Pion properties in the Bethe-Salpeter formalism with a separable kernel

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Motivation

The pion is the simplest quark-antiquark system. The small mass of the pion, compared to the masses of other mesons, allows the pion to play an important role in the description of nuclear dynamics.

There are many models to describe the pion:

- the model described by the QCD sum rules
- the model with a nonrelativistic potential and the relativistic model using the light-front formalism
- the Nambu-Jona-Losinio model
- the instanton model
- the model based on the Bethe-Salpeter equation with dressed quark and gluon propagators

In our work we used a model based on the relativistic covariant Bethe-Solpeter equation with a separable kernel.

Motivation

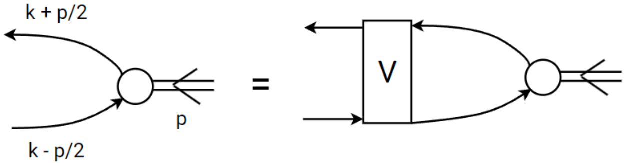
The study of pion properties has recently been actual because the transition form factor of the pion gives a contribution to the hadronic part of the anomalous magnetic moment of the muon (g-2). In addition, several experiments are planned to measure the charge and transition form factor of the pion in JLab: E12-22-003 [1], E12-06-101 [2], E12-19-006 [3], PR12-16-003 [4].

[1] Precision Measurement of the Neutral Pion Transition Form Factor.
[2] Measurement of the Charged Pion Form Factor to High Q2.
[3] Study of the L–T Separated Pion Electroproduction Cross Section at 11 GeV and Measurement of the Charged Pion Form Factor to High Q2.
[4] Determining the Pion Form Factor from Higher Q2, High -t Electroproduction Data.

Bethe-Salpeter equation for the bound state

The bound state equation for the vertex function is written as:

$$\Gamma_{\alpha\beta}(k;p) = i\int \frac{d^4k''}{(2\pi)^4} V_{\alpha\beta:\epsilon\lambda}(k,k'';p) S_{\lambda\eta}(k''+p/2)\Gamma_{\eta\zeta}(k'';p)S_{\zeta\epsilon}(k''-p/2)$$

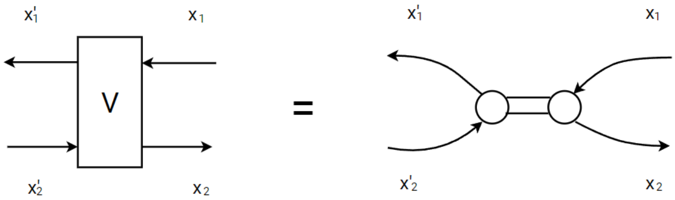


[5] Ito Hiroshi, Buck Warren, Gross Franz. // Phys. Rev. C. — 1991. — Vol. 43. — Pp. 2483–2498.

Separable kernel for the quark-antiquark interation

$$V_{\alpha\beta:\delta\gamma}(k',k;p) = \sum_{n',n}^{N} C_{n'n} \Delta^{n'}_{\alpha\beta}(k';p) \overline{\Delta}^{n}_{\delta\gamma}(k;p)$$

where N is the rank of the kernel, $C_{l'l}$ is a matrix of coefficients.



The function Δ is: $\Delta_{\alpha\beta}(k;p) = f(k^2, k\cdot p)\,\Omega_{\alpha\beta}$,
where Ω and $\overline{\Omega}$ are the matrixes, $f({k'}^2, k'\cdot p)$ и $f(k^2, k\cdot p)$ are the scalar functions of k', k and p momenta.

Pion vertex function

The general form of the pion vertex function:

$$\Gamma(k;p) = \gamma^5 \left[\Gamma_1(k^2;k\cdot p) + \frac{p_\alpha}{\mu}\gamma^\alpha\Gamma_2(k^2;k\cdot p) + \frac{k_\alpha}{\mu}\gamma^\alpha\Gamma_3(k^2;k\cdot p) + \sigma_{\alpha\beta}\frac{k^\alpha p^\beta}{\mu^2}\Gamma_4(k^2;k\cdot p)\right]$$

where μ is the pion mass.

In this work, the simplest case of the kernel for pion is considered:

$$f(k^2, k\cdot p) \to f(k^2) = 1/(k^2 - \Lambda^2 + i\epsilon), \qquad \Omega_{\alpha\beta} = \gamma^5_{\alpha\beta}$$

The kernel is:

$$V_{\alpha\beta:\delta\gamma}(k',k) = g\,\gamma^5_{\alpha\beta}\, f({k'}^2)\,\gamma^5_{\delta\gamma}\, f(k^2), \tag{1}$$

and the solution for the vertex $\pi q\bar{q}$ is

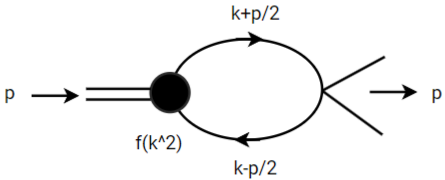
$$\Gamma(k) = N\gamma^5 f(k^2),$$

where N is the normalization constant determined from the charge conservation condition.

Pion constants

The weak pion decay constant:

$$f_\pi = -i4m\sqrt{3}N \int \frac{d^4k}{(2\pi)^4} \frac{f(k^2)}{((k-p/2)^2 - m^2 + i\epsilon)((k+p/2)^2 - m^2 + i\epsilon)}$$



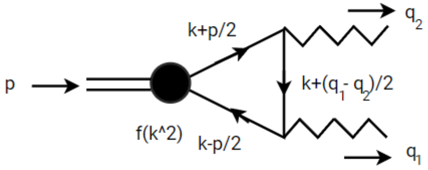
Pion constants

The two-photon decay amplitude:

$$M_{\pi\gamma} = G_{\pi\gamma}\epsilon_{\mu\nu\rho\sigma}\epsilon_1^{\mu}\epsilon_2^{\nu}k_1^{\rho}k_2^{\sigma},$$

where $G_{\pi\gamma}(p^2, q_1^2, q_2^2) = -\frac{4\sqrt{2}Nm}{\sqrt{3}}\int\frac{d^4k}{(2\pi^4)}$

$$\frac{f(k^2)}{((k+p/2)^2 - m^2 + i\epsilon)((k-p/2)^2 - m^2 + i\epsilon)((k+(q_1-q_2)/2)^2 - m^2 + i\epsilon)}$$



Pion constants

The two-photon decay width $\Gamma_{\pi^0 \to \gamma\gamma}$

$$\Gamma_{\pi^0 \to \gamma\gamma} = \frac{\pi\alpha^2}{4} m_\pi^3 G_{\pi\gamma}^2(m_\pi^2, 0, 0),$$

where $\alpha = \dfrac{e^2}{4\pi} = \dfrac{1}{137}$ is a fine-structure constant.

The radius from $\gamma^*\gamma \to \pi^0$ transition

$$r_{\pi\gamma}^2 = -6 \frac{F'_{\pi\gamma}(0)}{F_{\pi\gamma}(0)},$$

where $F'_{\pi\gamma}$ is the derivative of the transition form factor at zero value of the transfer momentum squared.

Pion constants

The squared charge radius of the pion r_π^2

The charge radius consists of two parts:

$$r_\pi^2 = (r_\pi^2)^{\mathrm{RIA}} + (r_\pi^2)^{\mathrm{int}}$$

$$(r_\pi^2)^{\mathrm{RIA}} = -6 \frac{d}{dq^2} F_\pi^{\mathrm{RIA}}(q^2)|_{q^2=0}$$

$$(r_\pi^2)^{\mathrm{int}} = -6 \frac{d}{dq^2} F_\pi^{\mathrm{int}}(q^2)|_{q^2=0},$$

which are defined by the derivative of the form factor at zero transmitted momentum squared.

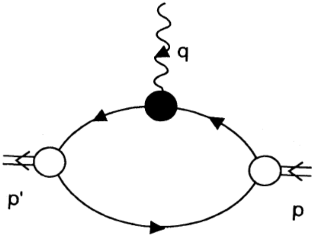
The pion form factors

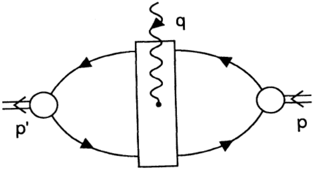
The pion transition form factor:

The process of $\gamma^*\gamma \to \pi^0$ transition can be considered as a cross channel to the two-photon pion decay

$$F_{\pi\gamma}(Q^2) = G_{\pi\gamma}(m_\pi^2, -Q^2, 0), \qquad Q^2 > 0.$$

The charge pion form factor:





The charge form factor

The charge form factor of the pion has the following form:

$$F_\pi(q^2) = F_\pi^{\mathrm{RIA}}(q^2) + F_\pi^{\mathrm{int}}(q^2)$$

$$F_\pi^{\mathrm{RIA}}(q^2)[p+p']^\mu = iN^2 \int \frac{d^4k}{(2\pi)^4}$$

$$\times \frac{\mathrm{Tr}[\gamma^5(\hat{p}'/2 + \hat{k}' + m)\gamma^\mu(\hat{p}/2 + \hat{k} + m)\gamma^5(\hat{p}/2 - \hat{k} + m)] f(k^2) f((k+q/2)^2)}{([p/2+k]^2 - m^2 + i\epsilon)([p/2+k+q]^2 - m^2 + i\epsilon)([p/2-k]^2 - m^2 + i\epsilon)}$$

$$F_\pi^{\mathrm{int}}(q^2) = \frac{iN^2}{P^2} \int \frac{d^4k}{(2\pi)^4} \frac{(4(k-p/2)\cdot P + P^2) f(k^2) f((k+q/2)^2)}{([p/2-k]^2 - m^2 + i\epsilon)}$$

$$\times \left(\frac{(m^2 + \mu^2/4 - k^2) f(k^2)}{([p/2+k]^2 - m^2 + i\epsilon)} + \frac{(m^2 + \mu^2/4 - (k+q/2)^2) f((k+q/2)^2)}{([p/2+k+q]^2 - m^2 + i\epsilon)} \right),$$

where $P = p + p'$.

Calculation results

Table of model parameters m, Λ and observations $f_\pi, r_{\pi\gamma}, r_\pi^2, \Gamma_{\pi^0\to\gamma\gamma}$

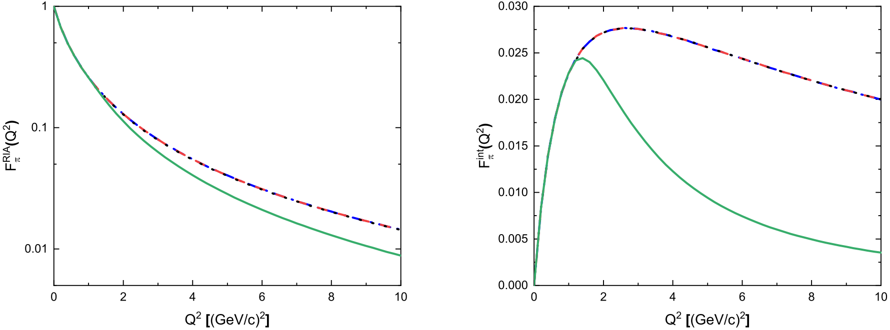
	m (MeV)	Λ (MeV)	f_π (MeV)	$r_{\pi\gamma}$ (fm)	r_π^2 (fm^2)	$\Gamma_{\pi^0\to\gamma\gamma}$ (eV)
I[1]	300.0	500.0	152.74	0.575	0.400	5.633
II[1]	300.0	750.0	175.00	0.541	0.308	5.269
III[1]	200.0	500.0	115.78	0.815	0.708	11.828
IV[2]	265.0	403.0	130.46	0.660	0.549	7.252
V[3]	260.0	550.0	143.37	0.639	0.459	7.249
exp			130.41(1)	0.659(4)	0.430(5)	7.57(3)

[1] Sets I-III are from [5].

[2] Set IV is fixed by fitting the only $f_\pi, r_{\pi\gamma}$ constants exactly.

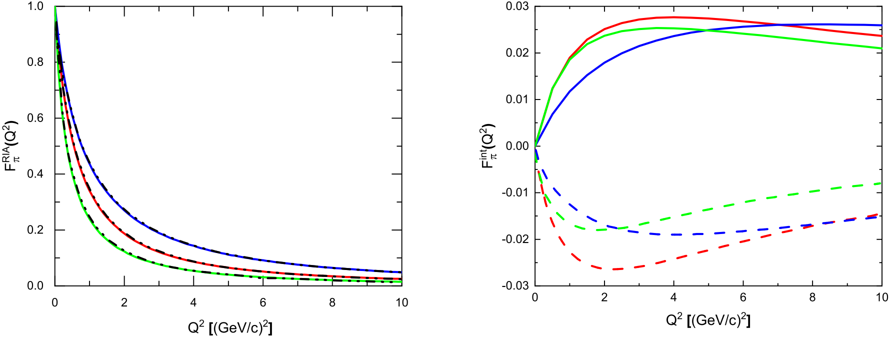
[3] Set V is fixed by fitting all $f_\pi, r_{\pi\gamma}, r_\pi^2, \Gamma_{\pi^0\to\gamma\gamma}$ constants by minimizing the χ^2 value (less than 10%).

Fig. 1. Contribution of RIA F_π^{RIA} (left panel) and the interaction current F_π^{int} (right panel) to the charge form factor of the pion



Solid green line — the Wick rotation method without considering poles, dashed black line — the Cauchy theorem method, dashed red line — the Feynman parameterization method, dashed blue line — the Wick rotation method with considering poles.

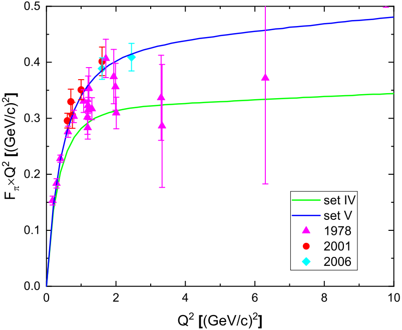
Fig. 2. Contribution of RIA F_π^{RIA} (left panel) and interaction current F_π^{int} (right panel) to the charge form factor of the pion

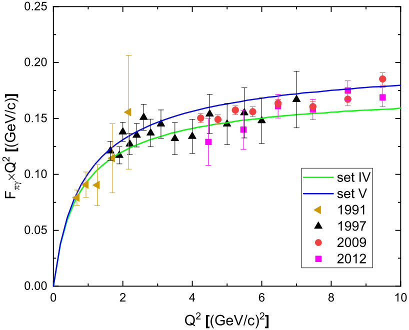


The three solid lines are the results of this report, the dots are the values given in the article [5]. The contribution of the RIA are the same, while the contribution of the interaction current turned are different.

[5] Ito Hiroshi, Buck Warren, Gross Franz. // Phys. Rev. C. — 1991. — Vol. 43. — Pp. 2483–2498.

Fig. 3. (Left panel) Charge pion form factor $Q^2 \times F_\pi$.
(Right panel) Transition $\gamma\gamma^* \to \pi0$ form factor $Q^2 \times F_{\pi\gamma}$





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Summary

– In this work, both the static constants and the dynamic electromagnetic properties of the pion were calculated using the amplitudes obtained by solving the Bethe-Salpeter equation with a separable kernel.

– The 4-dimensional integrals were calculated by three independent numerical methods (the Cauchy's theorem method, the Feynman parameterization method, and the Wick rotation method). The results of the calculation of all methods agreed with the accuracy of the statistical error.

– The influence of the additional poles that appeared inside the integration contour was also studied. Their contributions were found to be important.

– A set of parameters ($m = 260$ MeV, $\Lambda = 550$ MeV) was chosen to describe the pion constants that differed from the experimental data less than 10 % and also well described the pion form factors.

– One of the results is also the strong dependence of this model on the parameters. Future work will investigate the dependence on the type of interaction kernel function and adding parts of the general form of the pion vertex function.