# Extrapolation of four-dimensional gauge-Yukawa theories to ultraviolet

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We investigate four-dimensional renormalisible gauge-Yukawa theory with all possible dimension-4 operators which exhibit Asymptotic Safety. We calculated all $\beta$-functions and interacting ultraviolet fixed points at 2-loop for gauge, 1-loop for Yukawa and scalar fields.

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## Introduction

In this contribution, we discuss 4d gauge theories coupled to matter. The main reason for this consideration is related to the Standard Model (SM), which works very well for a wide range of phenomena. Moreover, it is a gauge theory that has a semi-simple gauge structure. Thus, it seems reasonable to be interested in the behavior of these sorts of theory. We know very well that such theories can be asymptotically free [1, 2]. It is interesting to find out if there are any other new possibilities for constructing ultraviolet (UV) complete theories.

Asymptotic safety (AS) [3, 4] suggests some extension of asymptotic freedom, where couplings in deep UV develop a fixed point (FP). Therefore, the theory remains interactive in high energies.

In this contribution, we consider an asymptotically safe model, calculate all $\beta$-functions for gauge, Yukawa, scalar fields, and find partly intercating fixed point and scaling exponents.

## Renormalisation group and UV fixed points

Firstly, let us consider the renormalization group equations (RGE). The running couplings flow is described by $\beta$-functions that will be entirely determined by field content and symmetries of particular theory

$$\frac{\partial \lambda_i}{\partial \log \mu} = \beta_i(\{\lambda\}). \tag{1}$$

In perturbation theory the $\beta$-functions are calculated through an expansion in terms of the couplings as

$$\beta_\lambda = c_1 \lambda^2 + c_2 \lambda^3 + ... \tag{2}$$

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| field | $SU(N_c)$ | $U_L(N_f)$ | $U_R(N_f)$ |
|---|---|---|---|
| $\psi_L$ | $N_c$ | $N_f$ | 1 |
| $\psi_R$ | $N_c$ | 1 | $N_f$ |
| $H$ | 1 | $N_f$ | $N_f$ |

*Table* 1. Model content with corresponding representations under gauge and global symmetry

and then these coefficients are determined by the particular theory of interest. The main reason to be using perturbation theory is that a lot of the heavy lifting has already been done [5–7] for general four dimensional field theories.

In particular in this contribution, we are interested in fixed points. This is points where the $\beta$-functions vanish

$$\beta_i(\{\lambda\}) = 0. \tag{3}$$

Thus, depending on what happens to trajectories when they come in the vicinity of these fixed points we can classify them in terms of UV $\lim_{\mu\to\infty}\lambda(\mu) = \lambda^*$ or IR $\lim_{\mu\to 0^+}\lambda(\mu) = \lambda^*$ theories. We are interested in ultraviolet fixed points, which will allow us to define QFT up to highest energies.

There are possible two options for the fixed point. Either all couplings are zero $\lambda^* = 0$ or all couplings are non-zero $\lambda^* \neq 0$ or some of couplings are non-zero at least now. In perturbation theory this is valid if we have small couplings $0 < |\lambda^*| \ll 1$. This means that the couplings must be much less than one. We will consider cases where they are non-zero.

There are necessary ingredients for perturbative asymptotic safety to be realised, see Ref. [8] for more details. In the next section we will consider a model in which it is possible to achieve AS.

## Model

Let us consider a four-dimensional, renormalizable QFT with $SU(N_c)$ gauge group and $U(N_f)_L \times U(N_f)_R$ global flavour symmetry. We have fermion and scalar fields, as listed in Tab.1 The corresponding Lagrangian consists of a gauge sector with field strength tensor $F_{\mu\nu}$, the coupling to the fermions via the covariant derivative $D_\mu$, the gauge fixing $\mathcal{L}_{gf}$ and ghost $\mathcal{L}_{gh}$ terms. The scalar and gauge sector interactions is mediated via the real chiral Yukawa couplings $y_i$. In the scalar sector, we have single-trace ($u$) and double-trace quartic couplings ($v$) and additional dimension-4 operators $\partial\mathcal{L}_4$. Traces in the Lagrangian run over both flavour and gauge indices.

$$\begin{aligned}
\mathcal{L} = &-\frac{1}{4}F^{A\mu\nu}F^A_{\mu\nu} + \mathcal{L}_{gf} + \mathcal{L}_{gh} + \mathrm{Tr}[\bar{\psi} i\hat{D}\psi] + \mathrm{Tr}[\partial^\mu H^\dagger \partial_\mu H] - m^2\,\mathrm{Tr}[H^\dagger H] \\
&- y_1(\mathrm{Tr}[\bar{\psi}_L H \psi_R] + \mathrm{h.c.}) - y_2(\mathrm{Tr}[\bar{\psi}_L H^\dagger \psi_R] + \mathrm{h.c.}) \\
&- y_3(\mathrm{Tr}[\bar{\psi}_L \psi_R]\,\mathrm{Tr}[H] + \mathrm{h.c.}) - y_4(\mathrm{Tr}[\bar{\psi}_L \psi_R]\,\mathrm{Tr}[H^\dagger] + \mathrm{h.c.}) \\
&- u\,\mathrm{Tr}[(H^\dagger H)^2] - v(\mathrm{Tr}[H^\dagger H])^2 - \partial\mathcal{L}_4,
\end{aligned} \tag{4}$$

$$\delta\mathcal{L}_4 = -s_1\left[\mathrm{Tr}(HHHH) + \text{h.c.}\right] - s_2\left[\mathrm{Tr}(HHH^\dagger H^\dagger)\right] - s_3\left[\mathrm{Tr}(HHHH^\dagger) + \text{h.c.}\right] = -\vec{\kappa}^{(4)}_{single}\cdot\vec{O}^{(4)}. \quad (5)$$

$$\begin{aligned}\delta\mathcal{L}_4 = &-d_1\left[\mathrm{Tr}(HH)\,\mathrm{Tr}(H^\dagger H^\dagger)\right] - d_2\left[\mathrm{Tr}(HH)\,\mathrm{Tr}(HH) + \text{h.c.}\right]\\ &- d_3\left[\mathrm{Tr}(HH)\,\mathrm{Tr}(HH^\dagger) + \text{h.c.}\right] - d_4\left[\mathrm{Tr}(HHH)\,\mathrm{Tr}(H) + \text{h.c.}\right]\\ &- d_5\left[\mathrm{Tr}(HHH)\,\mathrm{Tr}(H^\dagger) + \text{h.c.}\right] - d_6\left[\mathrm{Tr}(HH^\dagger H)\,\mathrm{Tr}(H) + \text{h.c.}\right]\\ &- d_7\left[\mathrm{Tr}(HH^\dagger H)\,\mathrm{Tr}(H^\dagger) + \text{h.c.}\right] = -\vec{\kappa}^{(4)}_{double}\cdot\vec{O}^{(4)}. \end{aligned} \quad (6)$$

$$\begin{aligned}\delta\mathcal{L}_4 = &\ t_1[\mathrm{Tr}(HH)\,\mathrm{Tr}(H)\,\mathrm{Tr}(H) + \text{h.c.}] + t_2[\mathrm{Tr}(HH)\,\mathrm{Tr}(H)\,\mathrm{Tr}(H^\dagger) + \text{h.c.}]\\ &+ t_3[\mathrm{Tr}(HH)\,\mathrm{Tr}(H^\dagger)\,\mathrm{Tr}(H^\dagger) + \text{h.c.}] + t_4[\mathrm{Tr}(H^\dagger H)\,\mathrm{Tr}(H)\,\mathrm{Tr}(H) + \text{h.c.}]\\ &+ t_5[\mathrm{Tr}(H^\dagger H)\,\mathrm{Tr}(H)\,\mathrm{Tr}(H^\dagger) + \text{h.c.}] = -\vec{\kappa}^{(4)}_{triple}\cdot\vec{O}^{(4)}. \end{aligned} \quad (7)$$

$$\begin{aligned}\delta\mathcal{L}_4 = &\ q_1[\mathrm{Tr}(H)\,\mathrm{Tr}(H)\,\mathrm{Tr}(H)\,\mathrm{Tr}(H) + \text{h.c.}] + q_2[\mathrm{Tr}(H)\,\mathrm{Tr}(H)\,\mathrm{Tr}(H)\,\mathrm{Tr}(H^\dagger) + \text{h.c.}]\\ &+ q_3[\mathrm{Tr}(H)\,\mathrm{Tr}(H)\,\mathrm{Tr}(H^\dagger)\,\mathrm{Tr}(H^\dagger) + \text{h.c.}] = -\vec{\kappa}^{(4)}_{quadruple}\cdot\vec{O}^{(4)}. \end{aligned} \quad (8)$$

In this work, we are interested in the planar (Veneziano) limit, where field multiplicities $N_f$ and $N_c$ are large and interactions are parametrically weak [9–13]. The advantage of the Veniziano limit is that it offers perturbative control, allowing expansions in a small parameter.

The model has 23 dimensionless couplings: gauge coupling $g$, the Yukawas $y_1$, $y_2$[1] and quartic scalar couplings $u$, $v$, $s_i$, $d_i$, $t_i$, $q_i$. One usually introduces a set of rescaled couplings

$$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}, \quad \alpha_{y_i} = \frac{y_i^2 N_c}{(4\pi)^2}, \quad \alpha_u = \frac{u N_f}{(4\pi)^2}, \quad \alpha_v = \frac{v N_f^2}{(4\pi)^2}. \quad (9)$$

$$\alpha_{s_i} = \frac{s_i N_f}{(4\pi)^2}, \quad \alpha_{d_i} = \frac{d_i N_f^2}{(4\pi)^2}, \quad \alpha_{t_i} = \frac{t_i N_f^3}{(4\pi)^2}, \quad \alpha_{q_i} = \frac{q_i N_f^4}{(4\pi)^2}. \quad (10)$$

This allows one to absorb all corrections with positive powers of $N_c$ and $N_f$ appearing in the $\beta$-functions for $\vec{\kappa}$ into the rescaled couplings given in Eqs. (9) and (10).

Moreover, the parameter $\epsilon$ becomes continuous in this limit, taking values in the entire range $\epsilon \in [-\frac{11}{2}, \infty)$. We are particularly interested in the regime $|\epsilon| \ll 1$, where we can control perturbativity.

A key feature of non-abelian gauge theories coupled to matter is that fixed point couplings $\alpha_i^*$ can be expanded as a power series in the small parameter $\epsilon$. In our case, this means the "conformal expansion" of $\epsilon$. The expansion

[1] We absorb $y_{3,4}$ in redefinition of fields

coefficients $\alpha_i^{(n)}$ are determined using perturbation theory, by performing a perturbative loop expansion up to order $n+1$ in the gauge and up to order $n$ in the Yukawa and quartic $\beta$-functions [4, 9–11]:

$$\alpha_x^* = a_{LO}\epsilon + a_{NLO}\epsilon^2 + a_{NNLO}\epsilon^3 + O(\epsilon^4). \tag{11}$$

It should be noted that generic $\beta$-functions have been calculated using `RGBeta` [14].

## Discussion

In Veneziano limit, one computes at 2(gauge) − 1(Yukawa) − 1(scalar) order

$$\alpha_g^* = \frac{26}{57}\epsilon, \quad \alpha_{y1,2}^* = \frac{4}{19}\epsilon, \quad \alpha_u^* = \frac{1}{19}\left(\sqrt{23}-1\right)\epsilon, \tag{12}$$

$$\alpha_v^* = -\frac{1}{19}\left(\sqrt{20+6\sqrt{23}} - 2\sqrt{23}\right)\epsilon, \quad \alpha_{s_i,d_i,t_i,q_i}^* = 0 \tag{13}$$

Given $\alpha^* = \alpha^*(\epsilon)$, one computes one IR-relevant eigendirection

$$\theta_g = -\frac{104\epsilon^2}{171} + \frac{33544\epsilon^3}{9747}, \quad \theta_{y_1} = \frac{6748\epsilon^2}{1083} + \frac{26\epsilon}{19}, \quad \theta_{y_2} = \frac{4\epsilon^2}{19} + \frac{4\epsilon}{19}, \tag{14}$$

$$\theta_u = \frac{8}{19}\sqrt{2\left(10+3\sqrt{23}\right)}\epsilon, \quad \theta_v = \frac{16\sqrt{23}\epsilon}{19}, \tag{15}$$

$$\theta_{s_1,d_1,d_2,d_4,d_5,t_1,t_2,t_3,q_1,q_2,q_3} = \frac{16\epsilon}{19}, \quad \theta_{s_2,s_3,d_6,d_7} = \frac{8}{19}\left(1+\sqrt{23}\right)\epsilon, \tag{16}$$

$$\theta_{d_3,t_4,t_5} = \frac{4}{19}\left(2 + \sqrt{2\left(10+3\sqrt{23}\right)}\right)\epsilon. \tag{17}$$

Therefore, we 1. calculated all $\beta$-functions for gauge, Yukawa, scalar fields; 2. found fixed points and scaling exponents. More details on the calculations can be found in a forthcoming work [15].

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