decay $B \to S^+S^-$

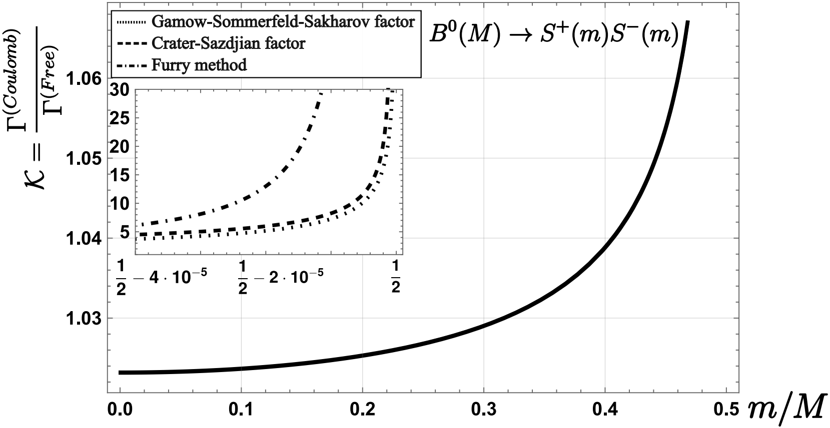


Figure: Comparison of three methods for accounting Coulomb interaction between particles.

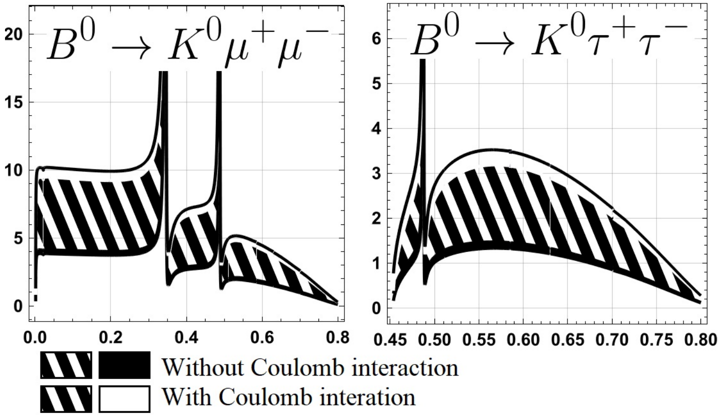
Part 2. Coulomb interation in $B^0_{s,d} \to \ell^+\ell^-$ and $B^0_{s,d} \to h^0\ell^+\ell^-$ decays.

Coulomb interaction in the decays $B_s^0 \to \ell^+\ell^-$

	$\mathcal{B}^{(exp)}$	$\mathcal{B}^{(free)}$	$\mathcal{B}^{(Coulomb)}$
$B_s^0 \to \mu^+\mu^- [10^{-9}]$	$\mathbf{3.83^{+0.44}_{-0.41}}$	$\mathbf{3.66 \pm 0.14}$	$\mathbf{3.75 \pm 0.14}$
$B^0 \to \mu^+\mu^- [10^{-11}]$	< 19	1.03 ± 0.05	1.05 ± 0.05
$B_s^0 \to e^+e^- [10^{-11}]$	< 940	1.77 ± 0.08	1.81 ± 0.09
$B^0 \to e^+e^- [10^{-13}]$	< 25000	4.99 ± 0.25	5.10 ± 0.26
$B_s^0 \to \tau^+\tau^- [10^{-8}]$	$< 6.8 \cdot 10^5$	4.61 ± 0.22	4.75 ± 0.23
$B^0 \to \tau^+\tau^- [10^{-9}]$	$< 2.1 \cdot 10^6$	1.28 ± 0.07	1.32 ± 0.07

Table: Branching ratio $\mathcal{B} = \Gamma_{B^0_{d,s} \to \ell^+\ell^-} / \Gamma^{(total)}_{B^0_{d,s}}$

Coulomb interaction in the decay $B^0 \to K^0\mu^+\mu^-$ and $B^0 \to K^0\tau^+\tau^-$

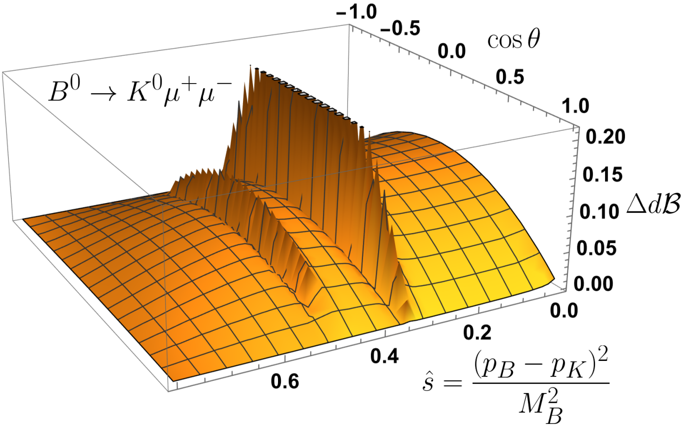


The differential decay width $10^7 \cdot \frac{1}{\Gamma^{(total)}} \frac{d\Gamma}{ds}$ as a function of s/M_B^2 where $s = (p_{B^0} - p_{K^0})^2$.

Coulomb interaction in the decay $B_s^0 \to h^0 \ell^+ \ell^-$

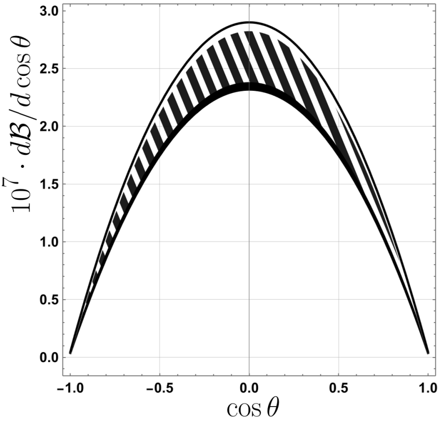
Decay	$\mathcal{B}^{(exp)}$	$\mathcal{B}^{(th,free)}$	$\mathcal{B}^{(th,coulomb)}$	Corr
$B^0 \to K^0 e^+ e^- [10^{-7}]$	$2.5^{+1.1}_{-0.9}$	3.64 ± 0.77	3.73	2.32%
$B^0 \to K^0 \mu^+ \mu^- [10^{-7}]$	3.39 ± 0.35	3.63 ± 0.77	3.72 ± 0.78	2.34%
$B^0 \to K^0 \tau^+ \tau^- [10^{-8}]$	-	5.0 ± 2.1	5.3 ± 2.2	5.75%
$B^0 \to \pi^0 e^+ e^- [10^{-8}]$	< 8.4	1.32 ± 3.0	1.35 ± 3.0	2.32%
$B^0 \to \pi^0 \mu^+ \mu^- [10^{-8}]$	< 6.9	1.31 ± 3.0	1.34 ± 3.0	2.34%
$B^0 \to \pi^0 \tau^+ \tau^- [10^{-9}]$	$-$	3.29 ± 0.73	3.45 ± 0.76	4.93%

Decay	$\mathcal{B}^{(exp)}$	$\mathcal{B}^{(th,free)}$	$\mathcal{B}^{(th,coulomb)}$	Corr
$B_s^0 \to \eta e^+ e^- [10^{-7}]$	–	4.24 ± 0.79	4.33 ± 0.80	2.32%
$B_s^0 \to \eta \mu^+ \mu^- [10^{-7}]$	–	4.22 ± 0.79	4.32 ± 0.80	2.34%
$B_s^0 \to \eta \tau^+ \tau^- [10^{-8}]$	–	6.7 ± 1.2	7.1 ± 1.3	5.64%
$B_s^0 \to \eta' e^+ e^- [10^{-7}]$	–	3.13 ± 0.58	3.20 ± 0.59	2.31%
$B_s^0 \to \eta' \mu^+ \mu^- [10^{-7}]$	–	3.11 ± 0.58	3.18 ± 0.59	2.34%
$B_s^0 \to \eta' \tau^+ \tau^- [10^{-8}]$	–	2.01 ± 0.36	2.15 ± 0.38	**7.0%**
$B_s^0 \to K^0 e^+ e^- [10^{-8}]$	–	1.42 ± 0.34	1.45 ± 0.35	2.32%
$B_s^0 \to K^0 \mu^+ \mu^- [10^{-8}]$	–	1.41 ± 0.34	1.44 ± 0.35	2.34%
$B_s^0 \to K^0 \tau^+ \tau^- [10^{-9}]$	–	2.49 ± 0.59	2.64 ± 0.62	6.02%



A graph of $\Delta d\mathcal{B} \equiv 10^7 \cdot \left(\frac{d\mathcal{B}}{d\hat{s}d\cos\theta}\right)_{\text{Coulomb}} - 10^7 \cdot \left(\frac{d\mathcal{B}}{d\hat{s}d\cos\theta}\right)_{\text{free}}$ as a function of $\hat{s} = (p_B - p_K)^2/M_B$ and $\cos\theta$, where $\theta = \angle(\mathbf{p}_{\ell^+}, \mathbf{p}_K)$ in the rest frame of the lepton pair.

Angle distributions in the decay $B^0 \to K^0\mu^+\mu^-$



Graph of the $10^7 \cdot d\mathcal{B}/d\cos\theta$ dependence on $\cos\theta$, where $\theta = \angle(\mathbf{p}_{\ell^+}, \mathbf{p}_K)$ in the rest frame of the lepton pair

Conclusion and results

- A method to account for the Coulomb interaction in lepton and semilepton decays of B-mesons was developed.
- Corrections to the decays were calculated $B^0_{s,d} \to \ell^+\ell^-$, $B^0 \to \{K^0, \pi^0\}\ell^+\ell^-$ and $B^0_s \to \{\eta, \eta', K^0\}\ell^+\ell^-$ (21 decays in total)
- For the $B^0_s \to \mu^+\mu^-$ decay, accounting for the Coulomb interaction improves the agreement between the CM predictions and the experimental data
- The Coulomb correction can be up to 7%, which is similar to the error of the form factors

Thank You for Your Attention!

The decay width for $B_s^0 \to \ell^+\ell^-$ is given by:

$$\Gamma_{B_s^0 \to l^+l^-} = \mathcal{K}^{(Furry)}_{B_s^0 \to \mu^+\mu^-} \cdot |D|^2 \frac{\sqrt{M^2 - 4m^2}}{8\pi}, \text{where} \tag{6}$$

$$D = \frac{iG_F}{\sqrt{2}} \frac{\alpha_{em}}{2\pi} \cdot V_{tb} V_{ts}^* f_{B_s^0} 2m C_{10A}$$

$$\mathcal{K}^{(Furry)}_{B_s^0 \to l^+l^-} = \frac{\Gamma^{(Coulomb)}}{\Gamma^{(free)}} = e^{\pi\alpha\mathcal{E}/p}$$

here $\mathcal{E}$ and p are the lepton's energy and momentum, respectively.

Coulomb interaction in the decay $B_s^0 \to K^0 l^+ l^-$

The differential decay width for $B_s^0 \to K^0 l^+ l^-$ is given by:

$$\frac{d\Gamma}{d\hat{t}d\hat{s}} = \frac{G_F^2 \alpha_{em}^2 |V_{tb} V_{ts}^*| M^5}{256\pi^5} (-\hat{\Pi}\beta_p + 2\hat{m}|C_{10A}|^2 \delta_p) \cdot \mathcal{K}^{(Coulomb)}, \quad (7)$$

$$\beta_p = \left| C_{9V} f_+(q^2) + 2MC_{7\gamma} s(q^2) \right|^2 + \left| C_{10A} f_+(q^2) \right|^2$$

$$\hat{\Pi} = (\hat{t} - 1)(\hat{t} - \hat{r}) + \hat{s}\hat{t} + \hat{m}(1 + \hat{r} + \hat{m} - \hat{s} - 2\hat{t})$$

$$\delta_p = \left(1 + \hat{r} - \frac{\hat{s}}{2}\right) |f_+(q^2)|^2 + (1 - \hat{r}) Re[f_+(q^2) f_-^*(q^2)] + \frac{\hat{s}}{2} |f_-(q^2)|^2$$

$$\mathcal{K}^{(Coulomb)} = exp\left(\frac{\pi\alpha}{\sqrt{1 - 4\hat{m}/\hat{s}}}\right)$$

General approach to accounting for Coulomb interaction

A general recipe that allows for the accounting of Coulomb interaction in decays, in which the final state contains an l^+l^- pair (and no more charged particles):

$$\langle l^+l^-H_2|O|H_1\rangle \to \langle l^+l^-H_2|O|H_1\rangle \cdot \frac{\Gamma(1-i\frac{\alpha\mathcal{E}_l}{2p_l})}{\Gamma(1+i\frac{\alpha\mathcal{E}_l}{2p_l})} exp(\frac{\pi\alpha\mathcal{E}_l}{2p_l})$$

O - operator representing an arbitrary combination of γ-matrices, momenta, as well as quark and lepton fields, $\mathcal{E}_l, p_l$ - energy and momentum of the charged lepton in the rest frame of the l^+l^- pair, $\Gamma(x)$ - Euler's gamma function, H_1, H_2 - neutral hadrons in the initial and final states respectively, $\alpha = \alpha_{em} \approx 1/137$.