Mixed inhomogeneous phase in rotating gluon plasma

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in collaboration with

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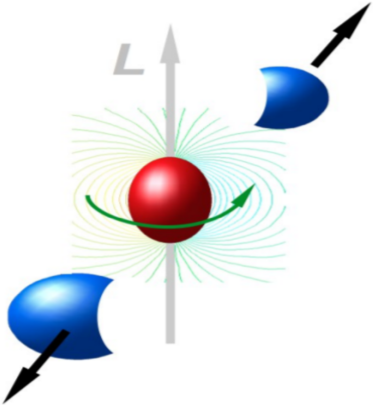
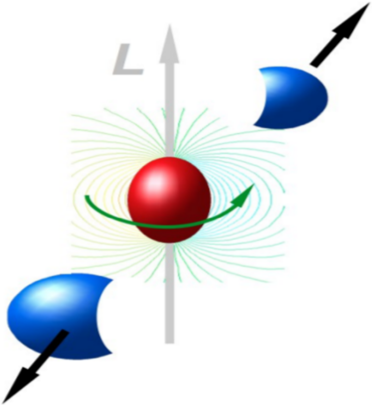
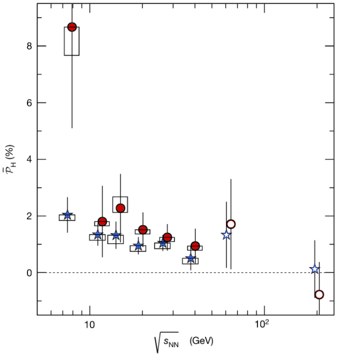
JOINT INSTITUTE
FOR NUCLEAR RESEARCH

- In non-central heavy ion collisions, the droplets of QGP with angular momentum are created.



- In non-central heavy ion collisions, the droplets of QGP with angular momentum are created.
- The rotation occurs with relativistic velocities.





[L. Adamczyk et al. (STAR), Nature **548**, 62–65 (2017), arXiv:1701.06657 [nucl-ex]]

$\langle\omega\rangle \sim 7$ MeV ($\sqrt{s_{NN}}$-averaged)

All* theoretical models assume rigid rotation, $\Omega \neq 0$.

- P. Singha, V. E. Ambrus, and M. N. Chernodub, (2024), arXiv:2407.07828 [hep-ph]
- Y. Chen, X. Chen, D. Li, and M. Huang, (2024), arXiv:2405.06386 [hep-ph]
- K. Mameda and K. Takizawa, Phys. Lett. B **847**, 138317 (2023), arXiv:2308.07310 [hep-ph]
- F. Sun, K. Xu, and M. Huang, Phys. Rev. D **108**, 096007 (2023), arXiv:2307.14402 [hep-ph]
- H.-L. Chen, Z.-B. Zhu, and X.-G. Huang, Phys. Rev. D **108**, 054006 (2023), arXiv:2306.08362 [hep-ph]
- and many others ...

Lattice study of the (averaged) critical temperature in gluodynamics and QCD: T_c increases with rotation

- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. **112**, 6–12 (2020)
- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, Phys. Rev. D **103**, 094515 (2021), arXiv:2102.05084 [hep-lat]
- V. Braguta, A. Kotov, A. Roenko, and D. Sychev, PoS **LATTICE2022**, 190 (2023), arXiv:2212.03224 [hep-lat]
- J.-C. Yang and X.-G. Huang, (2023), arXiv:2307.05755 [hep-lat]

Lattice study of the thermodynamics of SU(3) YM: $I < 0$ close to T_c, NBE(?) [See Talk by D. Sychev]

- V. V. Braguta, M. N. Chernodub, A. A. Roenko, and D. A. Sychev, (2023), arXiv:2303.03147 [hep-lat]
- V. V. Braguta, I. E. Kudrov, A. A. Roenko, D. A. Sychev, and M. N. Chernodub, JETP Lett. **117**, 639–644 (2023)
- V. V. Braguta, M. N. Chernodub, I. E. Kudrov, A. A. Roenko, and D. A. Sychev, Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]

Study of the rotating QCD properties

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The rotating system is not spatially homogeneous!

Inhomogeneity in rotating (Q)GP

Tolman-Ehrenfest effect: In a gravitational field the temperature is not a constant in space at thermal equilibrium.

In the co-rotating reference frame, effects of rotation are reduced to gravity, therefore:

$$T(r) = \frac{T_0}{\sqrt{1-\Omega^2 r^2}} = \frac{T_0}{\sqrt{1+\Omega_I^2 r^2}}\,. \tag{1}$$

TE law suggests that the rotation effectively heats the periphery, the local critical temperature decreases:

$$T_c^{TE}(r,\Omega) = T_{c0}\sqrt{1-\Omega^2 r^2} \qquad \Rightarrow \qquad [\text{ inhomogeneous phase in rotating (Q)GP }]$$

In the result, confinement is in the center, deconfinement is at the periphery (for *real* rotation):

- 2+1 cQED: M. N. Chernodub, Phys. Rev. D **103**, 054027 (2021), arXiv:2012.04924 [hep-ph]
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Qualitatively consistent results:

- S. Chen, K. Fukushima, and Y. Shimada, (2024), arXiv:2404.00965 [hep-ph]
- Y. Jiang, Phys. Rev. D **110**, 054047 (2024), arXiv:2406.03311 [nucl-th]

Lattice formulation of rotating gluodynamics

Observables are calculated on the lattice from first principles:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int DA\, \mathcal{O}[A] \exp\big(-S_G[A]\big)\,, \qquad \text{where} \qquad Z = \int DA\, \exp\big(-S_G[A]\big)\,. \tag{2}$$

The Euclidean action S_G in co-rotating reference frame is formulated in curved space,
[A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat]]

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega_I \\ 0 & 1 & 0 & -x\Omega_I \\ 0 & 0 & 1 & 0 \\ y\Omega_I & -x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix}, \tag{3}$$

where $r^2 = x^2 + y^2$, and the angular velocity is imaginary, $\Omega_I = -i\Omega$, to avoid the sign problem.

Lattice formulation of rotating gluodynamics

The gluon action is a quadratic function in angular velocity

$$S = \frac{1}{4g_0^2} \int d^4x \, \sqrt{g_E} \, g_E^{\mu\nu} g_E^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta} \equiv S_0 + S_1 \Omega_I + S_2 \frac{\Omega_I^2}{2} \, , \tag{4}$$

where

$$S_0 = \frac{1}{4g_0^2} \int d^4x F^a_{\mu\nu} F^a_{\mu\nu} \, , \tag{5}$$

$$S_1 = \frac{1}{g_0^2} \int d^4x \left[y F^a_{xy} F^a_{y\tau} + y F^a_{xz} F^a_{z\tau} - x F^a_{yx} F^a_{x\tau} - x F^a_{yz} F^a_{z\tau} \right] , \tag{6}$$

$$S_2 = \frac{1}{g_0^2} \int d^4x \left[r^2 (F^a_{xy})^2 + y^2 (F^a_{xz})^2 + x^2 (F^a_{yz})^2 + 2xy F^a_{xz} F^a_{zy} \right] , \tag{7}$$

Sign problem

- The sign problem is due to the linear terms ($S_1 \neq 0$)
- The Monte–Carlo simulation is conducted with imaginary angular velocity $\Omega_I = -i\Omega$.
- The results are analytically continued to real angular velocity, $\Omega^2 \leftrightarrow -\Omega_I^2$.

Causality restriction

- Analytic continuation is allowed only for bounded system with $\Omega r < 1$, i.e. $v_I^2 = (\Omega_I R)^2 < 1/2$
- Boundary conditions are important! (two different types of b.c.: open/periodic)

Lattice setup and observables

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Observables

The Polyakov loop is an order parameter,

$$L(x,y) = \frac{1}{N_z} \sum_z \mathrm{Tr}\left[\prod_{\tau=0}^{N_t-1} U_4(\vec{r},\tau)\right], \qquad L = \frac{1}{N_s^2} \sum_{x,y} L(x,y)\,. \tag{8}$$

In confinement $\langle L\rangle = 0$; in deconfinement $\langle L\rangle \neq 0$. $\langle L\rangle = e^{-F_Q/T}$
The local critical temperature is associated with the peak of the local Polyakov loop susceptibility

$$\chi_L(r) = \langle |L(r)|^2\rangle - \langle |L(r)|\rangle^2\,. \tag{9}$$

We use tree-level improved (Symanzik) lattice action; lattice size $N_t \times N_z \times N_s^2$; $R \equiv a(N_s - 1)/2$;
The temperature is $T = 1/N_t a$. It coincides with the temperature on the rotation axis T_0.

Inhomogeneous phases for imaginary rotation

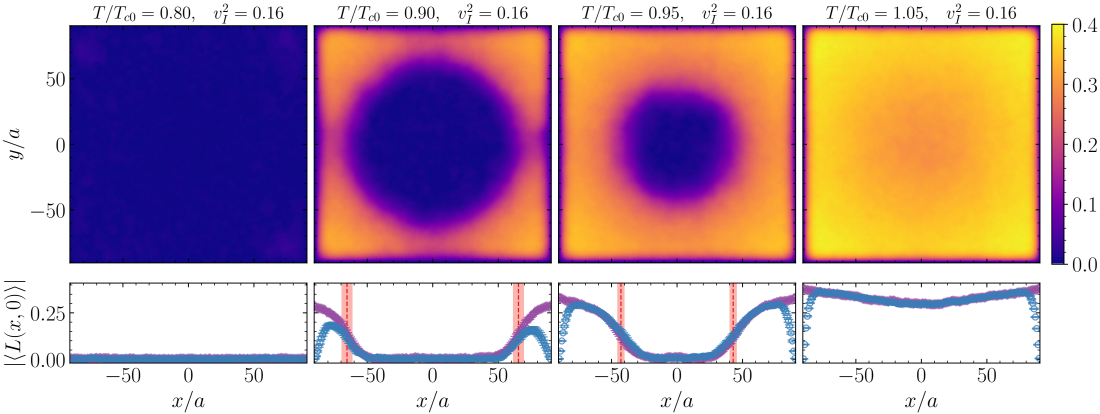


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed imaginary velocity at the boundary $v_I^2 \equiv (\Omega_I R)^2 = 0.16$ and different on-axis temperatures, $T = 1/N_t a$.

- As the (on-axis) temperature increases, the radius of the inner confining region shrinks.
- Boundary is screened.
- Local thermalization takes place; Phase transition occurs as a vortex evolution.

Inhomogeneous phases for imaginary rotation

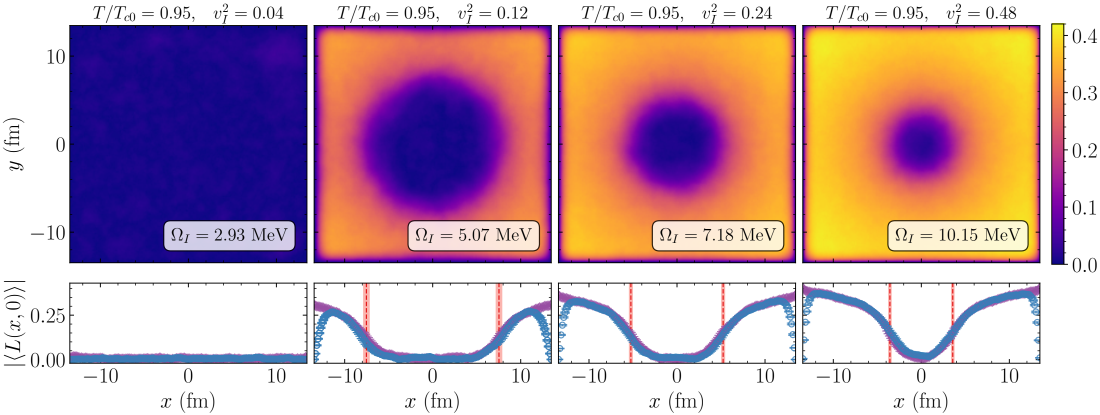
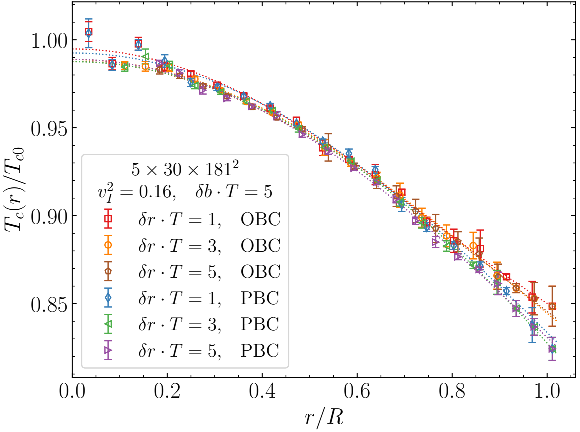


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed temperature $T = 0.95\, T_{c0}$ and different Ω_I; System size $R = 13.5$ fm.

- Mixed inhomogeneous phase may be observed for $T \lesssim T_{c0}$. For imaginary rotation, deconfinement appears at the periphery; confinement is in the central regions.
- The confinement region shrinks with the increase in Ω_I;

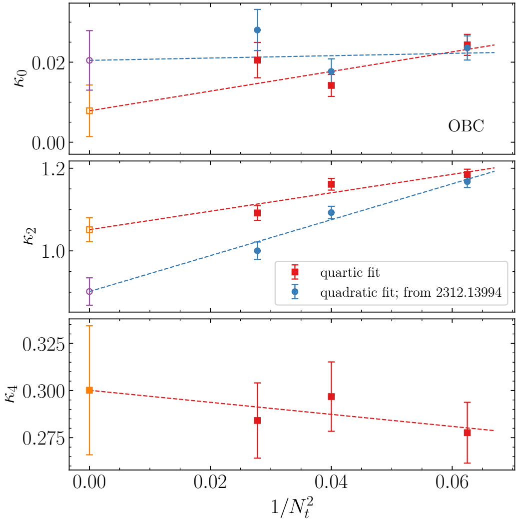
We split the system into thin cylinders of width δr and measure local critical temperature



- Results for different width $\delta r \cdot T = 1, \ldots, 5$ are in agreement.
- We discard δb layers adjacent to boundary.
- There is a minor difference on b.c. only at $r/R \sim 1$

We conducted simulations on the lattices of size $4 \times 24 \times 145^2$, $5 \times 30 \times 181^2$, $6 \times 35 \times 217^2$ for $v_I^2 = 0.04, 0.08, \ldots, 0.48$ and take continuum limit.

Local critical temperature: results



The local critical temperature decreases with imaginary angular velocity.

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - (\Omega_I r)^2 \left(\kappa_2 - \kappa_4 \left(\frac{r}{R}\right)^2\right) . \quad (10)$$

- The vortical curvature in continuum limit from quadratic fit ($\kappa_4 \equiv 0$, $r/R \lesssim 0.5$) is universal

$$\kappa_2 = 0.902(33) . \quad (11)$$

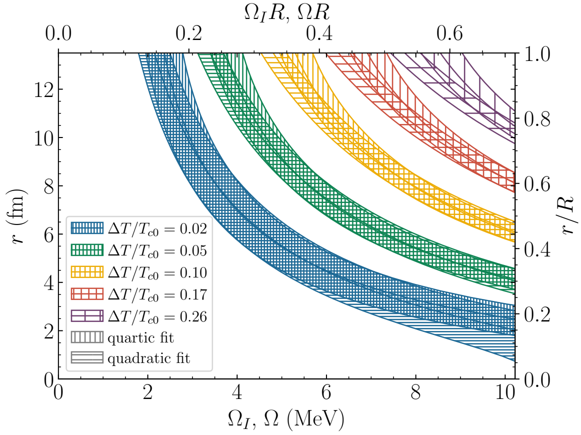
- From quartic fit (for OBC) we obtained

$$\kappa_2 = 1.051(29) , \qquad \kappa_4 = 0.300(34) , \quad (12)$$

  where κ_4 term is a finite volume correction;

- Analytic continuation: $\Omega_I^2 \to -\Omega^2$.

Condition for the mixed phase: Phase diagram



The critical distance r may be found from the following conditions:

- for imaginary angular velocity

$$T_{c0} - \Delta T = T_c(r, \Omega_I)\,,$$

(confinement in the center;
deconfinement at the periphery)

- for real angular velocity

$$T_{c0} + \Delta T = T_c(r, \Omega)\,.$$

(deconfinement in the center;
confinement at the periphery)

The diagram has the same shape for a given $\Delta T > 0$ (plot for $R = 13.5$ fm).

Decomposition of rotating action

The action of rotating gluons is a quadratic function in Ω_I,

$$S_G = S_0 + \lambda_1 S_1 \Omega_I + \lambda_2 S_2 \Omega_I^2 \,, \tag{13}$$

where we introduce switching factors λ_1, λ_2.

- The first operator S_1 is an angular momentum of gluons (in laboratory frame).
- The second operator S_2 is related to the chromomagnetic fields F_{ij}^2.

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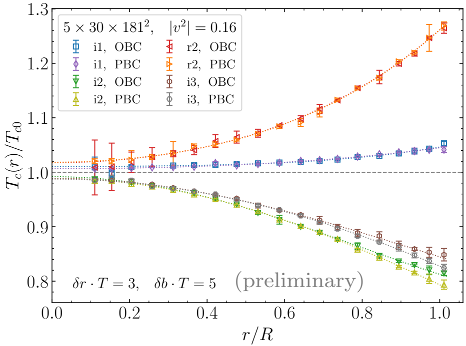

Mechanical and magnetic contributions to the moment of inertia originate from these operators.

- S_1 and S_2 have opposite influence on T_c.
- Contribution from S_2 dominates.
- The results resemble the decomposition of I [See Talk by D. Sychev]

Local approximation for inhomogeneous action

The homogeneous local action (in the vicinity of the point $x = r_0$, $y = 0$) is

$$S_G = \frac{1}{2g_0^2} \int d^4x \left[F^a_{x\tau} F^a_{x\tau} + F^a_{y\tau} F^a_{y\tau} + F^a_{z\tau} F^a_{z\tau} + F^a_{xz} F^a_{xz} + \right. \\ \left. + \left(1 + u_I^2\right) F^a_{yz} F^a_{yz} + \left(1 + u_I^2\right) F^a_{xy} F^a_{xy} - 2u_I \left(F^a_{yx} F^a_{x\tau} + F^a_{yz} F^a_{z\tau} \right) \right], \quad (14)$$

where $u_I = \Omega_I r_0$ is a local velocity. We measure $T_c(u_I)$ in this system:

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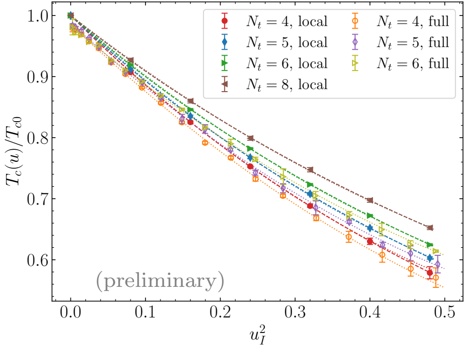


- The data are well described by the polynomial:

$$\frac{T_c(u_I)}{T_{c0}} = 1 - k_2 u_I^2 + k_4 u_I^4 \,. \quad (15)$$

- In continuum limit the coefficients are

$$k_2 = 0.869(31)\,, \qquad k_4 = 0.388(53)\,. \quad (16)$$

- The local critical temperature increases with real velocity $u = \Omega r$.
- There are no effects of b.c. and finite R.

Inhomogeneous phases in QCD (preliminary)

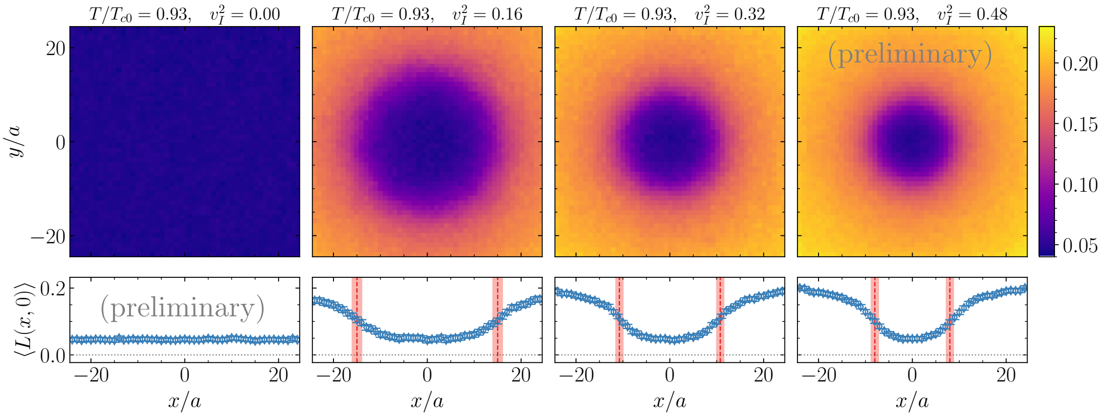


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $4 \times 20 \times 49^2$ at the fixed temperature $T = 0.93\, T_{c0}$ and different v_I; QCD with Wilson fermions (Iwasaki action), $m_\pi / m_\rho = 0.80$.

- Mixed inhomogeneous phase takes place also in QCD! (work in progress . . .)

Conclusions

- Using lattice simulation, we found a new mixed confinement-deconfinement phase in rotating SU(3) gluodynamics at thermal equilibrium.
- The local critical temperature increases for real rotation, and it is determined mainly by the local velocity of rotation $u = \Omega r$:

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + (\Omega r)^2 \left(\kappa_2 - \kappa_4 \left(\frac{r}{R}\right)^2\right), \qquad \text{[full system with OBC]}, \qquad (17)$$

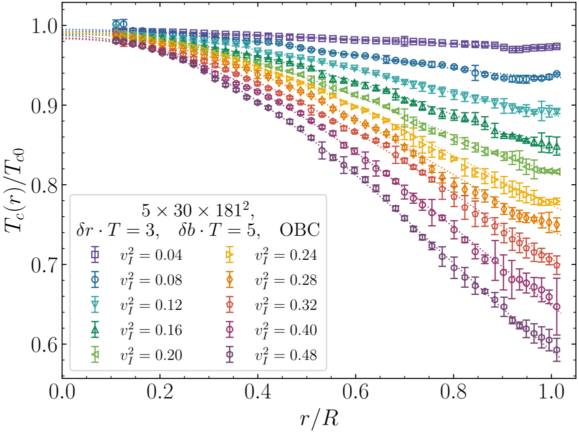
$$\frac{T_c(u)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4, \qquad \text{[local action]}, \qquad (18)$$

where κ_4 is sensitive to boundary effects.
- The local critical temperature on the axis $T_c(0)$ is T_{c0} with few percent accuracy. It is not TE.
- The rotation generates asymmetry in the action for chromomagnetic fields (S_2 dominates). The results in different regimes resemble decomposition of $I = I_{\text{mech}} + I_{\text{magn}}$. [See Talk by D. Sychev]
- Phase transition occurs as a evolution of vortex of new phase. We expect similar picture for QCD.
- Previous results for T_c should be understood as bulk-averaged values.

V. V. Braguta, M. N. Chernodub, and A. A. Roenko, Phys. Lett. B **855**, 138783 (2024), arXiv:2312.13994 [hep-lat]

Another details coming soon

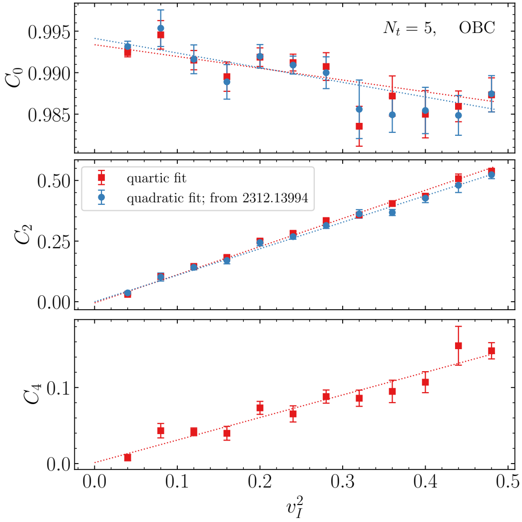
Thank you for your attention!



- The results in the whole region are well described by the quartic formula

$$\frac{T_c(r)}{T_{c0}} = C_0 - C_2\left(\frac{r}{R}\right)^2 + C_4\left(\frac{r}{R}\right)^4 . \quad (19)$$

- In the central region, $r/R \lesssim 0.5$, quadratic fit is sufficient ($C_4 = 0$).



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- We found numerically that

$$C_i(v_I^2) = a_i + \kappa_i v_I^2 . \quad (20)$$

- $T_c(0) \approx T_{c0}$ with few percent accuracy:
  - Effects of finite radius R.
  - Effects of averaging in layers of width δr.

Backup: Imaginary vs real rotation for different regimes

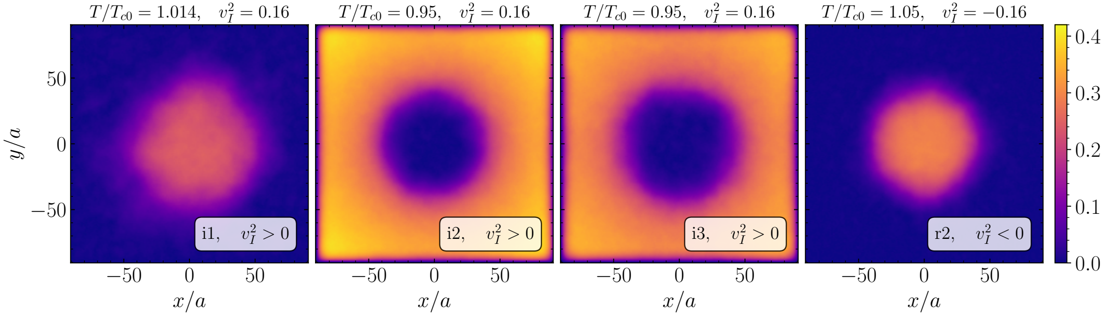
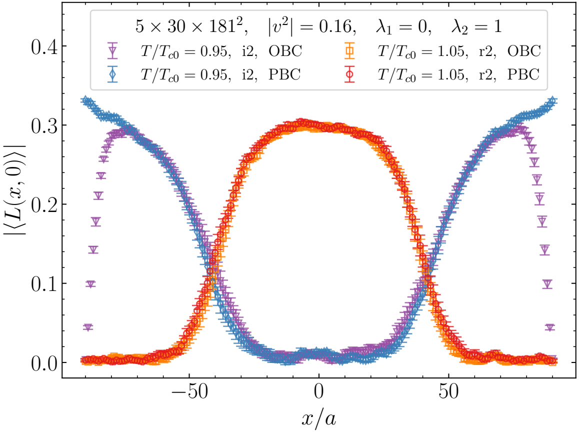


Figure: The distribution of the local Polyakov loop in x, y-plane for lattice size $5 \times 30 \times 181^2$, open boundary conditions (OBC) at fixed velocity $|v_I^2| = 0.16$ and different regimes.

- In the regimes i1 and r2, the rotation produces confinement phase in the outer region at $T > T_{c0}$. Regime r2 realizes real rotation for S_2 system.
- Phase arrangement is the same in i2- and i3-regimes. The radius of the inner region in regime i2 is slightly smaller, than in regime i3.

Backup: Imaginary vs real rotation for different regimes

The distributions of the Polyakov loop for real and imaginary rotation (S_1 term is omitted).



- (r2): $T = T_{c0} + \Delta T$ for real rotation $v^2 = 0.16$
- (i2): $T = T_{c0} - \Delta T$ for imaginary rotation $v_I^2 = 0.16$

Confinement ↔ deconfinement with approximately the same boundary.

Asymmetry in local action

The local action without the linear term is

$$S_G = \int d^4x \left[\beta \left((F^a_{x\tau})^2 + (F^a_{y\tau})^2 + (F^a_{z\tau})^2 + (F^a_{xz})^2 \right) + \tilde{\beta} \left((F^a_{yz})^2 + (F^a_{xy})^2 \right) \right], \quad (21)$$

where $\beta = 1/2g^2$ and $\tilde{\beta}/\beta = 1 - (\Omega r_0)^2 = 1 + (\Omega_I r_0)^2$.

External gravitational field generates asymmetry in the coupling constants of different components of the fields $(F_{\mu\nu})^2$, which influences the dynamics of gluons.

- $\tilde{\beta}/\beta > 1$ (imaginary rotation) $\Rightarrow$ T_c decreases.
- $\tilde{\beta}/\beta < 1$ (real rotation) $\Rightarrow$ T_c increases.

The rotation influences the dynamics of gluons, it is not TE.

Backup: Lattice size effects