Packing of disks with a power-law size distribution: correlations in real and reciprocal spaces

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We investigate analytically and numerically the spatial correlation properties of disk packings with power-law distributed radii [1, 2]. We study the corresponding structure factor, mass-radius relation, and pair distribution function of the disk centers. A toy model of dense segments, randomly placed in one dimension, is considered and solved exactly. It is shown theoretically in one dimension and numerically in one and two dimensions that such a dense packing exhibits fractal properties. It is found that the exponent of the power-law distribution and the fractal dimension coincide provided the packing is dense and random. The thermodynamic limit is considered when the total number of disks increases infinitely, while the mean density of the disk centers and the range of the size distribution are kept constant. It is shown that the fractal range in reciprocal space corresponds to the range of the size distribution in real space. The dependence of the structure factor on density is examined. As is found, the power-law exponent remains unchanged but the fractal range shrinks when the packing fraction decreases. In addition to random packing, we consider non-random packing algorithms, such as constant-pressure compression generated by the LAMMPS GRANULAR software package. As is shown, in this case the fractal exponent significantly deviates from the size distribution exponent. These findings reveal the fractal-like properties of the disk packings and can be applied to the analysis of small-angle scattering from such systems.

References

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